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Original Research Article

Integrating Mathematics, Art, and History: An Interdisciplinary Didactic Proposal on Fractals Through Hokusai's "The Great Wave off Kanagawa"

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ABSTRACT: We have developed a didactic design for high school students that focuses on Hokusai's artwork, The Great Wave off Kanagawa, for students' learning of notions related to fractals through an interdisciplinary approach. Our design is based on documental research, which included previous studies on mathematical analyses of fractals in paintings and Panofsky's method for art (especially painting) analysis. Experts in mathematics education, visual arts, mathematics, and history have validated our design and provided feedback that helped us to refine and improve it. We aim to enrich mathematics education, particularly by contributing to efforts to link mathematics and arts, by providing an educational framework that enhances interdisciplinary understanding and offers practical applications for educators and researchers. This interdisciplinary didactic proposal seeks to support learners' mathematical and critical thinking skills while integrating analyses that incorporate aspects of mathematics, art, and history to explore notions of fractals.

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The relationship between mathematics and art has been explored through different disciplines and approaches that reveal deep connections between the two fields. According to Hickman and Huckstep (2003), a correlation exists between mathematics education and art education, which justifies the integration of these subjects. On the one hand, the authors remark that "mathematics is an art, and, on the other, work in art that has a mathematical basis" (p. 1), which explains the correlation in terms of the mathematical profession. These authors highlight two main issues regarding instruction: (1) Methodological: should mathematics and art be taught in an integrated manner? and (2) teleological: what purpose underlies the inclusion of these disciplines in curriculum? Meanwhile, Jensen (2002) emphasizes the common objectives of both disciplines in attempting "to comprehend reality" (p. 45), reflecting on the structure of reality, which can be abstract or concrete, in mathematical or artistic works. Within this context, we have identified a convergence between art and mathematics in relation to the concept of fractal geometry. Mandelbrot (1982) suggested that "Clouds are not spheres, mountains are not cones, coastlines are not circles, and bark is not smooth, nor does lightning travel in a straight line" (p. 1), which emphasized the distinction between fractal and Euclidean geometries. Even before the development of fractal geometry, artists such as Mondrian aimed to represent nature by focusing on its smaller parts to gain a better understanding of the whole world. This approach has led to the mathematical study of dimensions in some of his works (Bountis et al., 2017). Furthermore, given the recent interest in an interdisciplinary approach to mathematics education, there is a need to design and develop projects that involve multiple areas of knowledge to promote student learning.

In the field of mathematics education, a literature review on the teaching and learning of fractal geometry in high school by Artigue et al. (2021) identifies that research focusing on approaches for teaching this geometry usually employs manipulative material, software or structured exercise sequences. Additionally, the review highlights that the main motivations for teaching fractals in these studies are either their esthetic appeal or their applications in other fields. However, none of the studies incorporate interdisciplinary activities. This underscores the need for projects that integrate multiple disciplines, enabling students to construct interrelated knowledge.

In countries such as Argentina, where the high school curriculum includes fractals in the last year, instruction focuses on developing an introductory understanding of the topic, and it serves as a review of key concepts covered throughout the student's secondary education, such as limits, sequences, and transformations, as well as providing a broader context for the geometry students have learned up to that point (Fusi, 2020).

Regarding the challenges in teaching and learning fractal geometry in high school, countries that have incorporated this topic into their curriculum report several difficulties. These include challenges in understanding the mathematical nature of fractals and their properties, such as self-similarity and fractal dimension, as well as operational difficulties with calculations involving rational numbers or numerical series (Karakuş, 2013 cited in Fusi, 2020). Additionally, there is a lack of teacher training in fractal geometry and a scarcity of resources for teaching fractal geometry in schools (Corica et al., 2024).

Garbin (2007) conducted a study with 77 university students who had taken differential and integral calculus courses. She investigates how students perceive fractal properties and identifies potential difficulties in their understanding. One result indicates that students encounter both mathematical and cognitive difficulties in constructing the concept of a fractal. Specifically, in the iterative visual construction of a fractal, students may represent initial steps (step 1, step 2, etc.), but they lack a representation of a final step, as no last step exists due to the infinite nature of fractals. This poses a conceptual challenge for students, as the fractal concept cannot be symbolized mathematically, and can only be conveyed verbally (Garbin, 2007).

Another difficulty identified among these university students (Garbin, 2007), and among high school students aged 15-17 (Garbin & Mireles, 2005) is the tendency to visualize fractal properties only partially, rather than comprehensively. As Garbin explains, "the properties that define the fractal set are not perceived simultaneously by the students. The only two properties that are visualized and considered simultaneously to give a definition of fractal are recurrence and self-similarity" (Garbin, 2007, p. 106). Furthermore, students find the concept of fractal dimension to be counterintuitive.

Considering the above, this study aims to contribute to the integration of visual arts and mathematics in interdisciplinary teaching at the high school level. The interdisciplinary approach challenges the usual idea that one discipline should be at the center and the others should only serve as motivations or aesthetic supports. Instead, it seeks to horizontally integrate the knowledge of all involved disciplines in the didactic proposal. The didactic design proposed in this work has been carefully constructed, considering mathematical, artistic, and historical elements. It particularly incorporates mathematical concepts of fractals, artistic analysis of the painting "The Great Wave off Kanagawa", and historical aspects of the Edo period, during which the painting was created.

