MECHANICAL BEHAVIOUR OF ELEMENTARY

CONSTITUENTS OF LAMINATES

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This paper concerns non linear mechanical behavior of composite laminates. Two constituents are defined : the elementary layer and the interface. This one is a zero thickness entity depending on the angle between the upper and the lower adjacent elementary layers.

Non linear models are built up for each constituent.

The layer is modelized by a coupled elasto-plastic and damage model.

The delamination is characterized by a damage model on the interface area.

Through an homogeneization method, the identification of the models is made with tests performed on angle-ply specimens.

At least some verifications are presented.

Introduction

For the multidirectional composite laminates the mechanical behaviour as well as the local phenomenon, delamination for example, depend on the stacking sequence and on the orientation of the layers /1/. This leads to research the behaviour of elementary constituents. They are the elementary layer and the interface.

The global behaviour of the material is obtained by an homogeneization method. Due to the unhomogeneity and the anisotropy of these materials, difficulties happen, for the identification of constituents models from the global measured values. An adequate choice in the stacking sequence of the specimen is required in order to measure easily the wished quantities. This work is supported by Aerospatiale which supplies us with the material a T300/914C composite laminate.

Experimental Method

Specimens

Specimens are cut out unidirectional and angle-ply coupons. Four types of angle-ply specimens are tested :

$$[5\ ,\ -5]_{2S}\ ,\ [10\ ,\ -10]_{2S}\ ,\ [22{,}5\ ,\ -22{,}5]_{2S}\ ,\ [45\ ,\ -45]_{2S}$$

All the specimens have been tested under a traction load, in or off orthotropic axes. The tests types are listed in table I.

Let θ the angle defined in figure 1.

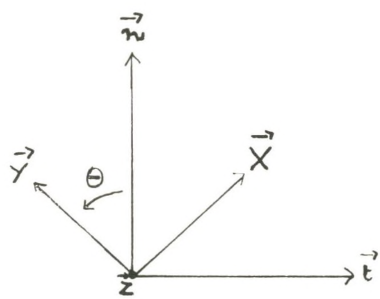

Figure 1 : definition of θ

where : - $\vec{X}$ is the fiber direction of the upper layer
- $\vec{Y}$ is the fiber direction of the lower layer
- $\vec{n}$ is the loading direction

rotation : R_0 is the reference $(0,X,Y)$, R the reference $(0,\vec{n},\vec{t})$

Table I. In and off axes types of tests

Laminates	θ (degrees)
$[45 , -45]_{2s}$	0 ; 33 ; 22,5 ; 15 ; 45
$[5 , -5]_{2s}$	0 ; 45
$[10 , -10]_{2s}$	0 ; 45
$[22,5 , -22,5]_{2s}$	0 ; 45
$[0 , 0]_{2s}$	45

Specimens shape is drawn on figure 2. On each specimen are stuck aluminium tabs to prevent material's deterioration by the loading heads.

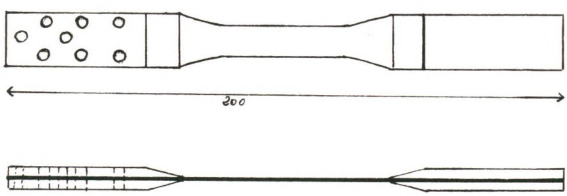

Figure 2 : Specimen's shape

Tests procedure

Loading heads have been designed to :
- ensure a good balancing of the specimen ;
- prevent bending effects ;
- ensure an homogeneous repartition of stresses.

The load is transmitted to the specimen by means of pins. This allows to have a weak compression of the specimen's heads.

Specimens have been applied monotonic tensile loadings or cyclic ones (unloading up to free state of stresses). The displacements rates used on the test machine are very low : 0,01 mm/mn in order to reduce the viscosity effects.

