ANALYSIS OF THERMOELASTIC LAMINATED COMPOSITE PLATES AND SHELLS ON THE BASIS OF 3-D ANALYTICAL SOLUTION

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SUMMARY: A mathematical approach for the analysis of three-dimensional problems of stress and strain state of thick laminated orthotropic plates under thermal loading is developed. The solution is based on the expansion of the sought displacements in double trigonometric Fourier series with a factor being a combination of unknown functions over the third variable. Both the real and complex roots of the characteristic equation as well as the type of expansion of the unknown functions through the thickness of the plate are defined. Results of the analysis of sandwich plates are presented and discussed.

KEYWORDS: orthotropic plate, thermal loading, thermoelastic state, characteristic equation, real and complex roots.

INTRODUCTION

Due to increased use of laminated composite plates and shells as primary structural elements the requirements of their analysis and design constantly moving to a significantly higher level. The understanding of behaviour of these structures under different loading conditions including such as thermal, static, dynamic and their combinations becomes of paramount importance for the safe design. A three-dimensional analytical solution is developed in the present paper for the analysis of thick laminated orthotropic plates under thermal loading. This solution can be used as a benchmark solution where the range of applicability of different higher-order theories can be identified.

CHARACTERISTIC EQUATION OF THE THERMOELASTIC 3-D PROBLEM

Let us consider a thick laminated orthotropic plate which is in the steady temperature field. The Cartesian orthogonal coordinate system x_1, x_2, x_3 (Figure 1) is used here. The expressions for the stresses in terms of strains and temperature are given by

$$\begin{aligned}
\sigma_{11} &= C_{11}e_{11} + C_{12}e_{22} + C_{13}e_{33} - \beta_1^\circ T \\
\sigma_{22} &= C_{21}e_{11} + C_{22}e_{22} + C_{23}e_{33} - \beta_2^\circ T \\
\sigma_{11} &= C_{31}e_{11} + C_{32}e_{22} + C_{33}e_{33} - \beta_3^\circ T \\
\sigma_{12} &= C_{66}e_{12}; \sigma_{13} = C_{44}e_{13}; \sigma_{23} = C_{55}e_{23}
\end{aligned} \tag{1}$$

where C are elastic constants; e_{11}, e_{22}, e_{33} are relative deformations; β_1^o, β_2^o, β_3^o are coefficients of thermal expansion given as

$$\beta_1^\circ = C_{11}\alpha_1^\circ + C_{12}\alpha_2^\circ + C_{13}\alpha_3^\circ$$
$$\beta_2^\circ = C_{12}\alpha_1^\circ + C_{22}\alpha_2^\circ + C_{23}\alpha_3^\circ \qquad (2)$$
$$\beta_3^\circ = C_{13}\alpha_1^\circ + C_{23}\alpha_2^\circ + C_{33}\alpha_3^\circ$$

Here, α_j° are the coefficients of linear thermal expansion in the direction of $x_j\,(j=1,2,3).$ axes respectively. Taking into account influence of the thermal field the equation of equilibrium for the orthotropic medium can be written as

$$\overline{C}\left(\frac{\partial}{\partial x}\right)u - M_T = 0 \qquad (3)$$

where $u = \left\| u_{1,}\ u_2,\, u_3 \right\|^T$ is the displacement vector and also we have

$$M_T = \left\| \beta_j^\circ \frac{\partial T}{\partial x_j} \right\|^T \qquad (4)$$

Assuming that the temperature field satisfies equation of the thermal conductivity (there are no internal heat sources), i.e.

$$\lambda_1 \frac{\partial^2 T}{\partial x_1^2} + \lambda_2 \frac{\partial^2 T}{\partial x_2^2} + \lambda_3 \frac{\partial^2 T}{\partial x_3^2} = 0 \qquad (5)$$

where λ_j are the coefficients of the heat conductivity tensor $(j=1,2,3).$

The boundary conditions on the external surfaces take the following form:

for $x_3 = b^{(n)} = -h/2:$

$$C_{55}^{(n)}\left(\frac{\partial u_3^{(n)}}{\partial x_1} + \frac{\partial u_1^{(n)}}{\partial x_3}\right) = -q_{13}^-;\ C_{44}^{(n)}\left(\frac{\partial u_3^{(n)}}{\partial x_2} + \frac{\partial u_2^{(n)}}{\partial x_3}\right) = -q_{23}^-$$
$$C_{13}^{(n)}\frac{\partial u_1^{(n)}}{\partial x_1} + C_{23}^{(n)}\frac{\partial u_2^{(n)}}{\partial x_2} + C_{33}^{(n)}\frac{\partial u_3^{(n)}}{\partial x_3} = -q_{33}^- \qquad (6)$$

