Back to Programme *Back to Topic*

APPLICATION OF FULL-FIELD MEASUREMENTS AND INVERSE IDENTIFICATION TO COMPOSITE MATERIALS

F. Pierron
LMPF, Arts et Metiers ParisTech, Rue St Dominique, BP 508
51000 Châlons en Champagne, France
fabrice.pierron@chalons.ensam.fr

SUMMARY

This paper presents an overview of applications of full-field deformation measurements and inverse identification to composites, with a focus on the Virtual Fields Method. Examples in orthotropic elasticity (in-plane, bending), non-linear behaviour, damage detection and high strain rate testing are given.

Keywords: Virtual Fields Method, Full-field measurement, Composites testing, Inverse identification

INTRODUCTION

The objective of the present paper is to give an overview of how full-field deformation measurements can be used through inverse identification routines to identify the mechanical behaviour of composites. It is mainly based on work produced over the years by the author and coworkers using a specific inverse identification procedure called the Virtual Fields Method (VFM). It is beyond the scope of the present paper to review the scientific literature on this subject in depth, a detailed account being available in [1,2]. The second reference also gives some details on the numerical procedures used to generate optimized virtual fields in linear elasticity.

HOMOGENEOUS LINEAR ELASTIC BEHAVIOUR

The principle of the VFM is to write the global equilibrium of the tested coupon using the principle of virtual work. This can be written as (in static and ignoring volume forces):

$$-\int_V \sigma_{ij}\varepsilon^*_{ij}dV + \int_{\partial V} T_i u^*_i dS = 0 \qquad (1)$$

where σ is the stress tensor, T the distribution of surface loads, u^* the virtual displacement field, ε^* the associated virtual strain field, V the volume of the solid and ∂V its boundary. Assuming a linear elastic behaviour with $\sigma_{ij} = C_{ijkl}\varepsilon_{kl}$, one can substitute the stress tensor into Eq. 1 and provided that the material is homogeneous, the C components can be taken out of the integrals to give:

$$-C_{ijkl}\int_V \varepsilon_{kl}\varepsilon^*_{ij}dV + \int_{\partial V} T_i u^*_i dS = 0 \qquad (2)$$

Therefore, for each choice of kinematically admissible virtual field, a linear equation can be established linking some of the elastic unknowns to integrals of the actual strains that can be evaluated if full-field measurements are available.

In-plane testing

For an in-plane orthotropic material, the previous equation can be written as:

$$-Q_{xx}\int_V \varepsilon_x\varepsilon^*_x dV - Q_{yy}\int_V \varepsilon_y\varepsilon^*_y dV - Q_{xy}\int_V (\varepsilon_x\varepsilon^*_y + \varepsilon_y\varepsilon^*_x)dV - Q_{ss}\int_V \varepsilon_s\varepsilon^*_s dV + \int_{\partial V} T_i u^*_i dS = 0 \qquad (3)$$

where the four Q parameters are the stiffness components to identify. It should be noted that this equation involves volume integrals of the strains. In practice, full field measurements are usually obtained on the specimen surface only, it is therefore necessary to make an assumption on the through-thickness distribution of the strains. In the present case, and as is usual for material testing, the strain distribution is assumed to be uniform through the thickness. The above equation then becomes:

$$-Q_{xx}\int_V \varepsilon_x\varepsilon^*_x dS - Q_{yy}\int_V \varepsilon_y\varepsilon^*_y dS - Q_{xy}\int_V (\varepsilon_x\varepsilon^*_y + \varepsilon_y\varepsilon^*_x)dS - Q_{ss}\int_V \varepsilon_s\varepsilon^*_s dS + \frac{1}{t}\int_{\partial V} T_i u^*_i dS = 0 \qquad (4)$$

where t is the specimen thickness. The choice of four independent virtual fields enables to build up a linear system where the Q components are the unknown. The inversion of this system yields the stiffness components. It should be noted that, as opposed to finite element model updating [3] which requires iterative FE calculations, this procedure is direct and therefore, very fast.

