

On the Determination of Fundamental Photographic Magnitudes.
By J. Halm, Ph.D. (Plate 13.)

In a paper entitled "A System of Photographic Magnitudes for Southern Stars" * reference was made to the fact, pointed out by Schwarzschild and others, that if light of different intensities I_1 and I_2 be exposed on a plate during the time-intervals t_1 and t_2 so that $I_1 t_1 = I_2 t_2$, the assumption of strict equality of the resulting actinic effects is not justified. Hence the so-called "law of reciprocity," according to which identical actions It produce the same actinic results, does not appear to be strictly applicable to photometric investigations based on the effect of light on photographic plates. No doubt the appreciable deviations from this law and the absence of a precise knowledge of their character have been partly responsible for the uncertainty attaching to astrophotometric results derived from the photographic method. This fact was recognised by Schwarzschild at the very outset of his important researches.† We owe to him one of the first and most fruitful experimental investigations into the character of the relation between the action of the light-source and the actinic effect produced on the plate, as well as a fundamental application of his experimental results to the problem of the photographic magnitudes of stars.

The modification of the law of reciprocity, at which Schwarzschild arrived, may be expressed as follows: Two sources of intensities I_1 and I_2 produce equal actinic effects, if the times of exposure t_1 and t_2 are so chosen that

$$I_1 t_1^p = I_2 t_2^p, \quad . \quad . \quad . \quad . \quad (1)$$

p denoting an exponent sensibly constant, *i.e.* independent of the intensity, within the range of Schwarzschild's experiments. Equation (1) may also be written in the equivalent form:

$$I_1^q t_1 = I_2^q t_2, \quad . \quad . \quad . \quad . \quad (2)$$

where

$$pq = 1.$$

The correctness of the "Schwarzschild law," as formulated by equation (1) or (2), was accepted in the determination of the photographic magnitudes of the south polar area published in my previous paper. Subsequent to this publication, however, I received a letter from Professor Eberhard of the Potsdam Observatory, in which he drew my attention to an experimental investigation made by Dr. Kron ‡ with the view of testing the

* *Monthly Notices*, lxxiv. p. 600.

† *Publikationen der v. Kuffner'schen Sternwarte Wien-Ottakring*, Band v.

‡ *Publikationen des Astrophysikalischen Observatoriums zu Potsdam*, No. 67: "Ueber das Schwaerzungsgesetz photographischer Platten."

constancy of the exponent q over a wider range of intensities, and also with reference to plates of different degrees of sensitiveness. The most important result of Dr. Kron's work is undoubtedly the establishment of the "Schwarzschild law" as a *special* representation of a more complicated relation between intensities and times of exposure, which is approximately correct only for great and small intensities. This limitation of the applicability of the Schwarzschild law necessitates a re-examination, in the light of Dr. Kron's experimental evidence, of the magnitudes obtained by the method advocated in my previous paper. It is, therefore, the object of the present communication, first to review the experimental data collected in Kron's paper, particularly those referring to the relationship between the action It of the light-source and its actinic effect on the plate, and secondly, to devise a practical method by which photographic magnitudes of stars, strictly based on these experimental facts, may be obtained.

§ 1. *Discussion of Dr. Kron's Experimental Results.*—Without entering into the detail of the work, it may suffice, for the immediate purposes of astrophotometric investigations, to start from the series of curves shown numerically in Table 17 and graphically on plate ii. of Kron's paper. These curves represent the relation between the action It and the intensity I (or rather between the logarithms of these quantities), for equal actinic effects (Curven gleicher Schwaerzung). The general character of the curves demonstrates at sight the incompetence of the Schwarzschild law to represent the observed facts over the whole range of intensities. Accepting equation (2) as the analytical expression of this law, it follows:

$$\log (It) = (1 - q) \log I + \text{const.}; \quad q = \text{const.} \quad . \quad . \quad (3)$$

Hence, plotting $\log (It)$ as ordinates against $\log I$ as abscissæ, the locus should be a straight line making an angle $\tan^{-1} (1 - q)$ with the axis of abscissæ. The observed curve, on the other hand, is represented in fig. 1 (Plate 13), which shows, as an example, Kron's observations made with plates of the trade-mark "Agfa-Diapositiv."

Apparently the Schwarzschild law can be considered approximately valid only at the extreme ends of the curve, *i.e.* for very large or very small values of I , where the curve reaches asymptotically the condition of the straight line expressed by equation (3). For a certain intensity, I_0 , the action It required to produce a desired actinic effect assumes a minimum value. Kron therefore defines this particular intensity as the *optimal* intensity of the plate. There is a particularly remarkable property of the It curves which will be utilised with advantage later on, *viz.* their symmetry with reference to the minimum point. It will also be noticed that in the vicinity of this point the law of reciprocity ($q = 1$) is fulfilled.

For practical purposes it is convenient to establish an empirical analytical relation between the action It and the intensity I .

The geometrical character of the curves suggested to Kron two alternative forms of empirical formulæ, viz. :—

$$\begin{aligned} \log It &= \text{const.} + a \sqrt{(\log I)^2 + b}, \\ \log It &= \text{const.} + \log (aI^\alpha + bI^{-\alpha}). \end{aligned}$$

From a physical point of view neither of these formulæ can at present claim preference ; their merits must be judged solely by their capability of representing the observations, and even in this respect their utilisation beyond the range of the latter can only be allowed with a certain reserve. According to Kron's calculations the former equation would seem to render somewhat better justice to the observations than the latter. But its very slight advantage in this respect is scarcely sufficient to warrant the rejection of the alternative formula, which would appear particularly useful in so far that it merges into the Schwarzschild equation (3) when extreme intensities are approached.

The figures comprised in Table I. demonstrate to what extent the second formula satisfies the observed facts. If we assume I_0, t_0 to represent the values of I, t at the minimum point and write

$$i = I/I_0 ; \tau = t/t_0,$$

the equation takes the form of that of a catenary symmetrical about the minimum point, which may be written in the form :

$$i\tau = \frac{1}{2}[i^\alpha + i^{-\alpha}] \quad . \quad . \quad . \quad (4)$$

The constants I_0, t_0, α can be determined from the observed data shown in the first and second column. By means of (4) the numerical values of $\log (It)$ may then be computed for the arguments $\log I$ of the first column ; they are shown in the fourth column, while the fifth column contains the differences (observed - computed). In addition two further rows of figures, headed $(m - m_0)$ and $\log \tau$, are exhibited, which will be useful later on. They are derived from the observed data of columns 1 and 2 in the following manner. According to fundamental photometric definition, the relation between magnitude m and intensity I is expressed by the formula

$$\log I = -0.40m + \text{const.} \quad . \quad . \quad . \quad (5)$$

If we assume the optimal intensity I_0 to be correlated with the magnitude m_0 , we have

$$m - m_0 = -2.5 \log i = -2.5 (\text{column 1} - \log I_0).$$

Further,

$$\log \tau = \log t - \log t_0 = (\text{column 2} - \text{column 1} - \log t_0).$$

Kron's investigation comprises four different brands of plates : two "slow" plates (Agfa-Diapositiv and Ideal-Diapositiv) and two "fast" plates (Schleussner and Seed 27). The material shown in Table I. has been somewhat condensed by combining into means three consecutive groups in each of the four series published in Kron's Table 17.

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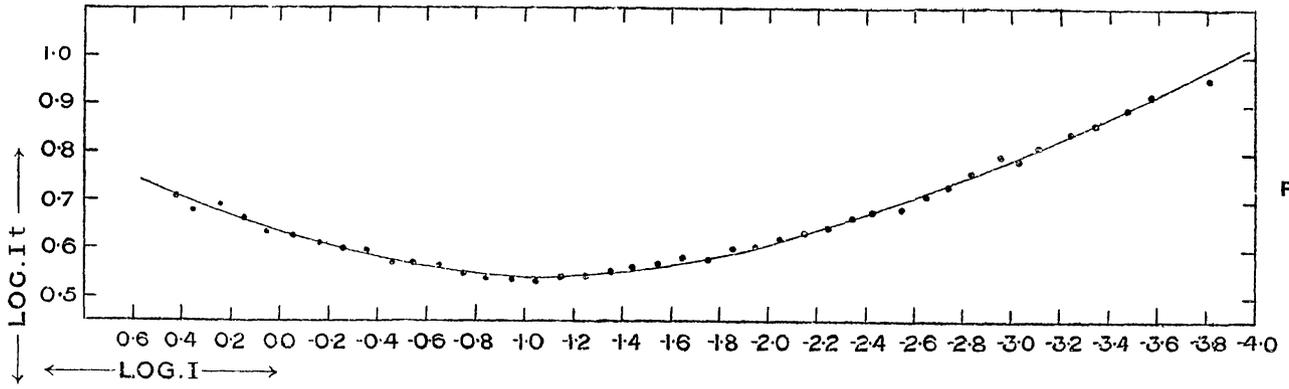


Fig. 1.