for $x_3 = b^{(0)} = h/2:$

$$C_{55}^{(1)}\left(\frac{\partial u_3^{(1)}}{\partial x_1} + \frac{\partial u_1^{(1)}}{\partial x_3}\right) = q_{13}^+;\ C_{44}^{(1)}\left(\frac{\partial u_3^{(1)}}{\partial x_2} + \frac{\partial u_2^{(1)}}{\partial x_3}\right) = q_{23}^+$$
$$C_{13}^{(1)}\frac{\partial u_1^{(1)}}{\partial x_1} + C_{23}^{(1)}\frac{\partial u_2^{(1)}}{\partial x_2} + C_{33}^{(1)}\frac{\partial u_3^{(1)}}{\partial x_3} = q_{33}^+ \qquad (7)$$

The boundary conditions for the heat conductivity on the external surfaces can be written as:

$$T\left(b^{(n)}\right) = f_1(x_1, x_2) \quad \text{when} \quad x_3 = h/2$$
$$T\left(b^{(0)}\right) = f_2(x_1, x_2) \quad \text{when} \quad x_3 = -h/2 \qquad (8)$$

where $f_1(x_1,x_2),\, f_2(x_1,x_2)$ are given functions which can be expanded in the double trigonometric Fourier series. An ideal thermal contact between the layers is also assumed and this can be expressed as

$$T^{(k-1)}\left(b^{(k-1)}\right) = T^{(k)}\left(b^{(k-1)}\right);\quad \lambda_3^{(k-1)}\frac{\partial T^{(k-1)} b^{(k-1)}}{\partial x_3} = \lambda_3^{(k)}\frac{\partial T^{(k)} b^{(k-1)}}{\partial x_3} \qquad (9)$$

A zero temperature is maintained at the lateral surfaces:

when $x_1 = 0, a_1$ and $x_2 = 0, a_2:$

(10)

$$T = 0$$

The distribution of the temperature over the plate can be expressed in the form of the double trigonometric Fourier series as:

$$T(x_1,x_2,x_3)=\sum_{m_1=1}^{\infty}\sum_{m_2=1}^{\infty}\overline{T}(x_3)\sin(\alpha x_1)\sin(\beta x_2) \tag{11}$$

where

$$\overline{T}(x_3)=F\exp(\gamma\pi x_3/h^{(k)})\,.$$

Relations (11) allow the boundary conditions to be satisfied on the lateral surfaces. Substituting expressions (11) into (5) leads to the following system of equations

$$\begin{aligned}
&A(\rho^2+t_{11})+Bt_{12}+D\rho t_{13}-F\alpha\beta_1^{\circ}=0\\
&At_{21}+B(\rho^2+t_{22})+D\rho t_{23}-F\beta\beta_2^{\circ}=0\\
&A\rho t_{13}+B\rho t_{23}+D(\rho^2+t_{33})-F\rho\beta_3^{\circ}=0\\
&t_{44}F=0
\end{aligned} \tag{12}$$

where *A*, *B* and *D* are unknown coefficients and F and ρ are known constants. In equation (12) we also have

$$t_{44}=-\lambda_1\alpha^2-\lambda_2\beta^2+\lambda_3\rho^2 \tag{13}$$

The homogeneous system of equations (12) has only one nonzero solution when the determinant of the system is equal to zero, that is

$$\begin{vmatrix}
\rho^2+t_{11} & t_{12} & \rho t_{13} & -\alpha\beta_1^{\circ}\\
t_{21} & \rho^2+t_{22} & \rho t_{23} & -\beta\beta_2^{\circ}\\
\rho t_{31} & \rho t_{32} & \rho^2+t_{33} & -\rho\beta_3^{\circ}\\
0 & 0 & 0 & t_{44}
\end{vmatrix}=0 \tag{14}$$

Equation (14) may be rewritten in the following form

$$\begin{vmatrix}
\rho^2+t_{11} & t_{12} & \rho t_{13}\\
t_{21} & \rho^2+t_{22} & \rho t_{23}\\
\rho t_{31} & \rho t_{31} & \rho^2+t_{33}
\end{vmatrix}t_{44}=0 \tag{15}$$

Taking into account that the problem is steady, equation (14) will correspond to the uncoupled thermoelastic problem. The system of equations (12) has eight roots κ_j and after determining these roots the expansion of the temperature and displacements through the thickness of the plate can be obtained.

