# From the Sachdev-Ye-Kitaev model to theories of strange metals and charged/rotating black holes

Subir Sachdev

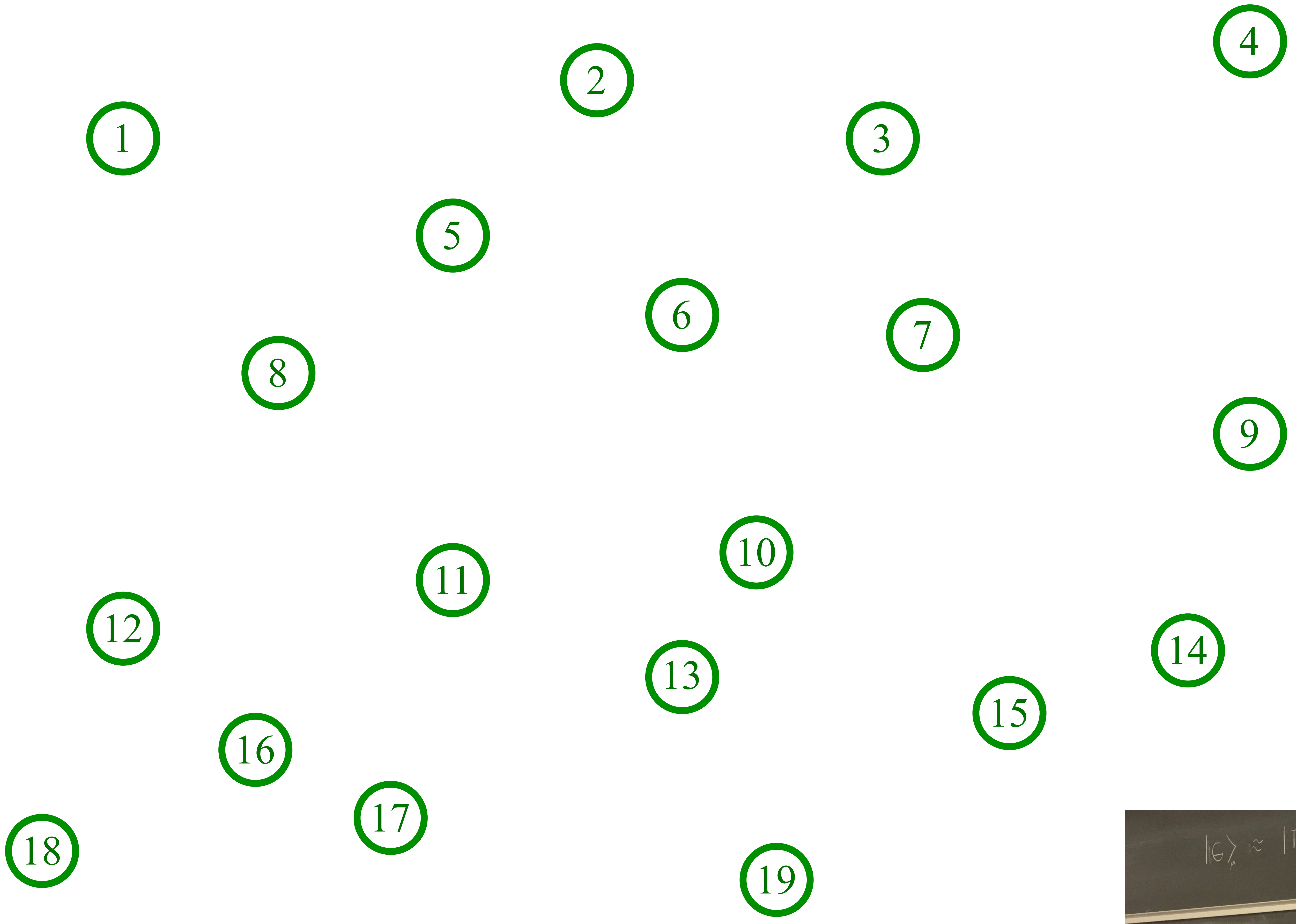
International Conference on Bose-Einstein Condensation, Superconductivity, Superfluidity and Quantum Magnetism
S. N. Bose National Centre for Basic Sciences, Kolkata
November 12-16, 2024

