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Art. XVII.—*The Failure of Cavities in Crystals and Rocks under Pressure;* by P. W. Bridgman.

It is a matter of geological importance to know at what depth in the earth's crust to expect open cavities. The problem is one of some difficulty, involving the complicated interplay of a number of factors. The character of the stress, the nature of the material, the effect of temperature and the element of time all must be considered. An experimental study of the problem must begin, therefore, under as simple and well-defined conditions as possible. The best known experimental work has been by Adams,[1] who subjected several minerals in the form of blocks pierced with holes to high pressures exerted by a steel plunger, and observed the pressure at which the hole collapsed. The most obvious criticism of these experiments is that the manner of applying stress is such that its character cannot be at all precisely specified, since the blocks were enclosed in shrunk-on jackets of mild steel, which yielded as the block was distorted. In an attempt to avoid this element of ambiguity I repeated the experiment of Adams under conditions such that the stress could be precisely specified. The cylinders of rock containing a cavity were directly immersed in a liquid, and stress applied by subjecting the liquid to a high hydrostatic pressure. The collapsing stress so obtained was considerably lower than that of Adams. These results are not yet published, and will be briefly described in the latter part of this paper. Their charac-

[1] F. D. Adams, J. Geol., 20, 97-118, 1912.

ter was not just what I had expected, and indicated that a systematic examination of the whole subject was necessary. Among other things, it was evident that a rock is too complicated a structure to give information on the various elements of the problem. The first experiments should obviously be performed on the homogeneous materials of which a rock is composed, that is, on individual crystals. But the preparation of individual crystals was at that time beyond my resources, and I allowed the matter to drop.

The whole problem was again forced on my attention by Dr. George F. Becker, who had for some time recognized its importance and had published results obtained by quite a different method. Dr. Becker's interest in the problem was so great that he was willing to undertake all the arduous work of superintending the preparation of the specimens, leaving to me the easy task of making the actual experiments. He procured an appropriation from the National Academy of Sciences, which, with the kind coöperation of Dr. Stratton, made it possible to secure the services of the optician of the Bureau of Standards, Mr. Clacey. He personally selected many of the specimens from the resources of the National Museum, which were at his disposal; specimens both of single crystals and of several rocks were prepared. Without his interest and assistance this paper would not have been possible.

This paper presents, then, results on the crushing of hollow cylinders of single crystals and rocks by the application of hydrostatic pressure to the external surface. The results are not of immediate geological applicability, because the conditions of the experiment are not duplicated in the field, but they suggest the nature of the effects to be expected under actual conditions. Apart from their geological interest, the results have an intrinsic interest from the points of view both of theory of elasticity and of the structure of crystals. Very few experiments have ever been made on the rupture of crystals; in fact the nature of the symmetry relations has not yet been worked out. From the point of view of the mathematical theory of elasticity the problem of the stress-strain relations in a crystalline cylinder under the conditions of the experiments seems never to have been discussed. I have been able to obtain the solution in several of the simpler cases. The general nature of these results will be used in this paper but the detailed solution is reserved for the following paper.

This paper also contains measurements on the density of several powdered minerals (quartz, feldspar, and talc) after subjection to 30,000 kg/cm^2. These measurements were directly suggested by the experiments on collapse of

FIG. 1. FIG. 2. FIG. 3.

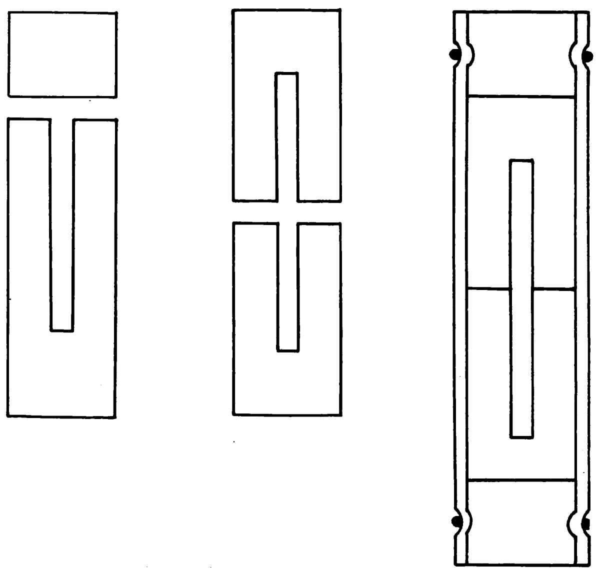

FIG. 1. Section of one form of specimen.
FIG. 2. Section of another form of specimen.
FIG. 3. Cylinder mounted in rubber tube with brass end pieces, ready for immersion in liquid and subjection to pressure.

cavities, and should allow a more significant geological interpretation to be attached to those results.

Details of Experiment.

The specimens were made in one of the two forms showed in figs. 1 and 2. The two parts were cut from contiguous parts of the same original crystal or rock. The two flat surfaces on which the two parts abut were made optically plane so that there should be as little distortion as possible when the two parts were pressed together.

This ensured that the stress conditions in a single uncut cylinder with an axial hole were reproduced as exactly as possible. The orientation of the two parts with respect to the original crystal was marked, and the two pieces were always, except at the very first, fitted together in the original orientation. This is an important point to which I will return. The outside surfaces of the cylinders were very nearly circular, and were polished. The inner holes were nearly but not quite cöaxial with the outer surface, and were nearly but not quite round. During the test the inner hole contained only air at atmospheric pressure, except for a small device to be described later. The outer ends of the cylinder were ground flat, but were not polished.

A piece of soft rubber tubing was slipped over the outside of the cylinders and was tied to brass end pieces as shown in fig 3. Between the brass end pieces and the crystal was a thin piece of hard rubber or red fiber, to equalize any slight inequalities in the surface of the brass. The combination was then immersed in a liquid in a heavy steel cylinder and stress applied by producing hydrostatic pressure to any desired amount in the liquid. The function of the soft rubber tubing was to freely transmit pressure to the specimen and at the same time to keep the liquid from the surface of separation of the two cylinders. It is evident that under these conditions the entire outer surface of the cylinders is exposed to the same hydrostatic pressure.

