MODELING OF DEFORMATION OF LAYERS IN THERMOPLASTIC COMPOSITES MANUFACTURED BY AUTOMATED FIBER PLACEMENT

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1 Abstract

The use of both thermoplastic composites and Automated Fiber Placement (AFP) as means of manufacturing is attracting more interests in recent years. One of the main differences between the thermoset and thermoplastic composites is that in thermoplastics it is necessary that temperature be close to the melting point to shape the desired profile. Combination of anisotropy, high temperature processing and non-isothermal cooling of the part in AFP manufacturing leads to internal residual stresses in the part. Due to high temperature and local pressure used in AFP the viscoelastic behavior of material and part changes from point to point. This can give rise to residual stresses and unwanted deformation.

2 Introduction

During the manufacturing process of a multilayer part, each layer behaves differently to the process-induced stress throughout time, and each layer will have different through-thickness strain accordingly. In the AFP manufactured part a difference in thickness of different layers is observed. In this research the behavior of layers during manufacturing process is studied in finite element analysis.

In a multilayer composite, where each layer is deposited upon the former layer, the pressure on each layer will follow some cyclic pattern. As the head of the AFP machine passes each point by adding new materials, and as the former layer is not completely solidified, the viscoelastic behavior will also follow the same pattern. The schematic figure of the fiber placement head and material is depicted in Figure 1. The through-thickness deformation of layers during manufacturing is mainly dominated by the pressure imposed on the part by the roller.

This imposed stress of the roller during manufacturing is the main focus of this study with the consideration of relaxation. Effect of number of roller passes and timing between each pass is studied. The effect of change in temperature and residual stress due to different coefficients of thermal expansions of constituents materials is to be subjected to further investigations in future works.

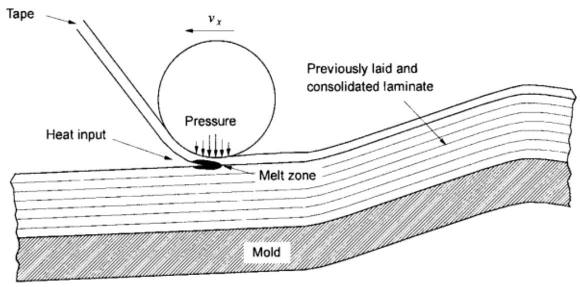

Fig 1 The schematic figure of tape deposition in fiber placement [2]

The result of this study shows that the residual strain or stress will converge to an approximate constant value.

3 Modeling Procedure

Sunderland, et. al studied residual stress induced by manufacturing process in thermoplastic composites, by taking into account the vicoealsticity and anisotropy of material [1]. The generation and relaxation of stress in this paper is considered, the numerical approach in the current research is similar to what that is used in the cited paper. The main reason that viscoelasticity and mainly relaxation is considered in process-induced stress is that since the material is manufactured in high temperature the matrix will be soft and viscoelastic. This will result in relaxation of the stress and considering thermoelastic analysis for the generation of stress would be over-estimated.

During the manufacturing and solidification of the part the behavior of the fibers does not change and it is the matrix that solidifies and behaves viscoelastically. Both time and temperature make a contribution to the deformation in the matrix, and composite. There are several models to describe relaxation behavior of polymers among those Prony series for the use of numerical analysis has several advantages [3]. The stress relaxation formula of the material using Prony series and is:

$$\sigma(t) = \mu_0 \varepsilon(0) + \sum_{j=1}^{N} \mu_j \exp\left(\frac{-t}{\tau_j}\right) \varepsilon(0) \tag{1}$$

in which the τ_j is relaxation time of each Maxwell element and depends on the viscoelastic behavior of the material. Time temperature superposition can be applied to large number of polymers and a shift factor for different temperature can be derived empirically. Relaxation behavior of the material is dependent on the temperature itself, in which the relaxation modulus shifts by *a* according to WLF equation:

$$\log a = \frac{-C_1\left(T - T_0\right)}{C_2 + (T - T_0)} \tag{2}$$

Where C_1 and C_2 are 17.44 and 51.5 are respectively, and they are evaluated empirically [4]. in case that T_g is used for the reference temperature of T_0. With the use of time-temperature superposition principle the behavior of matrix under the stress history, with effects of time and temperature is modeled.