EXPANSION OF THE TEMPERATURE THROUGH THE THICKNESS OF THE PLATE

Taking into consideration that the problem is steady the expansion of the temperature through the thickness of the plate can be written as follows

where $\overline{T}(x_3)=F_{14}\sinh f_4+F_{24}f_4\sinh f_4+F_{34}\cosh f_4+F_{44}f_4\cosh f_4$ (16)

Where F_{i4} expansion coefficients, and $f_4=\kappa_4\pi x_3/h^{(k)}$. After substituting (16) into the equation (5) the following expression for each layer *k* can be obtained

$$\begin{aligned}
&\left(-\lambda_1\alpha^2-\lambda_2\beta^2+\lambda_3\rho^2\right)\left(F_{14}\sinh f_4+F_{24}f_4\sinh f_4+F_{34}\cosh f_4+F_{44}\cosh f_4\right)\\
&+2\rho^2\left(F_{24}\cosh f_4+F_{44}\sinh f_4\right)=0
\end{aligned} \tag{17}$$

Taking into account that the first term of this expression contains product (13), and considering that $t_{44}=0$ it follows from (17) that

$$F_{24}\cosh f_4+F_{44}\sinh f_4=0 \tag{18}$$

Generalising the approach described above for the problems of statics, it should be noted that

equation (18) must be satisfied at any point within the layer. Thus, when $x_3 = 0$, that is on the midplane of the layer, we have $\sinh f_4 = 0, \cosh f_4 = 1$ and $F_{24} = 0$. When $x_3 \neq 0$, we get $\sinh f_4 \neq 0$ and $F_{44} = 0$ accordingly.
Finally, we obtain the expression of the temperature expansion within each layer

$$\bar{T}(x_3) = F_{14} \sinh f_4 + F_{34} \cosh f_4 \tag{19}$$

The expression (19) contains two unknown coefficients F_{14} and F_{34}, i.e. we obtain *2n* coefficients for *n* layers.

In order to determine them it is necessary to satisfy two boundary conditions on the external surfaces (8) and *2(n-1)* conditions of ideal thermal contact on the layer interfaces (9). Finally, we obtain *2+2(n-1)=2n* required equations.

The boundary conditions at the external surfaces are expressed through the coordinates of the top (*k = n*) and bottom (*k = 1*) layers, that is when $x_3 = -h^{(n)}/2$ and $x_3 = h^{(1)}/2$, respectively. They take the following form:

$$\begin{aligned} &\left(F_{14}^{(n)} \sinh f_4^{(n)} + F_{34}^{(n)} \cosh f_4^{(n)}\right)\sin(\alpha x_1)\sin(\beta x_2) = T(b^{(n)}) \\ &\left(F_{14}^{(1)} \sinh f_4^{(1)} + F_{34}^{(1)} \cosh f_4^{(1)}\right)\sin(\alpha x_1)\sin(\beta x_2) = T(b^{(0)}) \end{aligned} \tag{20}$$

The conditions of ideal thermal contact are given as follows

$$\begin{aligned} &F_{14}^{(k-1)} \sinh f_4^{(k-1)} + F_{34}^{(k-1)} \cosh f_4^{(k-1)} = F_{14}^{(k)} \sinh f_4^{(k)} + F_{34}^{(k)} \cosh f_4^{(k)}; \\ &\lambda_3^{(k-1)} \bar{f}_4^{(k-1)}\left(F_{14}^{(k-1)} \cosh f_4^{(k-1)} + F_{34}^{(k-1)} \sinh f_4^{(k-1)}\right) = \lambda_3^{(k)} \bar{f}_4^{(k)}\left(F_{14}^{(k)} \cosh f_4^{(k)} + F_{34}^{(k)} \sinh f_4^{(k)}\right); \\ &\bar{f}_4^{(k)} = \kappa_4 \pi / h^{(k)}; \qquad \bar{f}_4^{(k-1)} = \kappa_4 \pi / h^{(k-1)} \end{aligned} \tag{21}$$

Thus, the solution of the system of *2n* linear algebraic equations (20) and (21) allows us to find unknown coefficients of the expansion F_{14} and F_{34} for each layer and describe the distribution of temperature through the thickness. Having found the distribution of temperature for the entire plate (11), the stresses and displacements due to the thermal loading can be determined.