The cylinder in which pressure was produced was of chrome-nickel steel, 8 inches outside diameter and 1⅛ inches inside diameter. Pressure could be raised to 12,000 kg/cm^2 with this apparatus. The details of construction and methods of measuring pressure have been previously described.[2] A smaller cylinder in which a few tests were made permitted a maximum of 24,000 kg/cm^2.

The following measurements were made. The outside diameter of each piece at the two ends and the middle and at angular intervals of 30° was measured to 0·0001 inch with a micrometer. Six measurements of the outside length were also made with the same micrometer. The depth of the hole was measured with a depth gauge to 0·0001 inch; this measurement was always somewhat unsatisfactory because the bottom of the hole might be

[2] P. W. Bridgman, Proc. Amer. Acad., 49, 626-643, 1914.

irregular. The average diameter of the hole was measured by weighing the mercury which exactly filled it. By using the optically plane surface of the other piece of the crystal to rub off any mercury which might rise in the meniscus above the surface, it was particularly easy to get the hole always exactly full. All of these measurements were made at atmospheric pressure, after each exposure to pressure. The procedure was to expose the cylinder to a known pressure for a known time, take the apparatus apart, make all the measurements above, and set the apparatus up again and expose to a higher pressure. These measurements simply show, therefore, whether there has been any permanent deformation produced by the pressure. It would have been most desirable also to have made all these measurements while pressure was applied, and so obtain the strain for a given stress. But to do this would have been of excessive difficulty. By the use of a simple device it was possible, however, to obtain rough measurements of the diameter of the hole while under pressure. A little disc of solder mounted on a brass rod to facilitate handling and to keep it in position was turned in the lathe so as to be initially a push fit for the hole. This was placed in the hole during the application of pressure. The effect of pressure is of course to diminish the size of the hole. The solder disc offers inappreciable resistance, and because of its low elastic limit is permanently deformed to very nearly the minimum diameter reached by the hole. After application of pressure the disc may be removed and the minimum diameter obtained by direct measurements.

Collapsing tests of this kind were made on two specimens of quartz, two of tourmaline, and one each of calcite, barite, feldspar, andesite, porphyry and glass. Negative crystals of quartz were also tried. These experiments will now be described in detail.

Quartz.—The first specimen was in the shape of fig. 1, a long hollow cylinder with a shorter solid cap. The dimensions were: Outside length of hollow part 4·5 cm., external diameter 2·0 cm., inside diameter 0·36 cm. This cylinder was subjected to ten applications of pressure, beginning at 2000 kg/cm^2., and continuing at approximately equal intervals to 11,500 kg. Each application was for ten minutes. The first noticeable effect of pressure was at 3000 kg. and was a chipping off of slivers from the outside surface where the two pieces join. This was due to failure to orient the two pieces in their natural position. The proper orientation was found after a few trials, and after this the chipping almost entirely ceased, even up to the maximum pressure. The mathematical discussion shows that plane cross sections of the crystal warp under pressure, and explains therefore, the necessity for observing the original orientation. All later specimens were marked when cut from the original crystal so as to allow the correct orientation, but this precaution was not taken with the first specimen.

At higher pressures, beginning at about 6000, signs of failure at the interior appeared, increasing in intensity up to final complete rupture. These signs of failure were of two kinds. In the first place, there was a system of fissures in both the hollow piece and the cap. In the hollow piece these began as a set of cracks near the inner mouth of the hole, separated by an angular interval of 120°, corresponding to the symmetry of the crystal, and penetrating a short distance into the body of the crystal at right angles to the surface. At higher pressures these cracks became more numerous and penetrated to a greater depth, inclining toward the axis at the greater depths. This system of cracks had its counterpart in the solid cap, but the penetration here was always to only a slight depth. The cap also showed cleavage in the interior on surfaces at right angles to the axis, a phenomenon for which there was no counterpart in the hollow piece. These systems of cleavage planes are apparently connected in some way with end effects due to imperfect matching together of the two pieces. A mathematical discussion of these end effects is too difficult to be attempted.

Final failure of the crystal took place in a way entirely unconnected with these cleavage planes. At pressures of 7500 and higher minute flakes scaled off the inner surface, leaving it rough. This flaking proceeded at an accelerated rate at higher pressures, at 10,500 amounting to an enlargement of 3% of the volume. On the next application of pressure, to 11,500, however, the flaking off was so extensive that the upper end of the hole was eroded away to three times its original diameter, and the entire hole was tightly packed with a very fine quartz sand. The density of this sand was of course much less than that of the original solid quartz; so that when the hole had become sufficiently large, the sand was in a position to exert pressure on the interior walls, and in this way prevented further disintegrating action. The eroded cavity was very irregular in outline; for the greater part of its depth it was entirely unsymmetrical, being merely an angular extension on one side of the original circular cavity. This is as one would expect; an angle once formed at any part of the surface would be relatively unstable, and erosion would proceed more rapidly here. There was no apparent relation betwen the axis of the eroded cavity and the three systems of cleavage planes mentioned.

The outside of the cylinder was entirely unaffected by the erosion of the interior; there was no perceptible change of dimensions, either diameter or length, and the cracks were entirely confined to the interior. This is true of many of the other specimens tried. A photograph of this cylinder is shown in fig. 4.

The second specimen of quartz was cut in two symmetrical pieces as in fig. 2. The two parts were always so put together as to observe the original orientation in the crystal. It was subjected to pressures of 4000, 6000, 8000, 10,000, and 12,000 for 10 minutes each, experience with the first crystal having shown that applications of pressure at 1000 kg. intervals was not necessary. The results were very similar to those with the first specimen. The same system of funnel-shaped cracks separated by 120° was observed, but they were not so prominent, indicating that the fitting together was better, and the end effects, therefore, not so important. The same flaking-off of the inner surface was observed, first noticeable at 8000, and resulting in complete disintegration of the interior at 12,000. A photograph of one of these cylinders is shown in fig. 5. There was no permanent change of outside dimensions, no evidence whatever for flow, and the cracks did not reach to the outside. The density of the sand which packed the cavity was 1·43 against 2·65 of the original quartz; the interstital space was therefore nearly 50%.