From equation (1) we can define the characteristic relaxation function, which includes the time-independent deformation that is represented by μ_0 [4]. The relaxation function is as follows

$$C_{ij}(t) = \mu_0 + \sum_{j=1}^{N} \mu_j \exp\left(\frac{-t}{\tau_j}\right) \tag{3}$$

Thermo-mechanical stress in viscoelastic solid from an unstressed reference can be expressed by the equation:

$$\sigma(x,t) = F_{s=-\infty}^{s=t}\left[\varepsilon(x,s), T(x,s); x, t\right] \tag{4}$$

Where σ and ε are stress and strain at the place *x* and the time *t*. And T is the temperature at *x*. And F is is the function that takes into every small strain history and temperature history into account of stress at the given time and location. This equation can then be modified into equation 5 for the finite element analysis.

The constitutive equations for the finite element analysis can be expressed in a general convolution integral with regard to time-temperature superposition principle:

$$\sigma_i = \int_{-\infty}^{t} C_{ij}(T, t-\tau) \frac{\partial \varepsilon_j}{\partial \tau} d\tau \tag{5}$$

In which T refers to temperature and t to the time. And accordingly

$$\sigma_i = \int_{-\infty}^{t} \left[\mu_0 + \sum_{k=1}^{N} \mu_k \exp\left(\frac{-t}{\tau_k}\right) \right] \frac{\partial \varepsilon_j}{\partial \tau} d\tau \tag{6}$$

It should be noted that relaxation modulus, C, in equation three is a function of temperature at its core as the master curve would shift by different temperatures and using WLF equation.

For the numerical analysis the formulation that is used considers that the warpage-related displacement negligible [1] and the expression for the finite element equation for an element would be according to formula 6.

$$\sigma_i = \int_{v} \int_{-\infty}^{t} B^T C_{ij}(T, t-\tau) B \frac{\partial \varepsilon_j}{\partial \tau} d\tau dv \tag{7}$$

Where B is the strain-displacement transformation matrix:

$$[\varepsilon] = [B][u] \tag{8}$$

Using this formulation for viscoelastic we can calculate the effective properties of PEEK-Carbon composites with analysis of unit cell and finite element analysis. For this reason a 2D finite element model in microscopic scale is developed to achieve a better insight into the deformation behavior of the composite. In [5] the details of viscoelastic finite element modeling of materials are explained.

Boundaries of layers play a role on the behavior of the material in the micro-scale[6]. This effect is not immediately in the scope of this research and the focus is to determine the viscoelastic behavior of composites under certain manufacturing conditions. As a result a series of analysis has been done to reduce the effects of free edges and also the rigid edge at the bottom to find out the material properties under loading. To eliminate the effects of boundary conditions, the number of unit cells in both directions in the model was increased until a constant periodic behavior of single rectangular unit cell is reached. A similar stress to what it is in the manufacturing process, pressure from the AFP head, is imposed in the finite element process. The part is constrained to move at the bottom which will act as mold in the material, and the stress was imposed from the top of the model in finite element model.

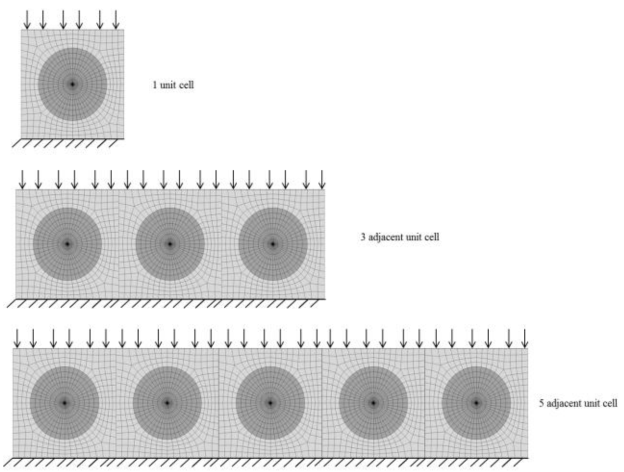

Fig 2 Periodic arrangement of 1, 3 and 5 adjacent unit cells in finite element modeling.