CALCULATION OF THE DISPLACEMENTS

In the case of the real roots we have

$$\begin{aligned} \bar{u}_1(x_3) &= -\left(A_{1j} \sinh f_j + A_{3j} \cosh f_j\right)/\alpha \\ \bar{u}_2(x_3) &= -\left(B_{1j} \sinh f_j + B_{3j} \cosh f_j\right)/\beta \\ \bar{u}_3(x_3) &= -\left(D_{1j} \cosh f_j + D_{3j} \sinh f_j\right)/\bar{f}_j \end{aligned} \tag{22}$$

and in the case of complex roots:

$$\begin{aligned} \bar{u}_1(x_3) &= -\left(A_{1j} P_{1j_j} + A_{3j} P_{3j}\right)/\alpha \\ \bar{u}_2(x_3) &= -\left(B_{1j} P_{1j} + B_{3j} P_{3j}\right)/\beta \\ \bar{u}_3(x_3) &= -\left(D_{1j} \bar{P}_{1j} + D_{3j} \bar{P}_{3j}\right)/\bar{f}_j \end{aligned} \tag{23}$$

Here P_{1j} and P_{3j} (*j=1, 2, 3*) are some coefficients which depend on geometrical parameters of the plate. The above expansion has 18 unknown coefficients of *A, B, D* type for each layer, i.e. *18n* sought coefficients for the whole layer package. The unknown coefficients are determined from the boundary conditions on the external surfaces and the conditions of the rigid contact on the interfaces. Finally, the system of linear algebraic equations can be derived.

In the case of the real roots we have the following equilibrium equations:

$$\begin{aligned}&\left\{\left[-\alpha/d_{1j}^2+\left(C_{11}\alpha^2+C_{66}\beta^2\right)/(\alpha C_{55})\right]A_{lj}+\alpha\left[(C_{12}+C_{66})B_{lj}+(C_{13}+C_{55})D_{lj}\right]/C_{55}\right\}\overline{\mu}_{lj}\\&-\alpha\beta_1^\circ\left(F_{14}\sinh f_4+F_{34}\cosh f_4\right)=0\\&\left\{\left[-\beta/d_{2j}^2+\left(C_{66}\alpha^2+C_{22}\beta^2\right)/(\alpha C_{44})\right]B_{lj}+\beta\left[(C_{12}+C_{66})B_{lj}+(C_{23}+C_{44})D_{lj}\right]/C_{44}\right\}\overline{\mu}_{lj}\\&-\beta\beta_2^\circ\left(F_{14}\sinh f_4+F_{34}\cosh f_4\right)=0\end{aligned}\tag{24}$$

In the case of the complex roots the equilibrium equations take the form:

$$\begin{aligned}&\left[\left(C_{11}\alpha^2+C_{66}\beta^2\right)A_{lj}/\alpha+\alpha(C_{12}+C_{66})B_{lj}+\alpha(C_{13}+C_{55})D_{lj}\right]/C_{55}\\&-A_{lj}R_{lj}/\alpha-\alpha\beta_1^\circ\left(F_{14}\sinh f_4+F_{34}\cosh f_4\right)=0\\&\left[\left(C_{66}\alpha^2+C_{22}\beta^2\right)B_{lj}/\beta+\beta(C_{12}+C_{66})B_{lj}+\beta(C_{23}+C_{44})D_{lj}\right]/C_{44}\\&-B_{lj}R_{lj}/\beta-\beta\beta_{21}^\circ\left(F_{14}\sinh f_4+F_{34}\cosh f_4\right)=0\end{aligned}\tag{25}$$

Here, we have three algebraic equations for an arbitrary point. These equations will be satisfied discretely for the system of points chosen within every layer. Besides, in order to satisfy equations (24) or (25) four points should be specified in the *k*-th layer. Consequently, we obtain twelve additional equations for each layer. The total number of equations for the plate, including additional ones, is *18n.*

The solution of the general system of algebraic equations, consisting of *6n* equations at the external surfaces when $x_3=b^{(n)}=-h^{(n)}/2$, $x_3=b^{(0)}=h^{(1)}/2$, equations at the interfaces, and also *12n* additional equations allows to determine the expansion coefficients A_{lj}, B_{lj} and D_{lj} for the laminated orthotropic plate made of *n* layers.

