Opportunities will be afforded to British manufacturers to obtain a firm footing all over the world in markets which have hitherto been entirely in the hands of the Germans, and, if these opportunities are grasped, an increase in the number of industrial factories may be looked for.

Naturally, money to finance new projects will be both scarce and dear, but, making allowance for this, we should be in a better position than our enemies or allies, whose countries have been devastated and many of their most important factories destroyed. This is a subject which should engage the most thorough consideration and careful attention of the personnel of the electrical industry in common with all other British manufacturers. We must face the future in a spirit of calm optimism; and, if we must "wait and see," let our waiting be watchful, and with Britain fully "waked up" the recapture of their former world trade by the Germans should be a "negative possibility."

In concluding these remarks, I should like to say a word or two about the great necessity for increasing the number of men in our Army and our Navy. We have, in the latter, men and ships scattered all over the world, ready for any and all contingencies. Our armies have also been widely distributed. Our colonies are not only able to look after themselves but have placed the best of their citizens at the disposal of the mother country. Naturally I am very proud of the part played by my native country, Canada, in this war. And now we have to send armies and reserves against countries which were supposed to be allies or at least neutral on the outbreak of war.

Those of us who are unable to go to the front must do our share at home. I have referred to the men and women working in overalls, but the men and women who cannot take an active part in this war must do their share by economizing in everything possible and by subscribing every penny they can save or have saved, rather than see the efforts of our brave soldiers and sailors rendered abortive.

ARMATURE COPPER LOSSES IN ROTARY CONVERTERS AND DOUBLE-CURRENT GENERATORS.

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SUMMARY.

This paper deals with the calculation of the copper losses in an armature carrying both continuous and sinusoidal current, and conversely the calculation of the output of such a double-current machine (as for instance a rotary converter) in terms of the output as a continuous-current machine for equal total armature copper losses.

If any specific conductor in a rotary-converter armature be considered, it is evident that the resultant current which it carries is the sum of two currents, one being sinusoidal, the other being constant over a half-cycle and then reversing to an equal and opposite value for the next half-cycle.

For a conductor midway between the slip-ring tappings, and assuming unity power factor, the continuous current reverses when the alternating current is passing through its zero value.

In a rotary converter the alternating and continuous currents are in opposite senses, since one tends to motor the machine while the other is being generated, so that the value of the current at any moment during the half-cycle may be represented by the expression

$$I_M \sin\theta - 1{\cdot}0 \qquad (1)$$

where I_M is the maximum value of the alternating current in terms of the continuous current.

The copper loss for an infinitely short time (which may conveniently be measured in terms of θ) is then

$$(I_M \sin\theta - 1{\cdot}0)^2 \, d\theta$$

and the total watts lost as heat per half-cycle may be obtained by integrating this expression over half a cycle, *i.e.* between the limits of $\theta - \pi$ and θ.

Then

$$h = \int_{\theta-\pi}^{\theta} (I_M \sin\theta - 1{\cdot}0)^2 \, d\theta$$

$$= \int_{\theta-\pi}^{\theta} (I_M^2 \sin^2\theta - 2 I_M \sin\theta + 1{\cdot}0) \, d\theta$$

$$= \Big[I_M^2 (\tfrac{1}{2}\theta - \tfrac{1}{4}\sin 2\theta) + 2 I_M \cos\theta + \theta \Big]_{\theta-\pi}^{\theta}$$

$$= I_M^2 \Big[\frac{\theta - (\theta - \pi)}{2} - \tfrac{1}{4}\{\sin 2\theta - \sin(2\theta - 2\pi)\} \Big] + 2 I_M [\cos\theta - \cos(\theta - \pi)] + \theta - (\theta - \pi)$$

$$= I_M^2 \, \pi/2 + 4 I_M \cos\theta + \pi$$

$$= \pi \{ I_M^2/2 + 1 \} + 4 I_M \cos\theta \qquad (2)$$

where h is the total watts lost per half-cycle for the given conductor. θ is of course equal to π in the above simple case, but, as will be shown later, it has other values for other conditions, hence it is preferable to keep it in the indeterminate form θ.

Consider now other portions of the armature. The current may still be represented by the expression ($I_M \sin\theta - 1{\cdot}0$) over the half-cycle between the reversals of the continuous current, but these reversals no longer take place at the zero values of the alternating-current wave; therefore the limits of integration are no longer 0 and π.

