MICROMECHANICS MODELING OF TENSILE/SHEAR BEHAVIOR AND CRACK DENSITY OF COMPOSITE MATERIALS

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ABSTRACT

The motivation of the study is that we evaluate the relationship between matrix crack and transverse crack and stiffness degradation of brittle matrix composites. We use the micromechanics model proposed by Curtin et al. for matrix crack and proposed by Okabe et al. and Gudmundson-Zang. for transverse crack. Moreover, we use the model proposed by Chow and Lu for calculation of shear stiffness degradation from the tensile stiffness degradation. Furthemore, the energy release rate is used as the calculation of damage evolution. We verify the micromechanics model of three dimensions by comparing to the result of Onodera et al.'s paper about transverse crack. Moreover, we validate the micromechanics model of matrix crack by comparing numerical analysis with test. This micromechanics model is implemented in Abaqus code. We check the marterial behavior with cracks in each direction.

1 BACKGROUND AND MOTIVATION

Composite materials have been widely applied to aircraft components [1] because of their high specific stiffness. By the characteristic of composite materials, it is possible to design the parts of product with considering the material constitution. This means that the property of composite materials can be controlled by changing raw materials such as fiber and matrix and the morphology such as unidirectional fiber sheet and woven fabric. In other words, it is possible to select proper raw materials and morphology to each part of product. However, the material development is costly and time-consuming because of manufacturing and testing composite materials repeatedly. Therefore, in terms of material development, it is desired to use numerical method. Many researchers have studied the numerical analysis methods for composite materials about micromechanics [2] and multiscale analysis based on homogenization method [3].

On the other hand, in terms of structural design, it is desired to consider the nonlinear material behavior for making the most of material property. The nonlinear material behavior is exhibited by damages such as matrix crack. By clarifying the relationship between damages and nonlinear material behavior, we can properly understand the mechanism and prevent excessive safety design. Moreover, in terms of damage inspection and measurement, it is desired to estimate the residual life from the observed damage condition. If the proper numerical analysis method is established, we will be able to estimate the limit state of design from material selection at the initial design stage.

The motivation of the study is that we evaluate the relationship between damages and stiffness degradation of composite materials. In this paper, we focus on matrix crack and transverse crack of brittle matrix composites. We consider the numerical analysis method of calculation of between tensile/shear behavior and crack density of composite materials.

2 OVERVIEW OF MICROMECHANICS MODELING

In order to apply the proper micromechanics modeling for the brittle matrix composites, we use the micromechanics proposed by Curtin et al. [4], which relates the matrix crack to the stiffness degradation in fiber direction. Moreover, we use the micromechanics proposed by Okabe et al. [5] or Gudmundson and Zang [6], which relates the transverse crack to the stiffness degradation in fiber orthogonal direction. With respect to shear behavior, we use the model proposed by Chow and Lu [7], which relates tensile stiffness degradation to shear stiffness degradation. We can calculate the tensile/shear behavior by using these models. We use the damage variable as stiffness degradation. Furthermore, the energy release rate is used as the calculation of damage evolution. The calculation procedure is shown in Fig. 1. When the arbitrary strain is given, the crack density, the damage variable and the stress are calculated. These detail micromechanics models are described afterwards.

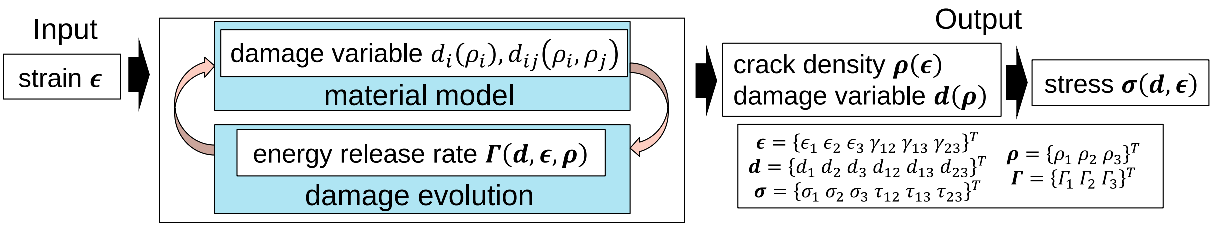


Figure 1: The schematic of calculation procedure

3 MATERIAL MODELING OF EACH DIRECTION

This chapter describes material modeling which is the relationship between the damage variable and crack density.