ANALYSIS OF SANDWICH PLATES

The approach developed above is used to analyse three-layered plates subjected to thermal loading. The material characteristics of the external layers *k=1,3* of the thickness of *0.25h* are as follows

$$\begin{aligned}&E_1=41.5\cdot10^3\,MPa;\quad E_2=E_3=18.2\cdot10^3\,MPa\\&G_{12}=G_{13}=G_{23}=6.83\cdot10^3\,MPa;\ \nu_{12}=\nu_{13}=\nu_{23}=0.257;\\&\lambda_1=1.2W/\left(m\ {}^\circ C\right);\ \lambda_2=\lambda_3=0.8W/\left(m\ {}^\circ C\right);\\&\alpha_1=8.07\cdot10^{-6}/C;\ \alpha_2=\alpha_3=16.32\cdot10^{-6}/C;\end{aligned}$$

The core layer (*k=2*) has thickness 0*.5h* and its properties are given as

$$\begin{aligned}&E_1=30.1\cdot10^3\,MPa;\quad E_2=E_3=26.5\cdot10^3\,MPa\\&G_{12}=G_{13}=G_{23}=4.28\cdot10^3\,MPa;\ \nu_{12}=\nu_{13}=\nu_{23}=0.108;\\&\lambda_1=1.0W/\left(m\ {}^\circ C\right);\ \lambda_2=\lambda_3=0.4W/\left(m\ {}^\circ C\right);\\&\alpha_1=12.7\cdot10^{-6}/C;\ \alpha_2=\alpha_3=14.37\cdot10^{-6}/C;\end{aligned}$$

The external surfaces of the plate are under the influence of the thermal field which has a sinusoidal distribution over the surface:

$$\begin{aligned}T\left(b^{(0)}\right)&=T_0\sin(\alpha x_1)\sin(\beta x_2)\\T\left(b^{(n)}\right)&=T_n\sin(\alpha x_1)\sin(\beta x_2)\end{aligned}$$

The amplitude values of the temperature are taken as: $T_0 = 50 C^{\circ}, T_n = 0$.

The influence of the relative dimensions of the plate (*a/h*) on the thermal stress state was considered. Figure 2 shows diagrams of the distribution of normal displacements u_3 / h and the temperature at the centre of the plate. These diagrams are plotted for the ratios *a/h = 2, 3, 5, 10*. When *a/h = 2* we have a considerably nonlinear character of the distribution of displacements and temperature through the thickness. Displacements at the heated surface are approximately 3.7 times larger than displacements at the unheated surface. The difference in displacements at the external surfaces decreases as the ratio *a/h* increases and at ratio *a/h=10* these displacements are practically constant through the thickness. The temperature distribution through the thickness of each layer is approaching the linear law as *a/h* increases and when *a/h = 10* the distribution law through the thickness is practically piecewise-linear.

The distribution of normal stresses $10^4 \sigma_{11} / E_1^{(1)}$ and $10^4 \sigma_{22} / E_1^{(1)}$ through the thickness is shown in Figure 3. In the case of thick plates with *a/h = 2, 3* the diagrams are nonlinear. The distribution of stresses in a layer is approaching the linear law as the ratio *a/h* increases. The maximum compressive stresses occur at the external heated surface. These stresses decrease as the ratio *a/h* increases. The stresses $10^4 \sigma_{11} / E_1^{(1)}$ on the unheated surface are almost constant and increase insignificantly as *a/h* increases while $10^4 \sigma_{22} / E_1^{(1)}$ changes its sign. In the case when *a/h=2* the compressive stresses take place in the top layer and become tensile when *a/h=10*.

The behaviour of the transverse normal stresses $10^6 \sigma_{33} / E_1^{(1)}$ at the centre of the plate and tangential stresses $10^4 \sigma_{12} / E_1^{(1)}$ $(x_1 = x_2 = 0)$ is shown in Figure 4. In the case of thick plates with *a/h=2, 3* the transverse normal stress changes its sign.

On the interface between layers 1 and 2 there is compressive stress, and on the interface between layers 2 and 3 we have tensile stress. The transverse normal stresses decrease as the ratio *a/h* increases and in the case of plates with *a/h=5, 10,* compressive stresses become predominant. The distribution behaviour within each layer of the stresses $10^4 \sigma_{12} / E_1^{(1)}$ is close to linear independently of the ratio *a/h* and their maximum values are close to each other.

Figure 5 shows the distribution of transverse shear stresses through .the thickness of the plate. Diagram $10^5 \sigma_{13} / E_1^{(1)}$ is plotted at the point $x_1 = 0, x_2 = a/h$, and diagram $10^5 \sigma_{23} / E_1^{(1)}$ at the point $x_1 = a/2, x_2 = 0$. In all the considered cases of *a/h* the maximum stresses $10^5 \sigma_{13} / E_1^{(1)}$ occur on the interface between the first and the second layers. These stresses decrease as *a/h* increases. The stresses change their sign twice within the second layer when *a/h=2*. In the case of thinner plates $10^5 \sigma_{13} / E_1^{(1)}$ change their sign in the second layer only once.