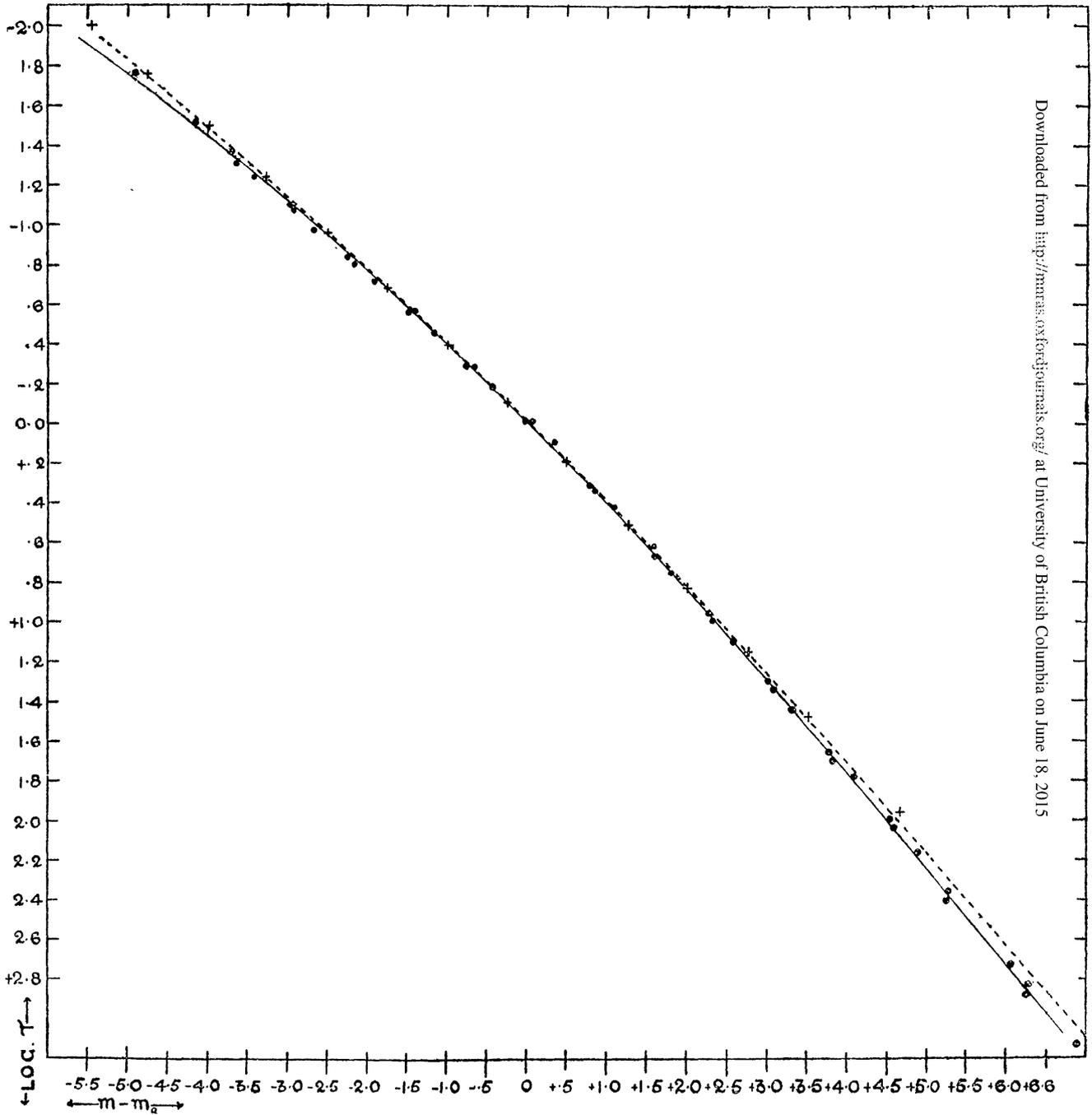


Fig. 2.

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TABLE I.

Agfa-Diapositiv. ($\alpha=0.26$; $\log I_0=8.880$; $\log t_0=1.670$.)

Observed.		No. of Obs.	Compd. log I_t .	O-C.	$(m-m_0)$	log τ .
log I .	log I_t .					
0.338	0.697	12	0.698	-0.001	-3.64	-1.311
0.050	0.643	30	0.649	-0.006	-2.92	-1.077
9.745	0.604	49	0.605	-0.001	-2.16	-0.811
9.449	0.573	82	0.575	-0.002	-1.42	-0.546
9.151	0.544	113	0.556	-0.012	-0.68	-0.277
8.848	0.538	163	0.550	-0.012	+0.08	+0.020
8.547	0.561	183	0.559	+0.002	+0.83	+0.344
8.248	0.588	169	0.581	+0.007	+1.58	+0.670
7.953	0.621	128	0.614	+0.007	+2.32	+0.998
7.654	0.663	74	0.658	+0.005	+3.07	+1.339
7.350	0.712	48	0.711	+0.001	+3.82	+1.692
7.047	0.780	22	0.773	+0.007	+4.58	+2.063
6.766	0.838	11	0.832	+0.006	+5.28	+2.402
6.377	0.922	5	0.922	0.000	+6.26	+2.875

Ideal-Diapositiv. ($\alpha=0.19$; $\log I_0=8.250$; $\log t_0=2.363$.)

0.431	0.786	6	0.787	-0.001	-5.45	-2.008
0.143	0.744	35	0.749	-0.005	-4.73	-1.762
9.844	0.705	47	0.711	-0.006	-3.98	-1.502
9.552	0.675	77	0.680	-0.005	-3.26	-1.240
9.250	0.651	112	0.653	-0.002	-2.50	-0.962
8.949	0.631	145	0.633	-0.002	-1.75	-0.681
8.647	0.615	162	0.619	-0.004	-0.99	-0.395
8.349	0.612	174	0.614	-0.002	-0.25	-0.100
8.053	0.624	137	0.615	+0.009	+0.49	+0.208
7.748	0.637	93	0.628	+0.009	+1.26	+0.526
7.452	0.652	56	0.639	+0.013	+1.99	+0.837
7.143	0.662	28	0.662	0.000	+2.77	+1.156
6.848	0.689	16	0.690	-0.001	+3.51	+1.478
6.385	0.708	10	0.744	-0.036	+4.66	+1.960

Schleussner. ($\alpha=0.235$; $\log I_0=6.880$; $\log t_0=2.157$.)

8.843	9.232	20	9.245	-0.013	-4.91	-1.768
8.547	9.186	26	9.193	-0.007	-4.17	-1.518
8.245	9.160	36	9.146	+0.014	-3.41	-1.242
7.946	9.132	44	9.105	+0.027	-2.67	-0.971
7.643	9.090	66	9.073	+0.017	-1.91	-0.710
7.342	9.046	88	9.050	-0.004	-1.16	-0.453
7.052	9.025	113	9.039	-0.014	-0.43	-0.184

TABLE I.—Schleussner. ($\alpha=0.235$; $\log I_0=6.880$; $\log t_0=2.157$)—*contd.*

Observed.		No. of Obs.	Compd. log I_t .	O - C.	$(m - m_0)$	log τ .
log I.	log I_t .					
6.750	9.022	132	9.038	- .016	+ 0.32	+ 0.115
6.452	9.038	116	9.048	- .010	+ 1.07	+ 0.429
6.157	9.075	97	9.069	+ .006	+ 1.81	+ 0.761
5.847	9.111	67	9.101	+ .010	+ 2.58	+ 1.107
5.551	9.146	48	9.141	+ .005	+ 3.32	+ 1.438
5.249	9.190	25	9.187	+ .003	+ 4.08	+ 1.784
4.915	9.237	19	9.245	- .008	+ 4.91	+ 2.165
4.362	9.346	9	9.354	- .008	+ 6.29	+ 2.827
Seed 27. ($\alpha=0.245$; $\log I_0=7.360$; $\log t_0=1.656$.)						
8.845	9.129	22	9.153	- .024	- 3.71	- 1.372
8.541	9.102	28	9.105	- .003	- 2.95	- 1.095
8.260	9.084	35	9.070	+ .014	- 2.25	- 0.832
7.949	9.056	46	9.059	- .003	- 1.47	- 0.549
7.658	9.027	65	9.022	+ .005	- 0.75	- 0.287
7.346	8.999	106	9.017	- .018	- 0.03	- 0.003
7.044	9.013	120	9.023	- .010	+ 0.79	+ 0.313
6.752	9.038	117	9.040	- .002	+ 1.57	+ 0.630
6.453	9.074	116	9.070	+ .004	+ 2.27	+ 0.965
6.151	9.112	95	9.109	+ .003	+ 3.02	+ 1.305
5.853	9.172	65	9.156	+ .016	+ 3.77	+ 1.663
5.546	9.231	35	9.211	+ .020	+ 4.53	+ 2.029
5.251	9.276	17	9.270	+ .006	+ 5.27	+ 2.369
4.940	9.328	10	9.335	- .007	+ 6.05	+ 2.732
4.613	9.403	5	9.405	- .002	+ 6.87	+ 3.134

A systematic "run" in the differences O - C, though occasionally noticeable, is scarcely important enough to invalidate the adoption of equation (4), especially if we remember that an error of 0.020 in log I, which is exceeded only in three instances, is equivalent to only 0.05 magnitudes.

A striking feature of the analysis of Kron's figures is the consistency in the values of the exponent α for different plates. The analysis indicates that for

"Slow" plates:	{	Agfa-Diapositiv . . .	$\alpha = 0.26$
		Ideal-Diapositiv . . .	0.19
"Fast" plates:	{	Schleussner . . .	0.24
		Seed 27 . . .	0.25

If it were not for the doubtlessly real exception presented in the "Ideal-Diapositiv" brand, the conclusion might be warranted that α has the same value for all plates. As matters stand, however, the assumption of a constant α , though highly probable,

especially if our attention is confined to "fast" plates such as are used in astrophotometric work, is certainly open to criticism. Fortunately, however, a method can be pointed out in the course of this discussion by which the exponent α may be determined from certain evidence furnished by the particular plate from which star magnitudes are determined.