3.1 Tensile behavior in fiber direction

The model proposed by Curtin et al. [4] is used to calculate the stress-strain curve from the matrix crack space and fiber breakage with matrix crack. In this paper, we use the matrix crack density instead of matrix crack space. The relationship between crack density ρ_1 and crack space x_1 is as follows.

$$\rho_1 = \frac{1}{x_1} \tag{1}$$

The relationship between stress and strain is as follows [4].

$$\epsilon_1 = \frac{\sigma_1}{E_c} + \frac{\alpha(\sigma_1 + \sigma_{th} + \sigma_{deb})\delta}{E_f}\rho_1 + \Delta\epsilon \quad \text{at } (1 \geqq 2\delta\rho_1) \tag{2}$$

$$\epsilon_1 = \frac{\sigma_1}{V_f E_f} + \frac{\alpha\sigma_{th}}{E_f} - \frac{\tau}{2E_f r\rho_1} + \Delta\epsilon \quad \text{at } (1 < 2\delta\rho_1) \tag{3}$$

E_c is elasticity modulus of composite material in fiber direction. E_f is elasticity modulus of fiber. V_f is fiber volume fraction. τ is the shear sliding stress between fiber and matrix interface. r is fiber radius. δ is the sliding length. α is material parameter. σ_{th} is residual thermal stress. σ_{deb} is debonding stress. $\Delta\varepsilon$ is the incremental strain by fiber breakage. For more information about the equations of these symbols, you will see the appendix. The damage variable d_1 is formulated as

$$d_1(\epsilon_1, \rho_1) = 1 - \frac{\sigma_1}{\epsilon_1 E_c} \tag{4}$$

Yarn strength σ_{UTS} is formulated by Curtin et al. as the approximation formula [4]. The equation is given as

$$\sigma_{UTS} = \frac{n+1}{n+2}\left(\frac{2n+2}{n(n+2)}\right)^{\frac{1}{n+1}} V_f \sigma_s \tag{5}$$

σ_s is Weibull's scale parameter of fiber breakage. n is Weibull's shape parameter of fiber breakage.

3.2 Tensile behavior in fiber orthogonal direction

The model proposed by Okabe et al. [5] or Gudmundson and Zang [6] is used to calculate damage variable d_2 from the transverse crack density ρ_2. These models are assumed that the cracks are surrounded by the materials. Okabe et al.'s model is as follows [5].

$$d_2(\rho_2) = 1 - P_2 \sum_{n=1}^{\infty} \frac{\tanh[(2n-1)P_1 b_2 \rho_2]}{(2n-1)^3 b_2 \rho_2} \tag{6}$$

P_1 and P_2 are as follows.

$$P_1 = \frac{\pi\lambda}{2} \tag{7}$$

$$P_2 = \frac{16}{\pi^3\lambda} = \frac{8}{\pi^2 P_1} \tag{8}$$

$$\lambda = \sqrt{\frac{E_2 - G_{23}(\nu_{23} + \nu_{12}\nu_{21})}{G_{23}(1 - \nu_{12}\nu_{21})}} \tag{9}$$

E_2 is transverse elasticity modulus of composite materials. G_{23} is shear elasticity modulus of composite materials. ν is Poisson's ratio of composite materials. b_2 is the length of crack. d_2 is the damage variable. On the other hand, Gudmundson-Zang model is as follows [5][6].

$$d_2(\rho_2) = \frac{\pi}{2} b_2 \rho_2 \sum_{n=1}^{10} \frac{a_n}{(1 + b_2\rho_2)^n} \tag{10}$$

a_n is the crack parameter described by Gudmundson and Zang's paper [6]. The difference of two models is that the stress and strain distribution around cracks are not considered in Gudmundson-Zang model but are considered in Okabe et al.'s model.

3.3 Model of relationship between tensile/shear behavior

The model proposed by Chow and Lu is used to calculate shear damage variable d_{ij} from tensile damage variable d_i and d_j.

$$(1 - d_{ij})^{-1} = \frac{1}{2}\left\{(1 - d_i)^{-1} + (1 - d_j)^{-1}\right\} \tag{11}$$

i and j are 1 or 2 or 3, $i>j$ and $i \neq j$. The tensile damage d_i is defined by the crack density ρ_i. Therefore, the shear damage d_{ij} is defined by the crack density ρ_i and ρ_j.

4 DAMAGE EVOLUTION BY THE ENERGY RELEASE RATE

The energy release rate is used to damage evolution. The equation is defined as

$$\Gamma_i = -\frac{\partial(U_\epsilon - W_i)}{\partial A_i} \tag{12}$$

i=1 is the fiber direction in which matrix crack occurs and i=2 or 3 is the fiber orthogonal direction in which transverse crack occurs. U_ε is the elastic strain energy. W_i is the work of external load. A_i is the matrix crack area. We consider the region surrounded by the pre-existing cracks in Fig. 2. The crack space before the new crack initiates is $2x_i$ (=ρ_i/2). The parameter t is crack position. We assume that the crack is periodic. When the new crack initiates, the crack space is x_i (=ρ_i). Then, the energy release rate is as follows.

$$\Gamma_i(x_i) = -\frac{U_\epsilon(2x_i(1-t)) + U_\epsilon(2x_i t) - U_\epsilon(2x_i) - W_i}{b_i w_i (1 - V_f \delta_{1i})} \tag{13}$$

b_i and w_i are the crack length. δ_{1i} is as follows.

$$\delta_{11} = 1 (i = 1), \delta_{1i} = 0 (i \neq 1) \tag{14}$$

The fiber area need to be excluded about i=1. The elastic strain energy U_ε is as follows.

$$U_\epsilon(x_i) = \frac{1}{2}bwx_i\left[\{\epsilon(x_i)\}^T\{\sigma(x_i)\} - \epsilon_1(x_i)\big(V_fT_f(x_i) - \sigma_1(x_i)\big)\right] \tag{15}$$