To eliminate the effects of boundaries in the behavior of the material, firstly in a row the number of unit cell was increased to reach a constant behavior (Figure 2). The graph in Figure 3 shows that the maximum displacement of the central unit cell in both *x* and *y* directions reach a constant value with seven adjacent unit cells. It can be derived number of five or more side by side unit cell is a good estimate to eliminate the effect free sides in the displacement of the single center unit cell.

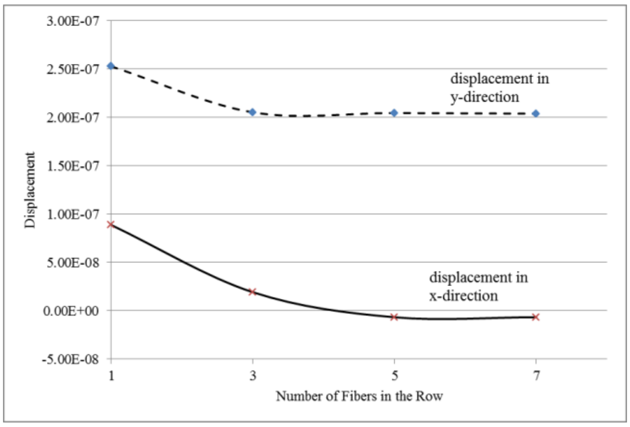

Fig 3 Maximum displacement of central unit cell with different number of side-by-side fibers.

The same approach is used to decrease the effects of top and bottom boundary conditions in the behavior of the unit cell. To achieve the behavior of unit cell in this specific type of stress, number of five unit cells is chosen and the number of unit cell rows was increased in the *y* direction. The maximum displacement and the strain in y direction of central unit cell in the middle row were measured until a constant value was achieved.

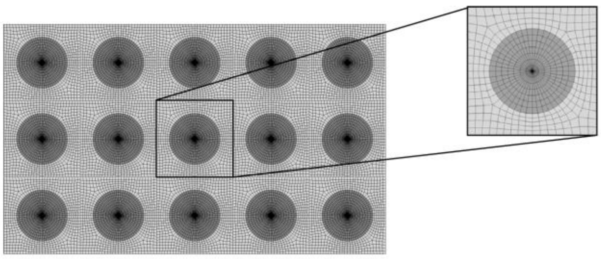

Fig 4 The five by three arrangement of unit cells and center unit cell.

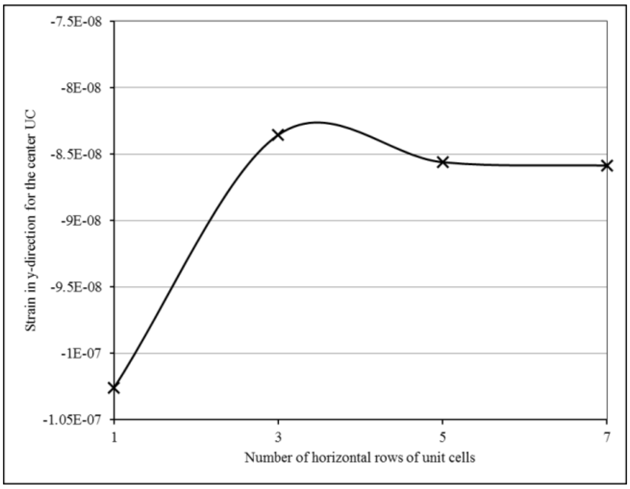

Fig 5 The strain in y direction for the center unit cell with increasing number of rows.

With the increase in the number of rows the maximum displacement was also increasing and diverging, this is mainly because the distance from the constant boundary condition or With these analyses one can propose that five by five series of unit cell attached together gives good result for the middle unit cell and the behavior of the center unit cell can be studied. The center cell in a five-by-five unit cell block is far enough from boundaries that we can tell that boundary conditions have no influence on the behavior of the center unit cell.