If the photometric relation (5) is introduced into (4), the latter equation may be written in the form

$$\tau = \frac{1}{2} [I_0^{0.4(1+\alpha)(m-m_0)} + I_0^{0.4(1-\alpha)(m-m_0)}] \quad (6)$$

or

$$\tau = \frac{1}{2} I_0^{0.4(1+\alpha)(m-m_0)} [1 + I_0^{-0.8\alpha(m-m_0)}].$$

For intensities considerably smaller than I_0 , *i.e.* for values of m considerably greater than m_0 , the bracket term on the right-hand side approaches unity, and hence

$$m = \text{const.} + \frac{2.5}{1+\alpha} \log \tau, \quad (7)$$

whereas the reciprocity law would require

$$m = \text{const.} + 2.5 \log \tau.$$

Consequently the Schwarzschild exponent, p , is equal to

$$1/(1+\alpha).$$

The mean value of p for the four brands of plates is therefore 0.810, and the extreme values are 0.794 and 0.840. Astrophotometric determinations of this quantity, on the other hand, appear to centre round a mean value 0.80, in excellent agreement with Kron's experimental result.

Provided that in astrophotometric work we are dealing with intensities much smaller than the optimal intensity of the plate, the method advocated by Schwarzschild and adopted in my previous paper is free from objection. We shall see, however, that this condition is fulfilled only for comparatively faint stars. For bright stars the constancy of p in Schwarzschild's equation cannot be assumed, and the relation between m and t must therefore be derived from the more general equation (6).

The importance of equation (6) for astrophotographic work makes it desirable to represent it in a tabular form. Consequently, values of $\log \tau$ were computed for every tenth of the argument ($m - m_0$) with three assumptions of α , *viz.* 0.30, 0.25, 0.20. The resulting figures are shown in Table II., which requires no further explanation. Further, to show how very satisfactorily equation (6) represents the observed data, I have plotted in fig. 2 (Plate 13) the values ($m - m_0$) and $\log \tau$ of Table I. as abscissæ and ordinates, marking the observed points by circular dots in the case of Agfa-Diapositiv, Schleussner, and Seed 27, which are satisfied by practically the same value of α , and by small crosses for the exceptional plate Ideal-Diapositiv. The smooth drawn-out curve shows the related co-ordinates ($m - m_0$) and $\log \tau$ taken from Table II. for $\alpha = 0.25$, while the dotted curve refers to the same co-ordinates computed with the assumption $\alpha = 0.20$.

TABLE II.

log τ .

Arg. $m - m_0$	$\alpha =$			Arg. $m - m_0$	$\alpha =$		
	0'30	0'25	0'20		0'30	0'25	0'20
-5'0	-1'674	-1'760	-1'837	-0'9	-0'347	-0'351	-0'354
-4'9	-1'645	-1'728	-1'803	-0'8	-0'309	-0'313	-0'315
-4'8	-1'615	-1'696	-1'769	-0'7	-0'272	-0'274	-0'276
-4'7	-1'585	-1'664	-1'734	-0'6	-0'234	-0'236	-0'237
-4'6	-1'556	-1'632	-1'700	-0'5	-0'196	-0'197	-0'198
-4'5	-1'526	-1'600	-1'665	-0'4	-0'157	-0'158	-0'159
-4'4	-1'496	-1'567	-1'631	-0'3	-0'118	-0'119	-0'119
-4'3	-1'466	-1'535	-1'596	-0'2	-0'079	-0'080	-0'080
-4'2	-1'436	-1'502	-1'561	-0'1	-0'040	-0'040	-0'040
-4'1	-1'406	-1'470	-1'526	0'0	0'000	0'000	0'000
-4'0	-1'376	-1'437	-1'491	+0'1	+0'040	+0'040	+0'040
-3'9	-1'346	-1'404	-1'456	+0'2	+0'081	+0'080	+0'080
-3'8	-1'315	-1'371	-1'421	+0'3	+0'122	+0'121	+0'121
-3'7	-1'284	-1'338	-1'386	+0'4	+0'163	+0'162	+0'161
-3'6	-1'253	-1'305	-1'351	+0'5	+0'204	+0'203	+0'202
-3'5	-1'222	-1'272	-1'315	+0'6	+0'246	+0'244	+0'243
-3'4	-1'191	-1'239	-1'280	+0'7	+0'288	+0'285	+0'284
-3'3	-1'160	-1'206	-1'244	+0'8	+0'331	+0'327	+0'325
-3'2	-1'129	-1'172	-1'209	+0'9	+0'374	+0'369	+0'366
-3'1	-1'097	-1'138	-1'173	+1'0	+0'416	+0'412	+0'407
-3'0	-1'065	-1'104	-1'137	+1'1	+0'459	+0'454	+0'449
-2'9	-1'033	-1'070	-1'101	+1'2	+0'503	+0'496	+0'491
-2'8	-1'001	-1'035	-1'065	+1'3	+0'547	+0'539	+0'532
-2'7	-0'969	-1'000	-1'028	+1'4	+0'592	+0'582	+0'574
-2'6	-0'936	-0'966	-0'992	+1'5	+0'636	+0'625	+0'616
-2'5	-0'903	-0'931	-0'955	+1'6	+0'681	+0'669	+0'659
-2'4	-0'871	-0'896	-0'919	+1'7	+0'726	+0'712	+0'701
-2'3	-0'838	-0'861	-0'882	+1'8	+0'772	+0'756	+0'743
-2'2	-0'804	-0'826	-0'845	+1'9	+0'817	+0'800	+0'786
-2'1	-0'771	-0'791	-0'808	+2'0	+0'863	+0'845	+0'829
-2'0	-0'737	-0'756	-0'771	+2'1	+0'909	+0'890	+0'872
-1'9	-0'703	-0'720	-0'734	+2'2	+0'956	+0'934	+0'915
-1'8	-0'668	-0'684	-0'697	+2'3	+1'002	+0'978	+0'958
-1'7	-0'634	-0'648	-0'659	+2'4	+1'049	+1'023	+1'001
-1'6	-0'599	-0'612	-0'621	+2'5	+1'096	+1'068	+1'045
-1'5	-0'564	-0'575	-0'584	+2'6	+1'144	+1'114	+1'088
-1'4	-0'528	-0'538	-0'546	+2'7	+1'191	+1'160	+1'132
-1'3	-0'493	-0'501	-0'508	+2'8	+1'239	+1'205	+1'175
-1'2	-0'457	-0'464	-0'469	+2'9	+1'287	+1'250	+1'219
-1'1	-0'421	-0'427	-0'431	+3'0	+1'335	+1'296	+1'263
-1'0	-0'384	-0'389	-0'393				

TABLE II.— $\log \tau$ —*continued.*

Arg. $m - m_0$	$\alpha =$			Arg. $m - m_0$	$\alpha =$		
	0'30	0'25	0'20		0'30	0'25	0'20
+3'0	+1'335	+1'296	+1'263	+7'1	+3'399	+3'265	+3'138
+3'1	+1'383	+1'342	+1'307	+7'2	+3'451	+3'314	+3'185
+3'2	+1'431	+1'388	+1'351	+7'3	+3'502	+3'363	+3'232
+3'3	+1'480	+1'434	+1'396	+7'4	+3'554	+3'413	+3'279
+3'4	+1'529	+1'481	+1'445	+7'5	+3'605	+3'463	+3'326
+3'5	+1'578	+1'528	+1'485	+7'6	+3'657	+3'512	+3'373
+3'6	+1'627	+1'575	+1'529	+7'7	+3'709	+3'561	+3'420
+3'7	+1'676	+1'622	+1'574	+7'8	+3'761	+3'610	+3'467
+3'8	+1'725	+1'669	+1'619	+7'9	+3'812	+3'660	+3'514
+3'9	+1'775	+1'716	+1'664	+8'0	+3'864	+3'710	+3'561
+4'0	+1'824	+1'763	+1'709	+8'1	+3'916	+3'759	+3'608
+4'1	+1'874	+1'810	+1'754	+8'2	+3'968	+3'808	+3'656
+4'2	+1'924	+1'858	+1'799	+8'3	+4'019	+3'858	+3'703
+4'3	+1'974	+1'906	+1'844	+8'4	+4'071	+3'908	+3'750
+4'4	+2'024	+1'953	+1'889	+8'5	+4'123	+3'958	+3'798
+4'5	+2'074	+2'000	+1'935	+8'6	+4'175	+4'007	+3'845
+4'6	+2'124	+2'048	+1'980	+8'7	+4'226	+4'056	+3'892
+4'7	+2'174	+2'096	+2'026	+8'8	+4'278	+4'106	+3'940
+4'8	+2'225	+2'144	+2'073	+8'9	+4'330	+4'156	+3'987
+4'9	+2'275	+2'192	+2'117	+9'0	+4'382	+4'206	+3'034
+5'0	+2'326	+2'240	+2'163	+9'1	+4'434	+4'255	+4'082
+5'1	+2'376	+2'288	+2'209	+9'2	+4'486	+4'304	+4'129
+5'2	+2'427	+2'336	+2'255	+9'3	+4'537	+4'354	+4'177
+5'3	+2'477	+2'385	+2'301	+9'4	+4'589	+4'404	+4'224
+5'4	+2'528	+2'434	+2'347	+9'5	+4'641	+4'454	+4'272
+5'5	+2'579	+2'482	+2'393	+9'6	+4'693	+4'503	+4'319
+5'6	+2'630	+2'530	+2'439	+9'7	+4'745	+4'553	+4'367
+5'7	+2'681	+2'579	+2'485	+9'8	+4'797	+4'603	+4'415
+5'8	+2'732	+2'628	+2'531	+9'9	+4'849	+4'653	+4'462
+5'9	+2'783	+2'677	+2'578	+10'0	+4'901	+4'703	+4'510
+6'0	+2'834	+2'726	+2'624	+10'1	+4'953	+4'752	+4'557
+6'1	+2'885	+2'774	+2'671	+10'2	+5'005	+4'802	+4'605
+6'2	+2'937	+2'832	+2'717	+10'3	+5'056	+4'852	+4'653
+6'3	+2'988	+2'872	+2'764	+10'4	+5'108	+4'902	+4'700
+6'4	+3'039	+2'921	+2'810	+10'5	+5'160	+4'952	+4'748
+6'5	+3'090	+2'970	+2'857	+10'6	+5'212	+5'002	+4'796
+6'6	+3'142	+3'019	+2'904	+10'7	+5'264	+5'052	+4'843
+6'7	+3'193	+3'068	+2'950	+10'8	+5'316	+5'102	+4'891
+6'8	+3'245	+3'117	+2'997	+10'9	+5'368	+5'152	+4'939
+6'9	+3'296	+3'166	+3'044	+11'0	+5'420	+5'202	+4'986
+7'0	+3'348	+3'216	+3'091				