Fractals and Their Properties

The Benoit Mandelbrot, a mathematician, and IBM researcher, coined the term "fractals", and developed the field of fractal geometry to describe complex shapes and structures found in nature. In his work *The Fractal Geometry of Nature*, Mandelbrot reflected about how mathematicians had disdained the study of nature's shapes. His motivation to develop first the notion and afterwards the theory of fractals was stated in terms of his observations of irregular and fragmented forms in the natural world, which were not adequately described by traditional Euclidean geometry:

> I conceived and developed a new geometry of nature and implemented its use in several diverse fields. It describes many of the irregular and fragmented patterns around us, and leads to full-fledged theories, by identifying a family of shapes I call fractals. (1982, p. 15).

Mandelbrot emphasized that many natural phenomena, such as coastlines, clouds, trees, and mountain ranges, exhibited self-similar patterns at different scales. For example, a coastline might appear jagged and irregular, yet when examined closely, smaller segments of the coastline resemble the larger whole. The properties of fractals identified by Mandelbrot are the following.

Self-similarity

It indicates that parts of an object resemble the whole, and this resemblance holds across varying scales, whether the object is magnified or reduced. Researchers such as Karakuş (2015) and Bembir (2019) have emphasized the relevance of iterative and recursive processes in understanding mathematically this property. Their work demonstrates that consistent patterns are evident across specified scales, revealing that phenomena observed at one scale can be replicated at another with slight modifications.

Peitgen et al. (1992) described iteration in fractal geometry as the repetitive application of a process where the result of one cycle feeds into the next. This recursive process is essential in producing the intricate patterns typical of fractals, observable at multiple scales. We inferred from their works that the self-similarity concept is related to iteration and scale notions, which are crucial in both natural and artificial structures and have significantly influenced the design of this research proposal.

Infinite Detail

It means that as we zoom in on any part of the fractal, we find more complex and infinite details, which often go down to arbitrarily small scales. In other words, fractals infinitely repeat patterns as they decrease in scale but are confined within specific ranges in practical terms. Eglash (1999) suggests that real-world objects exhibit fractal properties within certain limits, maintaining their fractal nature across defined scales.

Fractional Dimension

A defining property of a fractal is its non-integer dimension, which sets it apart from traditional geometric shapes such as lines (dimension-1), squares (planes, dimension-2), and cubes (dimension-3). Kern & Mauk (1990) provide an example, explaining that "...a one dimensional line can, in some fashion, be bent so many times that the line begins to fill space [cover an area]. Thus, the wiggly line has dimension greater than one." (p. 179)

The didactic proposal's design explicitly considered properties such as self-similarity and related mathematical notions like iteration and scale. These concepts were chosen for their potential to be effectively taught to students during artwork analysis. According to Karakuş (2015), "students could distinguish fractals from other patterns according to their characteristics, such as self-similarity and iteration" (p. 828). Additionally, the implicit connection between fractal dimension and infinity with notions of irregular shapes and the infinite was acknowledged.

Panofsky's Analysis of Artworks

It's important to note that the study of fractals is not limited to mathematics but extends to other disciplines like art and philosophy due to their presence in natural forms and art works. We focused on researching some theoretical foundations that would allow us to establish the elements corresponding to the development of an analysis of visual art works from the artistic discipline and to generate links between art, history, and mathematics. In this regard, we considered the work of Erwin Panofsky (1967), who suggested that contemplating an artwork requires considering it as a historical product that makes sense within the context of the prevailing aesthetic theories of its time, incorporating a comparative analysis that identifies regularities and singularities in each painting or artwork.

Panofsky developed an iconographic method that comprises three levels:

- Pre-iconographic level: This involves identifying the most basic visual elements of the work, including form, color, and composition, to understand their interrelationships and contribution to visual meaning.
- Iconographic level: This level involves identifying recurring symbols and themes in the work to understand their meanings within the historical and cultural context of the artwork's creation.
- Iconological level: This level involves understanding the deeper cultural and philosophical meanings of the work, analyzing how symbols and themes identified at the iconographic level relate to the cultural beliefs and values of the time (Cubero, 2010).

Interdisciplinary

Considering recent interest in mathematics education from an interdisciplinary perspective, it is crucial to develop projects that involve various areas of knowledge, thereby facilitating comprehensive student learning. Interdisciplinarity, as defined by Sriraman and Freiman (2011), is "the combination of multiple academic disciplines into a single activity, transcending traditional disciplinary boundaries" (cited in Ruíz-Rojas et al., 2020, p. 121). This request for a connection between knowledge from different disciplines is fundamental to an interdisciplinary approach. According to Roth (2020), "Interdisciplinarity denotes the fact, quality, or condition that pertains to two or more academic fields or branches of learning. Interdisciplinary projects tend to cross the traditional boundaries between academic disciplines" (p. 415). This quote highlights the integration between two or more academic fields or areas of knowledge and emphasizes an important aspect: it overcomes the conventional borders or limits that separate academic disciplines, thereby allowing a broader perspective in learning.

Borromeo Ferri (2019) indicates that an interdisciplinary approach is significant for students for three reasons. The first has to do with changing the perspective of mathematics; it is necessary for students to recognize the relationship between mathematics and other areas to change the perception that mathematics is just a series of repetitive arithmetic operations. The second reason points out the need to understand mathematical content through different disciplines. Lastly, this author discusses the networks of areas of knowledge, explaining that "students acquire skills and knowledge that can be networked through interdisciplinary learning" (p. 28). To achieve this, the boundaries between school subjects must be transcended so that students can recognize how the concepts and skills from one subject coexist in other areas of knowledge, which allows them to develop a deeper and more comprehensive understanding of the world around them.