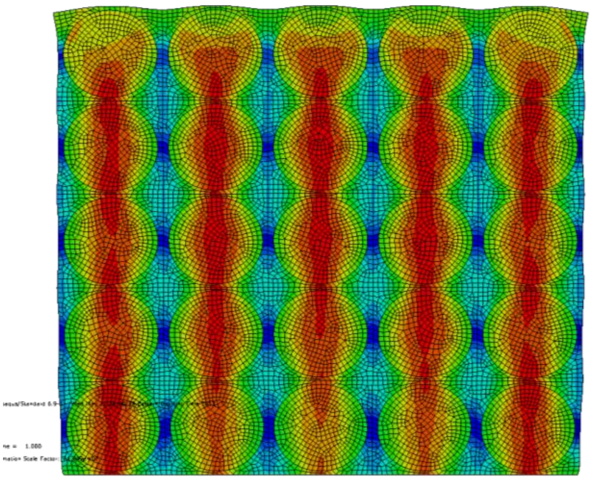

Figure 6 block of five-by-five unit cells analyzed in Abaqus

New periodic boundary conditions has been developed and compared to this model. On the subject of boundary conditions Sun and Vaidya developed a thorough modeling procedure which considers different types of loading that includes normal loading, transverse shear loading, and longitudinal shear loading [7]. For the case of residual stress it is very crucial to understand the types of loading that is applied on the material and choose the correct boundary conditions accordingly. Also, another important issue is the viscoelasticity, as the effects of free boundaries are more significant when we are dealing with viscoelasticity. With time, the strains would reside in a viscoelastic material, so small differences or errors in the analysis could accumulate in the material with time.

The homogenization process and finding the viscoelastic properties of the material is done by averaging the strains on boundary conditions of the central unit cell and calculating the stress on these boundaries and eventually finding the relaxation modulus of the Carbon/PEEK composites.

$$\bar{\varepsilon}_{ij} = \frac{1}{V}\int_V \varepsilon_{ij} dV \qquad (9)$$

where V is the total of volume of the Unit Cell and $\bar{\varepsilon}_{ij}$ is the average strain. ε_{ij} is the local strain and to avoid discontinuity between Unit Cells it should be continues on the boundaries with the neighboring Unit Cell, which the periodic boundary conditions comes into the picture, that will be discussed in the following. The average stress also would be,

$$\bar{\sigma}_{ij} = \frac{P_i}{a_j} \qquad (10)$$

where P_i is the force applied on the boundary condition, this case is axial compression force, and a_j is the surface area that force is applied to. The average modulus properties would be:

$$E_{ij} = \frac{\bar{\sigma}_{ij}}{\bar{\varepsilon}_{ij}} \qquad (11)$$

This is in the case of elastic, for the case of viscoelastic and creep a constant stress is applied and the strain is changing, so the relaxation modulus is going to be,

$$E_{ij}(t) = \frac{\overline{\sigma}_{ij}}{\overline{\varepsilon}_{ij}(t)} \tag{12}$$

This relaxation modulus has been used in the finite element formulation of viscoelastic material that is used in equation 7.

A novel periodic boundary condition has been used in this study that simplifies the procedure and decrease the computation time. In the case of viscoelastic finite element modeling the computation can be very inefficient, as with every change in the stress history the whole analysis should be repeated again. The boundary conditions used in this study acquired with a change in the system stiffness matrix of the elements. This change is designed in a way that local displacement on one boundary of element is equal to the one on the other side.

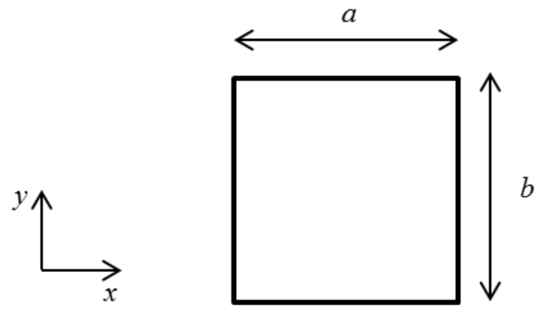

Figure 7 The schematic shape of a homogenized unit cell

In other words, considering Figure 7 we can conclude,

$$u_x(y) = u_{x+a}(y) \tag{13}$$

The system stiffness matrix is changed that the nodes on one side of the element is attached to the nodes on the other side as in depicted in Figure 8.