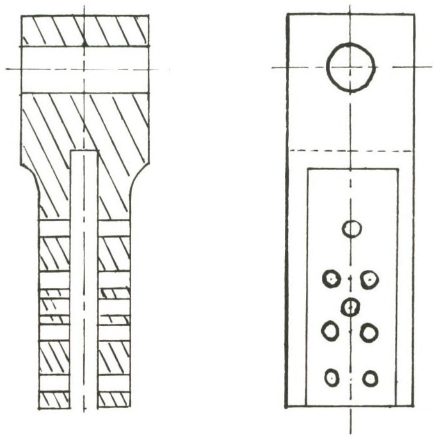

Figure 3 : Loading heads

Method of measure

High anisotropy for off axes test specimens induces bending and torsion effects. Because of the rather reduced dimensions of specimens, the free edge effects, such as delamination, may have some influence on measures. So for these cases, longitudinal and transversal strain gauges are fixed all around the same section in the middle of the specimen's length. (figure 4).

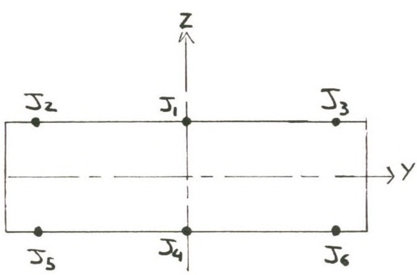

Figure 4 : Strain gauges arrangement

In order to define the traction force, when the induced effects are important, a calculation of the specimen is necessary. For in axis test specimens the repartition of strain gauges is simple ; J_1 and J_4 are only used.

Tests results

Let σ_n the stress, which is the applied force devided by the cross section. Let ε_n^n and ε_t the longitudinal and the transverse strains.

$$\sigma_n = f(\varepsilon_n) \qquad ; \qquad \sigma_n = g(\varepsilon_t)$$

f and g are plotted in each case of sollicitation.

. In fibres direction the material's behaviour is elastic and brittle. (figure 5).

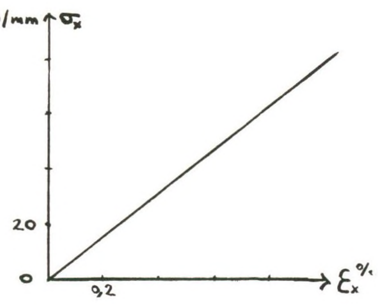

Figure 5 : Stress-strain curve for $[45, -45]_{2r}$ laminates loaded in a fibers direction.

The shear behaviour is obtained by an off axes test for a $[45, -45]_{2s}$ laminates with $\theta = 0°$.

Let T the rotation of $\pi/4$ around the Z axis (figure 1)

There : $T\sigma_R T = \sigma_{R_0}$

$$T\varepsilon_R T = \varepsilon_{R_0}$$

$$\tau_{R_0} = \frac{\sigma_n}{2}$$

$$\gamma_{R_0} = \varepsilon_n - \varepsilon_t$$

and $G^{*} = \dfrac{\sigma_n}{2(\varepsilon_n - \varepsilon_t)}$

On cyclic tests it is observed a deterioration of the material stiffness, the plastic deformations. (figure 6, 7). The value of the damage may be obtained by measuring the sloap between A and B on each cycle.

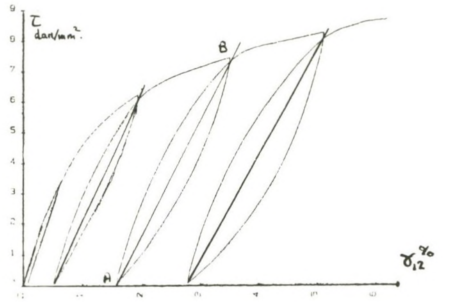

Figure 6 : Shear behaviour

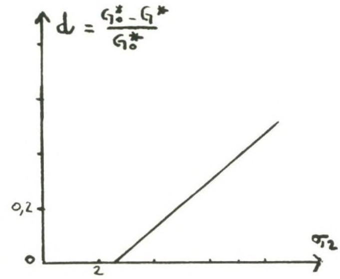

Figure 7 : Global shear damage

. In the case of an off axes tests for a $[\theta_0 , -\theta_0]_{2S}$ specimens with $\theta = 45°$ the effects of the induced bending and the torsion are important. (see figure 8).