Our educational proposal seeks to apply an interdisciplinary approach by integrating mathematics, art, and history. This approach allows students to recognize knowledge from disciplines that are normally taught separately, now integrated into a single proposal. In this way, students could explore and explain artistic ideas using mathematical arguments, enriched by the historical context. This approach not only broadens the understanding of each of the subjects involved but also promotes a shift in perspective on how mathematics is learned, by contextualizing and connecting it with other disciplines.

Method

To develop the didactic proposal presented in this article, we designed and followed a methodology consisting of five phases described below:

Phase 1: Documental Research on the Analysis of Art and Painting

After defining the initial problem and the research objective, we focused on investigating theoretical and methodological foundations that could help us establish the essential elements required for analyzing paintings from an artistic perspective and articulate connections between art, history, and mathematics.

Phase 2: Exploring Criteria for the Analysis of Paintings

After conducting the initial research, we analyzed multiple selected elements developed at each of the previous levels, based on Panofsky's work. This analysis aimed to identify aspects that could be incorporated into our didactic proposal by investigating a painting and its connection among the disciplines of art, history, and mathematics. We identified key aspects that we could further develop in our proposal, which include:

- Pre-iconographic level: This level deals with aspects related to meanings that students might assign to the artwork based on visible objects, such as shapes and figures, colors, and technical information like the name of the work, the artist's name, year of creation, style, and artistic movement.

- Iconographic level: At this level, we considered the meanings of the types of symbols recognized at the pre-iconographic level, using available literature, and determining their relationship to mathematical notions.
- Iconological level: Using Panofsky's framework, we sought to establish criteria, through available online information, to examine the cultural and historical meanings related to the symbols identified in an artwork, as well as the possible meanings the work had for the artist given their historical, social, and cultural context.

Phase 3: Selection and Analysis of a Painting of Interest

Considering our objective to familiarize students with fractal theory notions, we initially considered as potential artworks to be used in our design, paintings including Salvador Dalí's "The Face of War", Gustav Klimt's "Tree of Life", Pollock's works, and Hokusai's "The Great Wave off Kanagawa". However, we still needed to determine which artwork would best address the issue discussed above and achieve our objectives.

In our documental review of Mandelbrot's "The Fractal Geometry of Nature" (1982), particularly the section titled "Three Great Artists of the Past Illustrate Nature, and Thereby Bring the Reader to the Threshold of Fractals" (p. C2), we found direct references to visual artworks that exemplify fractal geometry. Among these, Hokusai's woodblock print "The Great Wave off Kanagawa" stands out as an example of the principles discussed in the text. Mandelbrot describes it as an artwork with "certain geometric shapes whose form is very irregular and very fragmented, and coined the term fractal to denote them" (Mandelbrot, 1982, p. C16), possibly because the book emphasizes how fractal geometry integrates mathematics and science to model a wide array of natural forms. Given this alignment with the principles and notions of fractal geometry, "The Great Wave off Kanagawa" emerged as the most suitable artwork to illustrate our research objectives.

Phase 4: Didactic Proposal Design

We proceeded with a comprehensive analysis of Hokusai's work, focusing on the key aspects outlined before. This step is intended to validate the significance of the elements we had initially considered. We focused on fractal notions employing Panofsky's method, where each member of our five-person research team examined and reflected on the relevant elements of each analytical level, and the fractal notions that could be recognize from the artwork. In designing the tasks, we aimed to foster inquiry and critical analysis, consistent with Sierpinska's (2004) emphasis on task problematization, which highlights the importance of exploring alternative formulations and reflecting on their impact on teaching, learning, and research outcomes. In addition, the analysis considered the possible answers that students would give to the tasks set out in the didactic proposal. The findings were collaboratively reviewed and synthesized to incorporate the collective insights of our team.

On the other hand, our didactic proposal is justified by integrated project-based learning in visual arts and mathematics (Portaankorva-Koivisto & Havinga, 2019). This approach of project-based learning suggests the integration of mathematics and visual arts in learning to understand phenomena and their ontological, contextual, and linguistic implications. By solving tasks through this approach, students can expand their knowledge of mathematical language, society, and culture, as well as their ability to interpret emotions, feelings, and communication processes.

Under this perspective, inquiry learning is an essential characteristic. In mathematics, it is fostered when "pupils have opportunities to work with non-routine mathematical problems or unfamiliar situations, individually or collaboratively, and thereby develop their own meaningful solutions" (Portaankorva-Koivisto & Havinga, 2019, p. 4). In art, because the knowledge implicit in a work is tacit, investigative learning arises when seeking to understand or interpret it, which requires distinguishing cultural relationships and symbols.

Our didactic proposal considers investigative learning, since the development of the mathematics involved is done through the analysis of the artistic work. In order to understand the work, the tasks contained in the proposal are intended for the student to investigate not only the work itself, but also the cultural and social aspects, with an integrative approach of the disciplines involved.