$\{\sigma\}$ is the stress vector of yarn. $\{\varepsilon\}$ is the strain vector of yarn. T_f is the fiber stress. b and w are the length except i direction. The equation is as follows.

$$b, w = 2x_j^0, 2x_k^0 (j, k \neq i, j \neq k) \tag{16}$$

$2x_j^0$ and $2x_k^0$ are the evaluation length that is no cracks in the j and k direction. $V_fT_f(x_i)-\sigma_1(x_i)$ is the elastic strain energy of fiber breakage and pull-out from matrix. Therefore, this effect needs to be excluded. $\{\sigma\}$ is calculated on the basis of the continuum damage mechanics.

$$\{\sigma\} = [M]^{-1}[D_0]\{\epsilon\} \tag{17}$$

$[M]$ is the damage effect matrix. The $[M]$ components are as follows.

$$\begin{gathered} M_{ii} = (1-d_i)^{-1}\ (i = 1,2,3), M_{qr} = 0(q, r = 1, \cdots, 6, q \neq r) \\ M_{44} = (1-d_{12})^{-1}, M_{55} = (1-d_{13})^{-1}, M_{66} = (1-d_{23})^{-1} \end{gathered} \tag{18}$$

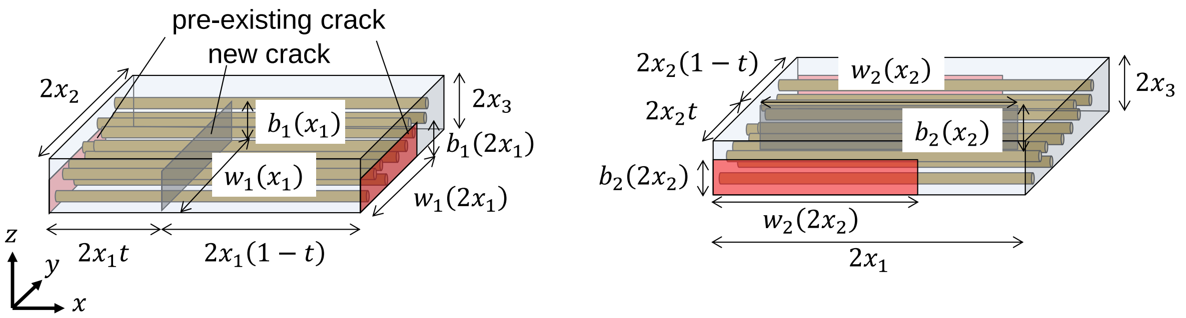
The condition of crack initiation is as follows.

$$\boldsymbol{\sigma} > \boldsymbol{\sigma_c} \tag{19}$$

$\boldsymbol{\sigma_c}$ is the stress criterion. The condition of crack growth is as follows.

$$\boldsymbol{\Gamma} > \boldsymbol{\Gamma_m} \tag{20}$$

$\boldsymbol{\Gamma_m}$ is the fracture energy of matrix. The crack density is determined when $\boldsymbol{\Gamma} = \boldsymbol{\Gamma_m}$ is satisfied. In this paper, the parameter t is 0.5 and the work of external load $W_i = 0$. These parameters have little effect on the stress-strain curve. On the other hand, these parameters slightly have an effect on the stress-strain curve. When you know the effect of these parameters, you can check the other paper [8].



(a) matrix crack (i=1) (b) transverse crack (depicted about i=2)

Figure 2: The definition of crack area and position in each direction

5 CRACK LENGTH MODEL

The crack gradually propagates in the matrix. The crack length of b_i and w_i is defined as

$$b_i = \min\left(b, B_ip_i(\rho_i)\right), b < B_i \tag{21}$$

$$w_i = \min\left(w, W_ip_i(\rho_i)\right), w < W_i \tag{22}$$

B_i and W_i are the length parameter of crack propagation. $p_i(\rho_i)$ is the crack propagation parameter and as follows.

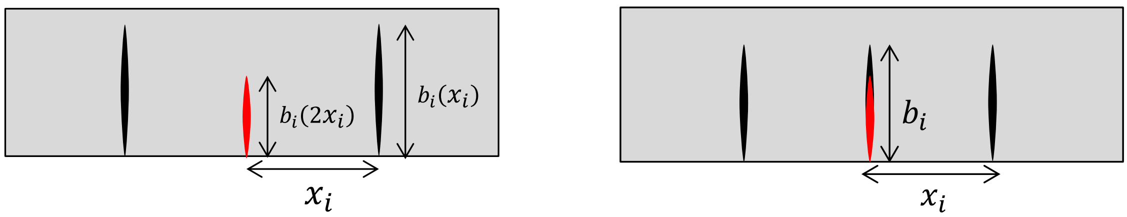
$$p_i(\rho_i) = 1 - \exp\left(-\left(\frac{\rho_i}{\rho_{pi}}\right)^{m_{pi}}\right) \tag{23}$$