The stress required to produce rupture of these crystals is much higher than might be expected, and the manner of rupture is apparently not like that contemplated in any theory of rupture, nor is it such as would be suggested by the elastic deformation before rupture. I have worked out mathematically the nature of the stress-strain relation,

FIG. 4.

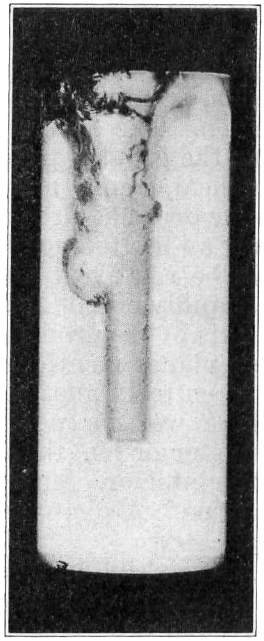

FIG. 5a.

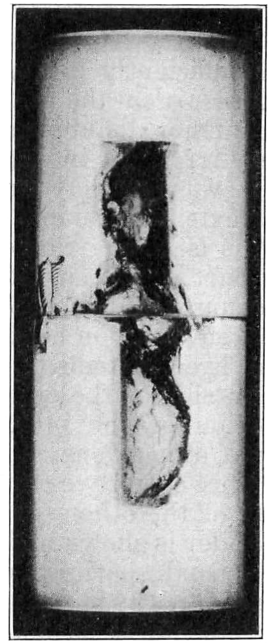

FIG. 5b.

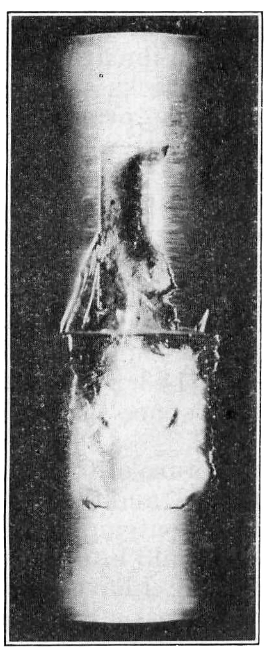

FIG. 5c.

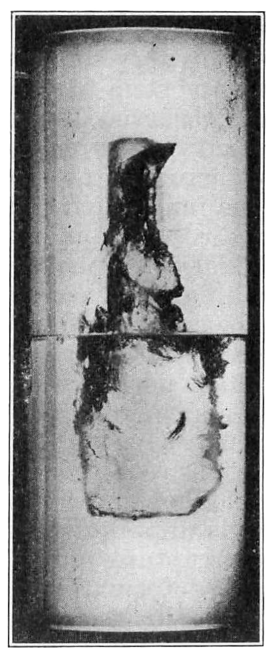

FIG. 4. The first quartz cylinder after failure under 12,000 kg.
FIG. 5 a, b, c. Three views of the second quartz cylinder after failure under 12,000 kg.

and shall assume here the results of the analysis, which will be given in detail in the following paper. Let us consider the ideal case of an infinitely long hollow cylinder subjected to hydrostatic pressure over its external surface. If the material is isotropic the system of stresses and strains is well known, and is very simple. Every plane cross section of the cylinder remains plane, and the distortion consists merely of a shortening of every radius without any change in the angle between any two radii. The most intense stress and strain are both at the inner surface. The stress at the interior is a compression on planes including the axis and the radius, amounting to approximately twice the external hydrostatic pressure. The amount of strain depends of course on the elastic constants; its nature is an elongation of the fibers along the radius at the inner surface (a paradoxical result) and a numerically much greater shortening of the circumferential fibers.

The elastic deformation in a crystal of quartz is much more complicated. The most important difference compared with the isotropic case is that plane cross sections do not remain plane, but become warped, the warping of course satisfying the conditions of symmetry and repeating itself every 120°. The warping with cylinders of the dimensions above, the inside diameter of which was 3·6 mm. and the outside diameter 2·0 cm., is a maximum at about 2·7 mm. from the axis. The shearing strain due to the warping is a maximum at the inner surface, however, and may amount to 35% of the maximum circumferential compression. In addition to the warping there are further distortions in the plane perpendicular to the axis. The radial displacement is not independent of the orientation in the crystal, but fluctuates with a period of 60°. The strain resulting from this fluctuation is only 4% of the maximum. Furthermore, there is a displacement along the circumference with a period of 60°, the resulting strain being only about 1% of the maximum. This complicated set of displacements results in a much more complicated system of stresses than in an isotropic solid. The most important of these additional stresses is a shearing stress along the axis in planes containing the axis and radius, rising at the inner surface to a maximum of 47% of the external pressure.

Numerical computation shows that with quartz of the dimensions above (inside diameter equal 1, outside diameter equal 5·5) using the values of the constants found by Voigt (see Love's Elasticity, page 157), the displacements, stresses and strains at the inner surface under an external pressure of 12,000 kg. are as follows: radial displacement 2·9% of the original radius, circumferential pressure 25,000 kg/cm^2, axial shearing stress 5700 kg., radial elongation 0·5%, circumferential compression 2·9%, shearing strain between radius and axis 0·97%. The inapplicability of the ordinary criteria of rupture is obvious. Under ordinary conditions of tests in one-sided compression, the crushing strength of quartz is about 1200 kg/cm^2. A stress 20 times as great, with corresponding larger values of shearing stresses and strains, was supported by the crystal above, and then rupture took place in a manner different from that to be expected.