§ 2. *Discussion of Properties of the Iso-Actinics.*—The curves shown in fig. 2 express the relationship between $\log i$ or $(m - m_0)$ on the one hand and $\log \tau$ on the other for *equal actinic effects*. Curves of this kind may therefore be termed “iso-actinics.” Kron’s and Schwarzschild’s experimental research and also the investigations of photographic magnitudes undertaken at this Observatory have established an important property of the iso-

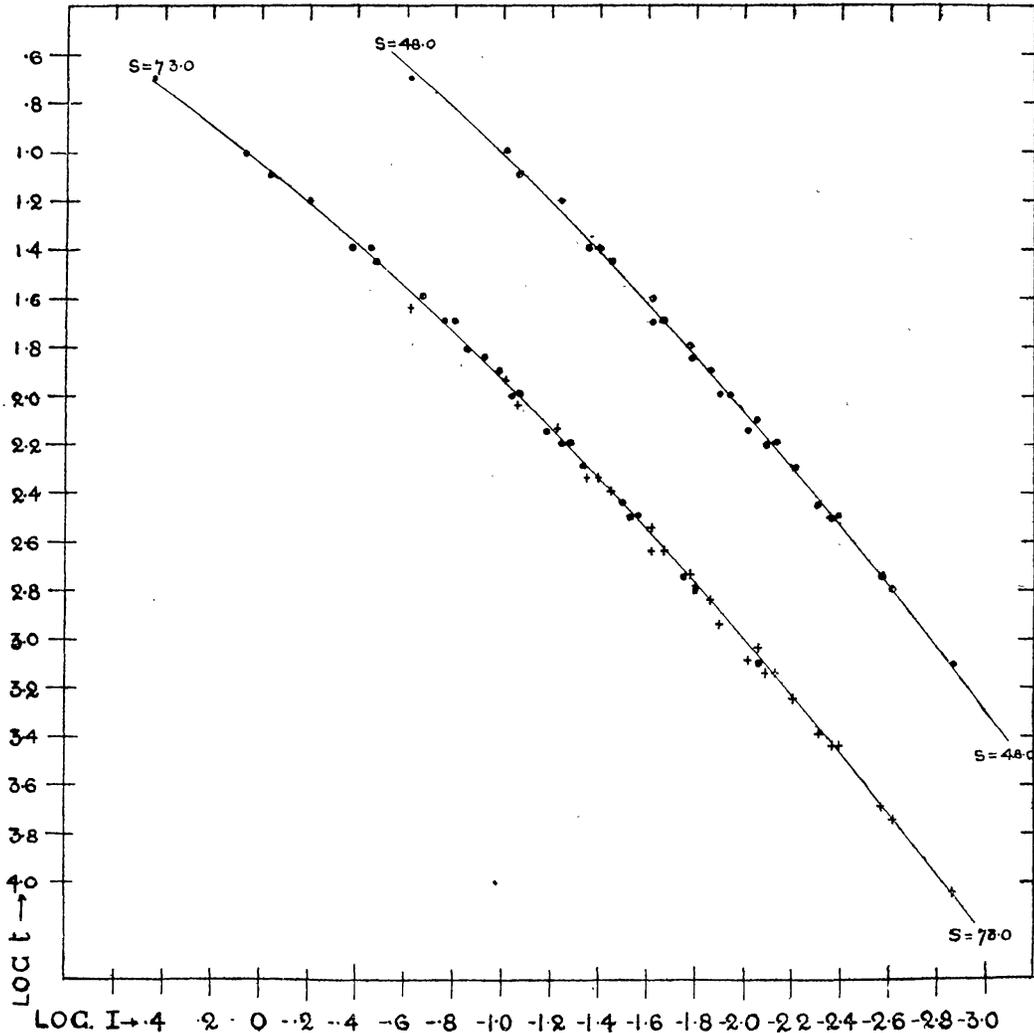


FIG. 3.

actinics which has proved of high value for the problem of determining fundamental photographic magnitudes. It appears that the iso-actinics belonging to different values of the actinic effects s are all similar in form and differ only in position by a constant shift parallel to the line of the $\log \tau$ co-ordinates. As an illustration the iso-actinics for the extreme cases of s (viz. $s = 73.0$ and 48.0) relating to Kron’s 3rd series of the Agfa-Diapositiv plates are exhibited in fig. 3. The two curves are shown separately, the dots marking the observed data; but the points of the curve ($s = 48.0$) have

Diameter.	log t .		$\Delta \log t$.
	Group I.	Group II.	
0.90	1.75	2.67	0.92
0.85	1.57	2.50	0.93
0.80	1.39	2.31	0.92
0.75	1.19	2.12	0.93
0.70	1.00	1.93	0.93
0.65	0.79	1.72	0.93
0.60	0.58	1.50	0.92

The constancy of $\Delta \log t$ throughout the whole range of diameters proves the correctness of the law expressed by equation (9).

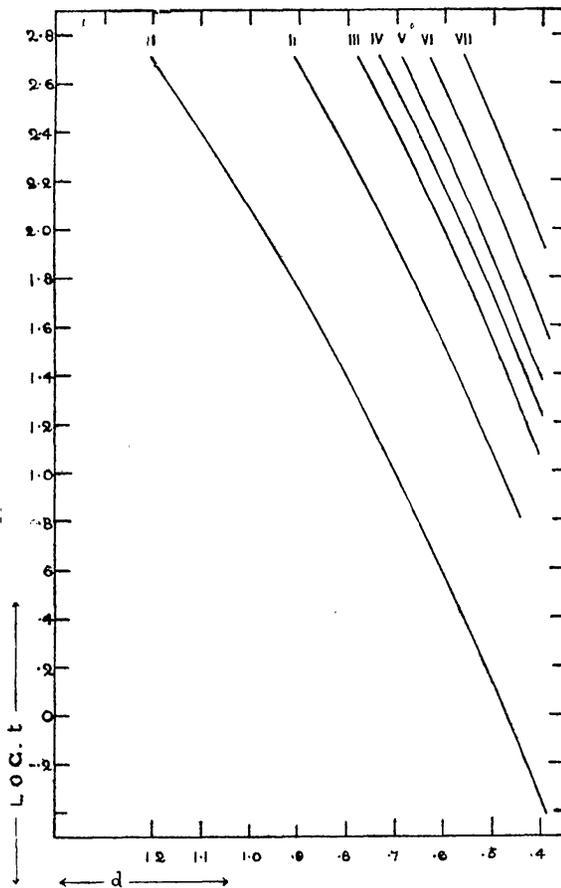


FIG. 4.

Owing to this property the curves of fig. 4 may be combined into one single curve, say that represented by Group I., on which subsequent calculations will be based.

§ 3. *Relation between the Intensities and Times of Exposure of Stars on any Plate and the Quantities i and τ .*—The material of a photographic plate, on which a number of stars have been exposed during varying lengths of time, may be examined in two

different directions. First, we may fix our attention on one star (or combined group of stars) of constant intensity I_c , and establish the relation between s (actinic effect) and $\log t$. This procedure is identical with that mentioned at the end of the last paragraph, and is graphically represented in fig. 4. Equation (9) may therefore be written

$$\log t = \phi(s) + \psi(\log I_c) \quad . \quad . \quad . \quad (10)$$

Or we may examine the actinic effects of objects of different intensity under the same conditions of exposure, *i.e.* $t = t_c = \text{const.}$ In this case equation (9) takes the form

$$\log t_c = \phi(s) + \psi(\log I) \quad . \quad . \quad . \quad (11)$$

Comparing equal actinic effects in both cases, we have the relation

$$\log t_c/t = \psi(\log I) - \psi(\log I_c) \quad . \quad . \quad . \quad (12)$$

But, in accordance with Kron's law, the intensity I exposed during time t produces the same actinic effect as the optimal intensity I_0 of the plate during time t_0 , where $t/t_0 = \tau$.

Hence, according to (9)

$$\log \tau = \psi(\log I) - \psi(\log I_0) \quad . \quad . \quad . \quad (13)$$

and in virtue of (12)

$$\log t_c/t = \log \tau + a \quad . \quad . \quad . \quad (14)$$

where a is a constant.

Provided that a can be determined, and also that the plate supplies means for ascertaining the exact value of the exponent a , the value of $\log \tau$ can be found for each star by means of (14), and the corresponding value of $(m - m_0)$ is then supplied by Table II. In order to find the absolute magnitudes of the stars, the further knowledge of the magnitude of at least *one* star is necessary. On the whole, therefore, three data are still required, *viz.* the constants a , α , and m_0 . We shall now proceed to show that at least two of these quantities, *viz.* α and a , are ascertainable from the internal evidence of the plate.

§ 4. *Determination of the Constant a .*—For the determination of this quantity it is required that the observations should be made with a wire grating in front of the object-glass.

The fundamental property of the wire grating consists in separating the light of a source into a series of components which, in the case of a star, are shown on the plate by a pattern containing a comparatively strong central disc flanked on either side by a row of fainter diffracted images. The principal central image will be henceforth denoted by the suffix p , the secondary diffracted images by the suffix s . Owing to colour dispersion the secondary images appear somewhat elongated, especially in the higher orders; with

the grating employed in the present investigation, however, the ellipticity of the lowest order image is so insignificant that the determination of its actinic effect by the diameter method is perfectly reliable. For this reason only the diffracted images of the lowest order will be utilised.

