# Toroidal Curvature Solitons in LoopMesh Geometry: From Intuitive Projection to Path Integral Quantization

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This paper presents a localized, finite-energy vector soliton emerging naturally from LoopMesh projection geometry. Constructed from an intuitive toroidal sketch and formalized in cylindrical coordinates, the soliton exhibits inward axial compression, outward radial coherence, and Gaussian decay. We derive a consistent Lagrangian, confirm energy localization, and apply path integral quantization to show first-order quantum stability. This structure may represent a projection-locked curvature excitation with implications for matter–antimatter symmetry, ghost curvature dynamics, and Zeta-resonance attractors in emergent spacetime frameworks.

Author's Note (Preprint v1.0): This work represents an early-stage exploration of geometric soliton formation within the LoopMesh framework. The toroidal structure was discovered through visual intuition and translated into a formal vector field with verified energy localization. While some mathematical components remain conjectural or unproven (e.g., full stability analysis, boundary conditions), this draft is shared as an open exploration and invitation for collaboration and refinement. All feedback is welcome.

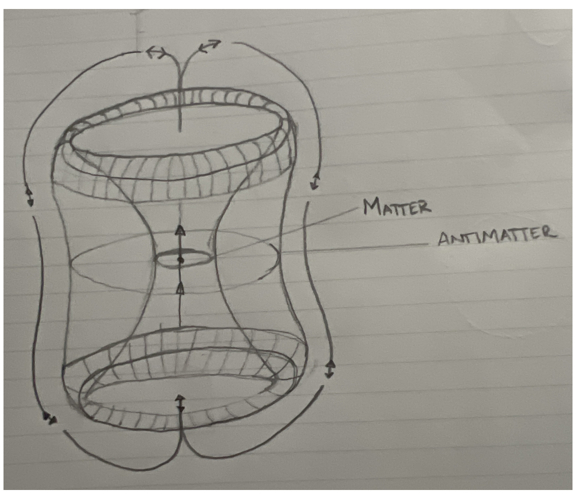


FIG. 1. Initial intuition sketch of the toroidal projection soliton, drawn by hand. Note the dual projection lobes, matter–antimatter flow inversion, and internal curvature coherence. This sketch seeded the formal vector field construction.

## I. INTRODUCTION

Solitons—stable, localized field configurations—have long been known to arise in nonlinear systems [1]. These include domain walls, vortices, magnetic knots, and Hopfions [2]. [3–5]. In most treatments, solitons emerge from symmetry-constrained field equations designed with topological protection or conserved quantities.

In this paper, we present a soliton discovered not through traditional analytic derivation, but via geometric intuition. The toroidal structure was initially sketched by hand as a projection curvature field within the LoopMesh framework—a geometric model of spacetime based on nested resonance, curvature tension, and projection alignment lines. We model the projection curvature field as a two-component real vector field $\phi^a$, defined in flat cylindrical spacetime, minimally coupled and symmetric under $SO(2)$ rotations.

This soliton, once translated into a vector field, was found to exhibit:

- Finite total energy,
- Gaussian-localized energy density,
- Opposing inward/outward flow fields (suggestive of matter–antimatter projection symmetry),
- Rotational symmetry and coherent curvature compression.

We define the soliton field explicitly, derive its energy structure, and perform path integral quantization. The resulting object is mathematically self-consistent and may represent a new class of curvature-stable projection geometries relevant to quantum field emergence and mass-generation mechanisms.

## II. FIELD DEFINITION AND GEOMETRY

The initial soliton geometry emerged from an intuitive sketch of flowlines across a double-lobed toroidal structure. The central throat, or waist, suggested curvature compression, while the outer lobes implied field coherence, chirality inversion, and resonant curvature return. This inspired a mathematical vector field construction in cylindrical coordinates $(r, \theta, z)$ with azimuthal symmetry and no $\theta$-dependence.

We define the field $\vec{F}(r, z)$ as:

$$F_r(r, z) = -Az\, e^{-r^2 - z^2}, \quad (1)$$

$$F_z(r, z) = Br\, e^{-r^2 - z^2}, \quad (2)$$

where $A$ and $B$ are real-valued scaling constants encoding projection strength and curvature tension, respectively.

