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sions as "attached to a carbon atom at the end of the chain," &c. &c., especially in text-books for students (who generally find it hard to distinguish fact from theory), and unaccompanied by any warning, or explanation of the real value of the expressions, is, I think, much to be deprecated. It behoves us ever to bear in mind that in science we deal with phenomena made up of an infinite number of infinitely small units, and that therefore any deficiency in careful measurement may lead to the supposition of a causal connexion where no such connexion exists, that even at the best we gain only approximations, and that from the most carefully conducted experiments we cannot tell what may happen in extreme cases (Principles of Science)*.

[To be continued.]

XXV. *On some Dynamical Conditions applicable to Le Sage's Theory of Gravitation.* By S. TOLVER PRESTON†.

1. **T**HE tendency of modern science is undoubtedly to look to the existence of physical conditions or processes in those natural phenomena to which the theory of "action at a distance" has been applied. The gravitation theory of Le Sage has therefore of late naturally received a considerable share of attention. Le Sage finds it necessary, as a basis to his theory, to lay down certain conditions, some of which cannot but be regarded as arbitrary. Thus (as given in the paper by Sir William Thomson, 'Philosophical Magazine,' May 1873) Le Sage assumes among other conditions:—

(1) That the *direction* of the streams of particles producing gravity is such that an *equal* number of particles are moving in *all* directions.

(2) That the streams are all *equally dense*; or the total assemblage of matter forming the streams is of the *same* density in *all* parts.

(3) That the *mean velocity* of the streams is everywhere the *same*.

2. These conditions cannot but be considered arbitrary. My object is to call attention to the fact (which, if it has been observed, would certainly appear to be deserving of more attention than it has received) that all these conditions which Le Sage, with the limited knowledge of his day, assumed to be arbitrary, are in reality inevitable deductions following

* For a fuller discussion of the constitutional formulæ of alcohols and acids &c., see *Die modernen Theorien*, 2nd edit. pp. 190-202.

† Communicated by the Author.

from the dynamical principles connected with the kinetic theory of gases, or that Le Sage unconsciously enunciated the inevitable principles of the kinetic theory—that, in short, all the conditions laid down in Le Sage's theory are perfectly satisfied by a gas whose particles are very minute, and consequently the mean length of path of whose particles is very great. In other words, it may be stated as a general proposition, that when two bodies are immersed in a gas at a less distance apart than the mean length of path of the particles of the gas, the two bodies will tend to be urged together. Thus all the arbitrary conditions of Le Sage's theory (and all the facts of gravity) would follow as inevitable deductions from the simple fundamental admission of the existence of matter in space, whose normal state is a state of motion.

3. The part of Le Sage's theory which most calls for explanation, and which he makes no attempt to explain, is (even if we allow as a purely arbitrary fact that the motion of his particles took place at one time uniformly or *equally* towards all directions) how this uniformity of motion of the particles could be kept up under the continual changes of direction resulting from the collisions of the particles against themselves and mundane matter. Now it has been proved mathematically by Professor Maxwell, in connexion with the kinetic theory of gases, that a self-acting adjustment goes on among a system of bodies or particles in free collision, such that the particles are caused to move *equally* towards *all* directions, this being the condition requisite to produce equilibrium of pressure. The method of calculating the rate of the above self-acting adjustment for any case is given in the Philosophical Transactions for 1866. This adjustment is of such a rigid character that, if by any artificial means the motions of the particles were interfered with and made to take place irregularly (*i. e.* unequally in different directions), the particles when left to themselves would in a very short time automatically return back to the above *regular* form of motion, *i. e.* so that an equal number of particles are moving in any two opposite directions. Thus it follows that when a system of particles are left in space with nothing to guide them, they will, by the rigid principles of dynamics adjust their motions in such a way as to be competent to produce the effects of gravity. In other words, the movement of streams of particles with perfect uniformity at all angles (which Le Sage assumed as a mere arbitrary postulate) is found to be the necessary consequence of dynamical principles; or the particles themselves adjust their motions so as to move in uniform streams in all directions; and, further, when any disturbance of the unifor-

mity of the motion of the particles takes place due to their collisions with mundane matter, the particles themselves read-just the uniformity of motion.