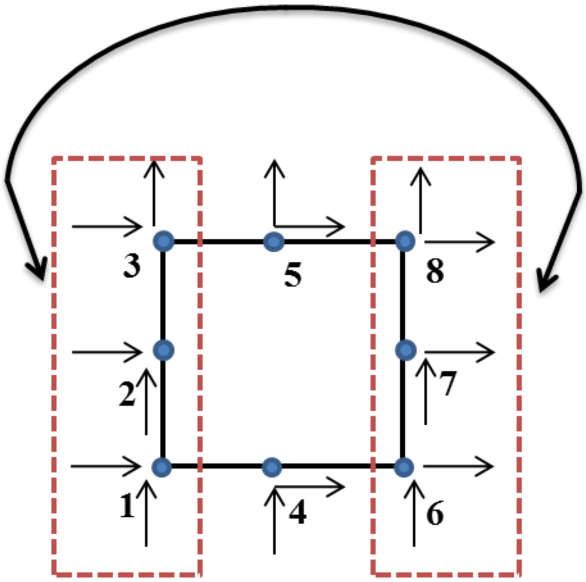

Figure 8 The attachment of the nodes for periodic boundary conditions depicted.

An eight-node two dimensional solid element with reduced integration was used for this study. Normally the element has sixteen degrees of freedom; however for the case of this type of periodic boundary conditions the element will have ten nodes. How nodes are attached to each other in the system stiffness matrix of the material is depicted in the Figure 8. The degrees of freedom are assembled in the stiffness matrix accordingly. If we assume that node *n* has vertical degree of freedom of *2n-1* and horizontal degree of freedom of *2n* then the system assembly will be according to the Figure 9. For instance, the degrees of freedom for a single element that goes from number one to number sixteen. The degrees of freedom from eleven to sixteen were assembled to the degrees of freedom one to six.

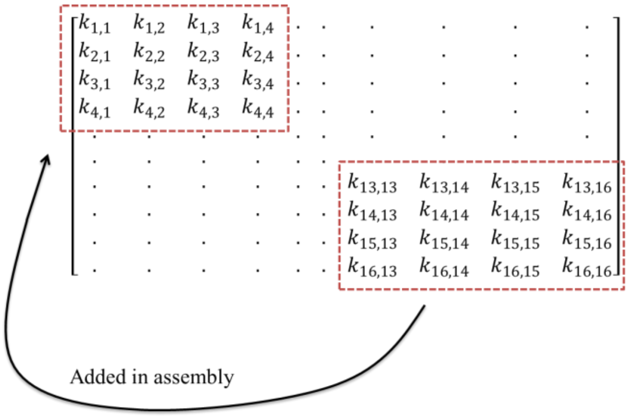

Figure 9 Depiction of assembly process of system stiffness matrix of a periodic boundary condition for one element

There are several studies on the subject of Unite Cell, Representative Volume Element and with an emphasis on periodic boundary conditions for the case of composite materials [7,8,9] That they used analytical method for periodic boundary conditions. In this study a somewhat simpler method is proposed and explained. Using this method can reduce computation time and avoid complications. In Figure 10, the maximum displacements of two different scenarios are depicted. The blue line shows maximum top displacement of the center element in the row of elements next to each other, and the red dashed line shows the top displacement of elements with the proposed periodic boundary conditions.

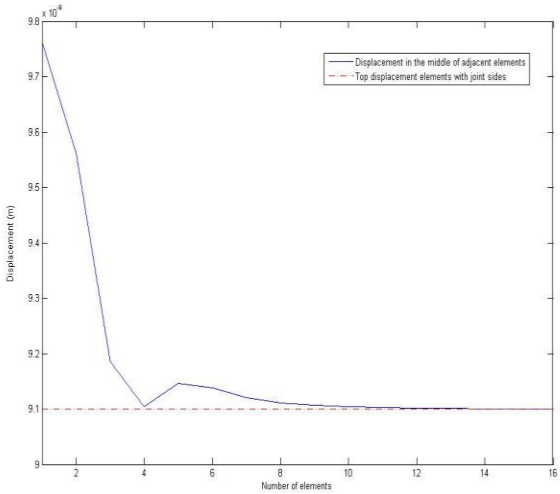

Figure 10 The top displacement in two different systems, blue line without periodic boundary conditions, the red line with proposed periodic boundary conditions.

