chief thing that is wrong with museums, national and provincial, is (as Bernard Shaw says of the poor) their poverty? WM. EVANS HOYLE.

National Museum of Wales, Cardiff,
March 15.

IN the timely and suggestive leading article on museums in NATURE of March 11 there are references to the Museum of Practical Geology that need explanation, not because they are incorrect, but because they are symptomatic of that forgetfulness of the fundamental purposes of this museum which has long been obvious in some quarters.

It is true that "the Museum of Practical Geology was a necessary concomitant of the Geological Survey," but this was not, and never has been, its sole *raison d'être*. It was founded as the Museum of Economic Geology—that is, of economic geology in its broadest aspects. It had, therefore, from its inception two functions to perform: (1) To serve as the storehouse and exhibition for all the concrete documentary material collected during the making of the geological maps—material of the greatest value as a demonstration of the facts of British geology and usefully employed for educational, industrial, and purely scientific purposes; and (2) to act as the national repository of material illustrative of all those mineral resources that form the basis of mining, metallurgical, and other industries.

The first of these functions is purely British in scope, the second is world-wide.

As regards overlapping with the Natural History Museum, there is none; and alternatively, as the lawyers say, if there is any it should cease, since the functions of the two institutions are clearly differentiated. The scheme of the geological and mineralogical departments of the Natural History Museum is academic, and that of the Museum of Practical Geology economic. On the other hand, the Imperial Institute in respect of its mineral exhibits does overlap the functions of the older institution. This is a question requiring attention in any scheme of reconstruction. WILLIAM G. WAGNER.

March 15.

Some Methods of Approximate Integration and of Computing Areas.

ENGINEERS and shipbuilders are continually requiring to find the area of a surface bounded by curved lines. If both the upper and lower boundaries are curved, it is a simple matter to divide the surface into two by a straight line, find the area of each part separately, and add them together.

Simpson's rule is almost universally used for this purpose, but a little consideration will show that a more accurate evaluation of the area can be obtained in most cases by using other rules.

We will consider an area contained by a base line, two vertical ordinates at the ends, and a number of intermediate ordinates placed at equal distances along the base line. If the base line be divided into m equal intervals, each of a length h, there will, of course, be $m+1$, or n, ordinates. When the height of these ordinates is known, and the value of h the interval also, an approximation to the value of the area can be obtained which increases in accuracy with the number of ordinates taken and measured, when the curve is of an anomalous shape.

(1) If the upper boundary be a straight line, an exact result will be obtained by merely the two end ordinates y_1 and y_2 and the length of the base line h; $A=\frac{1}{2}h(y_1+y_2)$.

(2) If the upper boundary be a parabola, an exact result will be obtained by bisecting the base line, and then

$$A=\frac{h}{3}\left(y_1+y_3+4y_2\right),$$

where h is half the base line.

This is Simpson's well-known rule: If any odd number of ordinates be taken, say 7, it is considered as a succession of three areas bounded above by three parabolas, *i.e.* the area from y_1 to y_3 is added to the area from y_3 to y_5, and this, again, is added to the area from y_5 to y_7. The formula used is then

$$A=\frac{h}{3}\left[\overline{y_1+y_7}+2\,\overline{y_3+y_5}+4\,\overline{y_2+y_4+y_6}\right].$$

If m denote the number of additional areas computed by this method, the general formula will take the form

$$A=\frac{h}{3}\left[\overline{y_1+y_{3+2m}}+2\,\overline{y_{1+2m}}+4\,\overline{y_2+y_{2+2m}}\right].$$

It should be especially noted that this formula must be used only when the number n of ordinates is odd and the number of intervals even. In the second and third terms the values 1, 2, 3, etc., are assigned successively to the symbol m, ending with that value of m which denotes the number of additional areas that are to be computed. The formula is based on the assumption that $y=a+bx+cx^2$, and gives the best possible approximation to the true area if only three ordinates are given.