The measurements of the change in internal dimensions already mentioned were not in conflict with what is to be expected from the mathematical analysis. The measurements were only rough, giving the order of the effect. Those on the second cylinder were much more consistent than those on the first, and gave a radial displacement of 2·7% under 12,000 kg. against 2·9% calculated above. The agreement is much better than could be expected. The accuracy of the measurements was of course not great enough to permit detection of the small oscillating effect superposed on the average radial displacement. The measurements of radial displacement furthermore showed that the relation between displacement and stress remains linear up to the point of complete rupture, thus justifying the use of the ordinary mathematical analysis of elasticity. This is of course only what is to be expected from a substance of such small viscosity as to show no flow under these very high stresses.

Tourmaline.—Both specimens were of approximately the dimensions of the second quartz specimen. They were cut perpendicularly to the axis. The first specimen was colored, ranging from light green through purple to dark brown, but was sufficiently transparent so that all the inner defects could be seen. The crystal was full of small striæ parallel to the axis, and also contained many surfaces of internal reflection, which were probably minute cracks. These were scattered at random throughout the crystal. This specimen was exposed to 3000, 5000, 7000, 10,000 and 12,000 kg. for 10 minutes each. At 12,000 there was appreciable flaking-off of the inner surface, similar to that of quartz at lower pressures. But there was no extensive erosion, so that tourmaline appears as a much stronger crystal than quartz. At 7000 and 10,000 minute longitudinal splinters were separated from the inner surface, but this ceased at 12,000. Very minute cracks were formed in the polished faces at the ends of some of the striæ. At 12,000 cracks at an angular separation of 120° had started, like those formed in quartz at 6000, but they were small, and there was none of the funnel-shaped appearance of the quartz. It was a surprise that there was no connection between the new cracks which appeared and the original flaws. There was also a system of cracks unlike those of quartz, cracks in planes perpendicular to the axis, reaching in some cases to the outer surface. One such crack was situated diametrically below the end of the hole. No phenomena of flow could be detected.

The behavior of this cylinder, as well as that of the quartz cylinder, was peculiar when compared with ordinary substances. Iron and copper, for example, when in the form of cylinders like this,[3] will also stand a much higher stress than indicated by ordinary compression tests, but they do it by a process of accommodation. Under stress they flow viscously until the grains have become properly arranged to stand a higher stress. But the quartz and tourmaline cylinders show no flow, and the only analogy to accommodation is flaking-off. I found it most difficult to see how flaking-off of the inner surface could result in enhanced resistance to rupture, and could think of no explanation except that the effect of time had been neglected. The same cylinders of tourmaline were therefore exposed again to 12,000 kg., this time for two hours. If tourmaline were like an ordinary metal, no effect would be produced by the second application of the maximum stress; there would be no further cracking or flaking-off, because the elastic limit had been raised by the previous application of pressure. But the effect was the exact opposite. Under the longer duration of stress the cylinder failed completely, being packed tightly full with an impalpable sand. The region of erosion of

[3] P. W. Bridgman, Proc. Amer. Acad., loc. cit.; Phys. Rev., 34, 1-24, 1912.

one of the cylinders, which was in form an elongated ellipse with pointed ends, extended entirely across the cylinder. The outer surface of this piece had in places been displaced bodily into the cavity. In the other piece, however, the erosion was much less extensive, and the outer surface had received no permanent change. In this second piece the most extensive erosion was not at the mouth of the hole but was nearer the bottom, showing that the flaking-off is not an end effect. This remark is confirmed by many other observations; the flaking-off has no relation to the end effect, and may take place at any point on the interior surface.

The second specimen of tourmaline was entirely opaque, so that no intimation could be obtained as to its internal structure; there were no flaws apparent in the original piece. The tests on this specimen were especially designed to find the effect of the element of time. It was exposed to 5100 kg. for 28½ hours, to 6630 for 46¼ hours, to 8170 for 38½ hours, to 9500 for 38½ hours, and to 12,200 for 40¼ hours. This specimen was much stronger than the first. Up to and including 9570 there was practically no effect. The edges of the mouths of the holes splintered a little, as they always do, because of imperfect register of the two parts, and there were a few barely perceptible flakes from the inside. At 12,200, however, the failure of the interior was complete, as it had been for the other specimen. There was no flow or permanent alteration of the outside. The average density of the sand with which the eroded cavity was packed was 2·15, against 3·09 for the original crystal. The interstitial space in the sand was therefore about 30%.

The change in internal dimensions of the cavity, found from measurements on the solder plugs, was at the rate of 0·65% per 12,000 kg. for the first, and 0·74% for the second specimen. Using Voigt's values for the elastic constants of tourmaline (see Love, page 157) the theoretical value is about 0·90%. Tourmaline varies greatly in its properties; the discrepancy may therefore be due to difference in the specimens. Here again, as for quartz, the relation between distortion and stress remained linear within the errors of measurement, which were large.

Calcite.—The interest in this material lies in its extraordinarily easily developed planes of cleavage, whereas both quartz and tourmaline show practically no such planes. One might expect the cleavage planes of calcite to bear some especial relation to its manner of rupture.

The specimen was in the form of a single cylinder with a cap, of the same dimensions as the first specimen of quartz. The axis of the cylinder was supposed to be parallel to the trigonal axis, as were quartz and tourmaline, but subsequent examination showed that the axis of the cylinder was inclined at about 7° to the axis of the crystal. The cleavage planes in the original piece were so strongly developed that at first it seemed of little use to attempt the experiment, but the results only confirmed the results with other specimens that original flaws have no effect on the manner of rupture under these special conditions. Apparently the friction produced by the high pressure is sufficient to prevent any slipping on original planes of fracture.

Three applications of pressure were made, to 500, 1000, and 1500 kg. for 10 minutes each. The most marked effect of pressure was a great increase in the number of internal cleavage planes. The increase was so great as to make the substance partially translucent instead of transparent. The development of cracks put an end to the experiment, the crystal falling apart in handling at atmospheric pressure while preparing for the next test at 2000 kg. The effect of stress on the cavity was not marked, but after 1500 there was unquestionably some flaking-off of the interior, just as for quartz and tourmaline at higher pressures. The material also showed a tendency to slip into the cavity along cleavage planes, but full accomplishment of this tendency was prevented by the geometrical configuration, the parts affording each other mutual support. The inner surface of the wall of the cavity, to a depth of several millimeters, was rendered entirely opaque, evidently because of the grinding together of the material as it was carried toward the center on convergent cleavage planes. The slip produced a definite figure on the plane surface, as shown in figure 6. Within the triangle the development of cleavage planes was much more pronounced than outside of it.