Over the years, this was applied to several situations. After a first attempt on a T-shaped specimen [4], most of the efforts were targeted at a bending/shear test based on the Iosipescu fixture [5,6]. This configuration was also the starting point of a study targeted at the optimization of the testing configuration. It was shown in [7] that by adapting the specimen length and the fibre orientation in a unidirectional specimen, a better compromise was reached to identify all the stiffness components.

The main motivation for the above is to obtain the complete in-plane orthotropic stiffness tensor of a composite from just one test instead of at least three for standard tension and shear tests. However, there are also situations where the geometry of the coupon does not allow the use of such standard tests. This was the case for the identification of the through-thickness stiffness components of a thick hoop wound glass-epoxy composite ring [8]. The idea here was to perform a simple diametric compression test on the full ring. By measuring the in-plane deformation on a 30° sector (see Fig. 2), it was shown that all the orthotropic stiffness components could be successfully identified in the cylindrical axes. However, it can be seen on Fig. 2 that two back-to-back cameras were used to measure the deformation of the ring on both its faces. This was found necessary because of the difficulty to align such a slender specimen in the testing fixture. The back-to-back strain fields were used to compute an average strain field to input in the VFM equation (this is equivalent to assuming a linear strain distribution through the thickness as opposed to a uniform one). Thanks to this refinement, very convincing results were obtained. This procedure is currently being applied to superconducting coils [9].

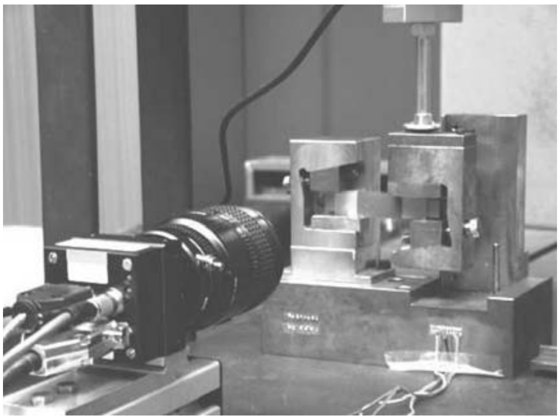

Figure 1 – Bending/shear test based on the ASTM double V-notch beam shear test standard [6].

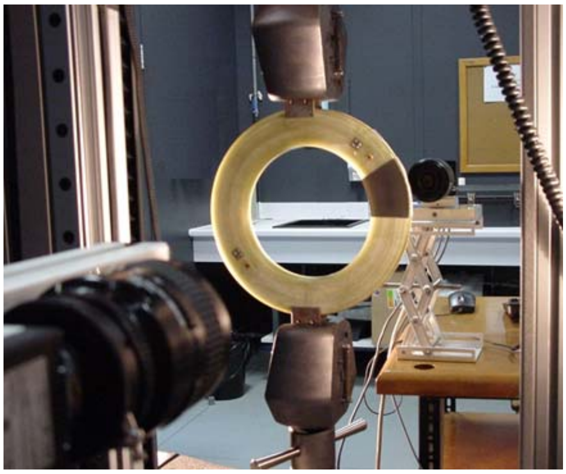

Figure 2 – Diametric compression test on a thick composite tube [8].

Bending tests

A significant amount of work was also devoted to the use of the same procedure to identify the bending stiffnesses of anisotropic composite thin plates. It was actually the historical application that led to the VFM in the first place [10]. In bending, Eq. 3 can be written as:

$$D_{xx}\int_S \kappa_x \kappa_x^* dS + D_{xy}\int_S (\kappa_x \kappa_y^* + \kappa_y \kappa_x^*) dS + D_{yy}\int_S \kappa_y \kappa_y^* dS + D_{ss}\int_S \kappa_s \kappa_s^* dS = \sum_{i=1}^{n} F_j w_j^* \qquad (5)$$