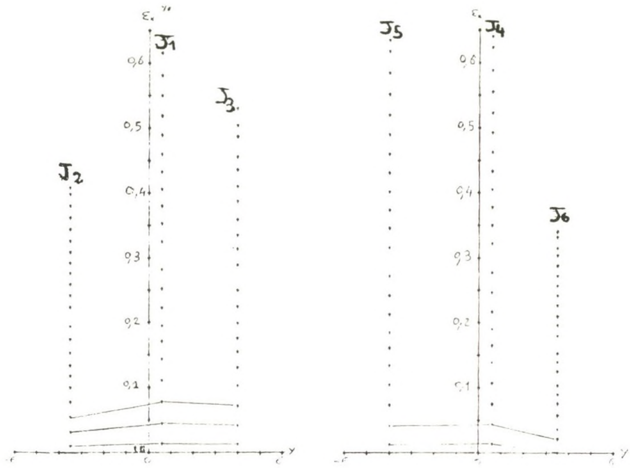

Figure 8 : Deformation values as a function of the gauges place on the width of the specimen, for a monotonic loading

Concluding remarks

The tests results imply that the off axes material's behaviour is elasto-plastic with damage. The layer's behaviour is then supposed of the same type.

In fiber direction no plasticity nor damage occur. The non linear behaviour is then only due to the matrix presence.

Elementary layer modelization

Initial values of Young's modulus, shear modulus, and Poisson's ratio of the layer are determined by two tensile tests performed on $[45, -45]_{2s}$ laminates for $\theta = 0$ and $\theta = 45$.

$E_1 = 148000$ MPa

$E_2 = 12000$ MPa

$G_{12} = 5600$ MPa

$V_{12} = 0{,}33$

εrupt = 0,9 % in the fibers direction.

Damage model

Damage phenomenon introduced by Katchanov /2/ is defined as progressive deterioration of loaded materials because of initiation and development of micro-cracks. It has been shown /3/ on quasi isotropic laminates that micro-cracks early appeared in the 90° orientated plies. These cracks are arranged in a parallel direction to the fibers. This crack development process is drawn on figure 9.

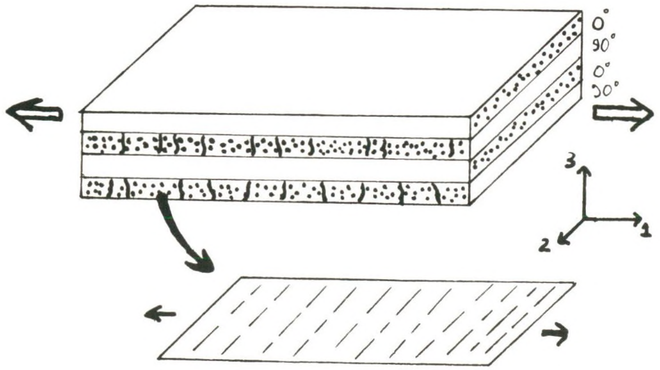

Figure 9 : Micro-cracks in the 90° orientated plies

Through these observations assumptions are made for the damage model :
- the damage comes from the matrix ;
- the direction of micro-cracks in any cases is parallel to the fibers.

Then the stiffness of the layer is obtained by homogeneization of the fibers and the matrix supposed damageable in the direction 2. The only transverse Young's modulus and the shear modulus of the layer are submitted to damage.

Let the elastic energy E_D

$$2E_D = \frac{\sigma_{11}^2}{E_1} - \gamma\sigma_{11}\sigma_{22} + \left[\frac{\sigma_{22}^2}{E_2} + \frac{\sigma_{12}^2}{G_{12}}\right]$$

the damage model leads to :

$$E_1 = E_1^0 \qquad \text{and} \qquad \gamma = \gamma_0$$

Hence

$$2E_D = \frac{\sigma_{11}^2}{E_1^0} - \gamma^0\sigma_{11}\sigma_{22} + \left[\frac{<\sigma_{22}>_+^2}{(1-\alpha d)E_2^0} + \frac{\sigma_{12}^2}{(1-d)G_{12}^0}\right] + \frac{<-\sigma_{22}>_+^2}{E_2^0}$$

d is the scalar damage variable.