It is assumed that the crack propagation obeys Weibull cumulative distribution of crack density ρ_i. m_{pi} is Weibull's shape parameter. ρ_{pi} is Weibull's scale parameter. Moreover, the crack length b_i is defined as the average of $b_i(x_i)$ and $b_i(2x_i)$ depicted by Fig. 3. The definition of crack length w_i is the same as that of b_i. The equations are as follows.

$$b_i = \bar{b}_i(x_i) = \frac{1}{L_i/x_i - 1}\left(\frac{L_i}{2x_i} b_i(x_i) + \left(\frac{L_i}{2x_i} - 1\right) b_i(2x_i)\right) \tag{24}$$

$$w_i = \overline{w_i}(x_i) = \frac{1}{L_i/x_i - 1}\left(\frac{L_i}{2x_i} w_i(x_i) + \left(\frac{L_i}{2x_i} - 1\right) w_i(2x_i)\right) \tag{25}$$

L_i is the evaluation length in i direction. L_i/x_i is the crack number in evaluation length direction.



(a) Crack length before and after the crack growth

(b) Average crack length before and after the crack growth

Figure 3: Description about crack length

6 CALCULATION PROCEDURE

The calculation procedure is shown in Fig. 4. The inputs are the strain $\boldsymbol{\varepsilon}$ at the current step and damage variable $\boldsymbol{d}^0$ and crack density $\boldsymbol{\rho}^0$ at the previous step. The equations described at 2-4 chapters are used at "Calculation" routine. The crack density $\boldsymbol{\rho}$ changes if $|\boldsymbol{\Gamma}\text{-}\boldsymbol{\Gamma}_m|$ is not under the residual criterion R, where $|*|$ is the Euclidean norm. The "Calculation" is carried out repeatedly, and finally the stress $\boldsymbol{\sigma}$, damage variable $\boldsymbol{d}$ and crack density $\boldsymbol{\rho}$ are obtained.

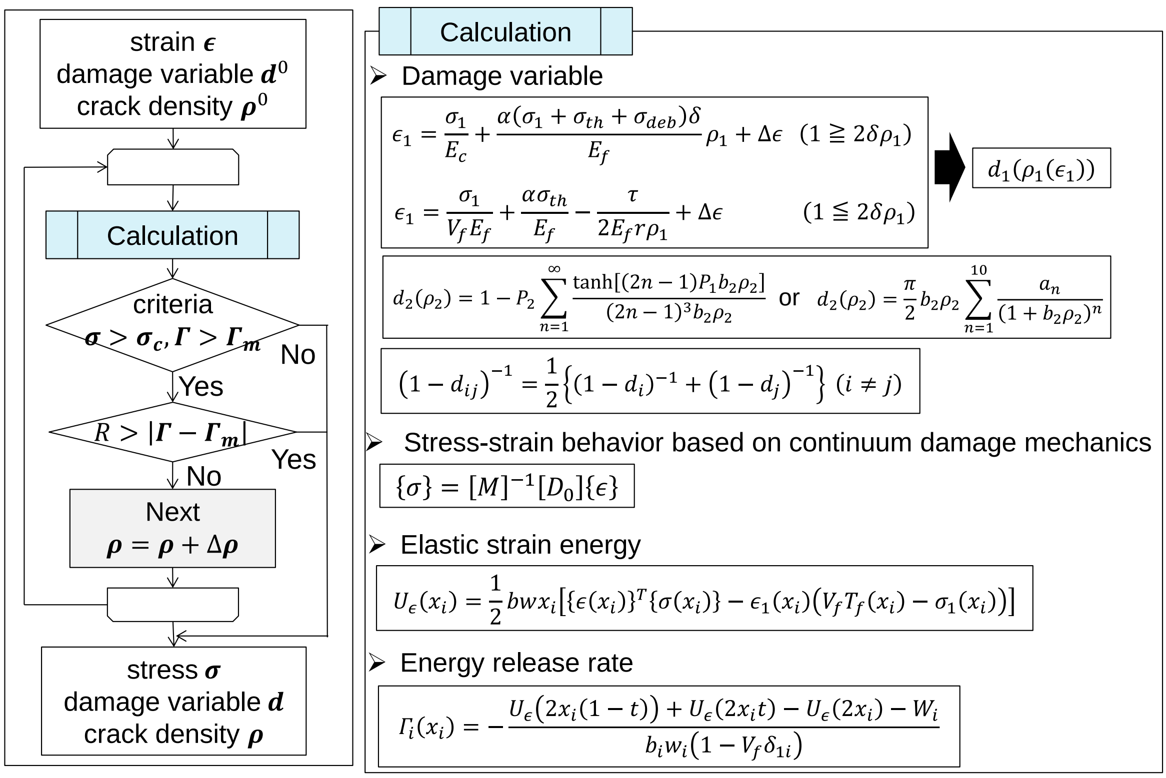


Figure 4: Calculation procedure

7 VERIFICATION AND VALIDATION OF MICROMECHANICS MODEL

We compared the numerical analysis of micromechanics model with that of some paper for verifying three dimensional calculation. Moreover, we compared the test with numerical analysis of the stress-strain behavior and crack density-stress behavior for validating the micromechanics model.

