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STRESS-STRAIN STATE OF AN ANNULAR PLATE

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Annotation: An axisymmetric plane stress state of a compressible, ideally elastic-plastic body is considered. The material is assumed to exhibit both elastic and plastic behavior under loading. The analysis focuses on the mechanical response of the body under these conditions.
Keywords: stress state, elastic-plastic body, mechanical response, axisymmetric

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НАПРЯЖЕННО-ДЕФОРМИРОВАННОЕ СОСТОЯНИЕ КОЛЬЦЕВОЙ ПЛАСТИНЫ

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Аннотация: Рассматривается осесимметричное плоское напряженное состояние сжимаемого, идеально упругопластического тела. Предполагается, что материал проявляет как упругое, так и пластическое поведение под нагрузкой. Анализ фокусируется на механическом отклике тела в этих условиях.
Ключевые слова: напряженное состояние, упругопластическое тело, механический отклик, осесимметричный

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1. Elastic State of an Annular Plate

In the region that remains in the elastic state, the components of the stress tensor, the radial component of the displacement vector, and the components of the strain tensor are determined using well-known formulas [1-3]:

$$\sigma_{r,\theta} = A \mp \frac{B}{r^2},\ Eu = (1-\nu)Ar + (1+\nu)\frac{B}{r},$$
$$E\varepsilon_{r,\theta} = (1-\nu)Ar \mp (1+\nu)\frac{B}{r^2}. \tag{1}$$

Here, r, θ are polar coordinates, ν is Poisson's ratio, and E is Young's modulus.

If the radial stresses are specified on the boundaries of the elastic region $a \le r \le b$ as $\sigma_r|_{r=a} = -p_a$, $\sigma_r|_{r=b} = -p_b$, then the values of A and B are determined by the following formulas [1]:

$$A = \frac{a^2 p_a - b^2 p_b}{b^2 - a^2},\ B = \frac{a^2 b^2 (p_a - p_b)}{b^2 - a^2}. \tag{2}$$

2. Conditions for the onset of the plastic region

Let us determine the values of the pressures p_a and p_b at which a plastic region begins to form within the elastic domain $a \le r \le b$, given that the yield condition is expressed as:

$$\max\{|\alpha_1\sigma_\theta + \beta_1\sigma_r + \gamma_1\sigma_z|, \dots, |\alpha_n\sigma_\theta + \beta_n\sigma_r + \gamma_n\sigma_z|\} = 2k. \tag{3}$$

In the principal axes σ_r, σ_θ of the stress tensor, the yield condition (3) defines a certain polygon, which serves as an approximation of the "experimental yield curve". For an isotropic body, the yield curve is symmetric with respect to the axis $\sigma_r = \sigma_\theta$.

The equation $\alpha_i\sigma_\theta + \beta_i\sigma_r = 2k$ defines a line passing through the i-th and $(i+1)$-th vertices of the yield polygon.

The solution of the system of equations

$$\begin{cases} \alpha_i\sigma_\theta + \beta_i\sigma_r = 2k, \\ \alpha_{i+1}\sigma_\theta + \beta_{i+1}\sigma_r = 2k \end{cases}$$

determines the coordinates of the $(i+1)$-th vertex of the yield polygon, formed by its adjacent sides numbered i and $(i+1)$:

$$\sigma_r = \frac{2k(\alpha_{i+1} - \alpha_i)}{\alpha_i\beta_{i+1} - \alpha_{i+1}\beta_i},\ \sigma_\theta = -\frac{2k(\beta_{i+1} - \beta_i)}{\alpha_i\beta_{i+1} - \alpha_{i+1}\beta_i}. \tag{4}$$

In the case of an axisymmetric plane stress state, the regimes (4) $(i = 1 \div n)$ corresponding to the vertices of the yield polygon cannot be realized in a two-dimensional domain. This assumption contradicts the equilibrium equation:

$$\frac{d}{dr}(r\sigma_r) - \sigma_\theta = 0.$$

An exception is the case when $\sigma_r = \sigma_\theta = \text{const}$, which does not contradict the equilibrium equation.

From the solution of (1) and (2), it follows that the regimes $\sigma_r = \sigma_\theta = \pm k$ are realized in a solid disk subjected to an external pressure $p_b = \mp k$ or in an annular disk under the action of pressures $p_a = p_b = \mp k$. In these cases, the annular region transitions entirely from the elastic state to the ultimate plastic state $\sigma_r = \sigma_\theta = A = \pm k$ when $|p_a| = |p_b| \le k$ and $a \ne 0$, or when $|p_b| \le k$ and $a = 0$.