Remark : The model takes into account the unilateral type of damage.

Let Y the associated variable of $\dot{d}$ in the dissipation expression :

$$Y = \div \frac{\partial E_D}{\partial d} = \frac{1}{2}\left[\frac{\alpha<\tilde{\sigma}_{22}>_+^2}{E_2^0} + \frac{\tilde{\sigma}_{12}^2}{G_{12}^0}\right]$$

$\tilde{\sigma}$ denotes the effective stress tensor

$$\tilde{\sigma} = \begin{pmatrix} \sigma_{11} & \dfrac{\sigma_{12}}{1-d} \\ \dfrac{\sigma_{12}}{1-d} & \dfrac{\sigma_{22}}{1-\alpha d} \end{pmatrix}$$

The standard model used yields to :

$$\left.\bar{Y}^{1/2}\right|_t = \sup_{\tau \leqslant t} \left[Y_\tau^{1/2} \cdot (1-d)\right]$$

The constitutive law of damage is :

$$d = \frac{<\bar{Y}^{1/2} - \bar{Y}_0^{1/2}>_+}{\bar{Y}_C^{1/2}} \qquad \text{if } d < 1 \quad ; \quad d = 1 \text{ otherwise}$$

Model of plasticity

The plastic behaviour of the layer is due to the matrix. No plastic deformation occurs in fibers direction. So the plastic criterion for the layer involves shear and transverse stresses. An isotropic hardening law is associated to the criterion.

Let f the yield function

$$f = s_{II} - R(p) - R_0$$

$R(p)$ is characteristic of the material ;
p is the cumulated plastic strain.

and s_{II} is defined by

$$s_{II} = \sqrt{\tilde{\sigma}_{22}^2 a^2 + \tilde{\sigma}_{12}^2}$$

a^2 is a constant.

For standard model, it does exist $\dot{\lambda}_p$ positive such that :

$$\dot{\tilde{\varepsilon}}^p = \dot{\lambda}_p \frac{\partial f}{\partial \tilde{\sigma}} \qquad \text{with} \qquad \dot{\tilde{\varepsilon}}^p = \begin{bmatrix} 0 & \dot{\varepsilon}_{12}^p(1-\alpha d) \\ \dot{\varepsilon}_{12}^p\ (1-\alpha d) & \dot{\varepsilon}_{22}^p\ (1-d) \end{bmatrix}$$

$$\dot{p} = \dot{\lambda}_p$$

$\dot{\lambda}_p$ is determined by the condition

$$\dot{f} = 0$$

Hence

$$\dot{\lambda}_p = \frac{\dot{s}_{II}}{\frac{\partial R}{\partial p}}$$

We assume : $R(p) = mp^n$

consequently :

$$\dot{\lambda}_p = \frac{\dot{s}_{II}}{nm^{1/n}(s_{II}-R_0)^{1-1/n}}$$

Let A a linear symmetrical operator such that

$$A(\sigma) = \begin{bmatrix} 0 & \frac{\sigma_{12}}{2} \\ \frac{\sigma_{12}}{2} & a^2\sigma_{22} \end{bmatrix}$$

owing to the relation

$$s_{II}^2 = \mathrm{Tr}\left[A(\tilde{\sigma}).\tilde{\sigma}\right]$$

it follows :

$$\frac{df}{d\tilde{\sigma}} = \frac{A(\tilde{\sigma})}{s_{II}}$$

and

$$\dot{\tilde{\varepsilon}}_p = \tilde{g}A(\tilde{\sigma})$$

where : $\tilde{g} = \dot{H}(s_{II}) = \dfrac{\dot{s}_{II}}{nm^{1/n} s_{II}(s_{II}-R_0)^{1-1/n}}$