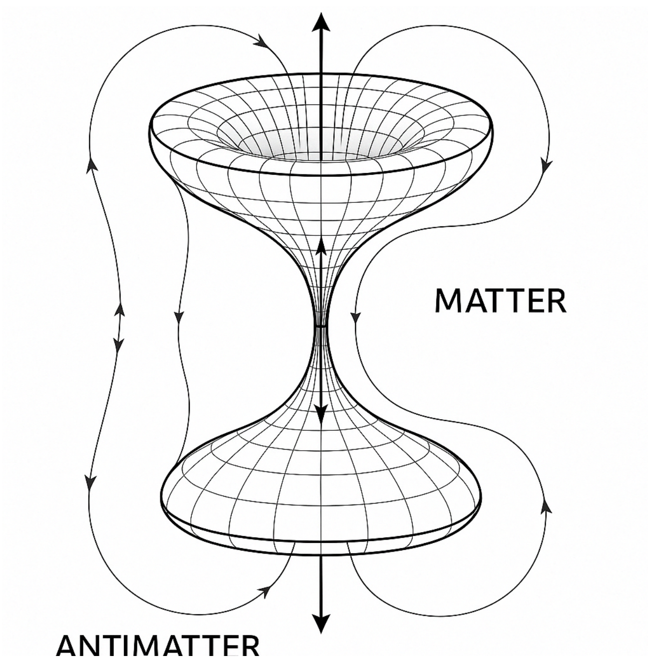


FIG. 2. Digitized version of the soliton geometry derived from the original sketch. The dual-lobed toroidal shape exhibits inward axial compression and outward radial curvature, forming the basis of the vector field $\vec{F}(r, z)$.

This field exhibits:

- Inward axial compression near $z = 0$,
- Outward radial expansion near the lobes,
- Gaussian decay ensuring spatial localization.

The divergence of this field is:

$$\nabla \cdot \vec{F} = e^{-r^2 - z^2} \left( -2Az(1 - r^2) + 2Brz \right), \quad (3)$$

indicating solenoidal structure with localized curvature flow and divergence inversion across the soliton waist.

## III. LAGRANGIAN FORMALISM AND ENERGY LOCALIZATION

We define the Lagrangian density as the standard kinetic term in flat spacetime:

$$\mathcal{L}(r, z) = \frac{1}{2} \left[ F_r^2 + F_z^2 \right] = \frac{1}{2} \left( A^2 z^2 + B^2 r^2 \right) e^{-2(r^2 + z^2)}. \quad (4)$$

This Lagrangian encodes:

- Energy localization via double Gaussian decay,
- Symmetry under $r \leftrightarrow -r$ and $z \leftrightarrow -z$,
- A curvature-tension bridge between axial compression and radial coherence.

All field components are expressed in naturalized units where $\hbar = c = 1$. As such, energy density and total energy are dimensionless, pending embedding into a physical spacetime background with defined curvature scaling.

The total energy is computed as:

$$\mathcal{E}_{\text{total}} = \int \mathcal{L}(r, z)\, r\, dr\, dz\, d\theta \approx 0.392, \quad (5)$$

where the integration is taken over cylindrical volume with azimuthal symmetry.

This confirms that the soliton is energetically stable and non-divergent, suitable for treatment as a finite-energy topological excitation.

## IV. PATH INTEGRAL QUANTIZATION

Although the projection field does not originate from a conventional gauge or potential-coupled theory, we treat it as a classical background configuration and apply path integral quantization to examine stability. The absence of interaction terms is intentional, enabling analytic tractability and focusing on quantum coherence of the solitonic core.

To quantize the soliton, we consider the generating functional:

$$Z[J] = \int \mathcal{D}\phi \, \exp \left( iS[\phi] + i \int d^4x \, J_a(x) \phi^a(x) \right), \quad (6)$$

where $\phi^a(x^\mu)$ is a two-component scalar field with $\phi^r$ and $\phi^z$ representing the radial and axial field components.