If the rotary converter has n slip-rings, the winding (considered as a 2-pole winding throughout) is divided into n sections, each subtending an angle of $2\pi/n$ on the armature. The leading end of such a section of the winding then comes under the brush when it has a phase angle of $-\pi/n$, and again when it has a phase angle of $\pi - \pi/n$, these values then being the limits of integration for that element of the winding, and $\theta = \pi - \pi/n$. Similarly for the trailing end of the winding $\theta = \pi + \pi/n$, and the value of θ varies uniformly between these two values over that section of the winding between the slip-rings.

If the alternating current is lagging in phase by the angle ϕ, the elements of the winding pass through the changes mentioned above, while the current is in phase earlier still by the angle ϕ; that is θ varies from $(\pi - \phi) - \pi/n$ to $(\pi - \phi) + \pi/n$ over the section of the winding between the slip-rings.

Consider now, as a basis of comparison, the effect of the continuous current alone.

The summation of the expression $(\text{current})^2\, d\theta$ over half a cycle, or through an angle π, is obviously equal to π, where the continuous current in each circuit is 1·0, as taken before. This of course corresponds to an output current of 2·0.

From the above, the heating of any given element of the armature can be compared with its heating by continuous current alone.

The heating or copper loss over a small element of the armature subtending an angle $d\theta$ at the centre is then, from Equation (2),

$$h\, d\theta = \left\{\pi\left(\tfrac{1}{2} I_M^2 + 1\right) + 4\, I_M \cos\theta\right\} d\theta.$$

The total heating over one section of the armature winding between the slip-rings is then the summation of the above value for h between the following limits:—

$(\pi - \phi) - \pi/n$, which may conveniently be called α,
and $(\pi - \phi) + \pi/n$, which may conveniently be called β.

Since there are n such sections on the armature, the total loss H may be written:—

$$\begin{aligned} H &= n\int_\alpha^\beta \left\{\pi\left(\tfrac{1}{2} I_M^2 + 1\right) + 4\, I_M \cos\theta\right\} d\theta \\ &= n\Big[\pi\,\theta\left(\tfrac{1}{2} I_M^2 + 1\right) + 4\, I_M \sin\theta\Big]_\alpha^\beta \\ &= n\left[\pi\,(\beta - \alpha)\left(\tfrac{1}{2} I_M^2 + 1\right) + 4\, I_M(\sin\beta - \sin\alpha)\right] \quad . \quad (3) \end{aligned}$$

Two trigonometrical simplifications are possible in this formula, namely:—

$$\begin{aligned} n(\beta - \alpha) &= n\Big[\{(\pi - \phi) + \pi/n\} - \{(\pi - \phi) - \pi/n\}\Big] \\ &= 2\pi \end{aligned}$$

and

$$\begin{aligned} \sin\beta - \sin\alpha &= \sin\{(\pi - \phi) + \pi/n\} - \sin\{(\pi - \phi) - \pi/n\} \\ &= 2\cos(\pi - \phi)\sin(\pi/n) \\ &= -2\cos\phi\sin(\pi/n). \end{aligned}$$

The resultant equation therefore becomes

$$H = 2\pi^2\left(\tfrac{1}{2} I_M^2 + 1\right) - 8\, I_M\, n\cos\phi\sin(\pi/n) \quad . \quad (4)$$

The corresponding loss as a continuous-current machine with the same continuous current is the loss per element π summed over the whole circumference, *i.e.* $2\pi^2$.

Dividing the value of H in Equation (4) by $2\pi^2$ and replacing I_M by $\sqrt{2}\, I$, where I is the virtual value of the alternating current, the equation becomes

$$W = H/(2\pi^2) = (I^2 + 1) - 0{\cdot}573\, I\, n \cos\phi \sin(\pi/n) \; . \quad (5)$$

where I is the virtual value of the alternating current in the armature winding (not in the mains) in terms of the continuous current in the winding, and W is the number of watts lost in terms of the watts that would be lost (I^2R loss) if the machine were acting as a plain continuous-current generator with the same current on the continuous side, n being the number of slip-rings.