Two plastic strains are defined

$$\dot{\varepsilon}_{12}^p = \dot{H}(s_{II})\,\frac{\tilde{\sigma}_{12}}{2(1-d)}$$

$$\dot{\varepsilon}_{22}^p = \dot{H}(s_{II})\frac{\tilde{\sigma}_{22}\,a^2}{(1-\alpha d)}$$

These relations lead to the expression of $\dot{p}$

$$\dot{p} = \sqrt{(1-d)^2\dot{\varepsilon}_{12}^{p2} + \frac{(1-\alpha d)^2}{a^2}\,\dot{\varepsilon}_{22}^{p2}}$$

Homogeneization

The aim of the homogeneization is to obtain relationship between global measured quantities and local quantities depending on the parameters introduced in the non linear constitutive law of the layers. The homogeneization only applies in the interior domain of the plate in which interface has no significant influence. /4/, /5/.

The classical laminates theory /6/, /7/ is built up with the assumptions of a plane state of stresses and a constant state of plane strains in the thickness.

The results of this theory are available in the inner part of the plate, that means at a distance of the thickness's order from the edges /8/.

The homogeneization method is based on the same hypotheses.

Let π the orthogonal projection operator on the layer's plane ; the plane constitutive law of the k^{th} layer is written in the form of :

$$K^k(\pi\varepsilon\pi) = \pi\sigma^k\pi$$

K^k_{ij} are defined by

$$K^k_{ij} = \overline{K}^k_{ij} - \frac{\overline{K}^k_{i3}\ \overline{K}^k_{j3}}{\overline{K}^k_{33}} \qquad ; \qquad (i,j) \in (1,2)$$

Each component of the stress tensor defined as the mean value of the corresponding components of the stress tensor of the layers. Relationships between global and local quantities are then deduced.

Identification of the Models

This identification use results of tensile test performed on a $[45\ ,\ -45]$ specimen loaded at $\theta = 0°$.

For this particular case of laminates the periodicity of the material is based on 2 layers, that the reason why the homogeneization involves only two adjacent layers.

The values of the components of the K^1 tensor in the R_0 reference are the following :

$K_{11} = 146500$ MPa

$K_{22} = 12000$ MPa

$K_{12} = 3900$ MPa

$K_{66} = 5600$ MPa

The equations are all written in the reference R_0 .

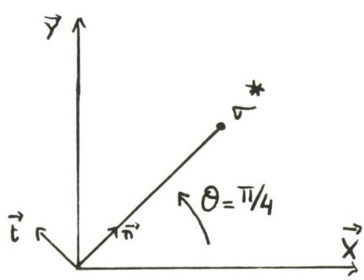

Figure 10 : Definition of references

Let σ^* the applied force divided by the cross section.

$$\sigma^*_R = \begin{bmatrix} \sigma^* & 0 \\ 0 & 0 \end{bmatrix}$$

consequently

$$\sigma^{\star}_{R_0} = \sigma^{\star}(n\bar{n})_{R_0} = \frac{\sigma^{\star}}{2}\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix}$$

up to now let us call $\frac{\sigma^{\star}}{2} : \tau^{\star}$

For the layer C_1 , the stress tensor is defined by :

$$\pi\sigma^1\pi = \begin{bmatrix}\sigma_{11} & \sigma_{12}\\ \sigma_{12} & \sigma_{22}\end{bmatrix}$$

and the Hooke tensor K^1

$$K^1 = \begin{bmatrix}K_{11} & K_{12} & 0\\ K_{12} & K_{22} & 0\\ 0 & 0 & K_{66}\end{bmatrix}$$

with respect of symmetry, we obtain for the layer C_2 :

$$\pi\sigma^2\pi = \begin{bmatrix}\sigma_{22} & \sigma_{12}\\ \sigma_{12} & \sigma_{11}\end{bmatrix}$$

and

$$K^1 = \begin{bmatrix}K_{22} & K_{12} & 0\\ K_{12} & K_{11} & 0\\ 0 & 0 & K_{66}\end{bmatrix}$$

the uniform strain tensor, accross the thickness is

$$\pi\varepsilon\pi = \begin{bmatrix}\varepsilon & \varepsilon_{12}\\ \varepsilon_{12} & \varepsilon\end{bmatrix}$$