The difference in magnitude between the secondary and the principal image ($m_s - m_p$) is known from the dimensions of the grating; it amounts in the present case to

$$m_s - m_p = 4^m \cdot 35.$$

Suppose now that the curve of equal intensity, represented by equation (10) and fig. 4, has been constructed from the principal images of a certain star by plotting the actinic effects (diameters) as abscissæ and $\log t$ as ordinates. Then, for the principal image of a star exposed during time t_c we obtain from (14)

$$\log t_c/t_p = \log \tau_p + \alpha \quad . \quad . \quad . \quad (15)$$

where $\log t_p$ is found by reading off from the curve which represents equation (10) with the argument s_p .

Also we find for the secondary image

$$\log t_c/t_s = \log \tau_s + \alpha \quad . \quad . \quad . \quad (16)$$

$\log t_s$ being obtained in like manner with the argument s_s .

Hence

$$\log t_p/t_s = \log \tau_s/\tau_p \quad . \quad . \quad . \quad (17)$$

and also

$$m_s - m_p = 4 \cdot 35.$$

Let us now assume a series of values for ($m_p - m_0$) such as are represented in the first column of Table III., and extract the corresponding values of $\log \tau_p$ from Table II., adopting the value of α determined by the method of the next paragraph. (Table III., as an example, is constructed on the assumption $\alpha = 0 \cdot 25$.) These values of $\log \tau_p$ are shown in the second column. Let us further construct in the same manner columns 3 and 4 for the secondary images, satisfying the condition $m_s - m_p = 4 \cdot 35$. Hence the differences between the figures of columns 4 and 2 will give $\log \tau_s/\tau_p$ shown in the 5th column.

But the particular value of $\log \tau_s/\tau_p$ satisfying the conditions of the plate and the selected star is known from equation (17). Entering with this value into the 5th column of Table III., we find directly the corresponding $\log \tau_p$ and $\log \tau_s$. Finally, from equations (15) and (16) we get

$$\left. \begin{aligned} \alpha &= \log t_c/t_p - \log \tau_p \\ &= \log t_c/t_s - \log \tau_s \end{aligned} \right\} \quad . \quad . \quad . \quad (18)$$

TABLE III.

$m_p - m_0.$	$\log \tau_p.$	$m_s - m_0.$	$\log \tau_s.$	$\log \tau_s/\tau_p.$
-5.0	-1.760	-0.65	-0.255	+1.505
-4.5	-1.600	-0.15	-0.060	+1.540
-4.0	-1.437	+0.35	+0.142	+1.579
-3.5	-1.272	+0.85	+0.348	+1.620
-3.0	-1.104	+1.35	+0.560	+1.664
-2.5	-0.931	+1.85	+0.778	+1.709
-2.0	-0.756	+2.35	+1.000	+1.756
-1.5	-0.575	+2.85	+1.227	+1.802
-1.0	-0.389	+3.35	+1.457	+1.846
-0.5	-0.197	+3.85	+1.692	+1.889
0.0	0.000	+4.35	+1.929	+1.929
+0.5	+0.203	+4.85	+2.168	+1.965
+1.0	+0.412	+5.35	+2.409	+1.997
+1.5	+0.625	+5.85	+2.652	+2.027
+2.0	+0.845	+6.35	+2.897	+2.052
+2.5	+1.068	+6.85	+3.142	+2.074
+3.0	+1.296	+7.35	+3.388	+2.092
+3.5	+1.528	+7.85	+3.635	+2.107
+4.0	+1.763	+8.35	+3.883	+2.120
+4.5	+2.000	+8.85	+4.131	+2.131
+5.0	+2.240	+9.35	+4.379	+2.139
+5.5	+2.482	+9.85	+4.628	+2.146
+6.0	+2.726	+10.35	+4.877	+2.151

The accuracy in the determination of α naturally depends on the accuracy of $\log t_s/t_p$, and also on the certainty with which this value can be located in Table III. Since the changes in $\log \tau_s/\tau_p$ are much more pronounced in the region of bright stars, we conclude that α must be determined by means of the *brightest* stars on the plate.

§ 5. *Determination of the Exponent α .*—A method exactly similar to that described in § 4 may be employed for the determination of α , only that in this case the *faintest* stars must be made use of. It was shown that for very small intensities Kron's law merges into Schwarzschild's law, which is expressed by equation (7), viz.:

$$m = \text{const.} + \frac{2.5}{1 + \alpha} \log \tau \quad . \quad . \quad . \quad (19)$$

Suppose now that a faint star has been exposed during a time t_c , sufficiently great to produce a secondary image whose actinic effect (diameter) can be measured with accuracy. Again entering

with s_p and s_s into the curve representing equation (10), we determine $\log t_p$ and $\log t_s$ as before. According to (17)

$$\log t_p/t_s = \log \tau_s/\tau_p.$$

But from (7) or (19)

$$m_s - m_p = \frac{2.5}{1 + \alpha} \log \tau_s/\tau_p = \frac{2.5}{1 + \alpha} \log t_p/t_s = 4.35.$$

Hence

$$\alpha = \frac{\log t_p - \log t_s - 1}{1.74} \quad (20)$$

Strictly speaking, the star employed in the determination of α should be of infinite magnitude. The value of α will always be too small when derived from stars of finite magnitude. We may, however, approximately represent the correct value by

$$\alpha = x \frac{\log t_p - \log t_s - 1}{1.74},$$

where $x > 1$. I have found that, using stars of the ninth magnitude, which may be supposed to be within the reach of this method, the factor x differs from unity only by 0.01.

§ 6. *Determination of the Constant m_0 .*—According to definition, m_0 represents the magnitude of a star whose incident intensity is equivalent to the optimal intensity of the plate. Its value for any plate may be determined from the known magnitude of any one star on that plate. If, however, the optimal intensity could be assumed the same for all plates, it would be sufficient to determine m_0 once and for all from one or more standard magnitudes, such as, for instance, the Pleiades, and to adopt this value for all magnitude investigations. The problem of determining absolute photographic magnitudes independently of extraneous photometric data would then be solved.

An experimental examination of the changes in the optimal intensity I_0 from plate to plate is therefore of the highest importance.

Kron's investigations throw some valuable light on this point. First of all, the data given in Table I. show most emphatically that I_0 cannot be assumed constant for plates of different degrees of sensitiveness. In Table I. we found for

Agfa-Diapositiv	$\log I_0 = -1.120$
Ideal-Diapositiv	-1.750
Schleussner	-3.120
Seed 27	-2.640

When, however, attention is confined to plates of the same brand, we arrive at the important result that the changes of I_0 from plate to plate are insignificant, as long as the plates are submitted

to a perfectly uniform treatment in developing, but that I_0 changes sensibly when the treatment is varied.

As a typical illustration we may select Kron's observations of the Agfa-Diapositiv plates. In Table 15 of his paper he publishes the co-ordinates $\log I$ and $\log It$ of the iso-actinics for seven series of observations, each series containing the mean results of a small number of plates against which the following details are marked in the ledgers:—

Series I.	Plates 259-263;	Developer: Rodinal 1:45;	6.5 mins.
II.	66-70	„ 1:40;	5 „
III.	71-75	„ 1:40;	5 „
IV.	99-91	„ 1:40;	5 „
V.	112-115	„ 1:40;	5 „
VI.	191-192	„ 1:40;	1.5 „
VII.	189-190	„ 1:40;	12.0 „

The iso-actinics constructed from Kron's figures are shown in fig. 5 with $\log I$ as abscissæ and $\log It$ as ordinates.

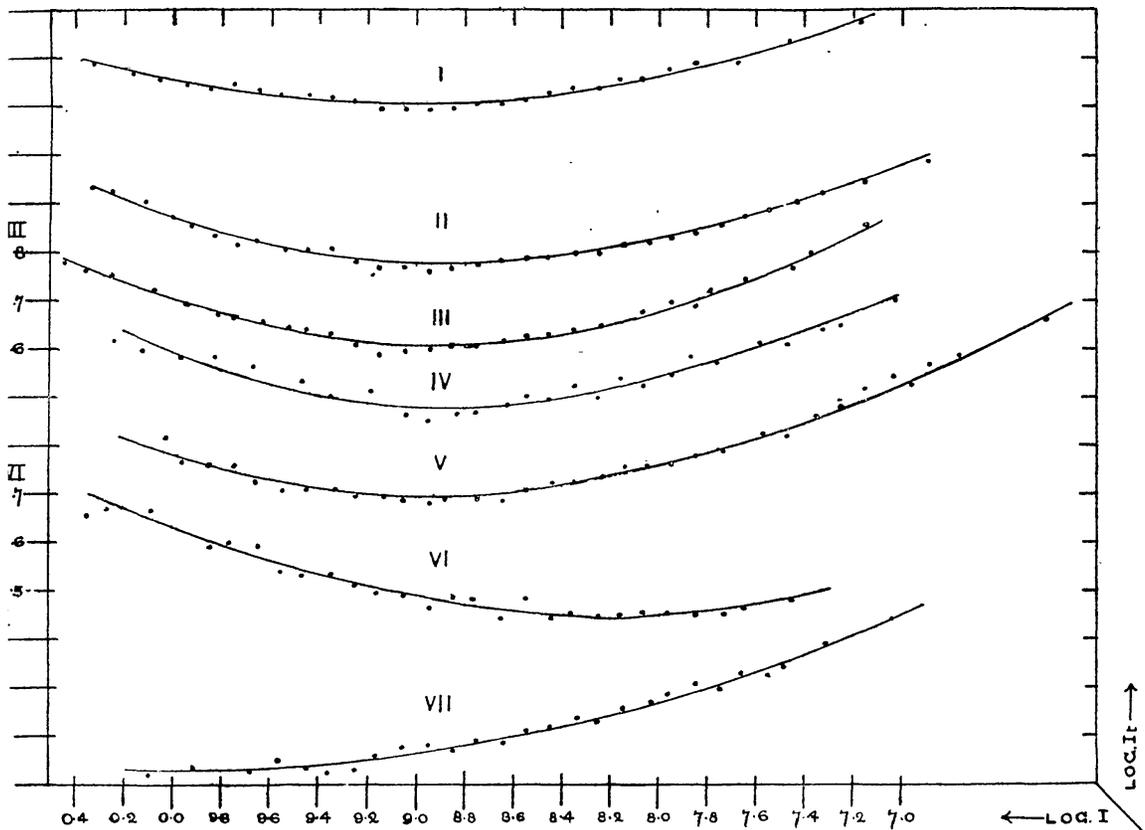


FIG. 5.