7.1 Verification of micromechanics model

We verify the micromechanics model by comparing to the energy release rate-crack density curve calculated by Onodera et al. [9]. Onodera et al. show the numerical analysis method of transverse crack using Okabe et al.'s model and Gudmundson-Zang model and predict accurately the transverse crack density of cross-ply composite materials. The material data is shown in Table 1. The length b_i, w_i of the paper are b_i=b and w_i=w. We can express b_i=b and w_i=w by $m_{pi} \approx \infty$, $\rho_{pi} \approx 0$ and $B_i = W_i \approx \infty$. Onodera et al. consider the thermal expansion strain. The transverse strain ε_2 is calculated as

$$\epsilon_2 = \epsilon_y^p + \epsilon_{th} \tag{26}$$

$$\epsilon_{th} = (\alpha_c - \alpha_2)\Delta T \tag{27}$$

ε_y^p is the external strain. α_c is the thermal expansion coefficient of $[0/90]_S$. α_2 is the transverse thermal expansion coefficient of lamina. ΔT is temperature difference between circumstance temperature in test and stress-free temperature in molding. Input strain is only ε_2 in Onodera et al.'s paper because of formulation in 1 dimension. On the other hand, micromechanics model of this paper is formulated in three dimensions. Therefore, we need to input not only ε_2 but also ε_1 (=-$\nu_{21}\varepsilon_2$) and ε_3 (=-$\nu_{23}\varepsilon_2$).

The result is shown in Fig. 5 when ε_y^p is 0.01 and ΔT is -140 K. The plots are the calculation with micromechanics model of this paper and the line is the result of Onodera et al.'s paper. As you can see, the result of calculation is completely coincides with that of Onodera et al.'s paper in the case of Gudmundson-Zang model. Therefore, we found out that there is no problem in this micromechanics model and in-house code. On the other hand, there seems to be the difference between the result of calculation and paper in the case of Okabe et al.'s model. The reason is that the distribution of stress and strain is considered in Onodera et al.'s paper whereas not considered in this calculation using the equation (6).

Table 1: Material data [9]

Symbol	Description	Property
E_1	Longitudinal elasticity modulus	143 GPa
$E_2 (= E_3)$	Transverse elasticity modulus	7.99 GPa
$\nu_{12} (= \nu_{13})$	Poisson's ratio in 12 direction	0.35
ν_{23}	Poisson's ratio in 23 direction	0.49
G_{12}	Shear elasticity modulus in 12 direction	3.96 GPa
G_{23}	Shear elasticity modulus in 23 direction	2.68 GPa
α_c	Thermal expansion coefficient of $[0/90]_S$	9.66×10^{-6} K^{-1}
α_2	Transverse thermal expansion coefficient of lamina	34.3×10^{-6} K^{-1}
b	Thickness of 90 degrees-ply	0.36 mm
w	Width of $[0/90]_S$	1 mm
m_p	Shape parameter of crack propagation	10^{20}
ρ_p	Scale parameter of crack propagation	10^{-20} /mm
$B(W)$	Length parameter of crack growth	10^{20} mm

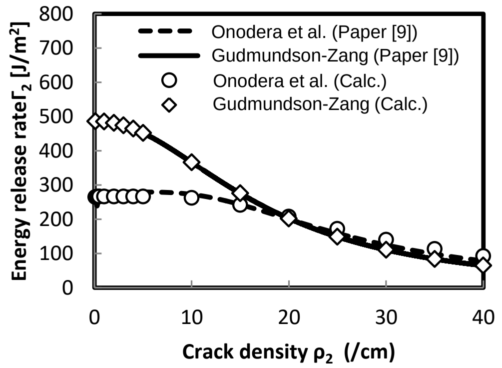


Figure 5: Energy release rate-crack density curve

7.2 Validation of micromechanics model

We will apply the micromechanics model for the brittle matrix composite laminate of stacking sequence $[0]_{12}$. This section shows the accurately prediction of matrix crack density of brittle matrix composites. The material data is shown in Table 2. This data is acquired by Pryce and Smith [10] and Blissett, Smith and Yeomans [11]. The longitudinal elasticity modulus of yarn E_c holds the rule of mixture. The equation is as follows.

$$E_c = V_f E_f + \left(1 - V_f\right) E_m \tag{28}$$

E_m is elasticity modulus of matrix. The value E_c of calculation is 124 GPa whereas that of test is 128±7 GPa. Therefore, we can use the result of the rule of mixture. The thermal residual stress σ_{th} is 112 MPa. The stress criterion σ_c is obtained from the knee point of stress-strain curve [10]. The fracture energy Γ_m is calculated from the fracture toughness value of the indentation test by Blissert et al. [11]. 46 J/m^2 is calculated by the method of Chantikul et al. [12].