Elastic calculation

Owing to the definition of the global stress tensor, and taking into account symmetries of the problem, we get

$$\tau^{\star} = \frac{\sigma_{11} + \sigma_{22}}{2} = \left(\frac{K_{11} + K_{22}}{2}\right).\varepsilon + K_{12}\varepsilon$$

$$\tau^{\star} = K_{66}\,\varepsilon_{12}$$

The homogeneous Hooke tensor $K^{\star}$ is :

$$K^{\star} = \frac{K^1 + K^2}{2}$$

A simple relation between σ_{22} and $\tau^{\star}$ is also obtained, and will be used further

$$\sigma_{22} = 2\tau^{\star}.\left(\frac{K_{11} + K_{22}}{K_{11} + K_{22} + 2K_{12}}\right) \qquad \text{(i)}$$

with the numerical values

$$\sigma_{22} \simeq \frac{\tau^{\star}}{5}$$

Identification of the damage model. The damage value is found thanks to cyclic tests. See figure 6. The mean behaviour is elastic on the cycle, so the plastic model isn't introduced here.

Let us remind

$$Y = \frac{1}{2}\left[\frac{\alpha\sigma_{22}^2}{(1-\alpha d)^2 E_2^0} + \frac{\tau^{2\star}}{(1-d)^2 G_{12}^0}\right] \qquad \text{(ii)}$$

the relation (i) enables us to write :

$$Y = \frac{\tau^{2\star}}{2(1-d)^2 G_{12}^0} \qquad \text{(iii)}$$

As a matter of fact, disregarding the first term of Y in (ii), the maximal error done is toward 2% ; and it decreases when damage grows up.

An intrinsic curve for the layer, $\overline{Y}^{1/2}$ as a linear function of d , may be drawn. $\overline{Y}^{1/2}$ denotes :

$$\overline{Y}^{1/2} = (1-d)\sqrt{2Y}$$

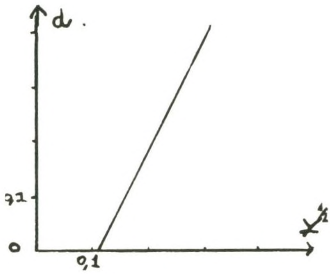

Figure 11 : Damage versus $\overline{Y}^{1/2}$. Intrinsic curve

The parameter α is identified with an other test performed on a $(\pm\theta_0)2s$ specimen ; θ_0 is such as σ_{22} hasn't negligeable effects.

Elasto-plastic calculation and identification

The plastic law is written with the effective stresses. All remarks of symmetry are available for these stresses.

The global effective stress tensor $[\tilde{\tau}^*]$ is the mean value of local effective stress tensor. That is yielding us to write :

$$\dot{\tilde{\tau}}^* = \frac{\dot{\tau}^*}{1-d} = 2G\left[\varepsilon_{12} - \dot{H}\,\tilde{\tau}^*\right]$$

Then the function $\dot{H}$ is known at any time. It would be possible to write all the global-local relationships and to get a^2 as a function of τ^*, ε . However tests results, give an accurate value of ε_{12} , but don't for ε . So it is more advisable to complete the identification on an other test performed on a $[\pm\theta_0]_{2s}$ specimen, for which the calculations are of the same type. The parameters of the hardening law are identified on the curve s_{II} as a function of p, when a^2 is get.

Verification of model

A numerical simulation of tensile tests is made. The constitutive law of the layers take into account the non linear model. The global behaviour of the material is obtained by homogeneization.

Up to now the identification hasn't be done when all induced effects are considered. It leads to rather good results for some cases. See figure 12.

Nevertheless, when induced effects are important (see figure 8), this calculation isn't sufficient.

