The classification of optical instruments

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THE CLASSIFICATION OF OPTICAL INSTRUMENTS

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ABSTRACT. Exception is taken to the classification of optical instruments by the signs of their powers, and an alternative division comprising five classes is proposed, based upon the separation of the four Gaussian constants into two groups according to their signs. This classification cannot be modified by the addition to the system of inverting prisms and the like, and the properties usually associated with the sign of the lens in reality depend upon its class according to the new system. Each class may have systems of positive or of negative power.

The elementary theory of optical instruments which we owe to Gauss has the great merit that attention may be given wholly to the events which take place in the object space and the image space, the method by which the rays are altered in direction and position, and even the position of the instrument which causes these changes, being unimportant. This treatment is highly advantageous when the purpose is to consider the general correlation between incident and emergent rays, but in the practical applications of the theory to the construction and use of real instruments the position of the instrument is of the greatest importance. This would be rendered more apparent if it became customary at a later stage of the theory to consider the position of the instrument in relation to the cardinal points; such a development leads naturally to a classification of instruments in which there are five groups, important special cases such as telescopes being regarded as borderline cases or as members of two groups. The division proposed is more fundamental than the usual separation of lens systems into positive and negative combinations, a distinction which is not generally correct as regards many of the properties usually associated with them.

The properties of a system, including the positions of the extreme surfaces relative to the cardinal points, are given by the four Gaussian constants A, B, C, D*, which satisfy the identity $BC - AD = 1$. The ordinary division depends wholly on the sign of A, the power of the system. The essential theoretical distinction between positive and negative lenses is indicated in Figs. 1 and 2 by means of three incident parallel rays identified by the letters P, Q, R: the position of the instrument is immaterial and both object space and image space are supposed to extend to infinity in every direction. In the classification now proposed the basis is the grouping of the constants according to agreements or disagreements of sign. Let

* These constants A, B, C, D are respectively $\kappa_{1,n}$, $\frac{\partial \kappa_{1,n}}{\partial \kappa_1}$, $\frac{\partial \kappa_{1,n}}{\partial \kappa_n}$, $\frac{\partial^2 \kappa_{1,n}}{\partial \kappa_1 \partial \kappa_n}$ in the notation used in calculations.

the constants of one sign be included in one bracket, and those of another sign in another bracket. There are then six possible groups

$$[ABCD],\ [ABC]\,[D],\ [A]\,[BCD],\ [AB]\,[CD],\ [AC]\,[BD],\ [AD]\,[BC],$$

it being of no consequence which bracket of a pair corresponds to a positive and which to a negative sign. An algebraically exhaustive list would include the groups $[ABD]\,[C]$ and $[ACD]\,[B]$, but these must be excluded since they require $BC - AD$ to be negative. The actual number of classes is one less than the number of these groups, for if a system is considered in the reverse direction B and C are interchanged. It follows that the two groups $[AB]\,[CD]$ and $[AC]\,[BD]$ correspond to a single class of instrument.

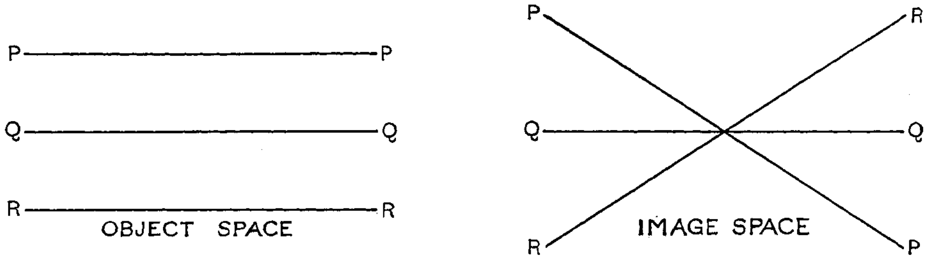

Fig. 1. Positive Lens.