The transverse shear stresses $10^5 \sigma_{23} / E_1^{(1)}$ reach their maximum in the core layer and they decrease as *a/h* increases. In the case of thick plates *a/h=2, 3* the sign is changing in the third layer and it is opposite to that in both the first and the second layers. In the case of plates with *a/h = 5, 10* the sign of stresses in the first layer is opposite to the second and third layers.

The thermoelastic analysis shows that in the case of thick plates $a \leq 5$ the plate should be treated as a three--dimensional structure. The orthotropic properties of the material influence significantly on the values and the character of distribution of the stresses through the thickness of the plate. For example, the normal stresses $10^4\sigma_{22}/E_1^{(1)}$ on the surface $x_3 = h/2$ are much higher than $10^4\sigma_{11}/E_1^{(1)}$ though $E_2^{(1)} < E_1^{(1)}$. This is because the surface $x_3 = h/2$ is heated by $20C^\circ$. The situation is opposite to that of the surface $x_3 = h/2$, that is $\sigma_{22} < \sigma_{11}$. Since $T_n = 0$ the contribution of the thermal factor to the stresses does not take place here and only relative deformations effect the stresses.

The appearance of tensile normal stresses σ_{33} is an important factor which promotes tearing off between the layers, and this can be checked by experiments. Of particular interest is the distribution of transverse shear stress through the thickness. The character of this distribution is far from the parabolic law used often in different applied theories.

CONCLUSIONS

The solution is developed for the three-dimensional problem of laminated orthotropic plates subjected to both static and thermal loading. The solution is based on the expansion of the sought displacements in double trigonometric Fourier series with a factor which is a combination of unknown functions over the third variable. Both the real and complex roots of the characteristic equation as well as the type of expansion of the unknown functions through the thickness are defined. An approach is proposed to determine the unknown coefficients by a discrete satisfaction of the equilibrium and heat conduction equations in a system of points through the thickness of the plate.

A solution technique is developed on the basis of the proposed approach for the analysis of plates made of orthotropic, transversally-isotropic and isotropic layers. The assessment of the accuracy of the proposed solution is done by a comparison of the obtained results when different number of terms is retained in the expansion. Some numerical results are also compared with test problems available in the literature.

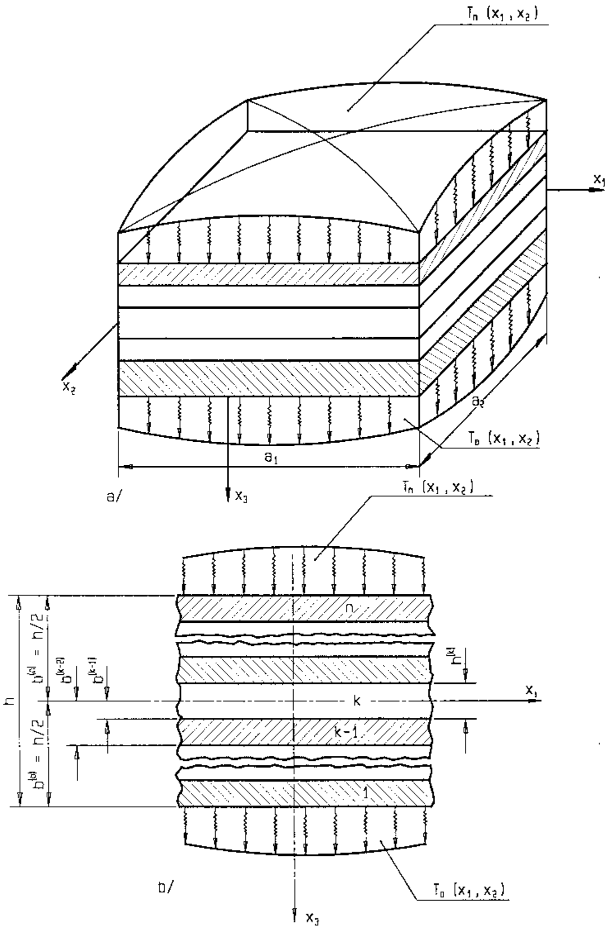

Figure 1 Laminated plate under thermal loading with the boundary conditions of the first kind at the external surfaces: a) general view; b) thickness structure