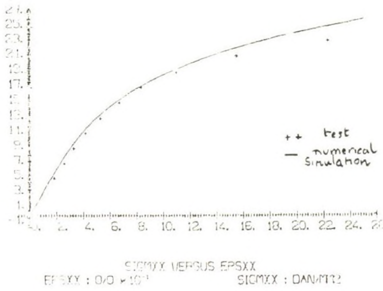

Figure 12 : Comparison between test curve and numerical one. $[\pm 45]_{2r}$ laminate loaded at 22,5°

Concluding remark. These verifications point out difficulties of identification, and necessity to know exactly induced effects.

Interface modelisation

The aim of the modelization is to take into account the discontinuity of mechanical properties from a layer to another.

The interface is introduced as a zero thickness entity depending on the orientation of the upper and lower adjacent layers.

The main effect occurring on interface area is the delamination. Fracture mechanics is the classical way to deal with this problem /9/, /10/. Beginning of delamination happens because of stresses concentration. But in laminates the very small thickness of the layers make stresses interacting from a layer to another. That's the reason why we use a continuous model founded upon the damage mechanics /11/, /12/. The initiation of delamination is calculated through an instability condition due to the constitutive law.

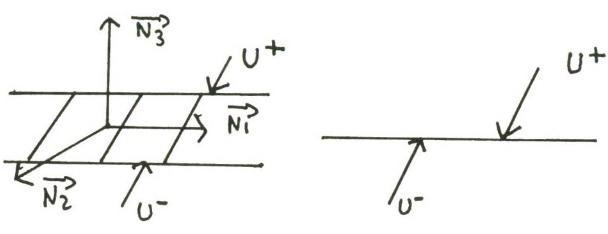

Figure 13 : Modelization of the interface

Let U^+ (resp U^-) the displacement of a point of the upper (resp lower) area of the interface, we note :

$$[\![U]\!] = U^+ - U^- = \begin{bmatrix} [\![U_1]\!] \\ [\![U_2]\!] \\ [\![W]\!] \end{bmatrix}_{(\vec{N}_1,\ \vec{N}_2,\ \vec{N}_3)}$$

A density surfacic energy E_D is choosen :

. Undamaged interface

$$2E_D = k\ [\![W]\!]^2 + k_1[\![U_1]\!]^2 + k_2[\![U_2]\!]^2$$

by introducing stresses :

$$2E_D = \underbrace{\frac{1}{k}\sigma_{33}^2}_{\text{traction-compression energy}} + \underbrace{\frac{1}{k_1}\sigma_{13}^2 + \frac{1}{k_2}\sigma_{23}^2}_{\text{shear energy}}$$

where k, k_1, k_2 are elastic constants.

. Damaged interface

$$2E_D = \underbrace{\frac{1}{k} <\sigma_{33}>_+^2}_{\text{compression energy}} + \underbrace{\frac{1}{k(1-d)} <\sigma_{33}>_+^2}_{\text{traction energy}}$$

$$+ \underbrace{\frac{1}{k_1(1-\gamma_1 d)}\sigma_{31}^2 + \frac{1}{k_2(1-\gamma_2 d)}\sigma_{32}^3}_{\text{shear energy}}$$

Let Y the associated variable of $\dot{d}$, according to the surfacic density of dissipation

$$Y = \frac{\partial E_D}{\partial d}$$

$$Y = \frac{1}{2k} <\tilde{\sigma}_{33}>_+^2 + \underbrace{\frac{\gamma_1}{2k_1}\tilde{\sigma}_{31}^2 + \frac{\gamma_2}{2k_2}\tilde{\sigma}_{32}^2}_{\text{coupling effect}}$$

the Y value introduces a coupling between shear stresses and normal stress.