where S is the surface of the plate, w* the virtual deflection field, F the normal forces applied to the plate at *n* points, κ the actual curvature fields, κ* the virtual curvature fields derived from w* and the D_{ij}'s the bending stiffness components to identify. One of the difficulties here is that the curvatures have to be measured. If deflection fields are obtained through stereo image correlation for instance, then a double spatial differentiation will be necessary to derive the curvature fields, which will dramatically amplify the effect of noise. Interferometric shearography could be an alternative to measure directly the slope fields at the specimen surface but quantitative shearography is not readily available. Another possibility is to use a deflectometry technique which consists in looking at the deformed image of a reference grid at the surface of the specimen [11]. It should be noted that in this case, the specimen surface has to exhibit excellent specular reflection with a very smooth finish. A coating can be applied on composite plates [12].

Very successful applications have been released in static bending, with different test configurations [13-15]. Again, one of the problems concerns the test configuration to use. Some recent efforts have been dedicated to the optimization of the tests [16,17].

NON-LINEAR BEHAVIOUR

The methodology presented above has also been extended to the identification of the non-linear behaviour in shear of a unidirectional composite. There are two cases depending on the parameterization of the stress-strain relationship. If it is linear with respect to the parameters, such as in [18]:

$$\sigma_s = Q_{ss}\varepsilon_s - K\varepsilon_s^3 \tag{6}$$

where K is a softening parameter, then Eq. 4 can be rewritten as:

$$-Q_{xx}\int_V \varepsilon_x\varepsilon_x^* dS - Q_{yy}\int_V \varepsilon_y\varepsilon_y^* dS - Q_{xy}\int_V (\varepsilon_x\varepsilon_y^* + \varepsilon_y\varepsilon_x^*)dS - Q_{ss}\int_V \varepsilon_s\varepsilon_s^* dS - K\int_V \varepsilon_s^3\varepsilon_s^* dS + \frac{1}{t}\int_{\partial V} T_i u_i^* dS = 0 \tag{7}$$

which is still a linear equation and the 5 parameters can then be identified from a single strain state thanks to the shear stress heterogeneity in the unnotched Iosipescu like specimen of Fig. 1 [19]. However, experimentally, the following model proved more realistic:

$$\sigma_s = Q_{ss}\varepsilon_s - K\varepsilon_s < \varepsilon_s - \varepsilon_s^0 >^+ \tag{8}$$

where $<.>^+$ indicates the positive part of the bracket contents and ε_s^0 is a strain threshold. This corresponds to a damage threshold and was the model developed in [20]. Obviously, the resulting VFM equation is now non-linear and the resolution is slightly different but the spirit remains the same. Details can be found in [6]. The results are represented on Fig. 3 where it can be seen that the identification is successful.

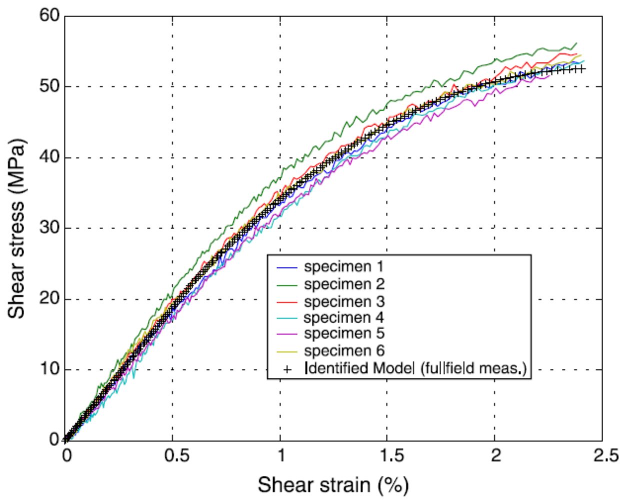

Figure 3 – Non-linear stress-strain curves corresponding to the model of Eq. 8, from standard Iosipescu tests (colour lines) and the Virtual Fields Method on the unnotched Iosipescu test (crosses) [6]