The other data is shown in Table 3. These parameters are calibrated as the numerical analysis of stress-strain behavior agrees with the test result. The reason of the parameters in Table 3 is described in the following. The shear sliding stress τ is calibrated as the infection point of stress-strain behavior of the numerical analysis agrees with the test result. The infection point is the stress of the crack saturation. The shear sliding stress τ by the various tests is shown by Lara-Curzio et al. [13]. The shear sliding stress τ is 10-19 MPa obtained by the indentation test and 14-19 MPa obtained by the push-out test. Therefore, 17 MPa in Table 3 is appropriate. Moreover, the shape parameter m_{pi}, scale parameter ρ_{pi} and the debonding stress σ_{deb} are calibrated as stress-strain nonlinear behaviour of the numerical analysis agrees with the test result. Furthermore, the scale parameter σ_s and the shape parameter n are calibrated as the composite strength of the numerical analysis agrees with 400 MPa of the test result. We predict the crack density-stress behavior with Table 2 and 3.

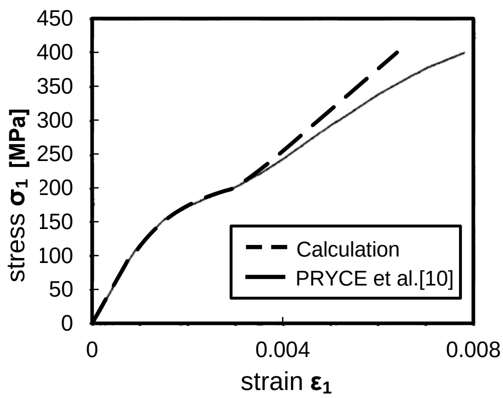
The results are shown in Fig. 6. In Fig. 6 (*b*), the dots mean the crack density of test and the line means the predicted crack density. The predicted crack density is coincided with the crack density of testing in Fig. 6(*b*). Therefore, we can predict the crack density-stress curve with the micromechanics model of this paper. On the other hand, we cannot agree the stress-strain curve of the numerical analysis with that of the test after the infection point which is the crack saturation. The stress-strain curve obeys the equation (3) after crack saturation. When the matrix does not supported the load, the longitudinal elasticity modulus is $V_f E_f$=66.5 GPa according to the rule of mixture. The predicted value of tangent modulus is 59.9 GPa in the range of 300-400 MPa in Fig. 6. On the other hand, the test value of tangent modulus is 39 GPa [10]. The reason of this difference is that the equation cannot express the many fiber breakage and the mechanism of the sliding between fiber and matrix with the fiber breakage.

Table 2: Material data

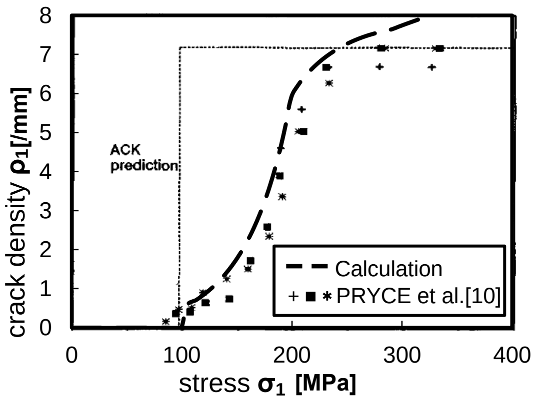
Symbol	Description	Property
E_f	Elasticity modulus of fiber	190 GPa [10,11]
α_f	Thermal expansion coefficient of fiber	3.3×10^{-6} K^{-1} [10,11]
E_m	Elasticity modulus of matrix	90 GPa [10,11]
α_m	Thermal expansion coefficient of matrix	4.6×10^{-6} K^{-1} [10,11]
V_f	Fiber volume fraction	0.34 [10,11]
ν	Poisson's ratio of composite laminate	0.24 [10]
ΔT	Temperature difference in process	1200 K [10]
r	Fiber radius	8 μm [10]
L	Specimen length	80 mm [10]
b	Thickness of composite laminate	2.1 mm [10]
w	Width of composite laminate	20 mm [10]
σ_c	Stress criterion of composite laminate	100 MPa [10]
Γ_m	Fracture energy of matrix	46 J/m^2 [11]

Table 3: Material data except Table 2

Symbol	Description	Property
τ	Sliding stress between fiber and matrix	17 MPa
m_p	Shape parameter of crack growth	0.8
ρ_p	Scale parameter of crack growth	5 /mm
σ_s	Scale parameter of fiber breakage	1750 MPa
n	Shape parameter of fiber breakage	2.5
Γ_i	Fracture energy of fiber/matrix interface	0.3 J/m^2
$B(W)$	Length parameter of crack growth	30 mm