The loading process in the elastic region is governed by an increase in the introduced equivalent stress. If the equivalent stress is chosen as the yield function, then the boundary at which the plastic zone begins to form during loading is determined by the value of the yield function for the elastic solution. If the elastic state is realized in the region $a \le r \le b$, then the equivalent stress, according to formulas (3) and (1), is given by:

$$\sigma_{eq} = \max\left\{\sigma_{eq}^{(1)}, \sigma_{eq}^{(2)}, \dots, \sigma_{eq}^{(n)}\right\}. \tag{5}$$

For certain values of the pressures p_a and p_b, a specific plasticity regime may begin to form at certain points within the region $a \le r \le b$. These points are determined by solving the equation:

$$\max_{a \le r \le b}\left(\sigma_{eq}\right) = 2k. \tag{6}$$

3. Points of plastic zone initiation

From the definition (5) of the equivalent stress, it follows that:

$$\frac{d\sigma_{eq}^{i}}{dr} = -2(\alpha_i - \beta_i)\frac{B}{r^3} = -2(\alpha_i - \beta_i)\frac{a^2 b^2 (p_a - p_b)}{b^2 - a^2}\frac{1}{r^3}.$$

Therefore, when $\alpha_i \ne \beta_i$ and $p_a \ne p_b$, the maximum value of $\sigma_{eq}^{(i)}$ can only be reached at the boundaries of the region ($r = a$ or $r = b$). In the case when $\alpha_i = \beta_i$ and $p_a = p_b$, the equivalent stress σ_{eq} reaches its maximum value simultaneously at all points within the region $a \le r \le b$.

For an isotropic body, all yield curves in the σ_r, σ_θ plane lie between the following triangles:

$$\begin{cases} \max\{2\sigma_\theta - \sigma_r, \sigma_\theta + \sigma_r, 2\sigma_r - \sigma_\theta\} & = \pm 2k, \\ \max\{2\sigma_\theta - \sigma_r, \sigma_\theta + \sigma_r, 2\sigma_r - \sigma_\theta\} & = \mp k \end{cases}$$

When $p_a \ne p_b$, both functions $2\sigma_\theta - \sigma_r$ and $2\sigma_r - \sigma_\theta$ are monotonic (one increasing and the other decreasing), and at the same time, they satisfy the condition:

$$\lim_{r\to\infty}(2\sigma_\theta - \sigma_r) = \lim_{r\to\infty}(2\sigma_r - \sigma_\theta).$$

Therefore, except for the cases when $p_a = p_b$ and $b \neq a$ or when $a = 0$, the plastic region in isotropic bodies will begin to form at the boundary $r = a$ during loading.

4. Conditions for transition to the plastic state

The transition to the plastic state begins at a certain value of the parameters that define the external loading. In the considered case, the external influences are the values p_a and p_b. The fulfillment of the plastic zone initiation condition (6) at the boundary $r = a$ requires a specific relationship between the external pressure p_b and the internal pressure p_a.

For definiteness, we assume that at the boundary $r = a$ (due to the reason mentioned above, plasticity regimes corresponding to the vertices of the yield polygon are not considered), the equivalent stress is given by:

$$\sigma_{eq} = \alpha_i \sigma_\theta + \beta_i \sigma_r. \quad (7)$$

Then, from condition (6), it follows that the pressures p_a and p_b are related by the equation:

$$p_a = 2\frac{\alpha_i b^2 p_b + (b^2 - a^2)k}{\alpha_i(a^2 + b^2) + \beta_i(a^2 - b^2)}.$$

Since, for the i-th plasticity regime to be realized at the boundary $r = a$, the following conditions must hold:

$$\begin{cases} \alpha_{i-1}\sigma_\theta + \beta_{i-1}\sigma_r \leq 2k \\ \alpha_i\sigma_\theta + \beta_i\sigma_r = 2k \\ \alpha_{i+1}\sigma_\theta + \beta_{i+1}\sigma_r \leq 2k \end{cases}$$

the pressure p_b at the boundary $r = b$ can vary within the range:

$$\min\{p_{b1}, p_{b2}\} \leq p_b \leq \max\{p_{b1}, p_{b2}\},$$

where

$$p_{b1} = k\frac{(\alpha_i - \alpha_{i-1})(a^2 + b^2) + (\beta_i - \beta_{i-1})(a^2 - b^2)}{(\alpha_{i-1}\beta_i - \alpha_i\beta_{i-1})b^2},$$

$$p_{b2} = k\frac{(\alpha_{i+1} - \alpha_i)(a^2 + b^2) + (\beta_{i+1} - \beta_i)(a^2 - b^2)}{(\alpha_i\beta_{i+1} - \alpha_{i+1}\beta_i)b^2}.$$

If, upon satisfying condition (7) at the boundary $r = a$, we express p_b as a function of p_a:

$$p_b = \frac{(b^2 - a^2)k}{\alpha_i b^2} + \frac{(a^2 + b^2)\alpha_i + (a^2 - b^2)\beta_i}{2\alpha_i b^2}p_a,$$

then the pressure values at the inner boundary must vary within the range:

$$\min\{p_{a1}, p_{a2}\} \leq p_a \leq \max\{p_{a1}, p_{a2}\},$$

where

$$p_{a1} = \frac{2k(\alpha_i - \alpha_{i-1})}{\alpha_{i-1}\beta_i - \alpha_i\beta_{i-1}},\ p_{a2} = \frac{2k(\alpha_{i+1} - \alpha_i)}{\alpha_i\beta_{i+1} - \alpha_{i+1}\beta_i}$$

For specific plasticity conditions, similar dependencies are provided in [4-11].

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