4. Le Sage imagined that the collisions of the particles disturbed permanently the uniformity of their motions, and therefore supposed these collisions to take place only at intervals of time very remote from each other. Thus he assumes " that not more than one out of every hundred of the particles meets another during several thousands of years ; so that the uniformity of their motions is scarcely ever disturbed sensibly." We now know that, so far from the collisions of the particles among themselves disturbing the uniformity of their motions, this is the very cause which corrects and maintains the uniformity of motion, or preserves the uniformity of motion in opposition to external disturbing causes. The assumption, therefore, of the above enormous interval of time between the collisions of the particles, though admissible, is by no means necessary. The only necessary condition is that the path of the particle should be a certain *length*, not that a certain time should be occupied in traversing it. The time taken by the particle in traversing its path depends on its velocity ; and this time might therefore be small, provided, under the conditions of the case, the velocity of the particle were high. Le Sage imagined that the collisions were detrimental, not only in destroying the uniformity of the motion of the particles, but also in destroying *vis viva* ; and he therefore supposed the collisions to take place as seldom as possible. This belief in the destruction of *vis viva* at collision was universal at the time of Le Sage ; and he therefore assumed that the gravific particles would finally come to rest, and gravity cease to exist. We now know that this is an error, and that motion is as naturally maintained among a system of particles as rest. Thus the one thing requiring to be admitted to account for all the effects of gravity is, that the universe is immersed in a gas the mean length of path of whose particles is great.

5. The other assumptions or postulates of Le Sage in connexion with his theory, viz. equal *density* in all parts of the streams of moving particles, equal *mean velocity* in all parts, follow no less as automatic consequences from the recent dynamical investigations connected with the kinetic theory of gases. Thus the conditions of Le Sage's theory become converted from a series of arbitrary assumptions or postulates, to a series of deductions following from the rigid principles of dynamics.

6. It forms a truly wonderful fact to consider, that a system

of bodies or particles left to themselves, with nothing to guide them but their own collisions (which might well be regarded as fulfilling all the essentials of a chaos), produces and maintains the most rigid system of order, such that the number of particles contained in unit volume of the system (taken anywhere) is equal, the mean velocity equal in all parts, the mean distance of the particles the same in all parts, and the particles are moving uniformly towards all directions in all parts. Such is the result produced by pure dynamics. In fact it may be said that leaving the bodies to themselves constitutes the most perfect system of control, for any interference whatever would disturb the regularity of the motions. This regularity of movement is not only naturally continued, but forcibly and automatically maintained against any disturbance,—such that if it were imagined that a system of bodies were purposely put in motion in the most chaotic manner possible, the motion would of itself in a short time become regular, or the whole would become a system of order and uniformity.

7. Clausius, as is known, has investigated a relation between the mean length of path of the particles of a gas and the diameter of the particles. From this investigation it follows that the mean length of path of the particle of a gas (*i. e.* the average distance which the particle moves before encountering another particle) increases in proportion as the square of the diameter of the particle diminishes. Thus by making the particle small enough, its mean length of path may be increased to any extent. No objection, evidently, can be made to this, for *à priori* one size of particle is just as likely as another. This minute size would render it possible for the particle to possess a high velocity without producing thereby disturbance or displacement among the molecules of ordinary matter; and this high velocity is necessary to accord with the observed facts of gravity. One velocity cannot be said *à priori* to be more likely than another. We must just be guided by the teaching of facts as to what the velocity is*.

* I have shown (*Phil. Mag.* June 1877) that a physical relation exists between the velocity of the particles of a medium constituted according to the kinetic theory and the velocity of propagation of a wave in the medium. Professor Maxwell has calculated (as given in postscript to the paper) the numerical value of this relation at $\frac{\sqrt{5}}{3}$. Thus it appears that if the velocity of propagation of a wave in any medium constituted according to the kinetic theory can be measured, then the velocity of the particles of the medium is given by dividing this velocity by $\frac{\sqrt{5}}{3}$. So, for example, the velocity of the molecules of air is given by dividing the
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8. It is an interesting fact pointed out by Sir William Thomson (Phil. Mag. May 1873) that the distance through which gravity is effective would depend on the distance through which the gravific particles move before being intercepted by collision with each other (which is equivalent to the mean length of path of the particles). By assuming the distance of the stars to be a multiple of the mean length of path of the particles, it would therefore follow that the stars do not gravitate towards each other—this satisfying the condition for the stability of the universe. The assumption of all the bodies of the universe gravitating towards each other is evidently quite inconsistent with stability (as already pointed out by Professor Challis). All that we require to admit is that the effects of gravity hold through as great distances as we have observed them.