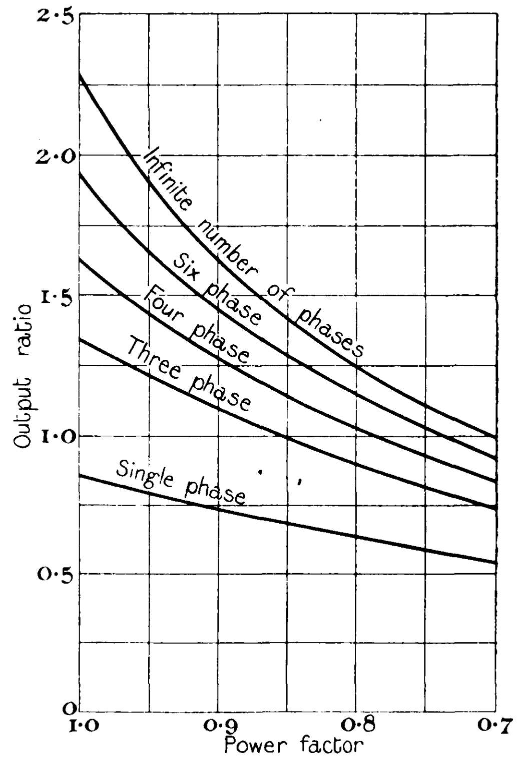

FIG. 1.

Here it may be noted that $\cos\phi$ is the only form in which ϕ enters into the equation; and since $\cos\phi = \cos(-\phi)$, it is immaterial whether ϕ, and hence the current, is leading or lagging. Further, the negative sign indicates that the two currents (C.C. and A.C.) flow in opposite directions. If they are in the same direction, as in a double-current generator, the sign must be changed to +, or the value of ϕ may be considered to be increased by π, which gives the same result.

For such a machine also it is more convenient to take the virtual value of the alternating current. We get therefore for a double-current generator

$$W = (I^2 + 1) + 0{\cdot}573\, I\, n \cos\phi \sin(\pi/n) \; . \; . \quad (6)$$

where I is the virtual value of the alternating current in the armature winding (not in the mains) in terms of the continuous current in the winding, and W is the number of watts lost in terms of the watts that would be lost if the continuous current alone were acting.

Conversely, it may be taken that given the value of I, n, and ϕ in Equations (5) and (6), the output obtainable with the same total armature loss is $\sqrt{1/W}$ times the output obtainable from the same armature as a plain continuous-current generator, both outputs being measured on the continuous-current side. Returning to Equation (5), which is a general equation for all values of I, it is evident that in a machine acting purely as a rotary converter the factor I is dependent on the factors n and ϕ, so that further simplification is possible.

The maximum voltage on the alternating-current side of a rotary converter is E sin (π/n), where E is the voltage on the continuous-current side. Hence the virtual alternating electromotive force is

$$0{\cdot}707 \text{ E} \sin(\pi/n).$$

For an efficiency of 100 per cent the energy is the same on both the continuous and alternating sides.

The energy on the alternating side

$$= n \times \text{I} \times \text{alternating voltage} \times \cos\phi,$$

and the energy on the continuous side

$$= 2\text{ E} \times \text{continuous current},$$

the currents in each case being measured in the windings.

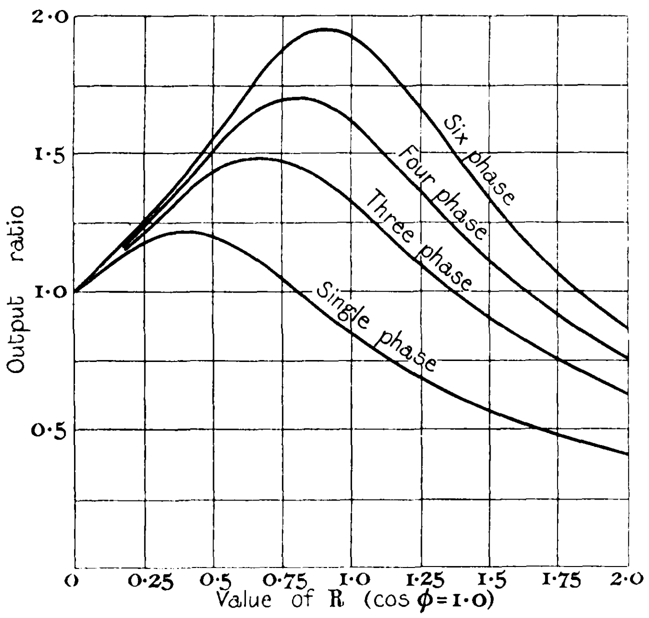

FIG. 2.