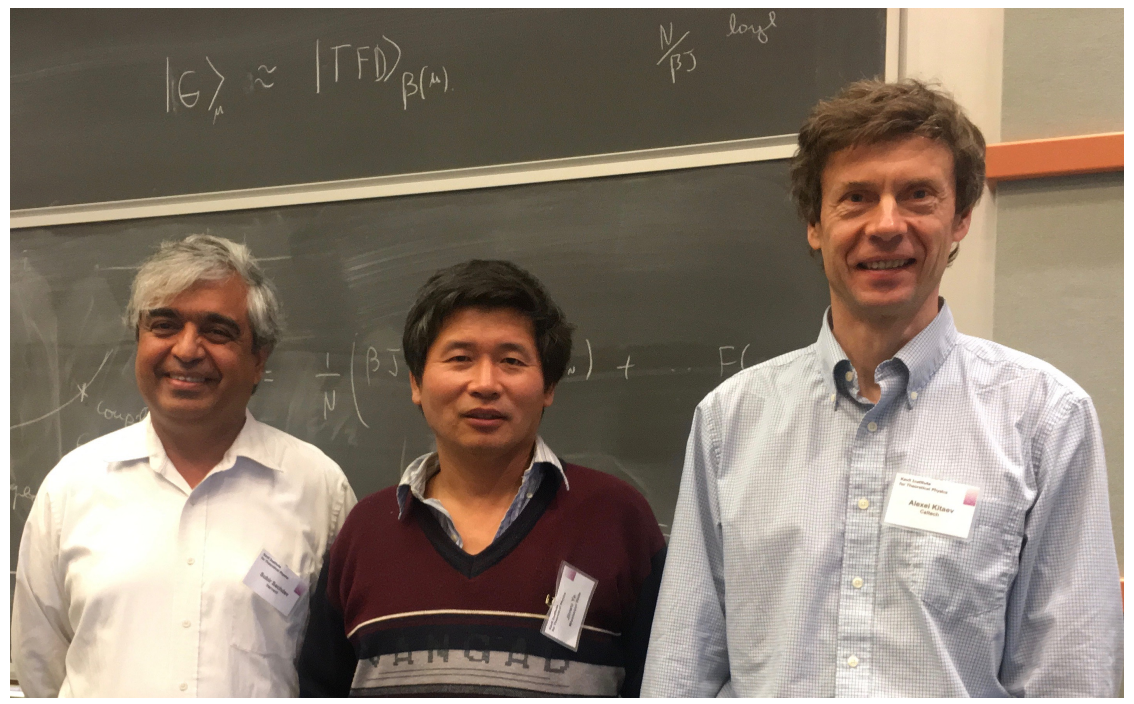
PHYSICS
HARVARD

# SYK model

# The SYK model

Sachdev, Ye (1993); Kitaev (2015)