9. The distance through which gravity has been observed to act is well known to be but an infinitesimal fraction of the distance of the stars. It may therefore well be that the mean length of path of the particles of the medium producing gravity may be but an infinitesimal fraction of this distance. The column of the gravific medium intercepted between two stars would therefore *on the whole* be at rest, just as a column of gas is at rest between two bodies a visible distance apart (*i. e.* a distance which is a large multiple of the mean length of path of the particles of gas). Le Sage appears to have assumed that the mean length of path of the gravific particles swept through the universe; or he assumed that streams of matter came from the depths of space and passed entirely through the visible universe into space beyond*. This assump-

velocity of sound in air by $\frac{\sqrt{5}}{3}$; so of other gases. Thus it appears that

the velocity of a wave in any medium constituted according to the kinetic theory (such as the velocity of a wave of sound in air) is solely dependent on and proportional to the velocity of the particles of the medium; and this velocity of the wave is independent of the density or pressure of the medium, or of any thing else excepting the velocity of its particles.

* Le Sage assumed that a continual supply of matter from without was necessary for the maintenance of gravity in the visible universe, and that all but a very small fraction of this supply passed through ineffectively and was dissipated again in ultramundane space—a means apparently quite disproportionate to the end in view. We observe that under the principles of the kinetic theory no such supply is necessary, but that all the conditions requisite for gravity may be fulfilled by a medium pervading the visible universe, and which is at rest *as a whole*; or the gravific medium within the bounds of the visible universe may be compared to the air within a receptacle, which is, as a whole, at rest; and therefore there is no more waste of the matter producing gravity than if mundane matter did not exist. If we imagine, in analogy, a being extremely small

tion cannot but be regarded as fantastic, and, as we observe, is by no means necessary. The mean length of path of the particles, so far from being comparable to the dimensions of the visible universe, may be but an infinitesimal fraction of the distance of two of its primary components. All we require to admit is that the mean length of path of the particles of the medium is at least as great as the very limited range through which gravity has been observed to act; or, in order to explain all the observed facts, it is sufficient to admit that the universe is immersed in a gas (or medium constituted according to the kinetic theory) the mean length of path of whose particles is so adjusted as to cause the *minor* or secondary portions of the universe to gravitate towards each other*. Under the simple conception of a variation in the diameter of the particles of a medium, the mean length of path of the particles (and with it the range of gravity) is capable of adjustment with precision to any range. It would probably be difficult to imagine any more simple condition as a mechanical means to an end than this.

10. It is a necessary condition to Le Sage's theory (in order

compared with the mean length of path of an air molecule, to be stationed in the centre of a receptacle containing air; then this minute observer would notice the air molecules passing continually in streams equally in all directions; and the observer, not being able to trace the beginning and end of the path of the molecules, would naturally imagine that the molecules were being supplied in streams from outside the receptacle, and, observing the continued regularity of the streams *equally* in all directions, would suppose that there must be some external mechanism supplying the streams of molecules equally and symmetrically. This is what would be supposed in the absence of the application of the principles of the kinetic theory to the case. On applying this theory, it is seen that the same result can take place without a supply of air; and that the adjustment of the streams of air molecules uniformly in all directions is automatic, or the inevitable result of dynamical principles.