(3) If, however, four ordinates be given, we may assume that $y=a+bx+cx^2+dx^3$, and the resulting formula based on this assumption,

$$A=\frac{3h}{8}\left[\overline{y_1+y_4}+3\,\overline{y_2+y_3}\right],$$

will give the best possible approximation if only four ordinates are given. This formula should be used only when the number of ordinates is $4+3m$, and it then becomes

$$A=\frac{3h}{8}\left[\overline{y_1+y_{4+3m}}+2\,\overline{y_{1+3m}}+3\,\overline{y_2+y_3+y_{2+3m}+y_{3+3m}}\right]$$

(4) If five ordinates be given, we shall obtain a more accurate result by assuming that y is a quartic function of x, and for 5, 9, 13, or $5+4m$ ordinates the following formula may be used:

$$A=\frac{2h}{45}\left[7\,\overline{y_1+y_{5+4m}}+14\,\overline{y_{1+4m}}+12\,\overline{y_3+y_{3+4m}}+32\,\overline{y_2+y_4+y_{2+4m}+y_{4+4m}}\right]$$

(5) Similarly, if $6+5m$ ordinates be given, y may be regarded as a quintic function of x, and the formula becomes

$$A=\frac{5h}{288}\left[19\,\overline{y_1+y_{6+5m}}+38\,\overline{y_{1+5m}}+75\,\overline{y_2+y_5+y_{2+5m}+y_{5+5m}}+50\,\overline{y_3+y_4+y_{3+5m}+y_{4+5m}}\right]$$

(6) Again, if $7+6m$ ordinates be given, y may be assumed to be a sextic function of x, and we then have the formula

$$A=\frac{h}{140}\left[41\,\overline{y_1+y_{7+6m}}+82\,\overline{y_{1+6m}}+216\,\overline{y_2+y_6+y_{2+6m}+y_{6+6m}}+27\,\overline{y_3+y_5+y_{3+6m}+y_{5+6m}}+272\,\overline{y_4+y_{4+6m}}\right].$$

In all these formulæ the first term consists of the sum of the first and the last ordinate. In (2), (3), (4), (5), and (6) the values 1, 2, 3, etc., are assigned successively to the symbol m in the following terms according to the number of ordinates. Thus if in (6) nineteen ordinates are given, $19=7+6m$, so $m=2$.

When $m=0$, the ordinates with m as a part of their subscript are omitted in all but the first term.

Example.—Suppose the base line be divided into six equal intervals $(h=\frac{1}{6})$, and the value of the ordinates be

$y_1=0$ $\quad y_2=0.5527708$
$y_3=0.7453560$ $\quad y_4=0.8660254$
$y_5=0.9428090$ $\quad y_6=0.9860133$
$y_7=1$

As seven ordinates are given, we may use any one of the rules (6), (3) which is adapted to $4+3m$ ordinates, and (2) which can be used when $3+2m$ ordinates are employed. We should expect to get a more accurate result when the higher-order formula is employed, and this we shall find to be correct. The values given refer to the quadrant of a circle, so that the true value is $\pi/4$, or 0·78539816 . . .

By method (6), putting $m=0$, the result is 0·7791866, *i.e.* 0·7972 per cent. too small.

By method (3) the result is 0·7758061, or 1·342 per cent. too small.

By method (2) the result is 0·777531, or 1·063 per cent. too small.

This result is curious, and shows that a small arc of a circle approaches more nearly to a small arc of a parabola than to a small arc of any cubic curve, but it will be noted that method (6) gives a much more accurate result.

We may, however, use a combination of the above rules; for instance, we may take five ordinates by rule (4) and the remaining two intervals by rule (2). As the first three ordinates increase more rapidly than the last three, we should naturally leave the last three to be dealt with by rule (2). In this way a result of 0·7784954, or a defect of 0·0069027, or an error of only 0·88667 per cent. is obtained. Had we reversed the order and used Simpson's rule for the first two intervals, the defect would have been 0·0078447, or an error of 1·0102 per cent.