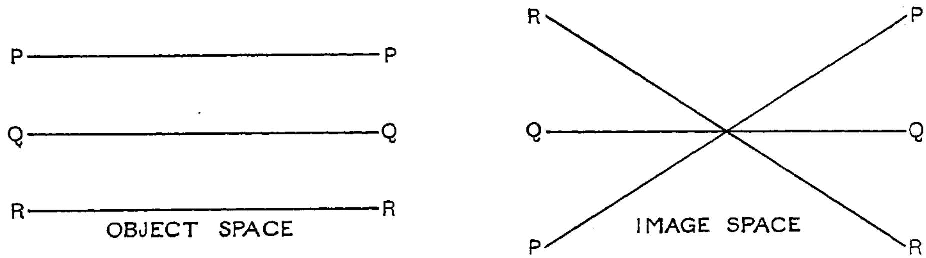

Fig. 2. Negative Lens.

The positions of pairs of conjugate points on the axis are given in terms of A, B, C, D by the relation

$$Axx' - Bx' - Cx + D = 0 \qquad \ldots\ldots(1),$$

where x is the distance from the first surface to the object point, and x' is the distance from the last surface to the image point, both considered positive when measured away from the system into the surrounding medium. It is at once evident from this equation that the positions of conjugate points depend upon the ratios of these constants, and that their relative signs rather than their absolute signs are important. The particular property affected by a change of all the signs without a change of magnitude of these four quantities is the inversion of the image of an object occupying a particular position, but in modern instruments the designer need pay little attention to this feature of a system as he has several devices which he may adopt to modify this result without altering the values of the four constants. The properties represented by the ratios of the constants on the other hand are fundamental and cannot be influenced by such devices as the introduction of reflecting surfaces.

From the equation connecting x and x' it is easy to see what properties are associated with each of the groups that have been described. Suppose that F (Figs. 3 to 8) is the point at which incident parallel rays are brought to a focus, that U is the image of the vertex of the first surface in the system, and that V is the vertex of the last surface. Then F, U, V are three points on the axis in the image space of which V is necessarily real while F and U are only real if they are encountered by a wave which has traversed the system after it has passed through V. There are six cases to consider. For U may be encountered by the wave before it reaches F or after it has passed through F, and V in either case may precede both,

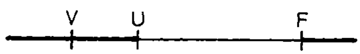

Fig. 3. [ABCD]

Fig. 4. [ABC] [D]

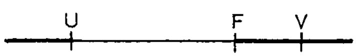

Fig. 5. [AB] [CD]

Fig. 6. [AC] [BD]

Fig. 7. [A] [BCD]

Fig. 8. [AD] [BC]

fall between or follow both U and F. If the points are placed in the order in which they are met by the wave, the following correspondences may be derived at once from equation (1):

$$\begin{aligned} VUF &= [ABCD], & VFU &= [AC]\,[BD], \\ UVF &= [ABC]\,[D], & FVU &= [A]\,[BCD], \\ UFV &= [AB]\,[CD], & FUV &= [AD]\,[BC]. \end{aligned}$$

The general properties of members of any class may be inferred either from equation (1) or from the order of the letters F, U, V if it is remembered that the image of the real part of the object space axis extends from F in the positive direction (through infinity if necessary) to U. Either method shows that in every group with the exception of the last, $[AD]\,[BC]$, there is a range of the axis in which a real image of a real object may be formed. The portion of the image axis which corresponds to the real object space axis is indicated by a thicker line in Figs. 3 to 8.

Among the special cases it will be noted that thin lenses, which have $B = C = 1$

and $D = 0$, are members, if the power is positive, of groups $[ABCD]$ and $[ABC]\,[D]$; if the power is negative they belong to groups $[A]\,[BCD]$ and $[AD]\,[BC]$. When the thicknesses are small without being exactly zero the groups are those mentioned second in each case. It is because the negative lenses ordinarily used belong to the class $[AD]\,[BC]$, and not because the power is negative, that all the images they form are virtual. It is not difficult to construct positive combinations which have this property.

Other special classes comprise the telescopes. For instance, prismatic binoculars, gun-sighting telescopes, and astronomical telescopes belong to either $[ABCD]$ or $[A]\,[BCD]$. On the other hand, Galilean binoculars belong to $[ABC]\,[D]$ or $[AD]\,[BC]$, and their undesirable properties are wholly attributable to the class to which they belong and have no connection with the erectness of the images they yield.