Phase 5: Validation of the Proposal

After designing the proposal, it was validated by four experts. These experts were an historian who specialized in Japanese culture, a mathematician who specialized in geometry, a Japanese art scholar, and an expert in mathematics education. Initially, they responded to a structured questionnaire with short-answer sections that aimed to gather preliminary feedback. Following this, modifications were suggested based on their insights. Subsequently, individual semi-structured interviews were conducted with each expert to delve deeper into the notions related to each discipline and consider additional aspects, such as the estimated time required to implement this proposal with high school students.

Design of the "Ukiyo-E Museum Curators" Proposal

The interdisciplinary "Ukiyo-E Museum Curators" proposal for high school students aims to explore the intersection of mathematics, art, and history through the masterpiece "The Great Wave off Kanagawa", highlighting its cultural significance as well as its potential to illustrate complex mathematical concepts such as fractals. This project emphasizes the relevance of fractals in understanding complex patterns and geometrical notions; and one of its objectives is to shed light on how mathematics can model real-world phenomena.

By involving students in curatorial roles, the project encourages them to actively engage and think critically while learning, fostering skills such as critical thinking and interdisciplinary analysis that are essential in contemporary mathematical education. This methodology advocates for curricular integration and project-based learning, which serves as a valuable tool in promoting advanced analytical skills and a deeper appreciation of the interaction between mathematics, art, and history emphasizing the interdisciplinary nature of the proposal.

Table 1 presents the aspects and criteria that were considered for the design of our didactic proposal, which aimed to incorporate and complement mathematics, art, and history concepts to promote interdisciplinary learning. The questions presented to students were designed to blend critical concepts from these disciplines, allowing for a more profound understanding of the topics through specific activities.

Section/Activity	Mathematical Concepts	Art Concepts	History Concepts (Social Sciences)
I. Visual and Technical Exploration of Painting.	Geometric shapes, dominant colors, measurements of objects, size, and dimensions.	Pre-iconographic stage. Visual analysis, composition techniques, artwork size and dimensions.	History and cultural origin of the artist. Artwork situated in history, culture, and artistic movement.

Section/Activity	Mathematical Concepts	Art Concepts	History Concepts (Social Sciences)
II. Meanings of Elements/Figures/Objects in the Painting. Activities 1 and 2.	Measurements of objects, regular and irregular shapes, proportions, and scales.	Iconographic stage. Artistic style and technique, symbolism in artistic elements. Meanings of objects and artistic composition.	Social and cultural influence on artistic style and objects in the artwork.
II. Activity 3	Self-similarity, geometric transformations (translation, rotation, homothety), ratios and proportions.	Pre-iconographic stage. Repetition of shapes, artwork structure. Artistic composition.	Meanings of objects from social, cultural, and anthropological sense.
II. Activity 4	Measurement of lengths, concept of fractal dimension, infinite, effects of the measurement scale.	Pre-iconographic stage. Measurement and composition.	—
III. Cultural and Historical Interpretation of the Observed Elements. III. Activity 1	—	Iconological stage. Biographical study of the artist, influence of personal events on art.	Impact of vital events on artistic creation.
III. Activity 2	Geometric and mathematical elements in art.	Iconological stage. Influence of technique and color on artistic perception.	Historical and cultural context of the creation of the work, relationship with other countries.
III. Activity 3	—	Iconological stage. Symbolic importance of Mount Fuji in Japanese art.	The cultural and spiritual significance of Mount Fuji and its associated legends.
III. Activity 4	Correlation of art and mathematics, and concepts of self-similarity, repetition, and proportionality. Mandelbrot's fractal definition.	Iconological stage. Artistic expression and technical style, analysis of the thematic and emotional depth of the artworks.	Cultural and historical perceptions of art, influence of individual artists' circumstances and their time.

Table 1. *Integrative Framework for Ukiyo-E Museum Curators Proposal*

Now we present a description of the sections that comprise the didactic proposal.

For example, by combining fractal notions with artistic analysis and historical context, the interpretation of "The Great Wave off Kanagawa" can be taken to a different level. This methodology proposes teaching individual and combined aspects of each discipline and enhancing them by exploring their interrelatedness. Panofsky's method for artwork analysis was used for artistic analysis. Concepts such as fractality as well as social science contexts were also considered in the design. This approach promotes a holistic and cohesive strategy that links the disciplines, fostering the enrichment of interdisciplinary learning.

Section I: Observing the Artwork

Questions in this section were designed to initiate the students in the visual and technical analysis of the artwork, promoting detailed observation that is essential in any artistic analysis, as in the pre-iconographic stage of Panofsky's proposal (1967). By delving into specifics such as the artist's cultural background, visible elements, shapes, colors, exhibition location, and the significance of certain elements, we aimed to foster an understanding of the artwork and its contextual relevance among students while also stimulating critical and analytical thinking. This section is presented in two parts: first, as a series of questions designed to prompt students to draw inferences based on their observations of the painting without prior knowledge; and second, as an art sheet to be completed by students during their initial investigation of the artwork.

Section II: Meanings of Elements/Figures/Objects in the Painting

The activities in this section aim at analyzing elements, figures and objects present in Hokusai's "The Great Wave off Kanagawa." The focus is on exploring their cultural, historical, and mathematical significance, emphasizing fractal concepts such as self-similarity, repetition, fractal dimension, and the infinite nature of fractals. The first activity involves researching and answering questions related to a collection of 36 paintings by the artist. Topics include the relationship with the ukiyo-e style, recurrent objects in the artworks, as well as the symbolic and geometric representation of elements like the wave and Mount Fuji. Students are asked to identify specific cultural symbols and geometric shapes and consider how these contribute to the interpretation of the painting. This exploration extends to understanding the shapes of water and clouds and their aesthetic and symbolic implications, as well as the direction and purpose represented in the boats and their occupants within the painting, reflecting on the repetition and self-similarity in these elements.