As it can be seen the displacement at the top of the central element after increasing the number of elements to ten elements converged to displacement of one single element with periodic boundary conditions. This can reduce the number of degrees of freedom to one sixth compared to one without the periodic boundary condition. And also is a proof that the new proposed periodic boundary condition gives accurate result. In the analysis this method is used for the finite element analysis.

3 Results

For increasing the number of fibers in one single tape in its thickness the number of elements with periodic boundary conditions increased on top of each other. The elements with homogenized viscoelastic behavior of Carbon/PEEK and periodic boundary conditions were employed as unit cells. Accordingly, deposition of another layer was achieved by adding number of elements on top of the deformed first layer. The act of roller was modeled with applying temporary pressure on top of each layer. The effect of different type of stress on displacements is studied.

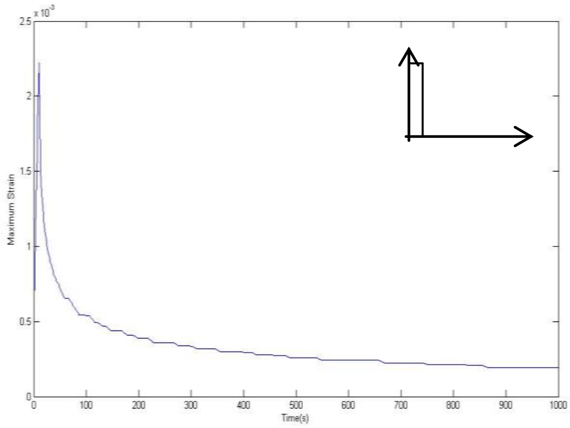

Figure 11 Displacement of one layer subjected to one-step stress with the type of stress depicted on the top right corner.

Figure 11 shows the creep behavior of the material subjected to one pass of the roller with step function as the stress.

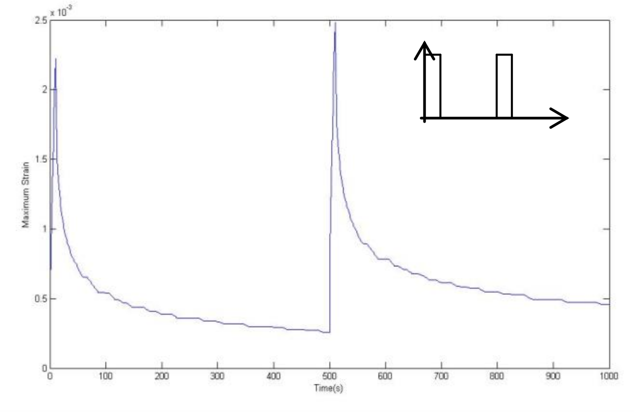

Figure 12 Displacement of one layer subjected to two-step stress with the type of stress depicted on the top right corner.

In Figure 12 the effect of passing by the roller twice on top of the first layer is shown. As it is shown the displacement that happening is sharp and instantly, this is mainly because that stress is applied as of a step function by the use of a ramp function for the stress, we will achieve a much smoother displacement behavior. Ramp function is also more realistic for the case of pressure applied on the material by the roller; this is depicted in the Figure 13 which shows the pick of the displacement is less sharp.

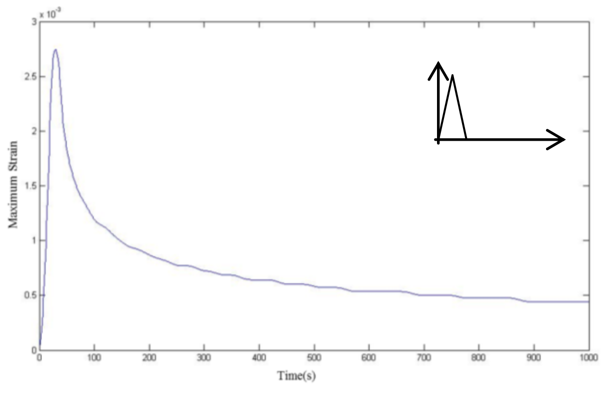

Figure 13 Displacement of one layer subjected to one-step ramp stress function with the type of stress depicted on the top right corner.

In the next step of analysis multilayer deposition of tapes on top of each other has been studied. The series of layers are deposited on top of each other and the effect of time and stress is studied.