Slip was accompanied by permanent change of dimensions, one of the few examples of it found. The outer diameter decreased from 0·7331 to 0·7310 inches, and the inner diameter by 2·3%. The slipping was in such a direction as to produce an increase of external length from

1·5894 to 1·6032 inches. The effect of slip on length was irregular, however, at 1000, there being a slight decrease instead of an increase. At 1000 the diameter was little changed, nearly all the effect coming betwen 1000 and 1500.

It was not possible to make measurements of the internal diameter under pressure with the solder plugs, the effect being too small. In spite of its easy cleavage and

FIG. 6.

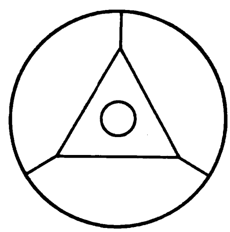

FIG. 6. Slip figure on plane face of calcite cylinder after exposure to 1500 kg.

mechanical softness, the elastic constants of calcite are high, its cubic compressibility, for example, being the same as that of quartz.

Apart from cleavage effects, these results show that in calcite there is present the same mechanism of rupture by flaking-off as was shown by quartz and by tourmaline.

Feldspar.—This was a white microcline feldspar, prepared in the same form and dimensions as the second specimen of quartz. Feldspar crystallizes in the triclinic system; quartz, tourmaline, and calcite are all trigonal. This material is of course opaque, but there were no evident flaws in it.

It was exposed to 3000 kg. for 40½ hours, to 5000 for 4 hours, and to 3000 again for 15½ hours. There was no noticeable change of appearance after the first application of 3000; no fissures appeared, and there was no change of dimensions, but a very minute quantity of fine dust had flaked off from the inner surface. After exposure to 5000 the disintegration of the interior was complete, the hole being packed with fine sand, as we have come to expect. The average density of this sand was 1·77 against 2·57 of the original crystal. The shape of the eroded region was more symmetrical than usual, being a well-marked rhombus in section, with evidently some connection with the crystalline system. The long axis of the rhombus was parallel to the direction of the white streaks in the original crystal. This substance is unusual in that no fissures whatever accompanied the erosion. Apparently, therefore, the mechanism of rupture by flaking-off is entirely independent of the mechanism of formation of cracks. As usual there was no change of external dimensions.

After exposure to 5000 it was exposed again to 3000, to see whether the flaking-off would continue at a lower pressure after having started at a higher pressure, and also to find whether loss of circular cross section in the cavity would result in easier rupture. There was practically no effect from the second application of 3000; a slight quantity of dust had flaked off the inner surface, but no more than on the first trial.

The change of internal diameter as measured by the solder plugs was 0·2% per 1000 kg., which is of the same order as for quartz. Apparently the elastic constants of feldspar have not been measured, so there is no chance to compare this value with calculated values.

Barite.—This was a translucent specimen with numerous flaws, which however did not affect the behavior under pressure. The specimen was of the form and dimensions of the second specimen of quartz. This was exposed to 1000 kg. for 4 hours, to 2000 for 17 hours, to 3300 for 6 hours, and to 11,400 for 39 hours.

The first two applications of pressure had no perceptible effect. At 3300, however, the cavity was partly disintegrated in the usual fashion, but not tightly packed with sand. The eroded region was a rhombus in section, rather more nearly square than for feldspar, and around this was another rhomboidal region, equal to the diameter in width, within which the original translucent material had changed to an opaque white like marble, evidently due to internal slip. There was no change of external dimensions.

It was evident that a pressure only slightly above 3300 would produce the usual complete disintegration, and occasion was therefore taken to answer a question suggested by much of the previous work, namely, what happens to the sand in the cavity when pressure is raised very far above the disintegrating pressure? It is hardly conceivable that interstitial spaces as large as 50 or 30% will persist, but on the other hand, will the powder fuse to a compact mass? The conditions here were exceptionally favorable for getting some idea of the answer to this question since pressure could be raised in the available apparatus to more than three times the disintegrating pressure. If fusing by pressure alone is ever to be expected, there is a good chance to find it here. For it is to be considered that the manner of formation of the sand, by automatic flaking-off of the solid material inside a closed cavity, ensures that the grains shall be clean. This is an important point, and one that is most difficult to obtain under ordinary conditions of experiment.

In an attempt to produce fusion of the powder, pressure was maintained at 11,400 for 39 hours. The crystal was thereby changed in appearance throughout the entire mass to the opaque white of marble, showing that slip had spread throughout the entire substance. At the inside there was a core of fine sand which had not fused together, but could readily be picked out with a wire. The average density of the entire cylinder was decreased somewhat, as had also the external diameter. In the marbleized region, there were no cracks except a few fine fissures near the core itself. Obviously, then, this material may suffer flow without losing its mechanical coherence, but if the particles have ever been separated too much, as when the grains of sand are formed, even a relatively high pressure will not cause them to fuse together. The core of sand had lost all rhomboidal shape, and was now ellipsoidal with angular ends. Small fissures radiated from the ends, in appearance extremely like lines of force about a bar magnet.

In spite of this experiment, the conviction is hard to escape that it must be possible to weld together the fragments of a solid merely by bringing them into contact, provided they are perfectly clean. In this connection Langmuir's[4] recent work on the significance of the rôle played by adsorbed layers of a gas only one molecule deep is most suggestive. The particles of sand formed from the barite appeared by microscopic analysis to be of the order of 0·001 mm. in diameter. A very rough calculation shows that the air originally in the cavity was much more than sufficient to cover all the grains of sand

[4] I. Langmuir, J. Amer. Chem. Soc., 38, 1145, 1916.

with molecules one layer deep. As far as any present evidence goes, it is not unreasonable to suppose that the sand formed by stress in a cavity originally entirely free from gas would ultimately be fused together again to a coherent mass.