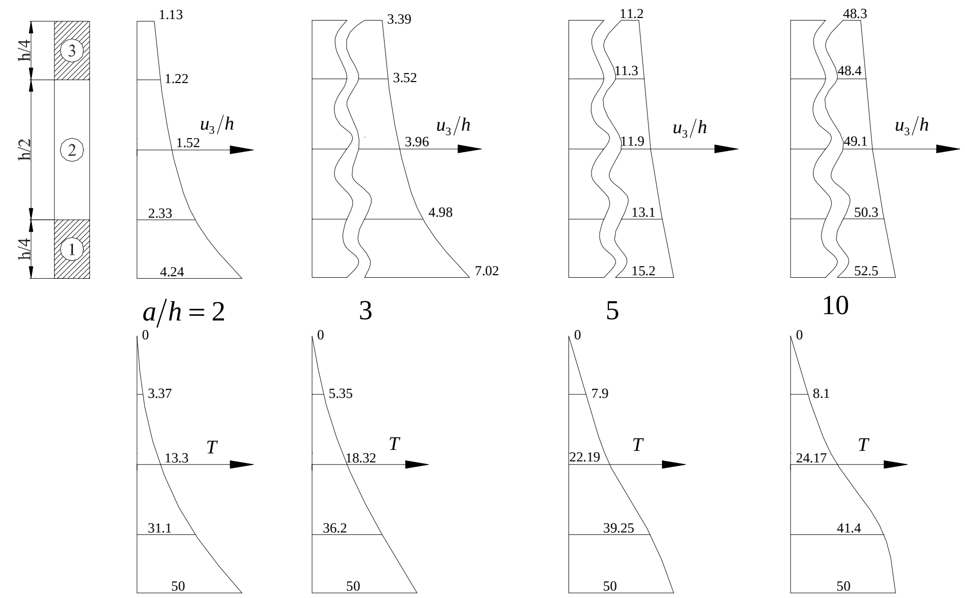

Figure 2 Distribution of displacements (u_3/h) and the temperature (T) at the centre of the square sandwich plate.

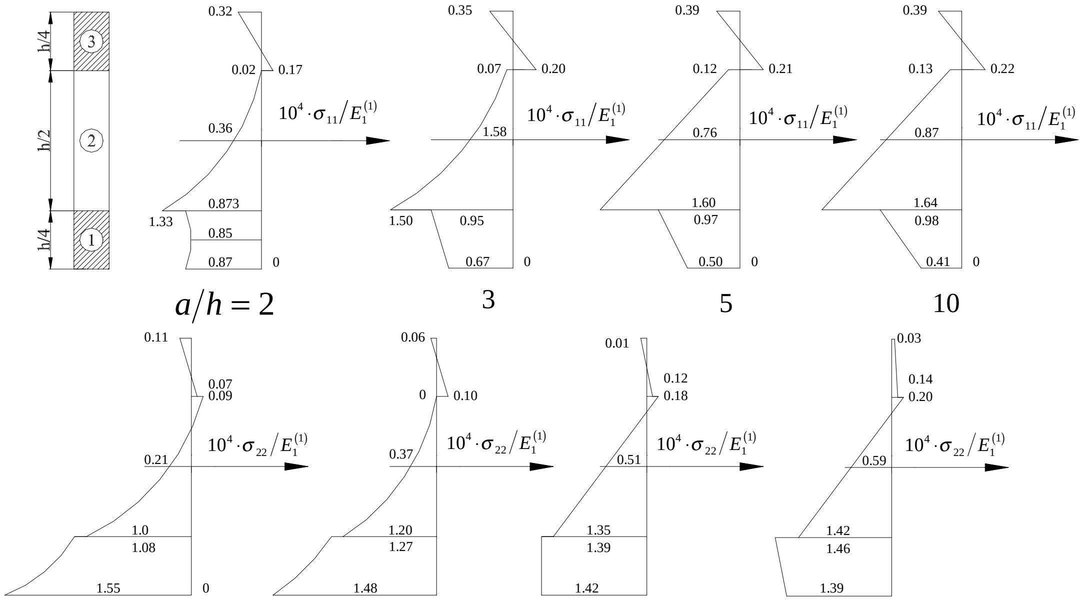

Figure 3 Distribution of normal stresses through $(10^4 \cdot \sigma_{11}/E_1^{(1)})$ and $(10^4 \cdot \sigma_{22}/E_1^{(1)})$ at the centre of the square sandwich plate.