Therefore the alternating current I in Equation (5) is, in terms of the continuous current,

$$= \frac{2\text{ E}}{n \times 0{\cdot}707 \text{ E} \sin(\pi/n) \times \cos\phi}$$

$$= \frac{2{\cdot}828}{n \cos\phi \sin(\pi/n)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Replacing I in Equation (5) by this value, the following result is obtained :—

$$\text{W} = \left(\frac{2{\cdot}828}{n\cos\phi\sin(\pi/n)}\right)^2 + 1 - 1{\cdot}62$$

$$= \left(\frac{2{\cdot}828}{n\cos\phi\sin(\pi/n)}\right)^2 - 0{\cdot}62 \quad . \quad . \quad . \quad . \quad . \quad (8)$$

Filling in the value for n (for 1, 3, 4, and 6 phases $n = 2$, 3, 4, and 6 respectively) the following equations are obtained :—

$$\textit{For single phase } \text{W} = \frac{2}{\cos^2\phi} - 0{\cdot}62 \quad . \quad . \quad . \quad (9a)$$

$$\textit{For three phase } \text{W} = \frac{1{\cdot}185}{\cos^2\phi} - 0{\cdot}62 \quad . \quad . \quad . \quad (9b)$$

$$\textit{For four phase } \text{W} = \frac{1{\cdot}0}{\cos^2\phi} - 0{\cdot}62 \quad . \quad . \quad . \quad (9c)$$

$$\textit{For six phase } \text{W} = \frac{0{\cdot}888}{\cos^2\phi} - 0{\cdot}62 \quad . \quad . \quad . \quad (9d)$$

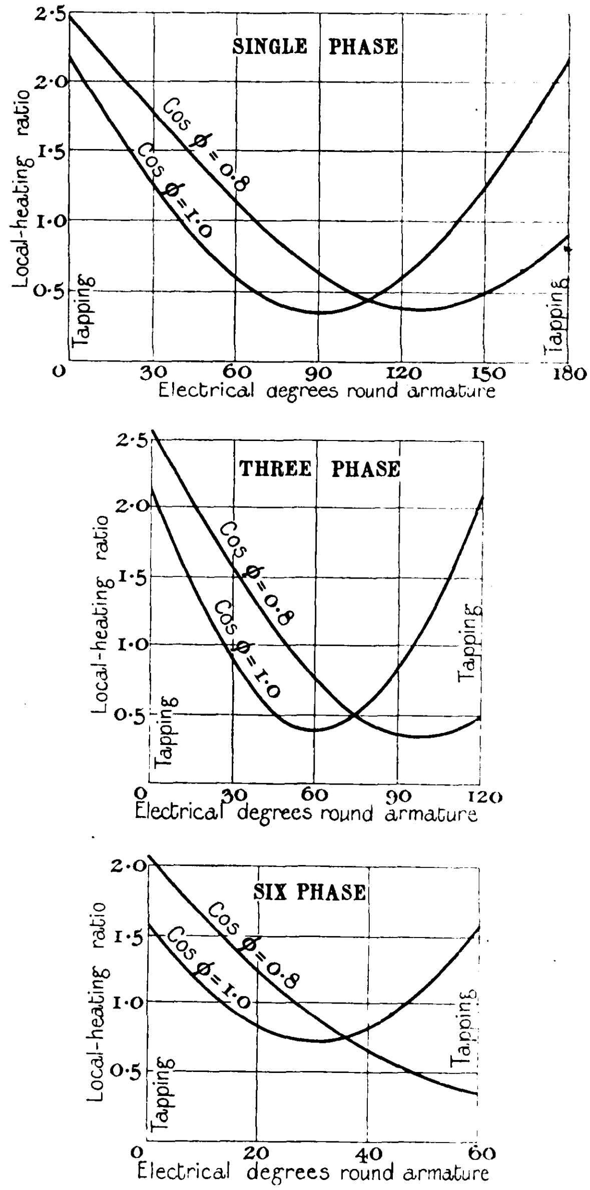

FIG. 3.

For an infinite number of phases π/n is extremely small, and $\sin(\pi/n) = \pi/n$, hence $n \sin(\pi/n) = \pi$, and

$$\text{W} = \frac{0{\cdot}81}{\cos^2\phi} - 0{\cdot}62 \quad . \quad . \quad . \quad . \quad . \quad . \quad (9e)$$

The above are the equations for plain rotary converters with no loss in conversion, W being the copper loss in terms of the copper loss that would occur if the machine were giving the same current on the continuous-current side as a plain continuous-current generator. The last equation shows that there is no great advantage to be gained by increasing the number of phases above six.