The second and third activities introduce the application of mathematical concepts to measure certain elements and analyze the painting. Students measure the dimensions of a print of the work, comparing it to the original dimensions displayed in the museum. This helps them understand the scale and proportions of the depicted elements. Additionally, they identify and analyze the geometric transformations of a figure that repeats in different sizes and positions within the image, highlighting the application of translation, rotation, and scaling. This exercise underscores the self-similarity and repetition inherent in fractal geometry, where similar patterns recur at progressively smaller scales, demonstrating infinite complexity. This activity not only reinforces the understanding of fractal geometry but also invites students to reflect on how differences in the measurement scale can profoundly affect the perception and measurement of dimensions, a crucial concept for understanding fractal dimension in art and nature. Through these activities, students engage with fractals' infinite and self-similar nature, gaining a deeper appreciation of how these mathematical principles manifest in this classical artwork.

Here is a description of examples of fractal notions that appear in section II:

Activity 1: "Objects of the Artwork" aims to provide a comprehensive understanding of Hokusai's artwork by delving into its thematic and symbolic elements within a collection of paintings by the artist. This activity centers on recognizing the relationship of these works to the ukiyo-e style. Participants are asked to analyze recurring objects across the series, such as Mount Fuji and the giant wave together with its crests, which are instrumental in grasping the thematic continuity and symbolic depth of Hokusai's art. By analyzing these elements, students could evaluate their significance and explore how they contribute to the narrative and aesthetic coherence of the series, gaining a deeper appreciation for the cultural and spiritual contexts depicted in these scenes.

Throughout this activity, participants also delve into the geometric and symbolic representation of key objects within "The Great Wave off Kanagawa". This involves a detailed examination of the wave's dynamic form and its interaction with other elements such as Mount Fuji, the boats, and their occupants (see Figure 1), which illustrate themes of nature's overwhelming power contrasted with human resilience. These insights establish a foundational context for subsequent activities, where the focus shifts to a mathematical analysis, specifically examining the fractal geometry evident in the repetitive and self-similar patterns of the wave.

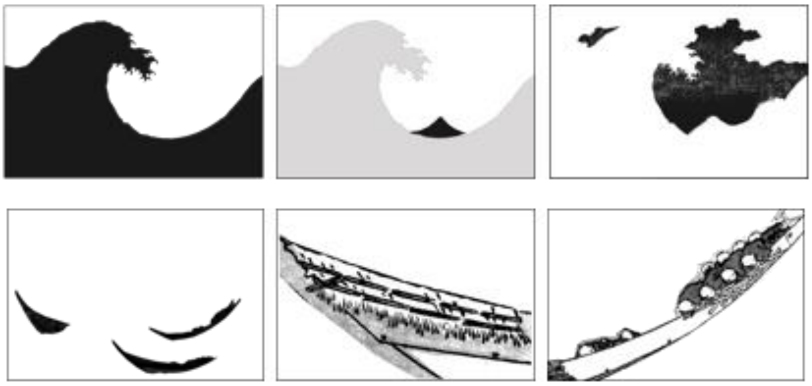

***Figure 1**. Figures in questions from Activity 1 Section II in the didactic proposal to analyze wave and its crests, Mount Fuji, clouds, and boats.*

Activity 2: Shapes and Measurements in the Work sets the foundational stage for engaging with fractal concepts. Students are tasked with identifying repeating figures that maintain the same shape across varying scales within the painting. This examination not only highlights the self-similarity inherent in the artwork but also introduces students to the idea of scale as they measure and analyze these repeating elements.

Questions included in the Activity 2 Section II ask students to identify a figure that repeats in the painting, having the same shape though not the same size, and to mark these figures. This activity introduces the concept of self-similarity, a key component of fractals, where a basic shape repeats at different scales throughout an object or design. By exploring how these shapes repeat in different sizes, students directly address the ideas of scale and transformation that are fundamental in fractal geometry. On the other hand, by comparing the dimensions of a print to the original artwork displayed in the museum, students can gain a practical understanding of how scale affects perception and depiction in art.

Activity 3: Repeated Shapes and Transformations build upon this foundation by focusing on the geometric transformations of the identified repeating figures. Students observe and document how these figures undergo translation, rotation, and scaling across different parts of the painting, exploring the mathematical operations that describe these transformations. This hands-on analysis helps students appreciate the pervasive nature of repetitive patterns and their fractal properties, enhancing their understanding of how consistent patterns can manifest differently depending on their scale and orientation.

We expect the students to answer activity 3.3 section II, by using rectangles to enclose and compare repeated figures on the artwork, where they engage with the concept of ratio, crucial for studying fractal dimensions (as seen in Figure 2). This methodical comparison fosters a deeper comprehension of the fractal description of objects, as students explore how the proportions between figures vary and what these variations reveal about the fractal nature of the artwork.