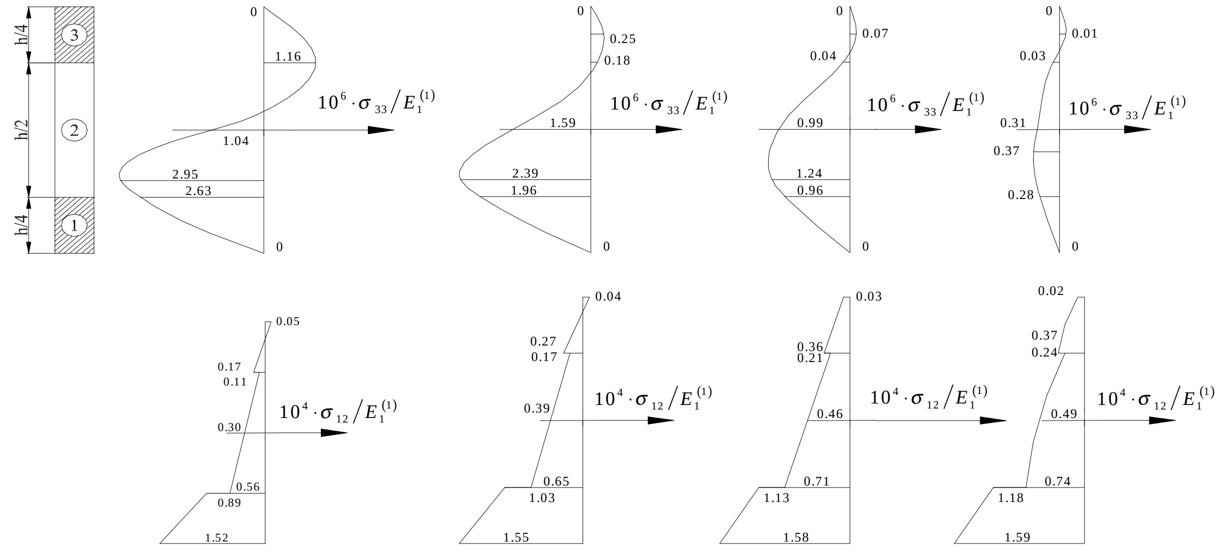

Figure 4 Distribution of transverse normal ($10^6 \cdot \sigma_{33} / E_1^{(1)}$) and ($10^4 \cdot \sigma_{12} / E_1^{(1)}$) stresses in the square sandwich plate.

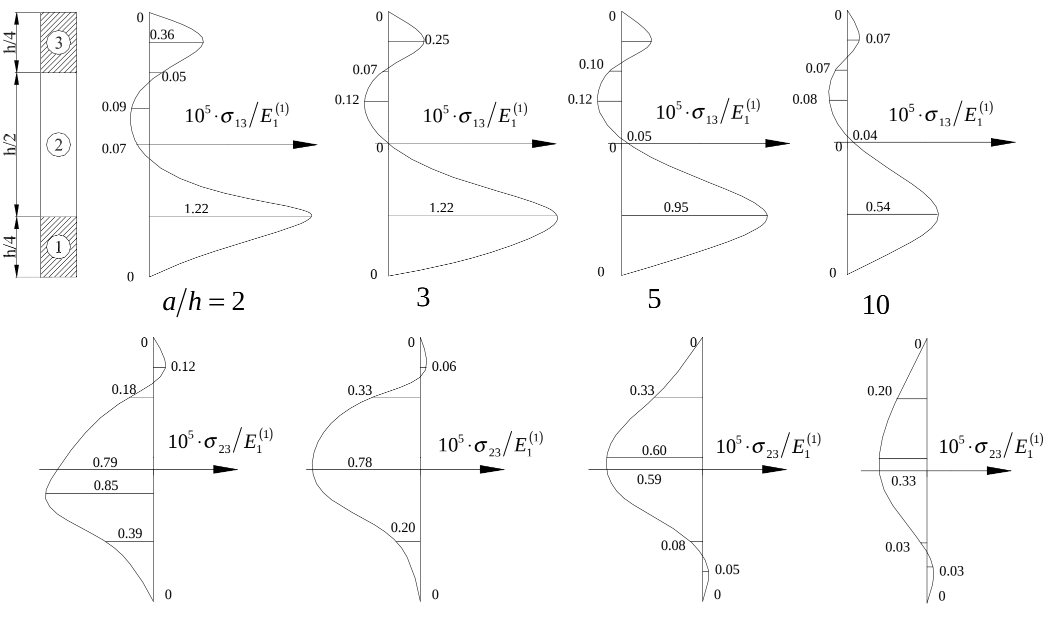

Figure 5 Distribution of transverse shear through ($10^5 \cdot \sigma_{13} / E_1^{(1)}$) and ($10^5 \cdot \sigma_{23} / E_1^{(1)}$) stresses in the square sandwich plate.