(a) Stress-strain curve



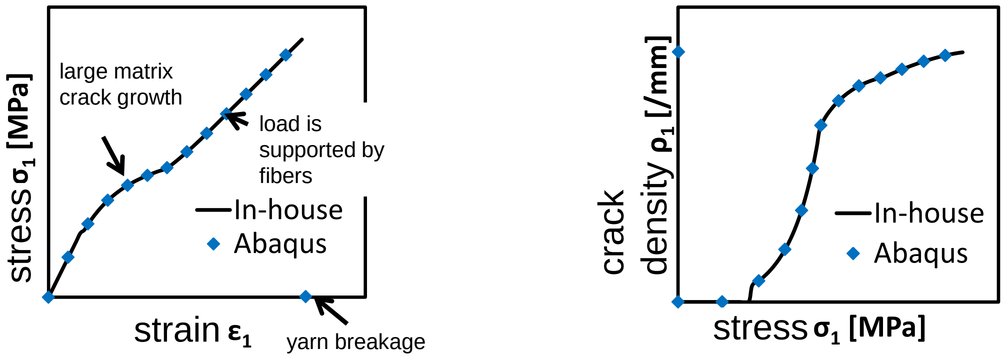
(b) crack density-stress curve

Figure 6: Comparison of the test with the numerical analysis

8 TENSILE/SHEAR BEHAVIOR AND IMPLIMENTATION OF ABAQUS

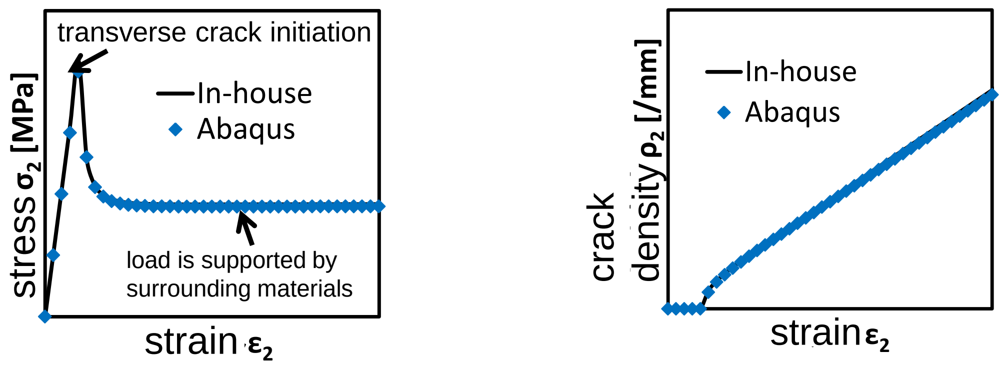
The input is 6 strain components and the output is 6 stress components in this micromechanics model. This means that this micromechanics model is implemented in the original material model of commercial FE software formulated by displacement method. We implement this micromechanics model in UMAT which is the subroutine of Abaqus. The following is the calculation example which is stress-strain curve and crack density-stress curve. The strain is loaded in each direction such as single tensile load and single shear load. The transverse strain by Poisson effect is also given to original in-house code as the input. On the other hand, the only single strain is loaded to one element in Abaqus code because the transverse strain by Poisson effect is calculated by solving the stiffness equation under boundary condition of uniaxial tension in Fig. 7-10. The lines are the results calculated by the original in-house code. The dots are the results calculated by implemented subroutine UMAT of Abaqus. As you can see, we found out that there is no problem in Abaqus code.

The Fig. 7 shows the tensile behavior in fiber direction. After the matrix crack initiation, the large matrix crack grows. After that, the external load is supported by the fibers. Finally, the composite material breaks at the stress calculated by the equation (5). The Fig. 8 shows the tensile behavior in fiber orthogonal direction. When transverse crack initiates, the stress is drastically decreased. The reason is that the elasticity modulus of brittle matrix is relatively high, which the elasticity modulus of brittle matrix is 90 GPa and the elasticity modulus of laminated composite is 124 GPa. After transverse crack initiation, the stress is nearly constant although the transverse crack grows. The reason is that the crack region is supported by the surrounding materials. The shear behavior shown in Fig. 9 and Fig. 10 is the same as the tensile behavior in fiber orthogonal direction because the transverse crack grows and the crack region is supported by the surrounding materials.



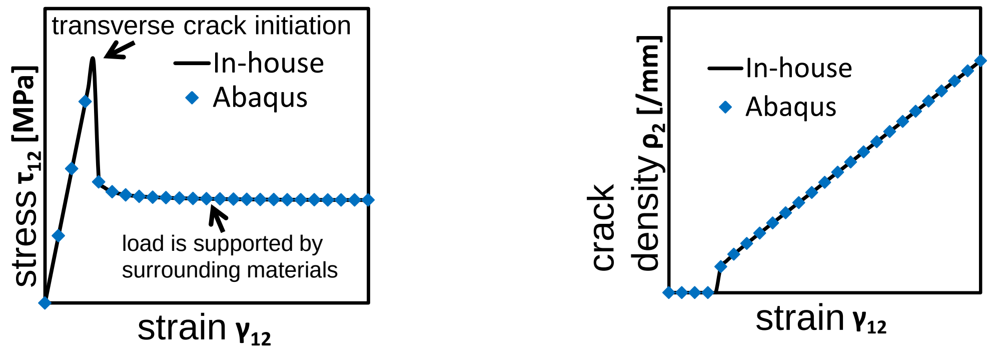
(*a*) Stress-strain curve (*b*) crack density-stress curve