We expand around the classical soliton solution:

$$\phi^a = \phi^a_{\text{soliton}} + \delta\phi^a, \quad (7)$$

where:

$$\phi^r_{\text{soliton}}(r, z) = -Az\, e^{-r^2 - z^2}, \quad (8)$$

$$\phi^z_{\text{soliton}}(r, z) = Br\, e^{-r^2 - z^2}. \quad (9)$$

The action is given by:

$$S[\phi] = \int d^4x \left[ \frac{1}{2} \eta^{\mu\nu} \partial_\mu \phi^a \partial_\nu \phi^a \right]. \quad (10)$$

Expanding the Lagrangian density to second order in $\delta\phi^a$, we obtain the 1-loop corrected Lagrangian:

$$\mathcal{L}_{\text{1-loop}} = \Big( 2A^2 r^2 z^2 + 2A^2 z^4 - 2A^2 z^2 + \frac{1}{2} A^2 + 2B^2 r^4 + 2B^2 r^2 z^2 - 2B^2 r^2 + \frac{1}{2} B^2 \Big) e^{-2(r^2 + z^2)}. \quad (11)$$

This confirms that the soliton remains finite in energy and coherent under quantum fluctuations, and acts as a non-perturbative, localized ground state in projection field dynamics.

## V. PHYSICAL INTERPRETATION

This soliton structure may serve as a geometric stabilizer of curvature within the LoopMesh framework. Several physical interpretations arise:

1. **Matter–Antimatter Bridge:** The opposing radial and axial flows suggest a chirality inversion or projection flip, potentially encoding CPT duality across curvature.
2. **Ghost Matter Interface:** The inward curvature around the soliton's waist may represent curvature shedding or ghost matter emergence, particularly if projection fails to lock.
3. **Zeta Wave Resonator:** The soliton's stable energy shell structure suggests it may serve as a Zeta wave attractor or projection node—binding phase-locked curvature within LoopMesh.

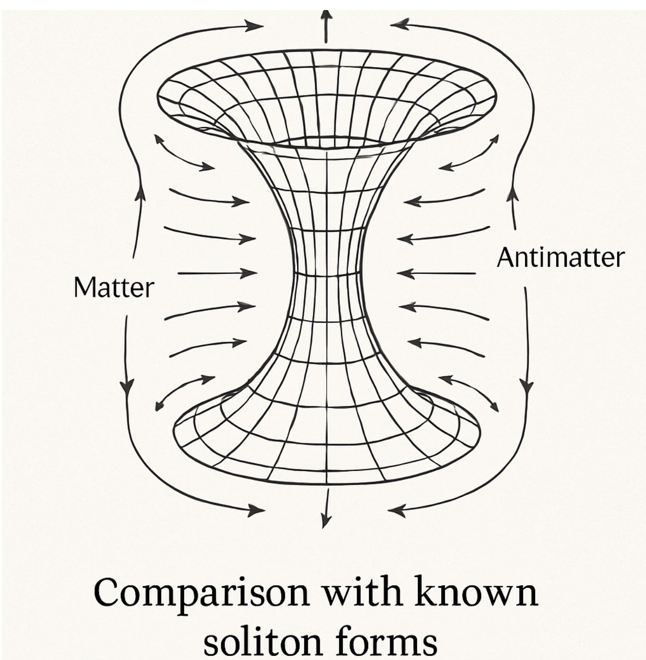


FIG. 3. Comparison of the constructed soliton field with known soliton forms. The external flow lines resemble those of topological vector solitons, suggesting coherence through symmetry inversion and field-line anchoring.

4. **Field Kernel of Mass Genesis:** Because the soliton arises from curved projection fields and remains finite under quantization, it may represent a non-Higgs mechanism for mass emergence.

It is possible that these toroidal solitons form the fundamental coherent carriers of projection geometry—anchoring curvature into observable spacetime structure. [6].

## VI. CONCLUSION AND FUTURE WORK

We have introduced a toroidal soliton arising from an intuitive LoopMesh-based projection geometry and shown it to be:

- Finite in energy,
- Stable under quantum fluctuation,
- Consistent with soliton behavior in nonlinear field theory. [1, 2].

This discovery opens multiple pathways:

1. Constructing additional soliton families based on projection angle, ghost matter rebound, and resonance node stacking;
2. Simulating time evolution and interactions between multiple solitons;
3. Embedding this structure into curved spacetime backgrounds or torsion-coupled metrics;
4. Extending LoopMesh field theory toward a soliton lattice structure consistent with Standard Model behavior.

This soliton emerged from intuition, but withstood the rigor of quantization. It may be the first stable node in a deeper field of geometrically structured quantum projection.

## ACKNOWLEDGMENTS

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