DAMAGE DETECTION

More recently, the above methodology was applied to the case of a damaged composite plate. The underlying idea is that if the present technology enables to locate a damage in a composite components that has undergone a mechanical impact (resulting in a complex combination of delamination, fibre breakage and matrix cracking in the impact area), these techniques, such as ultrasound [21], infrared thermography [22] or Lamb waves [23], to cite just a few, are not able to evaluate the local loss of mechanical properties, which is the relevant information for design engineers. Besides and thanks to the drastic improvement in computing power, a lot of efforts have been dedicated to the numerical simulation of impact damage in composites. However, the validation of these models requires the experimental determination of mechanical quantities, such as the in-plane or bending stiffnesses in the damaged area. This has mainly been achieved by simple tests on beams (tension, bending) but this process is very long and it is not possible to retrieve all stiffness components on the same beam.

The objective here is to identify stiffness components that vary within the tested plate. For instance, let us consider a flat composite plate that has undergone a local impact somewhere at its surface. This impact will have caused local damage and as a result, the local stiffness tensor will be affected, with principal components being reduced. As a first simplification, one can consider the damage as isotropic, ie, all stiffness components are decreased by the same ratio. One can therefore write:

$$\widetilde{D} = D^0(1 - f(x, y)) \tag{9}$$

where $\widetilde{D}$ is the bending stiffness matrix of the plate, D^0 the bending stiffness of the virgin composite and f(x,y) a function of the plate coordinates. In this case, D^0 is constant but $\widetilde{D}$ depends on the space variables because the stiffness components vary within the damage plate.

Clearly, the choice of f(x,y) is critical here. A first approach was to consider a polynomial for the f function. This would have good regularization properties even though the price to pay will be that the solution will not be purely local (ie, measurements from all the points influence the stiffness value at a particular position). However, it was shown that this solution was adapted to detect small stiffness variations (between 5 and 30%).

The resolution uses the same equation as that in Eq. 5. Knowing the D^0 components (obtained from the test on an undamaged plate of the same material), then Eq. 5 become a linear equation where the unknowns are coefficients of the f(x,y) polynomial. This is solved using optimized polynomial virtual fields. Details can be found in [24].

In bending of thin plates, the in-plane strains relate to the plate curvatures. Therefore, if full-field deflections were measured, two spatial differentiations would be necessary to obtain the curvatures, ie, the strains. This double differentiation will amplify the noise dramatically. But even worse is the fact that measuring a stiffness gradient is equivalent to measuring strain gradients, ie, third derivatives of measurements in this case. It was therefore chosen as a first step to use deflectometry [11] which enables to measure full-field slopes. Again, a coating is necessary to make the plate surface smooth and reflective [12]. Fig. 4 shows the experimental set-up.

As a first example, a 16 plies (2.56 mm thick) 190 x 140 mm^2 T300/914 carbon-epoxy unidirectional panel was manufactured and over a 50 x 50 mm^2 area, Teflon tape was inserted between the 14/15 and 15/16 plies interfaces. After curing, this part of the material was removed using a cutter, leaving a panel with a reduced thickness over this area. Measurements were performed on this panel. An 8^{th} degree polynomial was used for f, resulting in 44 unknowns to identify. The undamaged stiffnesses were obtained on an undamaged plate [24].