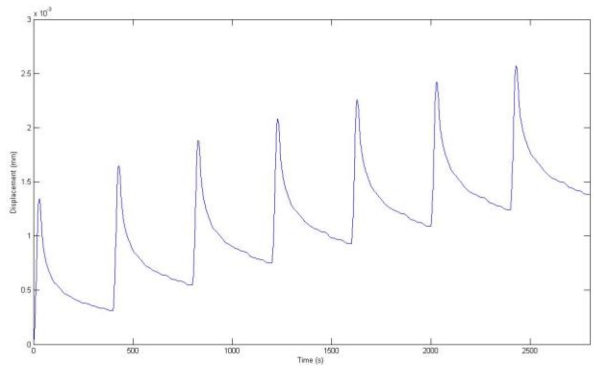

Figure 14 Increase in the displacement of first layer as the roller passes seven times.

Figure 14 shows the trend in increasing the displacement each time the roller passes; however, the interval in this graph between two imposing of stress is four hundred seconds. Obviously, with the increase in this interval the material have more time to relax and the trend will be with a slower trend, but the general trend will be increasing. Figure 15 shows the interval of eight hundred seconds between each passes of roller.

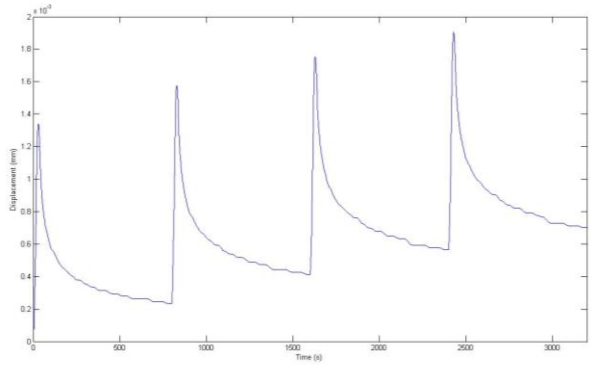

Figure 15 Increase in the displacement in the first layer with four passes of roller.

It is worthy that the relaxation trend remains the same after different types of stress scenarios. As the material behavior depends on the stress history at the same time the trend of relaxation will remain the same and eventually the residual strain or stress relaxes with the same trend but to different values. Figure 16 shows a relaxation behavior of the first tape after given enough time.

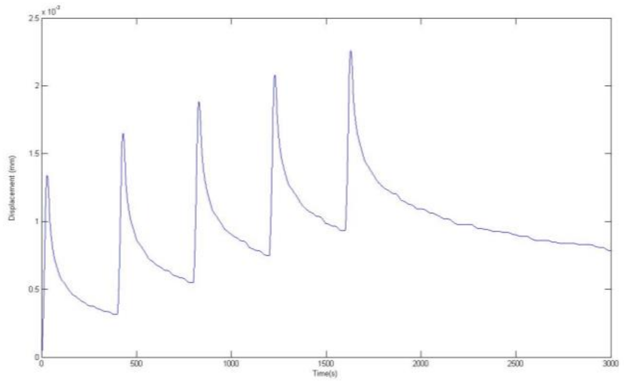

Figure 16 Relaxation of displacement after five times of roller passes.

As it is shown the decrease in the displacement after deposition of five layers is the same as the trend in the Figure 13 which is the relaxation of deposition of only one layer.

4 Conclusion

Knowing the behavior of unit cell, we can expand the model to the manufacturing simulation. The stress that is induced to the thermoplastic tapes has different effects on final part. One of these effects is inconsistent thickness of layers, in which the first layers are thinner and are introduced to more stress. These unwanted deformations can be analyzed and then considered in the design procedure. This observation is similar to results of this study that shows with each time deposition of a new layer on the material the first layer or all the layers underneath conforms and we have a different thickness in different layers. It is also noteworthy that the stress scenario plays an important role on the displacement of the part or through-thickness strain. As different parts require different manufacturing scenarios in Automated Fiber Placement, eventually these will create a different stress history. The effect of relaxation time can make big differences on the residual strains of final part. Effect of temperature is not considered in this study as it was assumed to be constant and subjected to further research. Considering these effects in the design will result in more precise parts, and parts with less residual stress or strain.

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