This completes the experiments on single crystals. As being of geological interest, several experiments were made on rocks and other substances.

Porphyry.—The two cylinders of this material were of approximately the same shape and dimensions as the second quartz. They were exposed to 1000, 2000, 3000, 4000, 6000, and 7000 kg. for 10 minutes each. Up to 4000 there was no perceptible effect; at 6000 a slight flaking off was perceptible at the mouth of the hole, and at 7000 failure was complete by flaking-off. The eroded regions were roughly triangular prisms, the prism in one piece being a prolongation of that in the other. There was no viscous flow of the outside of as much as 0·0001 cm. and no cracks whatever in the solid mass, even in the neighborhood of the eroded region. The mean density of the sand was only 0·31. Doubtless if the pressure had been continued longer, the sand would have become more tightly packed.

The elastic decrease of diameter of the inner hole was at the rate of 0·14% for 1000 kg. This is considerably less than that of quartz, showing that probably the mean elastic constants are considerably higher. Failure by erosion occurred at a lower stress than for quartz, however. Of course this is just as one would expect; a rock would have elements of weakness not possessed by an individual crystal.

The stress at the inside at the rupture point may be computed for an isotropic substance without knowing the elastic constants, and was approximately 14,000 kg/cm^2. Engineering tests on porphyry give a compressive strength under ordinary one-sided crushing tests varying from 1000 to 2600 kg/cm^2. The excess above the value computed from ordinary tests is therefore high, although not so extreme as for quartz.

Andesite.—This was a very fine-grained and perfect specimen, of the form and dimensions of the second specimen of quartz. Dr. Becker remarked that the results would be of particular significance because the composition of this rock is the mean composition of the crust of the earth.

Andesite was exposed to 1000, 2000, 3000, 4000, 6000, and 7000 kg. for 30 minutes each, then to 7000 again for 9 hours, and then to 8000 for 5½ hours. At 6000 the flaking-off of the interior was just perceptible. A most interesting effect in connection with this flaking-off was observed. The brass rod supporting the solder disc inside the cavity assumed a curious whitish appearance, suggesting chemical action with gaseous fumes. Examination with a high-power microscope showed, however, that the whitish coating was a covering of minute splinters of the rock, which had been projected with such violence when they had flaked off as to penetrate some distance into the solid brass and stick themselves in position.

Under the first application of 7000 the flaking-off was greater than at 6000, but still only slight. The brass rod was peppered with splinters as before. The second application of 7000 produced still more flaking-off, but not nearly as much as I had expected from the longer time interval. At 8000 the disintegration of the interior was complete. The eroded region was much larger than in porphyry, and was in shape an irregular pointed ellipse. There was no flow whatever of the outside, and no cracks in the solid mass, even in the immediate neighborhood of the hole. The average density of the sand was 1·14 against 2·69 of the original rock. The interstitial space was therefore more than 50%.

Microscopic examination of the sand showed that the pieces were most irregular in shape and of great range of size. Furthermore, coincidence in one piece of the light and dark material of which the rock is composed is extremely common, showing no tendency for each flake to be all of the same composition. It is evident, therefore, that the flaking-off is something which has no particular connection with structure, at least when it is on as fine a scale as in this rock. The same remarks apply to porphyry.

The compressive strength of basalt, which is closely allied to this fine-grained andesite, is given in engineering works as varying from 1000 to 3200 kg/cm^2. The compressive stress at the interior of this specimen when failure took place was 16,000 kg.

Granite and Limestone.—These materials formed the subject of the preliminary experiments mentioned in the introduction; they were much rougher in character than those just described. Both of these specimens were obtained for me by Professor R. A. Daly, who first aroused my interest in this subject.

The granite was gray granite, cylindrical in form, 2 inches long, 1 inch outside diameter, pierced for its entire length with a ¼-inch hole. The ends were closed with flat caps of hardened steel, a rubber tube was slipped over the whole, and it was completely immersed in a liquid and exposed to a pressure of 5000 kg. for 1 hour. The cavity was completely disintegrated into a closely packed sand, just as the other specimens already described. The outline of the eroded region was that of a pointed ellipse. The outside had also become slightly elliptical, thus showing some slight flow. There were no cracks in the non-eroded region.

The cylinder of limestone was of the same dimensions as the granite, and the manner of treatment and the results were the same. These two experiments, of course, gave no idea of the minimum pressure at which the cavity would close, but did show conclusively that Adam's value of 11,000 kg. for the pressure of collapse of granite is too high.

Negative Quartz Crystals.—One of the questions raised by these experiments is as to the effect of the plane of separation of the two parts of the specimen. It is not possible by any perfection of workmanship to secure such precise register of the cavities in the two parts of the specimen that one is exactly a continuation of the other, and disturbing effects at the surface are therefore to be expected. The mathematical solution would indicate infinite stresses and strains, that is rupture, in the neighborhood of any abrupt discontinuity, no matter how minute, and as a matter of experiment, the mouths of the cavities did always splinter somewhat. If experiments could be made with the natural cavities which occur in some minerals this objection would not be present.

Dr. Becker placed at my disposal a number of singularly perfect negative crystals of quartz, which he had selected with considerable labor from the resources of the National Museum. Some of these contained bubbles of gas, and were therefore adapted to this purpose. Two of these were exposed to 18,000 kg. under kerosene for 4½ hours. No effect whatever was observable in the neighborhood of the negative crystals, and there was no diminution in the size of the bubble, indicating that there had been no permanent change of dimensions. A disturbing question naturally arises here: did not the liquid in the cavity afford important support to the walls from the inside, after the external pressure had slightly decreased the size of the cavity? But such cavities are usually filled with CO_2, and in this case the support would be quite inappreciable, because of the high compressibility of this gas. However, even if the cavity had been initially entirely filled with water, a simple computation shows that the maximum internal pressure would have been of the order of 1000 kg., which may be neglected in comparison with 18,000.