Conversely the value of $\sqrt{1/W}$ gives the output obtainable from a rotary converter for the same total armature copper loss, in terms of the continuous-current output which would give that copper loss, both outputs being measured on the continuous-current side (see Fig. 1).

In certain cases it may happen that the rotary converter is doing mechanical work (either positive or negative) in addition to its converting functions, as for example when a rotary converter has coupled to it a booster, which may be either raising the volts (boosting) or lowering the volts (motoring). In this case either Equation (5) may be used, or if by R we denote the ratio of the actual alternating current on the rotary-converter slip-rings to the current for one-to-one power conversion, Equations (9*a*), (9*b*), (9*c*), and (9*d*) become

$$\textit{Single phase } W = \frac{2R^2}{\cos^2\phi} + 1 - 1\cdot62R \quad . \quad . \quad . \quad (10a)$$

$$\textit{Three phase } W = \frac{1\cdot185R^2}{\cos^2\phi} + 1 - 1\cdot62R \quad . \quad . \quad (10b)$$

$$\textit{Four phase } W = \frac{R^2}{\cos^2\phi} + 1 - 1\cdot62R \quad . \quad . \quad . \quad (10c)$$

$$\textit{Six phase } W = \frac{0\cdot888R^2}{\cos^2\phi} + 1 - 1\cdot62R \quad . \quad . \quad (10d)$$

These figures are easily obtained by substituting I R for I in Equation (5), and if R be taken negative, the conditions for a double-current generator are obtained—this being an alternative form to Equation (6), with which it agrees.

Curves showing the values of $\sqrt{1/W}$ obtained are given in Fig. 2, and while $R = 1\cdot0$ only applies to the theoretical condition of an efficiency of 100 per cent, these curves show the correction to be applied when the machine is motoring from the alternating-current end (R slightly greater than unity) or from the continuous-current end (R slightly less than unity).

From these curves it is noticeable that to keep the rotary-converter losses down, and hence the output up, it pays to keep the alternating current less than the simple transformation ratio. Thus if the machine is supplied with an alternating-current booster in series with the armature, it pays to let the booster reduce the alternating voltage, *i.e.* to motor (if the set is motoring from the alternating-current side).

The local distribution of the heating around the armature between the slip-ring tappings can be calculated from Equation (2), θ being varied between its limits of $(\pi - \phi) - \pi/n$ and $(\pi - \phi) + \pi/n$.

Fig. 3 shows some typical curves for converters with a one-to-one power-transformation ratio.

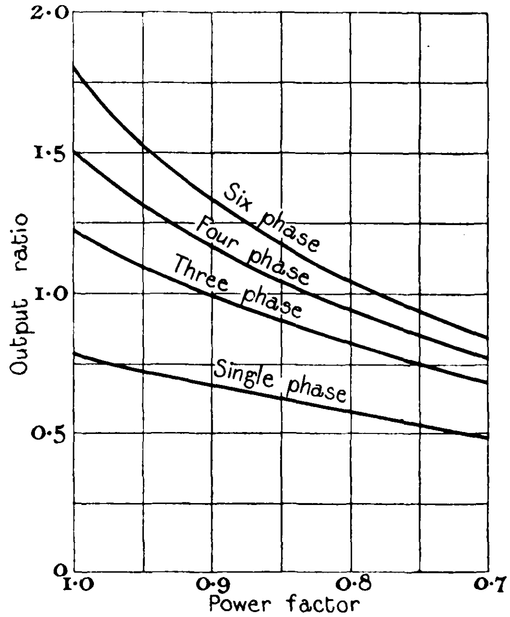

FIG. 4.

The actual margin to be allowed in the output of the rotary converter depends of course on the ratio of the iron losses to the copper losses, and on the quality of the ventilation, etc. It must be therefore left to the judgment of the individual designer. As a suggestion for practical use, the curves shown in Fig. 4 have been plotted. These allow, first, for the alternating current to be increased 5 per cent above the theoretical value ($R = 1\cdot05$) in order to allow for driving the machine, and secondly, a margin of 5 per cent in output below the value thus obtained in order to allow for unequal armature heating.