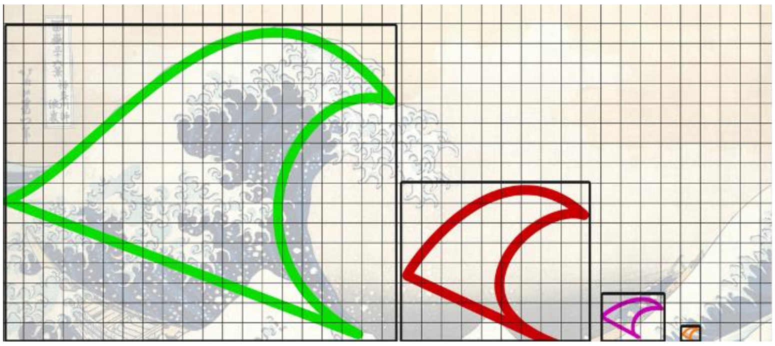

Figure 2. *Expected answer for question II 3.3 using self-similarity, iteration, and scale.*

Activity 4: Length of the Wave's Edge further expand on these concepts by introducing students to the notion of the infinite. In these activities, students measure the edge of the wave using segments of decreasing lengths. As the measurement tool becomes finer, the total measured length increases, illustrating the fractal dimension concept where the detail and complexity of an object can seem endless as one zooms in further. This activity not only demonstrates the counterintuitive aspects of fractal measurements but also provides a visual and practical understanding of how fractals depict an infinite complexity (Figure 3), bridging the gap between abstract mathematical theories and tangible artistic representations.

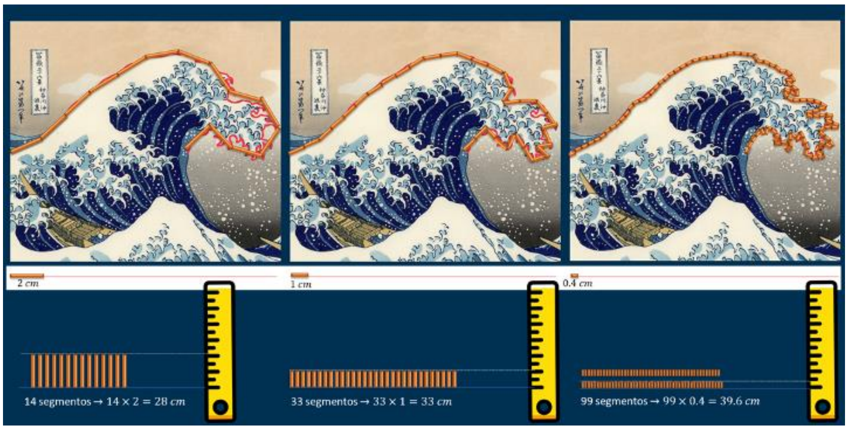

Figure 3. *Expected answer for question II 4.4 using notions of infinite and fractal dimension*

Section III: Cultural and Historical Interpretation of the Observed Elements

Questions from activity III 1, *Hokusai, Life and Work*, aim at helping students understand how Hokusai's personal and professional context influenced his art. By studying Hokusai's biography and the circumstances surrounding the creation of his work, students can relate historical and cultural aspects to artistic interpretation.

By answering questions from activity III 2, *Historical and Cultural Circumstances*, students investigate the environment in which Hokusai worked, which in turn can help them better understand how external factors can influence artistic creation. This analysis assists students in situating the work within a broader context of cultural and mathematical interactions, promoting a richer understanding of how art and science mutually influence each other.

Section IV: Report of the Analysis

The final task of compiling and presenting a final report synthesizes the students' learning and allows them to articulate their understanding of the work from multiple disciplinary perspectives. This activity culminates the learning process, enabling students to demonstrate their ability to integrate and apply interdisciplinary knowledge.

Expert Validation of the Didactic Proposal

Expert validation through assessment by individuals with expertise in a particular topic is commonly defined as "an informed opinion from individuals with a background in the topic, who are recognized by others as qualified experts, and who can provide information, evidence, judgments, and assessments" (Escobar-Pérez and Cuervo-Martínez, 2008, p. 29). The primary function of expert judgment is "to eliminate irrelevant aspects, incorporate those that are essential, and/or modify those that require it" (Robles & Rojas, 2015, p. 124).

The purpose of the current validation was to explore the internal coherence of the proposal and identify issues to address. The experts consulted in this process provided several perspectives that demonstrated the potential for interdisciplinary integration of a didactic proposal of this nature, as well as its feasibility and inherent challenges. We believe that these insights will be valuable for the continued development of the proposal.

Integration of Knowledge from Focus Disciplines

The didactic proposal presented to the experts emphasizes the interdisciplinary integration of mathematics with arts, history, and culture through pictorial analysis. The participation of these academics revealed that this sequence not only incorporates important cultural and aesthetic features touching upon various fields of knowledge, but also serves as a platform for exploring how fractal notions intersect with different disciplines in teaching.

The consulted mathematician with expert knowledge in fractals, identified in the proposal an opportunity to explore mathematical concepts in a real-world context, highlighting how this approach can lead to a better understanding of the structure and composition of natural forms. He pointed out the relevance of using concrete examples like the wave edge measurement to illustrate abstract concepts such as infinity and fractal dimensions, as specified in his comment during the interview: "...this teaches about the concept of increment and the notion of infinity in fractal terms". His approach underscores the educational importance of connecting theory and practice.