Some parts of the original quartz crystals remote from the negative crystals were smoky in appearance, and under the microscope, before the application of pressure, were seen to contain minute bubbles of quite irregular shape. In these regions a positive effect was produced by pressure, the appearance being that the interior of the bubbles had been eroded and tightly packed with sand, exactly as had the larger cavities in the cylindrical specimens. The same effect had been previously observed in a small region in the end of the second quartz specimen.

The conclusion must not be drawn that the flaking-off process is an end-effect connected with the unavoidable surface of separation, for this flaking-off was repeatedly observed at all points of the cavity, irrespective of the distance from the mouth. For instance, the fine splinters referred to in the experiment on andesite were distributed quite uniformly over the entire length of the brass wire. The cracks, however, which frequently appear, and are usually especially prominent near the surface of separation, may in the majority of cases be intimately connected with imperfect joining.

With respect to flaking-off, the conclusion seems to be that the state of polish of the surface is an important factor; if the polish is complete down to molecular dimensions, as it was for the negative crystals, the tendency to flake off is much less than if there are grosser irregularities in the surface structure. The polish must be very complete indeed to produce an appreciable effect; no artificial polish that could be applied to the interior of the specimens above had any effect. The first specimens were left rough after drilling the hole, but the later ones were polished.

Glass.—I have already referred to former experiments in which a sealed glass capillary had been exposed to 24,000 kg. without effect, while cavities in copper have been squeezed out of existence by 10,000 kg. Some essential difference between crystalline and non-crystalline materials suggested itself, which it was the partial purpose of these experiments to examine. A control specimen was therefore made of optical glass, exactly like the second specimen of quartz, and it was subjected to the same sort of treatment. It was first exposed to 5100 kg. and pressure immediately released, with no effect whatever. This confirmed the result with the capillary tube, that stresses much higher than those reached in ordinary compression could be sustained if the material is so arranged as to afford itself mutual support. After this test at 5100 it was a serious question how to proceed. Previous experience had shown that glass is exceedingly likely to receive some sort of internal strain under pressure so that on the second application of pressure it is much more likely to rupture than on the first. If a complete series of tests were made, as with quartz, there was danger that this effect would obscure the results, whereas if pressure were at once raised to the maximum and rupture found to have been produced, there would be no way of telling at what pressure rupture had occurred. I optimistically chose the second method and lost.

Pressure was raised to 12,000 kg. and immediately released. Rupture was complete. Failure to reproduce the result found with the capillary was doubtless due to incomplete alignment of the two halves. The results were nevertheless instructive in that the manner of rupture was entirely different from that of quartz. The entire mass of glass was filled with haphazard cracks, many of them running through to the outside. Each crack was curved and changed direction many times in a complicated fashion, quite unlike the simple cracks in the crystals. These cracks were more numerous at the inner wall, where they interpenetrated each other so extensively as to produce an apparent erosion of the interior like that of the minerals. But the eroded fragments were large instead of an impalpable powder, and in places the original polish of the walls of the cavity was still intact. It is therefore probable that the mechanism of rupture by flaking off was entirely absent in this specimen of glass.

Density of Compressed Sand.

The results already described show that in all probability large cavities cannot exist at considerable depths in the earth's crust, but the walls will disintegrate, packing the cavity with fine sand. The question as to the existence of microscopic cavities in this sand and the average density of material which has been ruptured was still untouched, and I therefore made a few further simple experiments in an attempt to partially answer this question.

Quartz, orthoclase feldspar, and talc were experimented on. These were reduced to sand in a mortar, and the quartz and feldspar were separated into various sizes with sieves of 40, 60, 80, and 120 meshes to the inch, and then cleaned by washing them with HCl and distilled water and dried by moderate heating in the air. The powder was placed in thin cylindrical copper boxes with copper covers, 1·5 cm. diameter and 1·5 cm. high, and subjected to compression between pistons of hardened steel in a hydraulic press. Lateral expansion was prevented by a very heavy ring of chrome-vanadium steel. The intensity of pressure in these experiments was 30,000 kg/cm^2. The pressure was not strictly hydrostatic, but this was no objection in view of the negative nature of the results. Under the conditions there must have been considerable grinding past each other of the grains, which is more conducive to fusion than pure normal pressure.

As far as fusion goes, the results were entirely negative. Talc, as well as quartz and feldspar of different sizes, pure and mingled together could not be fused to a homogeneous mass. All of these materials, however, could be pressed together into a cake coherent enough to handle; it was a surprise that the cake of talc was not especially more coherent than that of quartz. The average density of the cake was determined in the standard way by weighing under water, and is of some significance. The quartz ranged in density from 2·572 to 2·584, the original material being 2·65. There was a tendency for the cakes formed from originally larger grains to be of higher density, probably because a larger proportion of the total volume was occupied by unfractured material. The smallest density of all, 2·572, was that from a mixture of two sizes of sand, one passing through 40 but not 60, and the other through 120. The fragments into which the sand was reduced after compression were entirely irregular and of all sizes from a few hundredths to fractions of a thousandth of a millimeter. It is therefore obvious that any calculation of the density to be expected from the average density of closely piled spheres is inappropriate.

A single experiment on pure feldspar gave a density of 2·47, on pure talc 2·616, and on a mixture of equal parts by weight of quartz and feldspar sand, 2·520. The initial densities of feldspar and talc were 2·57 and 2·76. The average interstitial spaces were, therefore, 2·6, 3·9, and 5·2% in quartz, feldspar, and talc respectively. The high value for talc was a surprise, but it receives possible explanation from the remarks of the next paragraph.