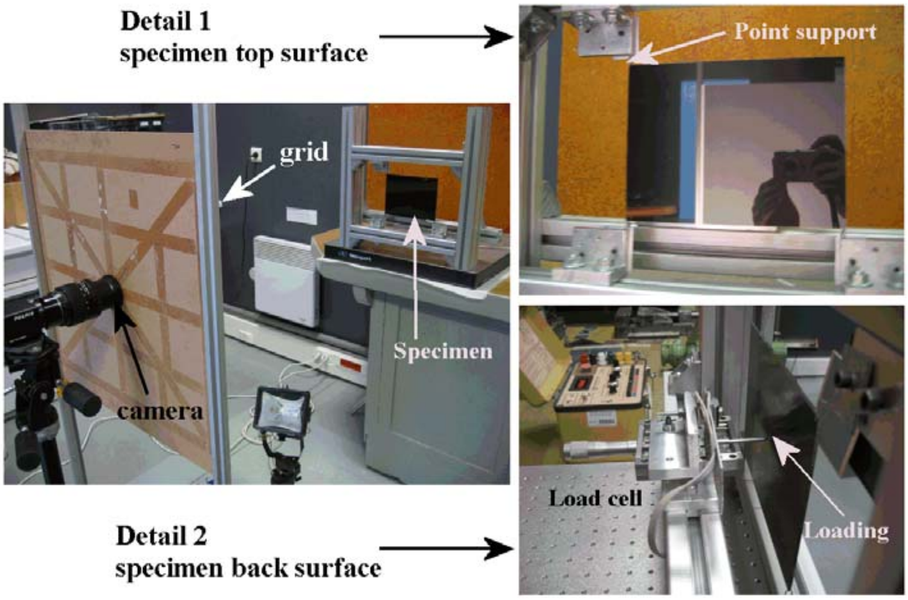

Figure 3 – Deflectometry set-up [24]

Fig. 4 shows the test configuration. A load of 6.5 N was applied.

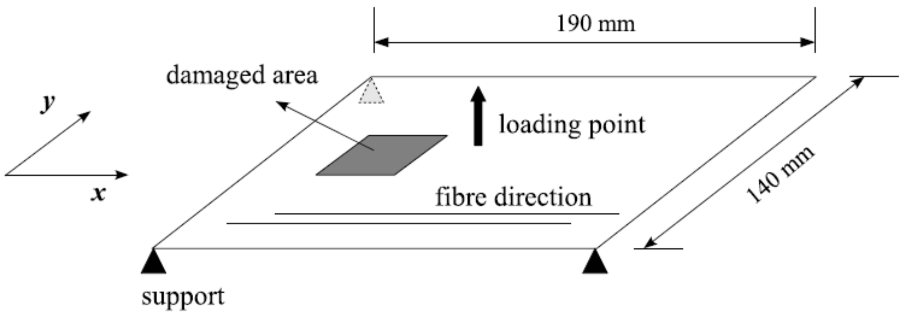

Figure 4 – Test configuration [24]

Fig. 5 is a plot of the f function from the identified parameters. It can be seen that the technique enables not only to detect the damage location but also to give a good estimate of the stiffness reduction (theoretical stiffness profile in blue on Fig. 5). One can also see that the polynomial description is not ideal. However, for a real impact damage, the transition between the undamaged and damaged zones will be much smoother that that used here and the polynomial description should perform better.
Discrete parameterization was tried [25] but did prove very unstable. Some sort of regularization should be added. Another strategy would be to use several load cases. This is presently underway.