In Y expression two terms appear, one for the first opening mode, due to the tension energy, the second one for mode II and mode III, due to the shear energy. By the construction of a standard model the damage d may be defined as :

$$d(t) = L\left[\sup_{\tau<t} Y(t)\right]$$

Where L is a characteristic function of the interface, which defines an undamaged domain, we assume :

$$\begin{cases} d = \sup\limits_{\tau<t} \dfrac{Y}{Y_C} & \text{if } d < 1 \\ d = 1 & \text{otherwise} \end{cases}$$

Identification is done through classical parameters of fracture mechanics ; G_{IC} , G_{IIC} , G_{IIIC} ; which are limit values of the energy release rate of mode I, II, III obtained for a unit value of d .

Identification model

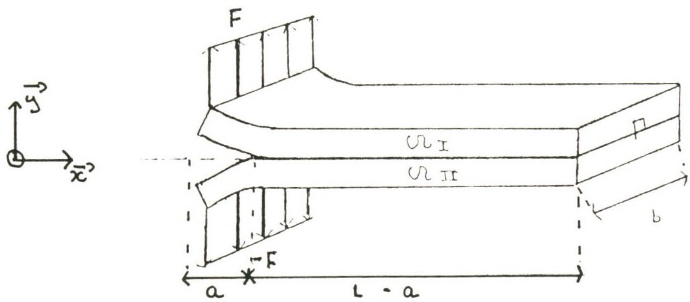

Figure 14 : Specimen's shape

- a is the length crack
- F is the applied force
- b is the width of the specimen

Pure mode I calculation with damage model leads to simple relations between the different parameters.

The field of displacements in Ω_I and Ω_{II} are choosen as

$$\vec{U} = (u(x) - yv(x))\vec{x} + w(x)\vec{y}$$

The variational formulation of the problem is

$$\forall\, \vec{U}^* \quad ; \int_{\Omega_I \cup \Omega_{II}} T_r \left[\sigma\varepsilon(\vec{U}^*)\right] d\Omega + \int_\Gamma 4k(1-d)[\![w]\!]\,[\![w^*]\!]\,d\Gamma = 2Fw^*(0)$$

Equilibrium equations and boundary conditions are deduced, just as the continuities of v and w at the crack tip which lead to relation between F , k , $[\![w]\!](a)$.

The instability is defined as :

$$\frac{dF}{dw(0)} = 0$$

This condition is equivalent to the relation $[\![w]\!](a) = \left(\frac{2Y_C}{k}\right)^{1/2}$

Using calculation of G_{IC} from the definition of G_I :

$$G_I = \frac{F}{2b}\left(\frac{d(2w(0))}{da}\right)$$

We obtained for a sufficient length of crack :

$$G_{IC} = 2Y_C$$

It is worth noting that G_{IC} is independent of the length crack a .

When there is no crack, it has been shown that the critical value of F only depends on Y_C .

Hence Y_C is an intrinsic characteristic of the interface.

Main interests of this model are to be able to calculate the initiation of the crack so as well as the front of delamination.

Experimental results are taken in different studies /13/.

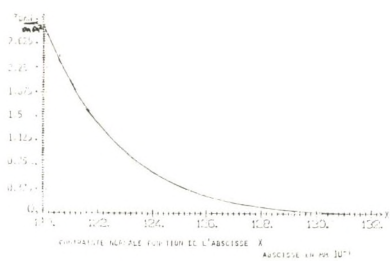

Figure 15 : Normal stress versus longitudinal coordinate

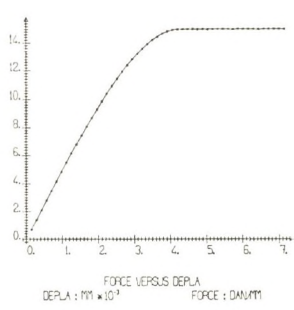

Figure 16 : Force versus displacement for the initiation of delamination

CONCLUSION

Even with simple sollicitations, it has been shown that induced effects and delamination have to be taken into account in the treatment of tests results. So identification isn't easy. For the inner part of a structure a layer's model of damage and plasticity is built up. On the other hand, a continuous damage model of the interface is built up for the peripheric domain where delamination occurs.

The introduction of fatigue in the models hasn't been treated at present and is one of the possible extension of the work.

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