Comparing first those series in which the plates were developed under identical conditions as regards time and strength of the

developer (Series II.-V.), the constancy of I_0 appears to be remarkably pronounced. We find from the curves:—

Series II.	.	.	.	$\log I_0 =$	-1.20
III.	.	.	.		-1.10
IV.	.	.	.		-1.20
V.	.	.	.		-1.14
Mean	.	.	.		-1.16 ± 0.02

On the other hand, a pronounced change with the duration of development is noticeable in Series VI. and VII., pointing to the conclusion that I_0 increases when the time of development is increased, and vice versa.

Provided, therefore, that precaution be taken to develop the plates under exactly the same conditions of time, concentration, and temperature of the developer, Kron's experiments appear to warrant the conclusion that m_0 may be assumed constant for plates of the same manufacture. We shall see later on that this result is strongly supported by star observations.

The question as to whether the constancy of m_0 , even with the precautions referred to, can be relied on to such an extent that modern demands on the accuracy of magnitude determinations are satisfied, cannot be decided without further research. In experiments of the past the influence of the duration and mode of development has been seriously underrated, and for this reason we can scarcely claim to possess material fulfilling the necessary requirements. We also know next to nothing about the degree of homogeneity of the promiscuous brands of plates which have been used in astrophotographic work. The results of Kron certainly emphasise the necessity of the utmost care in the selection of plates for magnitude investigations and in their treatment in the developer.

Doubtless, therefore, of the three quantities a , α , and m_0 the last one is the most evasive, so far as changes from plate to plate are concerned. Not only is it affected by such changes of the optimal intensity as are indicated in fig. 5, but also by the varying conditions of definition and atmospheric absorption. Nevertheless, there are strong indications in favour of the view that difficulties in these various directions may eventually be overcome, and that *photographic* magnitudes, based on exact photometric principles, may be derived, independently of any data supplied by the results of *visual* photometry.

In connection with the changes of I_0 due to development, as shown in fig. 5, an explanation may now be given of a peculiar phenomenon mentioned in the previous paper. It was pointed out there that, if the Schwarzschild law be assumed and the exponent p be determined for each plate separately, the values of this quantity exhibit changes from plate to plate which correspond with similar fluctuations in the intensity of the actinic effect produced by the same star.

The adoption of the Schwarzschild law within a certain range of magnitudes implies that the Kron curve is considered a straight line inclined at an angle to the axis of abscissæ, whose tangent is derived from (4) by differentiation with respect to $\log i$. Considering equation (3), we may write:—

$$(1 - q) = \frac{p - 1}{p} = \alpha \frac{i_m^\alpha - i_m^{-\alpha}}{i_m^\alpha + i_m^{-\alpha}} \quad (21)$$

where i_m represents the “mean” value of i for the interval considered. As an illustration of the changes which p undergoes when i_m is varied, I computed in Table IV. from (21) the p 's for values of $\log i_m$ at regular intervals, assuming $\alpha = 0.25$.

TABLE IV.

$\log i_m$.	p .	$\log i_m$.	p .
0.0	1.00	-1.6	0.85
-0.2	0.97	-1.8	0.84
-0.4	0.95	-2.0	0.83
-0.6	0.92	-2.2	0.82
-0.8	0.90	-2.4	0.82
-1.0	0.88	-2.6	0.82
-1.2	0.87	-2.8	0.81
-1.4	0.86	-3.0	0.81

Suppose now that the same stars have been observed on different plates; then $I_m = \text{const.}$ for all the plates, and hence i_m varies inversely with I_0 . But, according to fig. 5, I_0 increases with the duration of development. Hence, in agreement with Table IV., strongly developed plates show a smaller value of p than feebly developed plates. Since it may reasonably be assumed that the actinic effect of the same star is greater on the former than on the latter, we conclude that p derived from a plate on which the actinic effect d (diameter) or s (Schwaerzung) is abnormally large, is abnormally small, and vice versa. Hence the numerical relations mentioned in the previous paper, viz.:—

$$\begin{aligned} p &= 0.821 - 0.30(d - 1.01) && \text{(Cape focal diameters)} \\ p &= 0.829 - 0.028(s - 9.2) && \text{(Schwarzschild's extra focal} \\ &&& \text{“blackening”)} \end{aligned}$$

are qualitatively explained.

§ 7. *Summary of Method.*—In the practical application of the method the following scheme of observation and reduction may now be adopted:—

1. A series of exposures of varying lengths, denoted by $t_c, t'_c, t''_c \dots$, are taken with a wire grating in front of the object-glass, the dimensions of the grating being so selected that measurable actinic effects of the secondary images of stars equal to or fainter than ninth magnitude are obtainable.

2. The diameters of all the principal and secondary images are measured by micrometer.

3. The diameters of the principal images for various lengths of exposure are collected into groups, each group consisting of stars of approximately the same magnitude.

4. The corresponding values of d and $\log t$ for each group are plotted as abscissæ and ordinates and a smoothed curve is drawn through the observed points.

5. The curves of the individual groups are combined into one single curve by appropriate shifts in the direction of the $\log t$ co-ordinate. The resulting curve, which establishes the empirical relation between d and $\log t$ for constant magnitude and which extends over the complete range of diameters, will be referred to as the A-curve.

6. With the diameters d_p and d_s (belonging to the same time of exposure t_c) of the principal and secondary images of a number of the *faintest* possible stars, the A-curve is entered and the corresponding values of $\log t_p$ and $\log t_s$ are ascertained. The exponent α is then obtained from the relation

$$\alpha = 2.5x \frac{\log t_p - \log t_s}{\Delta m} - 1 \quad (x = 1.01 \text{ approx.})$$

where Δm signifies the difference of magnitude between secondary and principal image derived from the dimensions of the grating.

7. With the diameters d_p and d_s (belonging to the same time of exposure t_c) of the principal and secondary images of some of the brightest stars the A-curve is entered and the corresponding values of $\log t_p$ and $\log t_s$ are ascertained. Then with the argument

$$\log t_p/t_s = \log \tau_s/\tau_p$$

a table similar to Table III., but based on the previously determined α and Δm , is entered and the corresponding values of $\log \tau_p$ and $\log \tau_s$ are extracted. Finally, for each star and exposure t_c the constant α is determined by the relation

$$\alpha = \log t_c/t_p \cdot \tau_p = \log t_c/t_s - \log \tau_s.$$

8. The quantities α and α being known, the calculation of the magnitude of every star can be proceeded with. These calculations may be based on the principal as well as the secondary images, but only the former will be here referred to.

In the case of every star, with the diameters d_p for all the observed times of exposure $t_c, t'_c, t''_c \dots$ the A-curve is entered, the corresponding values $\log t_p, \log t'_p \dots$ are extracted and the logs of the ratios $t_c/t_p, t'_c/t'_p \dots$ are computed. Apart from accidental errors, these logs should be identical; their mean is

therefore taken as the $\log t_c/t_p$ for the star in question. We then obtain

$$\log \tau_p = \log t_c/t_p - a,$$

and entering with this quantity into Table II., the figures of which must be based on the derived value of a , we obtain $(m - m_0)$ for every star of the plate.

9. Finally, if the magnitude of one star be known from other sources, m_0 may be determined and the magnitudes m obtained by the addition of m_0 .

The magnitudes thus derived are in strict accordance with the photometric law experimentally established by Kron for photographic plates in general.

The consistency in the values of a , which satisfy the individual series of each of the four brands of plates examined by Kron, leads to the conclusion that, at least as long as the same brand of plate is employed, this exponent may be safely assumed constant for all plates. Its evaluation might therefore be entrusted to special observations made in accordance with No. 6, and the derived value might then be accepted for observations on other plates of the same manufacture. The calculations would be considerably simplified owing to the fact that the same Tables II. and III. could be used throughout. From the experience gained in the course of these investigations, no hesitation is felt in recommending $a = 0.25$ as a value most likely to satisfy the conditions of such plates as are employed in this class of work. Nevertheless, it is important to show that the astronomical method discussed in § 5 offers the means for determining this quantity independently of the evidence adduced by laboratory experiments.

§ 8. *Determination of the Photographic Magnitudes of Stars in the Pleiades by the Preceding Method.*—As an example in illustration of the preceding method, I propose to reduce the results of three plates containing the stars of the Pleiades. Magnitudes of these stars, based on the Schwarzschild law, were already communicated in the paper referred to. It is therefore particularly interesting to see what improvement can be effected by the introduction of the more correct Kron law. Schwarzschild had already pointed out that his photographic results show unmistakable systematic differences when compared with the visual photometric results obtained at Potsdam. We shall find that these differences lose entirely their systematic character when Kron's law is adopted.