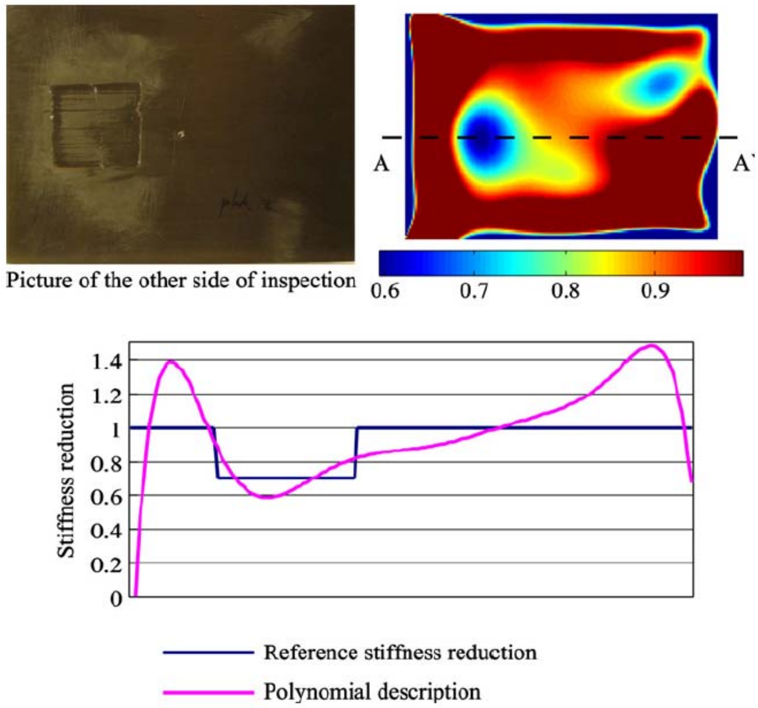

Figure 5 – Stiffness reduction identification

DYNAMIC TESTING

The last application reviewed in this paper is a very recent one. It consists in recording images in a fast transient deformation process as obtained on a Hopkinson bar type of set up. As these tests usually result in very high rates of deformation, full-field measurements have to be performed at frame rates of the order of several thousands of frames per second. Two main types of fast cameras are available on the market. The first category uses a single sensor. This exhibits the same imaging features as that of the usual CCD cameras but because of issues related to data transfer, full spatial resolution can only be obtained for frame up to about 5000 fps, typically. Higher frames rates can be reached by compromising on the spatial resolution but then quantitative full-field deformation cannot be achieved in a satisfactory manner. The alternative consists in a multi-sensor technology. Typically, 16, 32 or 64 sensors are spatially distributed within the camera and a rotating device (mirror or shutter) sends the light to the different sensors. The frame rates obtained with this technology reaches up to several millions of fps at a full spatial resolution of 1Mpixel typically. A good summary of the relative performances of such cameras can be found in [26].

Tests were performed using a tensile Hopkinson bar fixture on open hole quasi-isotropic carbon-epoxy specimens. A Cordin 550-62 camera was used to capture images during the deformation process. Details of the experimental procedure can be found in [27]. The deformation fields were obtained through the grid method, with a 5 lines/mm grid bonded onto the specimen. Because of contrast and lighting issues, the frame rate could not be set above 300.000 fps, which was sufficient to capture about 6 images before the first surface crack appeared on the top 45° ply. A first interesting finding was that the deformation was not symmetrical probably because of the test fixture, with an additional in-plane bending moment resulting in cracks appearing preferentially on one side of the specimen. It must be noted that just before the first surface crack, strain rates up to 5000 s^{-1} were recorded near the hole.

However, the purpose here is to show how quantitative stiffness data can be obtained from such measurements. Basically, rewriting the principle of virtual work in dynamics for this quasi-isotropic material, the following equation is obtained:

$$
\begin{aligned}
&-Q_{xx}\int_V\left(\varepsilon_x\varepsilon_x^* + \varepsilon_y\varepsilon_y^* + \frac{1}{2}\varepsilon_s\varepsilon_s^*\right)dV - Q_{xy}\int_V\left(\varepsilon_y\varepsilon_x^* + \varepsilon_x\varepsilon_y^* - \frac{1}{2}\varepsilon_s\varepsilon_s^*\right)dV \\
&+\int_{\partial V}\left(T_x u_x^* + T_y u_y^*\right)dS = \rho\int_V\left(a_x u_x^* + a_y u_y^*\right)dV
\end{aligned}
\qquad (9)
$$

where a is the acceleration field and ρ the material density. In static (Eqs. 2 and 3), it is clear that if no force is measured, it is possible to select virtual fields that will zero the terms containing T but in this case, only systems of the type AQ=0 can be obtained. This means that only stiffness ratios (for instance, Poisson's ratio) can be identified, not actual stiffness values. In dynamics, the right hand-side term comes to the rescue and when enough acceleration forces are present, it is possible to discard the force term and still balance out the internal virtual work with the acceleration virtual work. This is very powerful; it is basically using the acceleration forces as a volume distributed load cell (provided that the density is known, of course). This idea has been used previously for vibration tests where stiffness and damping was identified on thin plates regardless of the boundary conditions [28,29]. The processing of these results is still underway but Fig. 6 shows the identification of Poisson's ratio for several load steps before the first surface crack. It is reasonably stable and not far from the target. To the best knowledge of the author, this is the first time that quantitative use is made of measurements obtained with this type of ultra high speed camera.