On the other hand, the consulted expert artist emphasized the significance of selecting "The Great Wave off Kanagawa" due to its rich visual composition and implicit notions of proportions and geometry. He believes that this intersection between mathematics and art enables a deep understanding of artistic techniques and reveals how mathematical concepts can be comprehended through "geometric shapes" and "visual structures". The artist stated that the artwork displays "curved, triangular shapes, but within that image, many are inverted, enlarged, or distorted... it is a repetition of many shapes, of the same shape", which confirms his recognition of the proposal's fractal notions of self-similarity, scale, and transformation.

From the viewpoint of the expert in history and sociology, the proposal holds potential to delve into the cultural dialogue that Hokusai's artwork embodies. The artwork's selection, which shows the exchange between the Eastern and Western countries, can spark discussions on how mathematical and aesthetic concepts have been influenced and evolved by cultural factors. The historian highlights the importance of "recognizing the mutual influence between Asian and European cultures in the development of art and mathematics". This could offer a historical dimension to the study of mathematics and art, contextualizing learning and enriching it in the process.

The interdisciplinary proposal has been recognized for its richness in Mathematics Education by the expert in the field, who however raised concerns regarding the potential risks of information overload. She recommended a more analytical and less accumulative approach, which involves a careful consideration of the balance of content from the disciplines involved. The mathematics educator emphasized a significant aspect of the proposal: exploring fractal notions such as self-similarity, iteration, scale, infinite and irregular shapes. This approach contrasts with traditional mathematics education, which usually emphasizes well-defined measurements and figures. The expert highlighted this distinction during the interview and discussed how this innovative treatment of fractals could reshape conventional teaching methodologies, as expressed:

> Usually in mathematics we look for something with specific measurements, with a 'perfect' figure and well-defined parameters, but in this case analyzing a work where the limits are not required to model the figure... the contour should no longer be an exact curve, but it is transformed to a very small degree of approximation. It may not even be modellable by a simple function... there is no simple function to model that contour. However, this does not negate the existence of a pattern and a figure from which an analysis can be conducted, abstracting an abstract notion from the concrete, and I think that could be worked on.

The consulted experts not only identified concepts related to their specific disciplines, but also recognized notions related to other fields. For example, the mathematician noted that the design includes sociological questions about "what was happening in Japan at that time". Likewise, the historian, who also has studies in Philosophy, mentioned that the proposal design addresses issues related to the Philosophy of Art Aesthetics as well, which makes part of programs in Fine Arts or Philosophy.

Feasibility of the Proposal

According to experts, the didactic proposal "Ukiyo-E Museum Curators" is considered feasible for implementation in a classroom setting with an interdisciplinary approach. Regarding the educational level, experts noted that the proposal could be implemented within the last year of high school (ages 17 to 18) or the first year of university.

One of the key aspects of the educational proposal was its innovative approach to integrating content from seemingly unrelated disciplines such as mathematics, art, and history. This approach, in contrast to traditional methods that often fragment disciplines, was identified by experts as a coherent and meaningful way to provide an enriching educational experience for students. By encouraging students to explore connections between different areas of knowledge, this proposal offers an opportunity for them to develop a deeper and more holistic understanding of the subject.

One of the aims of designing the proposal and integrating content from different areas was to promote interdisciplinary work among teachers. Experts pointed out that implementing a proposal of this type required the participation of several teachers. The mathematician commented:

> We tend to divide things, history over there, art over there, sciences over there, and the beauty of the proposal is that there is a union. So, you are working on a unified and very interesting concept with many things of great value for the student's learning. I would call the teachers and tell them: we have this project... we need universal teachers. They have specialized too much, and then the math teacher does not know history, and the history teacher does not know art...

The expert in mathematics education pointed out important considerations regarding the proposal's implementation. She said that it was important to "avoid inquiring", which seemed to contradict the nature of this proposal, since it is based on inquiry. However, when she was asked to expand on this idea during the interview, she explained that an activity like this should not appear as a "long list of questions" that students look up on the internet and answer. Instead, reflection while searching for answers should be encouraged, which involves rethinking some questions and combining several of them into one that stimulates more thoughtful responses in line with the intention of the proposal.

The mathematics education expert proposed two suggestions: a) to include a video that explains how this print is made step by step to gain insight into highlighted figures that it contains, and b) to encapsulate several historical and cultural questions into one and present this as a request to analyse the artwork. The expert also recommended that the activity related to the perimeter of the wave could be developed using pencil and paper and complemented with digital technology such as the "zoom" tool that allows visualizing the repetitions of the forms. This recommendation could be implemented to enhance the proposal.

Appraisal for the Proposal

The evaluated educational proposal received positive feedback for its innovative and interdisciplinary approach. The proposal fosters a comprehensive understanding of mathematics, particularly the concepts of fractals, and effectively integrates art, history, and culture. The experts' varied perspectives highlight the importance of designing an educational proposal combining depth and breadth in its content by considering the disciplines involved. Such an approach would ensure that students not only acquire fragmented knowledge from different fields but also combine it in an intersectional manner and apply it in real-life contexts, which would help them develop their critical and analytical thinking.