Unfortunately, the longest exposure on these plates is not sufficient to produce measurable secondary images of stars fainter than the seventh magnitude, owing to the considerable step in magnitudes between the principal and secondary images. We shall therefore accept $a = 0.25$ from Kron's investigation. Each plate contains nine exposures t_c, t'_c, \dots ranging from $9^m.3$ to 3^s , of which, however, in the case of the faintest stars, only the two longest show measurable diameters in the principal images.

As regards the A-curve, its construction has already been dis-

cussed in § 2, where it is graphically shown in fig. 4. It need only be mentioned that the curve of Group I., supplemented by the transposed curves of the other six groups, has been finally adopted.

For the evaluation of the constant a the seven brightest stars on the plate were used. First the values of $\log t_c/t_p$, $\log t'_c/t'_p \dots$ were determined. Their internal agreement may be shown in the case of the brightest star η (Alcyone):

Exposure 1 ; $\log t_c/t_p = -0.25$	Exposure 6 ; $\log t_c/t_p = -0.34$
2 -0.27	7 -0.32
3 -0.32	8 -0.33
4 -0.35	9 -0.31
5 -0.41	Mean -0.322

Next for each star $\log t_p/t_s$ was found in accordance with § 7, No. 7. Table V. contains these quantities and the number of images on which they depend.

TABLE V.

Star.	$\log t_c/t_p$.	$\log t_p/t_s$ = $\log \tau_s/\tau_p$.	No. of Images.
η	-0.322	2.006	7
b	$+0.033$	2.020	6
f	-0.026	2.028	5
c	$+0.134$	1.968	5
d	$+0.221$	2.020	4
e	$+0.278$	1.977	4
h	$+0.560$	1.980	4
Weighted means	$+0.078$	2.001	

Entering with $\log \tau_s/\tau_p = 2.001$ into Table III., the corresponding value $\log \tau_p = 0.441$ is found.

Hence

$$a = \log t_c/t_p - \log \tau_p = -0.363 \pm 0.095.$$

A considerable uncertainty in the individual values of $\log \tau_s/\tau_p$ is of course not surprising. The p.e. of a may perhaps appear large, but we shall see presently that an error of this amount is thrown almost entirely into the constant m_0 , and therefore is of no consequence. It must also be added that the plates were taken under extremely bad atmospheric conditions, and consequently show images much below the average quality.

After having computed the quantities $\log t_c/t_p$, $\log t'_c/t'_p \dots$ for all exposures and taken the means for each star, we find, by subtracting a from these means, the values of $\log \tau_p$, and entering into Table II. (for $a = 0.25$) the corresponding $(m - m_0)$. The figures representing these quantities are given for every star in Table VI.

TABLE VI.

Star (Bessel's Notation).	$\log \tau_p$.	$(m - m_0)$.	Star.	$\log \tau_p$.	$(m - m_0)$.	Star.	$\log \tau_p$.	$(m - m_0)$.
η	+0.041	+0.10	12	+1.837	+4.16	33	2.355	+5.24
b	+0.396	+0.96	24	+1.827	+4.13	9	2.425	+5.38
f	+0.337	+0.82	s	+1.790	+4.06	1	2.393	+5.32
c	+0.497	+1.20	19	+1.880	+4.25	30	2.458	+5.45
d	+0.584	+1.40	29	+1.963	+4.42	18	2.541	+5.63
e	+0.641	+1.54	4	+2.018	+4.54	27	2.596	+5.74
h	+0.923	+2.18	22	+1.996	+4.49	13	2.596	+5.74
g	+1.166	+2.72	10	+2.098	+4.70	21	2.750	+6.05
k	+1.363	+3.15	39	+2.153	+4.82	2	2.766	+6.08
			17	+2.051	+4.61	15	2.796	+6.15
34	+1.416	+3.26						
l	+1.535	+3.52	37	+2.165	+4.84	36	2.833	+6.22
p	+1.561	+3.57	31	+2.201	+4.92	8	2.716	+5.98
32	+1.709	+3.88	20	+2.277	+5.08	25	2.733	+6.02
38	+1.755	+3.98	23	+2.217	+4.95	11	3.085	+6.74
			7	+2.305	+5.14			

We may here examine how far the resulting magnitudes are affected by an error in α of the amount of the p.e. stated above. Let us therefore assume for α three values: -0.458 , -0.363 , and -0.268 , and with the corresponding values of $\log \tau_p$ let us take $(m - m_0)$ from Table II. It will be sufficient to do this for the three stars η , p , 11 which mark the beginning, middle, and end of the series. We find:—

		$\alpha =$			II.-I.	II.-III.
		-0.458 .	-0.363 .	-0.268 .		
		I.	II.	III.		
η	$m - m_0 =$	+0.34	+0.10	-0.14	-0.24	+0.24
p		+3.77	+3.57	+3.37	-0.20	+0.20
11		+6.93	+6.74	+6.54	-0.19	+0.20

The almost perfect constancy of the differences II.-I. and II.-III. justifies the above assertion, that errors in α of the assumed amount are almost entirely thrown into the constant m_0 , but do not sensibly affect the scale.

§ 9. *Comparison of Photographic with Visual Magnitudes.*—The stars discussed in the preceding paragraph have been visually examined with great accuracy at Potsdam. In a comparison between visual and photographic magnitudes the correction for "colour" must be taken into account. As a measure for this correction we may conveniently use Hertzsprung's "effective wave-lengths," determined by means of the grating method, and we may be allowed to assume that the correction to magnitude, on account of colour, is proportional to the effective wave-length. The zero-point of this correction may be fixed at $\lambda = 4100$. Each

star of Table VI. thus leads to an equation of condition of the form

$$(m - m_0) - \text{Potsdam vis.} = a + b(\lambda - 4100) \quad (22)$$

from which the constants a and b are to be found by a least squares solution.

Instead of the individual stars we shall discuss the means of the nine groups into which Table VI. has been subdivided. For these groups the following data are available:—

TABLE VII.

Group.	$(m - m_0)$ m	Potsdam Visual Mag. m	Effective Wave-length (Hertzprung).
I.	+0.90	3.95	4101
II.	+2.40	5.48	4098
III.	+3.64	6.81	4118
IV.	+4.20	7.12	4129
V.	+4.63	7.54	4141
VI.	+4.99	7.96	4144
VII.	+5.40	8.40	4141
VIII.	+5.95	8.82	4145
IX.	+6.24	8.74	4264

A method of least squares solution of equations of the type (22) leads to the following values:—

$$a = -3.09; \quad b = +0.0036.$$

The Cape photographic system is thus represented by

$$(m - m_0) + 3.09,$$

and the Potsdam photographic system by

$$\text{Potsdam visual} + 0.0036(\lambda - 4100).$$

In Table VIII. the results are finally collected in such a form that direct comparisons may be made between the Potsdam photographic system on the one hand and the Cape system C_2 of the present paper, based on Kron's law, on the other. In addition, the Cape system C_1 of the previous paper, based on Schwarzschild's law, and also Schwarzschild's Ottakring observations S are stated and compared with Potsdam.

TABLE VIII.

Group.	P.	C_2 .	C_1 .	S.	$C_2 - P.$	$C_1 - P.$	S - P.
I.	3.95	3.99	3.99	4.07	+0.04	+0.04	+0.12
II.	5.47	5.49	5.41	5.41	+0.02	-0.06	-0.06
III.	6.87	6.73	6.65	6.65	-0.14	-0.22	-0.22
IV.	7.22	7.29	7.22	7.13	+0.07	0.00	-0.09
V.	7.69	7.72	7.66	7.61	+0.03	-0.03	-0.08
VI.	8.12	8.08	8.04	7.97	-0.04	-0.08	-0.15
VII.	8.55	8.49	8.47	8.42	-0.06	-0.08	-0.13
VIII.	8.99	9.04	9.05	8.95	+0.05	+0.06	-0.04
IX.	9.33	9.33	9.35	9.41	0.00	+0.02	+0.08

The differences $C_2 - P$, $C_1 - P$, and $S - P$ are graphically represented in fig. 6. It appears that the adoption of Kron's law leads to a highly satisfactory systematic agreement between magnitudes derived from photographic and visual methods. The assumption of the Schwarzschild law, on the other hand, introduces a slight but marked curvature, which must be ascribed to the fact that this law is not the correct expression for the photometric relations involved in the problem. The discrepancies between the Kron and Schwarzschild law become much more strongly accentuated when magnitudes outside those of Table VIII. are considered. Thus a star whose magnitude, according to the Kron law, is $0^m \cdot 00$, would be $+0^m \cdot 72$ accord-

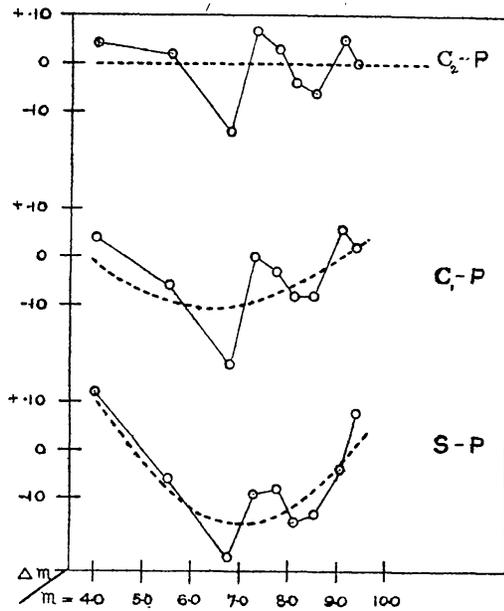


FIG. 6.

ing to Schwarzschild's law, and again a star of magnitude $14^m \cdot 00$ according to the Kron law would be $14^m \cdot 37$ according to the Schwarzschild law.