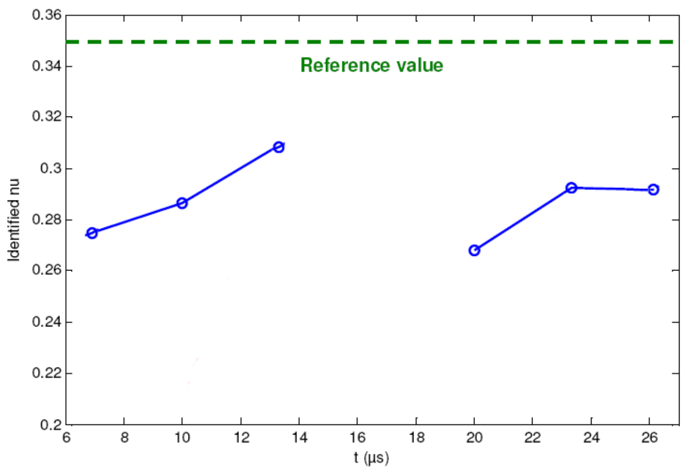

Figure 6 – Identified Poisson's ratio

CONCLUSION

The present paper has briefly reviewed the different applications of the Virtual Fields Method to composites in the recent past. The versatility and potential is clear though the subject is still in its infancy. On the long term, the objective is to renew the materials testing procedures based on full-field optical deformation measurements that are now more or less readily available in a great numbers of mechanical testing laboratories. The target is also to address complex problems such as high strain rate testing or heterogeneous materials to help validate numerical simulations with more advanced experimental tools.

References

1. Pierron F, and Avril S. Inverse problems in experimental solid mechanics, Chapter 6.194.23, *EOLSS encyclopedia*, www.eolss.org.
2. Grédiac M, Pierron F, Avril S, and Toussaint E. The Virtual Fields Method for extracting constitutive parameters from full-field measurements: a review. *Strain*, 42:233-253, 2006.
3. Lecompte D, Smits A, Sol H, Vantomme J, and Van Hemelrijk D. Mixed numerical-experimental technique for orthotropic parameter identification using biaxial tensile tests on cruciform specimens. *International Journal of Solids and Structures* 44(5):1643-1656, 2007.
4. Grédiac M, Pierron F, and Surrel Y. Novel procedure for complete in-plane composite characterization using a single T-shaped specimen, *Experimental Mechanics* 39(2):142-149, 1999.
5. Pierron F, and Grédiac M. Identification of the through-thickness moduli of thick composites from whole-field measurements using the Iosipescu fixture, *Composites Part A: Applied Science and Manufacturing*, 31(4):309-318, 2000.
6. Chalal H, Avril S, Pierron F, and Meraghni F. Experimental identification of a nonlinear model for composites using the grid technique coupled to the virtual fields method, *Composites Part A: Applied Science and Manufacturing*, 37(2):315-325, 2006.
7. Pierron F, Vert G, Burguete R, Avril S, Rotinat R, and Wisnom M. Identification of the orthotropic elastic stiffnesses of composites with the virtual fields method: sensitivity study and experimental validation, *Strain*, 43(3):250-259, 2007.
8. Moulart R, Avril S, and Pierron F. Identification of the through-thickness rigidities of a thick laminated composite tube, *Composites Part A: Applied Science and Manufacturing*, 37(2):326-336, 2006.
9. Nunio F, Védrine P, Kim J-H, and Pierron F. Experimental and numerical characterization of mechanical properties of superconducting windings using the virtual fields method, *2008 Applied Superconductivity Conference*, 17-22 August 2008 in Chicago, Il., USA.
10. Grédiac M. Principe des travaux virtuels et identification, Comptes Rendus de l'Académie des Sciences. Vol. II 309:1–5, 1989.
11. Surrel Y. Deflectometry: a simple and efficient noninteroferometric method for slope measurement. In *Xth SEM International Congress on Experimental Mechanics*, 2004.
12. Kim J-H, Pierron F, Grédiac M, and Wisnom M. A procedure for producing reflective coatings on plates to be used for full-field slope measurements by a deflectometry technique, *Strain*, 43:138-144, 2007.
13. Grédiac M. On the direct determination of invariant parameters governing the bending of anisotropic plates. *International Journal of Solids and Structures*, 33:3969–3982, 1996.