In conclusion, it may be stated that if the nature of the curve is unknown a more accurate result will always be obtained by using the highest-order formula that can be used with the given number of ordinates. If two different formulæ are used, it has just been shown that the most accurate result is obtained when the higher-order formula is used for that part of the curve in which the variation of the ordinates is the greater. If the curve be a parabola, an absolutely accurate result is obtained by using only three ordinates by means of method (2).

It may be thought that plotting the curve and estimating its area mechanically by means of a planimeter will be always the best and speediest method to adopt, but this is by no means the case. It often takes far less time to calculate, say, thirteen ordinates and to use method (6) than to trace the curve.

A. S. PERCIVAL.

Westward, Newcastle-upon-Tyne.

An Electronic Theory of Isomerism.

THE interesting suggestions made by Mr. W. E. Garner in NATURE for February 19 with regard to a possible explanation of the isomerism of certain organic compounds may be examined from a different, but perhaps simpler, point of view by employing the "ring electron" or "magneton" of Mr. A. L. Parson. The electron is looked upon as a circular anchor ring of negative electricity rotating about its axis at a high speed, and therefore behaving like a small magnet. In connection with atomic and molecular numbers I have directed attention elsewhere to the "rule of eight," according to which a difference of 8 or a multiple of 8 is frequently found between the numbers of the unit electric charges associated with analogous atoms or molecules. In the theory of the "cubical atom" put forward by Prof. Gilbert N. Lewis and developed by Dr. Irving Langmuir, one of the most stable configurations for the atomic shell is that in which eight electrons are held at the corners of a cube. The single bond commonly used in graphical formulæ involves two electrons held in common by two atoms (Fig. 1); the double bond implies that four electrons are held conjointly by two atoms (Fig. 2) Or if the *pair* of electrons be regarded as the most stable grouping of all, it may be, as Lewis and Langmuir suggest, that the pairs of electrons held in common by two atoms are drawn closer

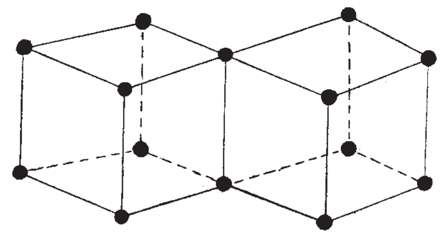

FIG. 1.

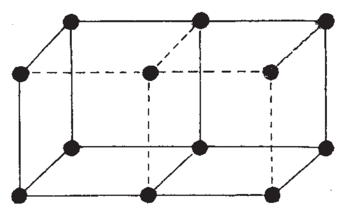

FIG. 2.

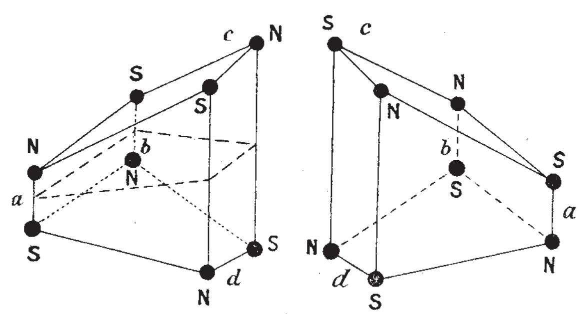

FIG. 3.

together by the magnetic attraction between them. Dextro- and lævo-rotatory forms of a compound might then be represented as mirror images as in Fig. 3. The letters N and S in this diagram may be taken to represent the polarity of the external face of the ring electron.

Mr. Garner suggests the possibility of the existence of a large number of optical isomerides amongst organic compounds, but the view here put forward does not lead to that conclusion; on the contrary, it seems to give exactly the same number of isomerides as the ordinary structural formulæ. It is true that it is possible to reverse *in the diagram* the magnetic polarity of one or more pairs of electrons, but even if the arrangements so obtained were stable, it is doubtful whether they would represent different isomerides. It would not be possible to explain the phenomenon of free mobility about a single bond