That the two laws should differ so much in the case of faint stars, although in that region the Kron law merges into the Schwarzschild law, is of course due to the fact that for the magnitudes of the column $C_1 p$ was determined from the values of $\log t_s/t_p$ of the brightest stars, viz. :—

$$p = \frac{4 \cdot 35}{2 \cdot 5 \times 2 \cdot 001} = 0 \cdot 870,$$

whereas the Kron law adopted for column C_2 approaches for faint stars a Schwarzschild equation whose $p = \frac{1}{1 + \alpha} = 0 \cdot 800$. Hence there results a considerable difference in scale. The example

shows how unsafe it is to use the Schwarzschild law over a wide range of magnitudes.

§ 10. *Corrections to the Magnitudes of the South Polar Area.*— Since the magnitudes published in my previous paper are based on the Schwarzschild law, corrections will be required to reduce them to the Kron law. In this instance the principal and secondary images of the three brightest stars, $-89^\circ, 47$; $-89^\circ, 53$; $-89^\circ, 34$, may be utilised for the determination of the constant a . We find for the mean of these stars, by combining the results of the six plates:—

$$\log \tau_s/\tau_p = 2.108 \pm 0.021; \log t_c/t_p = 0.030.$$

But, according to Table III., the value of $\log \tau_p$ corresponding with $\log \tau_s/\tau_p$ is

$$\log \tau_p = 1.530.$$

Hence

$$a = 0.030 - 1.530 = -1.500.$$

For the determination of m_0 the photometric magnitude of $-89^\circ, 47$ (σ Octantis) = $5^m.73$ has been adopted. The observations of this star show

$$\log t_c/t_p = -0.290.$$

Hence

$$\log \tau_p = -0.290 + 1.500 = 1.210,$$

and from Table II.

$$m - m_0 = +2.81,$$

$$m_0 = 5.73 - 2.81 = 2^m.92.$$

The agreement of m_0 with the value $3^m.09$ found for the Pleiades is surprisingly good, and seems to corroborate the conclusion drawn from Kron's experiments that the optimal intensity is constant for plates of the same brand. (In both cases Ilford-Monarch plates were used.) The agreement must, however, be considered somewhat fortuitous, since according to Table III. the p.e. ± 0.021 of the quantity $\log \tau_s/\tau_p$ would correspond with a probable error in m_0 of approximately $\pm 0^m.7$.

With the derived data we may now compute for a number of magnitudes the corresponding $(m - m_0)$ and $\log \tau_p$ according to Kron's law. These quantities are shown in the first three columns of Table IX.

On the other hand, the magnitudes in the previous paper were derived from the Schwarzschild equation:

$$m = 6.32 + 2.05 \log t_c/t_p = 6.32 + 2.05 (\log \tau_p + a),$$

or

$$m = 3.25 + 2.05 \log \tau_p \quad . \quad . \quad . \quad (23)$$

Substituting $\log \tau_p$ into this equation, the figures of the fourth column are found, which therefore represent the magnitudes according to the Schwarzschild law.

TABLE IX.

Kron Law.			Schwarzschild Law.	Correction
$m.$	$m - m_0.$	$\log \tau_p$ (Table II.)	(Equation (23.) $m.$	($k - s$).
m	m	m	m	m
5.00	2.08	0.881	5.06	-0.06
6.00	3.08	1.331	5.98	+0.02
7.00	4.08	1.801	6.94	+0.06
8.00	5.08	2.278	7.92	+0.08
9.00	6.08	2.764	8.92	+0.08
10.00	7.08	3.255	9.92	+0.08
11.00	8.08	3.749	10.93	+0.07
12.00	9.08	4.245	11.95	+0.05
13.00	10.08	4.742	12.97	+0.03

The corrections of the last column are to be applied to the magnitudes of the stars of the south polar area in my previous paper in order to reduce them to the photometric conditions expressed by Kron's law.

§ 11. *Influence of Variations in α and m_0 on Derived Magnitudes.*

—Until confirmed by further experiments, the assumption $\alpha = 0.25$ for all plates, on which the preceding calculations are based, must be accepted with reserve. Kron's investigation shows a noticeable departure from this value (viz. $\alpha = 0.19$) in the case of the Ideal-Diapositiv plates, and although his "fast" plates are remarkably concordant, the presence of such an exception cannot be overlooked. It is, therefore, important to examine to what extent the derived magnitudes are changed, if for instance $\alpha = 0.20$ be substituted for $\alpha = 0.25$. The computations differ from those of § 8 only in so far that Tables II. and III. have to be used for the adopted value of α .

With regard to m_0 , it was remarked that its determination depends on the accuracy with which $\log \tau_s/\tau_p$ of the bright stars can be ascertained. The p.e. of this quantity in the case of the Pleiades may be estimated at ± 0.02 , so that discrepancies amounting to 0.06 are certainly possible. We shall, therefore, recompute the magnitudes of the nine groups on the assumption that $\log \tau_s/\tau_p = 1.94$ instead of 2.00, retaining $\alpha = 0.25$.

The two solutions will be denoted by C_α and C_{m_0} respectively. They are shown in Table X., together with C_2 of Table VIII., the zero points being so chosen that the magnitudes of the first group agree in all the series.

TABLE X.

Group.	$\log t_c/t_p$ (Observed.)	C_2 (Table VIII.)	C_a .	C_{m_0} .	$C_a - C_2$.	$C_{m_0} - C_2$.
I.	+0.010	3.99	3.99	3.99	0.00	0.00
II.	+0.660	5.49	5.44	5.55	-0.05	+0.06
III.	+1.232	6.73	6.68	6.83	-0.05	+0.10
IV.	+1.496	7.29	7.25	7.40	-0.04	+0.11
V.	+1.700	7.72	7.68	7.84	-0.04	+0.12
VI.	+1.870	8.08	8.05	8.20	-0.03	+0.12
VII.	+2.071	8.49	8.48	8.62	-0.01	+0.13
VIII.	+2.338	9.04	9.05	9.17	+0.01	+0.13
IX.	+2.480	9.33	9.35	9.47	+0.02	+0.14
X.	+5.294 (assumed)	15.00	15.28	15.16	+0.28	+0.16

To the nine groups, which embrace the actually observed stars, a tenth group is added showing the differences between the three assumptions for stars of the 15th magnitude, if in all cases the same quantity $\log t_c/t_p = +5.294$ be assumed as the observed quantity. It appears that a star, whose magnitude according to assumption C_2 is 15.00, shows a magnitude 15.28 under C_a and 15.16 under C_{m_0} . Differences of this amount are doubtless of significance in fundamental investigations of star magnitudes. They demonstrate the necessity of determining the quantities a and m_0 with a high degree of accuracy; particularly the former quantity, since the magnitude "scale" of the fainter stars depends largely on it. Whether a sufficient accuracy is practically attainable remains to be shown by further experiment. So far the results of observation are encouraging, and probably a stricter treatment of the plates in the process of developing may result in an even greater consistency of the results.

§ 12. *Changes of a and m_0 from Plate to Plate.*—It was pointed out in my previous paper (pp. 611–12), that changes from plate to plate are on the whole insignificant to such a degree that results derived from means of only two to three plates are highly concordant among themselves. A reliable test for this assertion is afforded by the quantities $\log t_c/t$ derived for the same stars from different plates. It was shown that these quantities differ from the $\log \tau$ of Table II. by a constant. Hence, if the constants a and m_0 of Kron's equation have the same values for different plates, the quantities $\log t_c/t$ referring to the same stars on different plates should be brought into agreement by adding appropriate constants to their values on each plate. Thus, if the constants be so chosen that the quantities $(\log t_c/t + \text{const.})$ agree for one particular star, they should, in the absence of changes in a and m_0 , likewise agree for all the other stars. To what extent the plates agree in this respect is demonstrated by the following values of $(\log t_c/t + \text{const.})$, which comprise series of three and four plates specified at the heads of their respective columns in Table XI. The observations refer to

groups of stars of the south polar area arranged in order of magnitude, the constants being so chosen that $(\log t_c/t + \text{const.}) = 0$ for the first group.

TABLE XI.

Group.	Mean Magnitude of Group. m	log $t_c/t + \text{const.}$		
		Three Plates with Grating.	Three Plates with Grating.	Four Plates with- out Grating.
I.	6.8	0.00	0.00	0.00
II.	8.9	0.99	0.98	1.02
III.	10.0	1.55	1.54	1.58
IV.	10.7	1.90	1.88	1.89
V.	11.0	2.06	1.99	2.06
VI.	11.4	2.25	2.18	2.22
VII.	11.7	2.44	2.37	2.39
VIII.	12.0	2.57	2.52	2.56

The close agreement between the three series appears to justify the assumption that the constants a and m_0 of Kron's equation do not essentially differ from plate to plate. It was pointed out in my previous paper that the assumption of their constancy may be extended even to observations taken with different instruments (*l.c.*, pp. 611-12). On the whole, there is strong evidence in favour of the view that over a range of about 6 to 7 magnitudes the values $m_0 = 3^{m.0}$ and $a = 0.25$ may be adopted universally, at least under the optical conditions on which the discussed observations are based. It is probable, however, that m_0 may vary with the aperture of the telescope.

If the universality of m_0 and a should be confirmed by further investigations, the problem of photographic magnitudes would assume an extremely simple character. It would be sufficient, then, to take a series of exposures—no grating being required in this case—and to determine the A-curve, *i.e.* the relation between actinic effect and time, in accordance with the instructions of § 7. This curve will then enable us to determine the quantities $\log t_c/t$ for every star. Now, if the magnitude m_x of one particular star be known, we enter with argument $(m_x - 3.0)$ into Table II. and note the corresponding value of $\log \tau_x$. Then

$$a = \log t_c/t_x - \log \tau_x$$

and this quantity subtracted from the values of $\log t_c/t$ for every star will lead to the corresponding values of $\log \tau$, which in turn directly determine m by means of Table II.

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