Discussion

The didactic proposal outlined here underscores the value of integrating mathematics, art, and social studies to enhance students' understanding of advanced mathematical concepts, such as fractals. By focusing on "The Great Wave off Kanagawa" as a subject of mathematical, artistic, and social analysis, this framework aims to make the principles of self-similarity and infinite complexity in fractals—articulated by Mandelbrot (1982) and Peitgen et al. (1992)—more accessible and meaningful to students. This approach diverges from traditional teaching methods, which often separate mathematical concepts from their cultural and aesthetic contexts, as noted by Miàs (2017) and Borromeo Ferri (2019).

The proposal also emphasizes the potential for students to create interconnected knowledge, fostering critical and flexible thinking—an essential component of contemporary education, as suggested by Portaankorva-Koivisto and Havinga (2019). This aligns with research advocating for the relevance of mathematics in non-traditional contexts, promoting a broader understanding of geometry beyond the confines of isolated abstract concepts. This supports the growing need for projects that bridge multiple disciplines, as highlighted in prior studies, contributing to a richer understanding of the world (Sriraman & Freiman, 2011; Roth, 2020).

Expert validation recognizes this proposal as an invaluable educational tool. Its design aims to cohesively integrate mathematics, art, and social sciences, enabling students to engage with content across various disciplines without relegating any single area to a supplementary role. By encouraging students to identify mathematical concepts within an artistic context enriched by historical and social factors, the proposal fosters meaningful multidisciplinary development and holistic learning.

For effective implementation, it is recommended to establish a timeframe of three months. This duration will allow for a comprehensive exploration of each theme while simultaneously addressing cognitive load considerations. Institutional support for interdisciplinary collaboration among educators is essential. Facilitating necessary communication channels for joint reviews, adaptations, and feedback will be crucial in ensuring responsiveness to the diverse educational needs of students. This endeavor requires a commitment from all stakeholders involved.

To enhance student comprehension, the incorporation of multimedia resources is advisable. These might include visual aids that highlight patterns in Hokusai's works, as well as step-by-step videos demonstrating the creation of The Great Wave off Kanagawa. Such resources would facilitate an intuitive understanding of fractal patterns and relevant mathematical concepts. By concentrating on key thematic areas during each session, we can mitigate the risk of information overload and render interdisciplinary content more accessible and engaging for students.

Given the interdisciplinary nature of this initiative, professional development sessions for educators are of paramount importance. The training focused on the integration of mathematics, art, and history will better prepare educators to guide students through complex, interconnected subject matter. Additionally, implementing a feedback mechanism that allows students to share their insights will support continuous adjustments to the curriculum, thereby ensuring it meets educational standards and effectively addresses student needs.

We acknowledge that this proposal does not provide a depth examination of the concepts of fractal dimension and infinity complexity. This limitation is primarily attributed to the challenges identified in the introduction, particularly the difficulties students encounter in understanding these abstract concepts within a constrained timeframe, given their existing mathematical knowledge (Garbin, 2007; Karakuş, 2013). In this sense, our proposal is in line with Fusi (2020), who points out that, given the complexity that fractal geometry can present, a "notion of fractal" may be the most relevant thing to teach in high school, thus preventing students from seeing the concept as unattainable. To this end, it is suggested that the proposed activities should be based as much as possible on a constant treatment of the concrete. While this does not diminish the proposal's effectiveness in conveying fractal concepts, including the foundational aspects of these two concepts, it does highlight an opportunity for future research to delve deeper into these advanced topics. We recommend that subsequent proposals and projects of this nature focus more intently on fractal dimension and infinity complexity, developing targeted activities and methodologies designed to enhance understanding of these complex concepts for high school students.

Furthermore, fostering collaboration with educational researchers, teachers, and institutions may yield valuable insights and promote the effective dissemination of strategies developed through this project. These initiatives would not only enhance students' understanding of fractal geometry but also deepen their appreciation of the interplays between mathematics, art, and history, ultimately resulting in a more integrated learning experience. Adopting such an approach will empower educators to implement similar interdisciplinary methodologies across various educational contexts, thereby broadening the impact of this proposal.

Afterword

The Open Review process conducted by PhD Camilo Rodríguez-Nieto has been instrumental in improving the coherence, clarity, and academic rigor of our manuscript. His recommendations prompted us to focus on the specific educational challenges that students face when engaging with the abstract and complex mathematical concepts associated with fractals. Furthermore, his advice to refine the Introduction by updating our literature review and articulating the issue of fractal comprehension in educational contexts encouraged a deeper examination of the conceptual and operational obstacles faced by students. This refinement has solidified the foundation of our study and underscored the relevance of our proposed approach within the field of mathematics education.

In addition, PhD Rodríguez-Nieto's insights led to meaningful advancements in the Discussion section. His suggestions motivated us to conduct a thorough comparison of our arguments and didactic proposal with existing research, enabling us to deeply clarify the novel contributions of our interdisciplinary approach. By contextualizing our findings within the broader literature, we have effectively emphasized how integrating mathematics with social and artistic frameworks could enrich students' understanding of fractals. This approach situates mathematical concepts within a more tangible and culturally relevant context.

Through this process, we aim to highlight not only the originality of our work but also its potential impact on the theoretical and practical dimensions of mathematics education.

The reflective engagement with PhD Rodríguez-Nieto has proven invaluable for our authorship team. The Open Review process has created a constructive and fertile environment for growth, aligning our research more closely with the needs and perspectives of the mathematics education community. We are deeply appreciative of the opportunity to develop our ideas within this framework, and the insights gained will undoubtedly inform our future research endeavors.

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