Figure 7: Tensile behavior in fiber direction



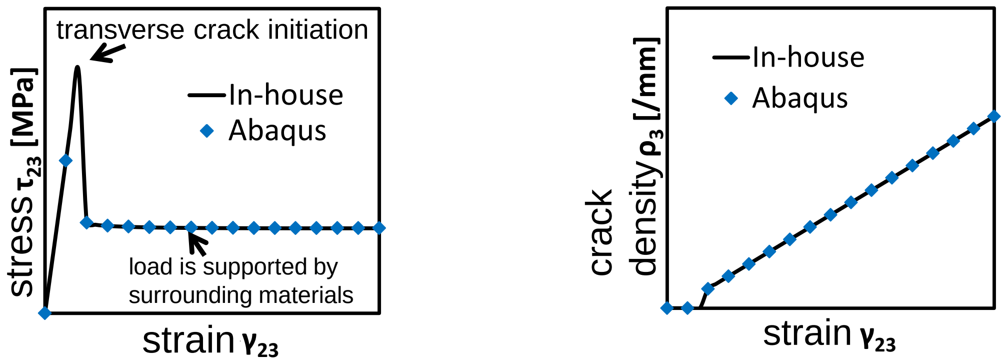
(*a*) Stress-strain curve (*b*) crack density-stress curve

Figure 8: Tensile behavior in fiber orthogonal direction



(*a*) Stress-strain curve (*b*) crack density-stress curve

Figure 9: Shear behavior in 12 direction



(*a*) Stress-strain curve (*b*) crack density-stress curve

Figure 10: Shear behavior in 23 direction

9 CONCLUSIONS

In order to calculate matrix crack and transverse crack density of brittle matrix composites, the micromechanics modeling was formulated in three dimensions. In other words, we can calculate the stress and crack density from the arbitrary strain. First of all, we compared the energy release rate-transverse crack density of Onodera et al.'s paper [9] with that of the micromechanics model of this paper to verify the proper micromechanics model and in-house code. As a result, the result of calculation is completely coincides with that of Onodera et al.'s paper in the case of Gudmundson-Zang model. Therefore, we found out that there is no problem in this model and in-house code. Moreover, this micromechanics model is applied to brittle matrix composite laminates of stacking sequence $[0]_{12}$ to predict the matrix crack density. As a result, the matrix crack density-stress curve is accurately predicted using the parameter calibrated by stress-strain curve. Finally, this micromechanics model is implemented in ABAQUS and we check whether the shear behavior can be calculated. In the future, this model will be applied to the composite materials such as the cross-ply laminates and the woven fabric for the validation.

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APPENDIX

δ is the sliding length as follows.

$$\delta = \frac{\alpha r}{2\tau}(\sigma_1 + \sigma_{th} - \sigma_{deb}) \tag{A1}$$

α is the material parameter as follows.

$$\alpha = \frac{(1 - V_f)E_m}{V_f E_c} \tag{A2}$$

σ_{th} is the residual thermal stress as follows.

$$\sigma_{th} = -V_f E_f \Delta T(\alpha_f - \alpha_m) \tag{A3}$$

$\varDelta T$ is the temperature difference between circumstance temperature in test and stress-free temperature in molding. α_f is the thermal expansion coefficient of fiber. α_m is the thermal expansion coefficient of matrix. σ_{deb} is the debonding stress as follows.

$$\sigma_{deb} = \frac{1}{c_i}\left(\frac{\varGamma_i E_m}{r}\right)^{\frac{1}{2}} \tag{A4}$$

$\varGamma_i$ is the fracture energy of fiber/matrix interface and c_i is the material property described by Hutchnison and Jansen's paper [14] as follows.

$$c_i = \frac{\sqrt{(1 - \nu^2)(1 - V_f)}}{2V_f} \tag{A5}$$

ν is Poisson's ratio of composite material. $\varDelta\varepsilon$ is the incremental strain by fiber breakage as follows.

$$\varDelta\epsilon = \frac{1}{E_f}\left(T_f - \frac{\sigma_1}{V_f}\right)(2\delta\rho_1) \quad (1 \geqq 2\delta\rho_1) \tag{A6}$$

$$\varDelta\epsilon = \frac{1}{E_f}\left(T_f - \frac{\sigma_1}{V_f}\right) \quad (1 < 2\delta\rho_1) \tag{A7}$$

T_f is the fiber stress and calculated from the nonlinear equation. The equations are as follows.

$$\frac{\sigma_1}{V_f} = T_f(1 - q(z_0, T)l_s\rho_1) \tag{A8}$$

$$q(z_0, T_f) = 1 - \exp\left[-\frac{\left\{1 - \left(1 - \frac{z_0}{l_s}\right)^n\right\}\left(\frac{T_f}{\sigma_s}\right)^{n+1}}{n + 1}\right] \tag{A9}$$

q is the fiber breakage probability and n is Weibull's shape parameter and σ_s is Weibull's scale parameter. z_0 is the length as follows.

$$z_0 = \delta \ (1 \geqq 2\delta\rho_1), \quad z_0 = \frac{1}{2\rho_1} \ (1 < 2\delta\rho_1) \tag{A10}$$

l_s is the sliding length as follows.

$$l_s = \frac{rT_f}{2\tau} \tag{A11}$$