14. Grédiac M, and Vautrin, A. Mechanical characterization of anisotropic plates: experiments and results. *European Journal of Mechanics A: Solids*, 12:819–838, 1993.
15. Avril S, Grédiac M, Pierron F, Surrel Y, and Toussaint E. Deflectometry and virtual fields for identification of static plate bending stiffnesses, In *Xth SEM International Congress on Experimental Mechanics*, 2004.
16. Syed-Muhammad K, Toussaint E, Grédiac M, Avril S, and Kim, J-H. Characterization of composite plates using the virtual fields method with optimized loading conditions, *Composite Structures*, 85(1):70-82, 2008.
17. Syed-Muhammad K, Toussaint E, and Grédiac M. Optimization of a mechanical test on composite plates with the virtual fields method, Structural and Multidisciplinary Optimization, 38(1):71-82, 2009.
18. Swansson SR, Messick M, and Toombes GR. Comparison of torsion tube and Iosipescu in-plane shear test results for a carbon epoxy composite, *Composites*, 16(3):220-224, 1985.
19. Chalal H, Meraghni F, Pierron F, and Grédiac M. Direct identification of the damage behaviour of composite materials using the virtual fields method, *Composites Part A: Applied Science and Manufacturing*, 35(7-8):841-848, 2004.
20. Ladevèze P, and Le Dantec E. Damage modeling of the elementary ply for laminated composites, *Composites Science and Technology*, 43(3):257–68, 1992.
21. Preuss TE, and Clark G. Use of time-of-flight C-scanning for assessment of impact damage in composites, *Composites*, 19(2):145–148, 1988.
22. Steinberger R, Valadas Leitão TI, Ladstätter E, Pinter G, Billinger W, and Lang RW. Infrared thermographic techniques for non-destructive damage characterization of carbon fibre reinforced polymers during tensile fatigue testing, *International Journal of Fatigue*, 28(10):1340–7, 2006.
23. Su Z, Ye L, and Lu Y. Guided Lamb waves for identification of damage in composite structures: a review, *Journal of Sound and Vibration*, 295(3–5):753–780, 2006.
24. Kim J-H, Pierron F, Wisnom M, and Syed-Muhamad K. Identification of the local stiffness reduction of a damaged composite plate using the virtual fields method, *Composites Part A: Applied Science and Manufacturing*, 38(9):2065-2075, 2007.
25. Kim J-H, Pierron F, Wisnom M, and Avril S., Local stiffness reduction in impacted composite plates from full-field measurements, *Composites Part A: Applied Science and Manufacturing*, submitted, 2009.
26. Tiwari V, Sutton MA, and McNeill SR. Assessment of high speed imaging systems for 2D and 3D deformation measurements: Methodology development and validation. *Experimental Mechanics*, 47:561-579, 2007.
27. Moulart R, Pierron F, Hallett S, and Wisnom M. Full-field strain measurements at high rate on notched composites tested with a tensile Hopkinson bar, *ICCM17*, 27-31 July 2009 in Edinburgh.
28. Giraudeau A, and Pierron F. Identification of material stiffness and damping in vibrating plates using full-field measurements, *7th international congress on "Modern practice in stress and vibration analysis"* (Institute of Physics & British Society for Strain Measurements), 8-10 September 2009 in Cambridge (UK).
29. Giraudeau A, Guo B, and Pierron F. Stiffness and damping identification from full field measurements on vibrating plates, *Experimental Mechanics*, 46(6):777-787, 2006.

Back to Programme ***Back to Topic***