The powder after compression has been referred to as in the form of coherent cakes; this requires considerable qualification. It was never possible to obtain a single coherent button of the dimensions of the containing box, but this button always broke up spontaneously into laminæ, slightly cupped, the faces of the laminæ at right angles to the direction of pressure. These laminæ varied from small scales a few tenths of a millimeter thick to comparatively large plates 2 mm. thick and nearly 1·5 cm. in diameter. Density determinations were made on the largest coherent pieces. Spontaneous break-up of the buttons into laminæ was always a comparatively slow affair, and might occupy 10 or 15 minutes, the button slowly puffing up with a crawling simulation of life. The force involved in this break-up was considerable, and was frequently sufficient to pull apart the copper box.

This phenomenon is not what one would at first expect, elastic recovery from stress being immediate, and it seems to me of considerable importance for an interpretation of the other results. A plausible explanation seems to me as follows: Under the intense stress, comminution into minute fragments proceeds so far that there are few actual voids left, but many spaces that would be void under no stress are closed by the elastic deformation of the walls. The majority of such spaces are probably lens-shaped, like the erosion cavities in the crystals above, and these are squeezed flat by the pressure. But on the surface of each grain there is a film of adsorbed air, which is squeezed extremely thin by the pressure, but nevertheless keeps the walls of the cavities from actual molecular contact, for otherwise there would be welding of the grains. When pressure is released, the lens-shaped cavities tend elastically to recover their form, but are prevented by the air films, which in such excessively thin layers will act like an extremely sticky and viscous glue. The slow viscous yield of these thin films of air accounts for the visibly slow break-up into laminæ.

This explanation suggests that the figures given above for the densities of compressed powders must be considerably less than the density when actually under pressure, and that for example the interstitial space in quartz sand under high pressure may possibly be considerably less than 2·6%. It is certain that 2·6% represents an upper limit. I made an attempt to find the actual volume of the sand while under pressure from the dimensions of the copper box, but unsuccessfully.

When these considerations are applied to geology, there are further complicating elements. In a cavity surrounded by solid walls the process of erosion by flaking-off may well be stopped long before the sand has acquired the mean pressure of the surrounding rock; this would allow a larger interstitial space than might be computed from the mean pressure.

Discussion and Summary.

Cavities in the materials dealt with in this paper, which may be broadly characterized by the property of brittleness, exhibit a method of failure under high compressive stresses not shown by ductile materials like the metals. This method consists in the shooting-off of minute fragments with considerable violence from the walls of the cavity. The frequency, and probably the velocity, of projection varies with the pressure, the rapidity of disintegration becoming greater at higher pressures. This mode of disintegration is shown both by rocks and by single crystals; in rocks the splinters show no relation to the boundaries between chemically homogeneous parts of the mixture, and in the crystals there is no obvious connection with the crystalline symmetry. The rate of change of speed of disintegration with pressure may vary greatly from substance to substance, being comparatively small for quartz and high for tourmaline and andesite.

The phenomenon of rupture by flaking-off is independent of other phenomena accompanying high stress. Some substances develop cracks at the same time that they erode; the number of cracks may be great as in calcite, or small as in quartz. Or the erosion may be accompanied by no cracks whatever, as in feldspar, porphyry and andesite. The substance may show no viscous flow during erosion, or it may flow like granite and baryte. The formation of cracks was never in these tests the cause of final rupture, except with glass. Cracks are probably in many cases due to the attempt of the solid to slip bodily into the cavity, but such slip can never go far before it is stopped by the mutual supporting action of the walls. Such slip may be prominent in a substance with easy cleavage, or slight as in quartz. It is probable that the cracks in quartz and calcite were essentially the same in character, one being merely more prominently developed than the other.

Flaws in the original specimen are apparently so tightly closed by pressure that they play no part in fracture.

Rupture by flaking-off is not even suggested by any mathematical theory of rupture, and probably cannot be in the nature of things. Mathematical theory treats the material as mathematically homogeneous, whereas we probably have to do here with a phenomenon of molecular agitation. All the observations are consistent with the view that a microscopic splinter flies off when its kinetic energy of temperature agitation has by chance become sufficiently higher than the average. The tendency to fly off will evidently be higher when the stress is high, and if the stress is high enough the process of disintegration will become rapid enough to be appreciable. Thermodynamics is familiar with something similar when the vapor pressure of a liquid is increased by an increase of pressure acting on the liquid phase. One expects that there will be no sharp point at which spontaneous disintegration suddenly begins, but that the effect continues over a range of pressure. This is confirmed by the experiments above. On the other hand, as with all such effects, there must be a pressure at which the effect practically ceases, so that at low pressures the chance of disintegration is of the same order, for example, as the chance that a pail of water will freeze on a red hot stove.

Consistently with this view one would expect the character of the surface to have an important effect. If the surface is perfectly smooth, as in the negative quartz crystals, or if as in glass there is a protecting covering of water 200 molecules deep like a jelly in which there may be surface tension effects, one expects very slight disintegrating tendency. Any artificial cavities, on the other hand, must contain large irregularities and be favorable to the effect.

This paper mentions the results of a new mathematical analysis of the effect in crystals of hydrostatic pressure applied as in these experiments. It appears that the new phenomena introduced by crystalline structure are not prominent enough to lead one to expect rupture because of them, and that in most cases an approximate solution may be obtained by treating the crystal as isotropic with mean values of the elastic constants.

The stresses which these brittle materials stand are many times higher than would be predicted by ordinary compression tests. If one neglects the flaking-off effect, which is entirely uncontemplated in mathematical theory, stresses at least 20 times higher than those of ordinary compression tests may be reached without rupture. At the same time the possible stresses are very appreciably lower than those found by Adams. His results were affected by the unknown action of shrunk-on steel jackets.

Attempts to weld together finely powdered quartz, feldspar, and talc failed up to 30,000 kg/cm^2. There is, however, no evidence that such welding would not take place if the adherent film of air could be entirely removed; this is a matter of extreme experimental difficulty. The amount of interstitial space in compressed powders has been measured, but caution must be used in inferring from these figures the density of a compressed sand while actually under pressure. The results of these collapsing tests makes it extremely probable, however, that minute crevices, at least large enough for the percolation of liquids, exist in the stronger rocks at depths corresponding to 6000 or 7000 kg/cm^2., and possibly more.

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