* If two particles of a gas be conceived gradually to increase in size and mass, they will gradually lose their translatory motion and finally come practically to rest. If now the two enlarged particles be at a distance apart less than the mean length of path of the remaining particles of the gas, these two particles will gravitate towards each other under the dynamic action of the other particles of the gas which pass them in streams (the opposed faces of the two enlarged particles being sheltered from the streams). This condition of things probably occurs when a vapour condenses, when the gradually enlarging water vesicles which happen to be at a distance apart less than the mean length of path of the other molecules of the vapour will be driven together by the dynamic action of these molecules, thus accelerating condensation. Thus possibly effects of "gravity" may be imitated by gaseous matter on molecular scale, as is observed in the large scale of the universe. It may be observed that these are simply applications on a smaller scale of the dynamical principles of Le Sage's theory, and therefore the deductions hold if the premises are admitted to be possible.

that gravity may be sensibly proportional to mass) that the total volume of free space in a substance, in the form of interstices between the molecules or in their structure, must be great compared with the total volume of matter contained in the molecules themselves. Le Sage assumed the molecules of substances to have a sort of open structure in the form of cages with wide interstices. This condition of free interstices would be equally satisfied by assuming the molecules to be small relative to their mean distance, or on the condition of the vortex-ring atom theory, without any necessity for making the above somewhat fantastic assumption of cage-structure.

11. It is necessary to assume that the particles producing gravity are in very close proximity compared with molecules, otherwise the particles would be unable by their motion to produce a perfectly equable pressure upon the molecules of matter. It might be thought that, because the particles of the gravific medium are so close, and the molecules of ordinary matter relatively far apart, therefore the quantity of matter in the form of gravific particles enclosed in a given volume of space must be very great compared with the quantity of ordinary matter that that same volume of space would contain—or, in other words, that there must be a relatively enormous quantity of matter in the form of gravific particles. This by no means follows; for although the gravific particles may be very close, the relative quantity of matter in them may be very small, provided the particles themselves are small. Indeed by simply conceiving an extreme degree of subdivision, the particles pervading a given volume of space may by continued subdivision be conceived to be brought into as close proximity as we please; and though the space itself is large, the total quantity of matter thus used may be conceived as small as we please. No consequence how minute the size (or mass) of a particle may be, the effect produced by its motion remains as great, provided its velocity be adequately augmented. The minute size is the very condition adapted to a high velocity; and this minute size is at the same time the necessary condition for a long mean path. Thus we may observe that the mechanical conditions of the problem fit into each other. The matter of the gravific medium is in such a finely subdivided state, and its motion so rapid, that its presence necessarily eludes detection. The pressure (termed "gravity") due to the motion of the particles of the gravific medium is no more difficult of realization than the pressure due to the motion of the molecules of air. If the motion of the molecules of air be unrecognized by the senses, how much more must this be the fact with the minute gravific particles; indeed it is difficult

to see what mechanical objection can be urged against this realization of the problem, which is extremely simple.

12. The theory of "action at a distance" being rejected, which is necessary in order to explain the facts at all, the effects of gravity can in principle be referred to only two conceivable causes. The tendency of two molecules of matter to approach each other can be referred (1) to a motion possessed by the molecules themselves disturbing the equilibrium of pressure of the medium between them; (2) to a motion possessed by the medium itself (in the form of streams or currents) acting upon the molecules. The first of these two conditions appears to be inadmissible, from the fact that we cannot interfere with or modify gravity at will, whereas we can very readily interfere with or modify the motion of the molecules of matter (as by adding or subtracting heat, for example). It therefore would appear that gravity must be due to some motion that we cannot interfere with, *i. e.* to a motion in the external medium which we cannot handle or which is beyond our control. Only one conclusion appears therefore to be possible here; and therefore it would seem that the theory of Le Sage can scarcely be regarded as a mere hypothesis, but rather as an irresistible deduction which is forced upon us in the absence of any other conceivable inference. Certainly, if simplicity be a recommendation, the theory needs no recommendation on that ground.

London, July 1877.

XXVI. *The Finite Integrals of certain Partial Differential Equations which present themselves in Physical Investigations.* By the Rev. S. EARNSHAW, M.A.

To the Editors of the Philosophical Magazine and Journal.

GENTLEMEN,

THE Astronomer Royal, in his treatise 'On Sound and Atmospheric Vibrations,' has drawn particular attention to equations of the following general form,

$$\frac{d^2u}{dt^2} = \frac{d^2u}{dx^2} + \frac{a}{x} \frac{du}{dx}, \quad \dots \dots \dots (1)$$

and has expressed an opinion that equations of this class cannot be approached by any one general method of attack. The equation has been solved by the Astronomer Royal himself in a finite form for the two cases $a=0$ and $a=2$; but in other cases he has had recourse to infinite series, which, it is observed,