The adoption of this classification will lead naturally from the Gaussian treatment of rays to the consideration of entrance and exit pupils, an important branch of geometrical optics which has not until lately, in this country at all events, received a proper amount of attention. It would be convenient to have short descriptive titles for each group, but the writer wished to have the opportunity of hearing the views of other workers before making any proposals of this kind.

DISCUSSION

Mr A. Whitwell asked whether it would not be necessary to work out a system from its constructional data in order to see to which class it belonged.

Commander T. Y. Baker (communicated):

Mr Smith's proposal for a classification of optical systems in accordance with the signs of the four Gaussian constants is an exceedingly good one. So far we have had no such classification except as regards the optical power, i.e. the coefficient A. As the author of the paper points out, the physical peculiarities of optical systems that are of vital importance to the designer are dependent quite as much or even more upon the signs and magnitudes of B, C, and D, and a classification on the lines that Mr Smith suggests would, I think, be of more than purely theoretical interest.

I doubt very much, however, whether the nomenclature of classification that Mr Smith advocates will be practically satisfactory because such a symbol as $[AB]\,[CD]$ conveys no meaning at all apart from the equation

$$Axx' - Bx' - Cx + D = 0$$

and this equation must always be borne in mind together with the convention of the signs of x and x'. I suggest that a more practical method is to classify optical systems according as the principal foci and exit pupils are real or virtual.

Mr Smith shows in Figs. 3 to 8 the positions of the exit pupil U and back focus F in each of his six classes. Now the distance $VU = D/B$ and the distance $VF = C/A$ so that the exit pupil and the back focus are real only when D and B, and A and C are of the same sign respectively. In just the same way if light passes through the system in the reversed direction we get a real front focus and a real "reversed exit pupil" only if B and A, and D and C, respectively, are of the same sign. Hence Mr Smith's classification can equally well be expressed in terms of the reality or otherwise of the two principal foci and the two exit pupils; as thus:

	[ABCD]	[AD] [BC]	[AC] [BD]	[AB] [CD]	[ABC] [D]	[A] [BCD]
Back focus	real	virtual	real	virtual	real	virtual
Exit pupil	real	virtual	real	virtual	virtual	real
Front focus	real	virtual	virtual	real	real	virtual
Reversed exit pupil	real	virtual	virtual	real	virtual	real

It is clear from a glance at these groups that consideration of the reality or otherwise is unnecessary for more than three of the four points as in no group could there be three real images and one virtual or three virtual and one real. The classification is consequently known completely by the reality or otherwise of three of these four points and I think without question the two foci and the exit pupil are the three that are the most practical to use. There are two alternatives, namely real or virtual, for each of three positions so that in all there are eight possibilities but two of them are such that the equation $BC - AD = 1$ could not be satisfied. These two arrangements are

(1) Back focus real, front focus virtual, exit pupil virtual.

(2) Back focus virtual, front focus real, exit pupil real.

Neither of these arrangements is possible in any optical system.

The author, in reply to Mr Whitwell, said that in the case of a concrete instrument the class would be determined by seeing if the images of a distant object were real or virtual for both light directions, and also by making the same test for the image of the first surface in the complete instrument.

The classification exhibited in Commander Baker's table agrees with that suggested in the paper, and is based upon identical principles, with the one exception that Commander Baker proposes to use the exit pupils instead of the images of the extreme surfaces in the remainder of the system. As the exit pupil depends upon the relative sizes of various diaphragms, this alteration seems undesirable, if indeed it does not render the proposed basis impracticable for classifying instruments. Apart from this change the table exhibits the properties of the classes very clearly.

The notation used in the paper is sufficiently concise and convenient to serve during a preliminary discussion of the present proposals. If they are adopted the construction of convenient names and a suitable notation will doubtless require fuller consideration. The difficulty anticipated in remembering the equation and the sign conventions does not in reality arise, for there is a generally accepted convention which governs the sign of the power A, and the signs of the other quantities follow automatically since they may be derived by simple differentiation. It suffices then to note that when A, B, C, D are all positive the critical images are all real. A change in the sign conventions for x and x' will alter the signs of terms in the equation, but will not affect the signs of the four Gaussian constants which appear as coefficients in the equation or the classification based on them.