Pick a set of random positions

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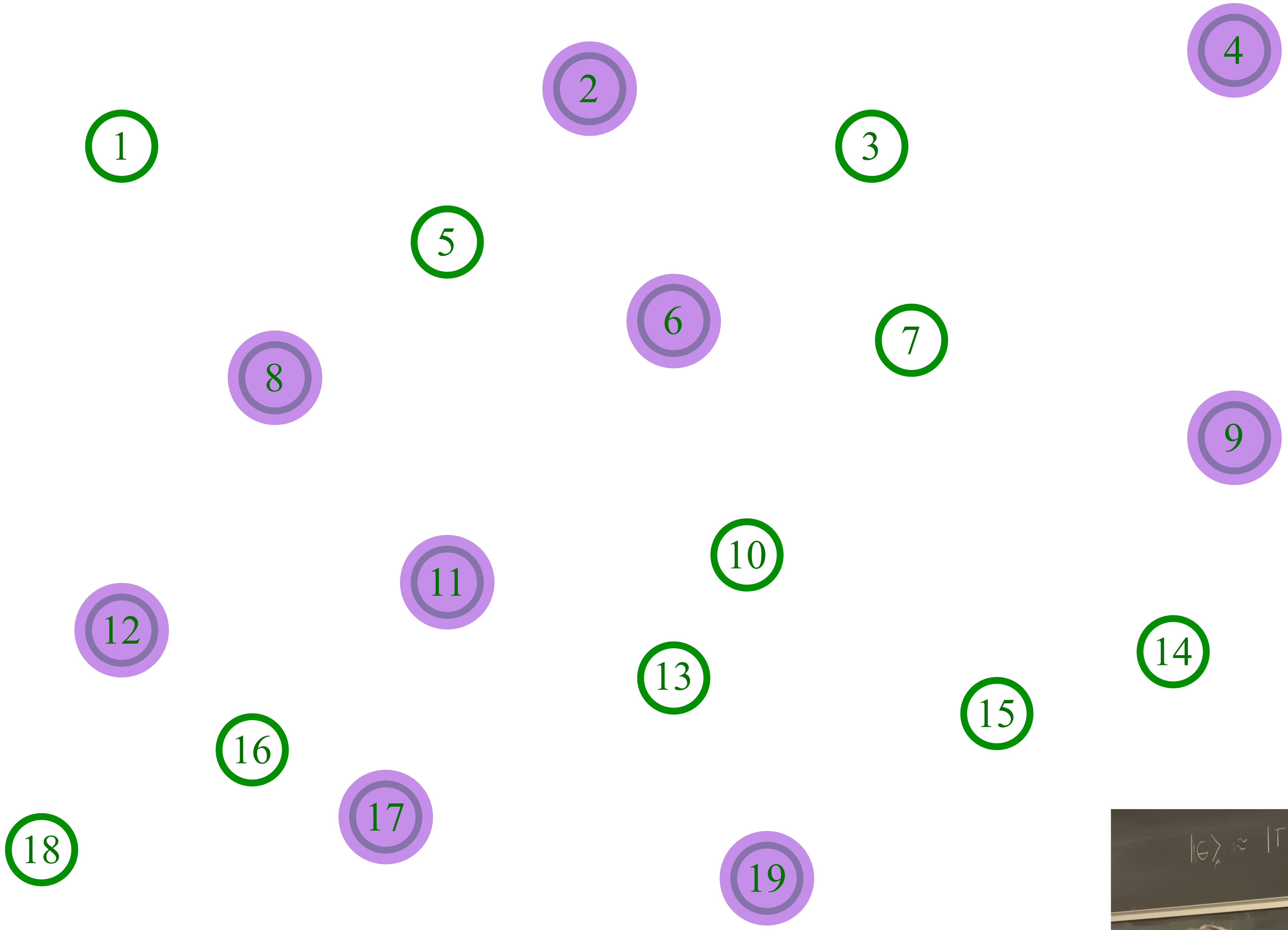
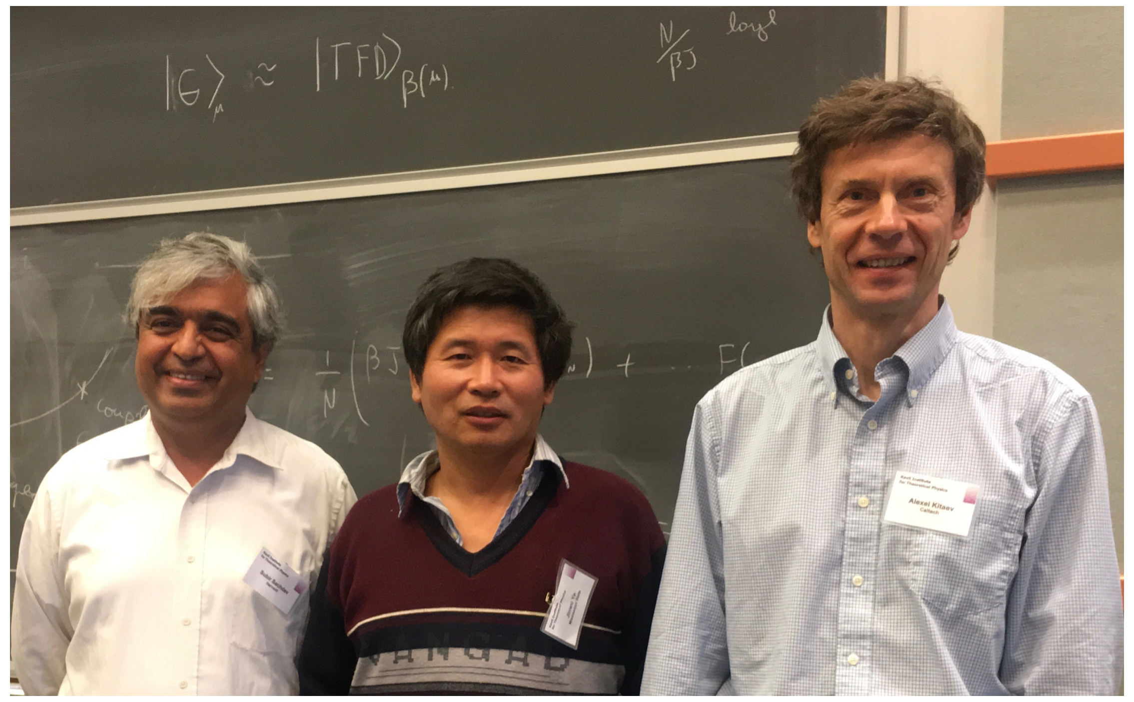



Place electrons randomly on some sites



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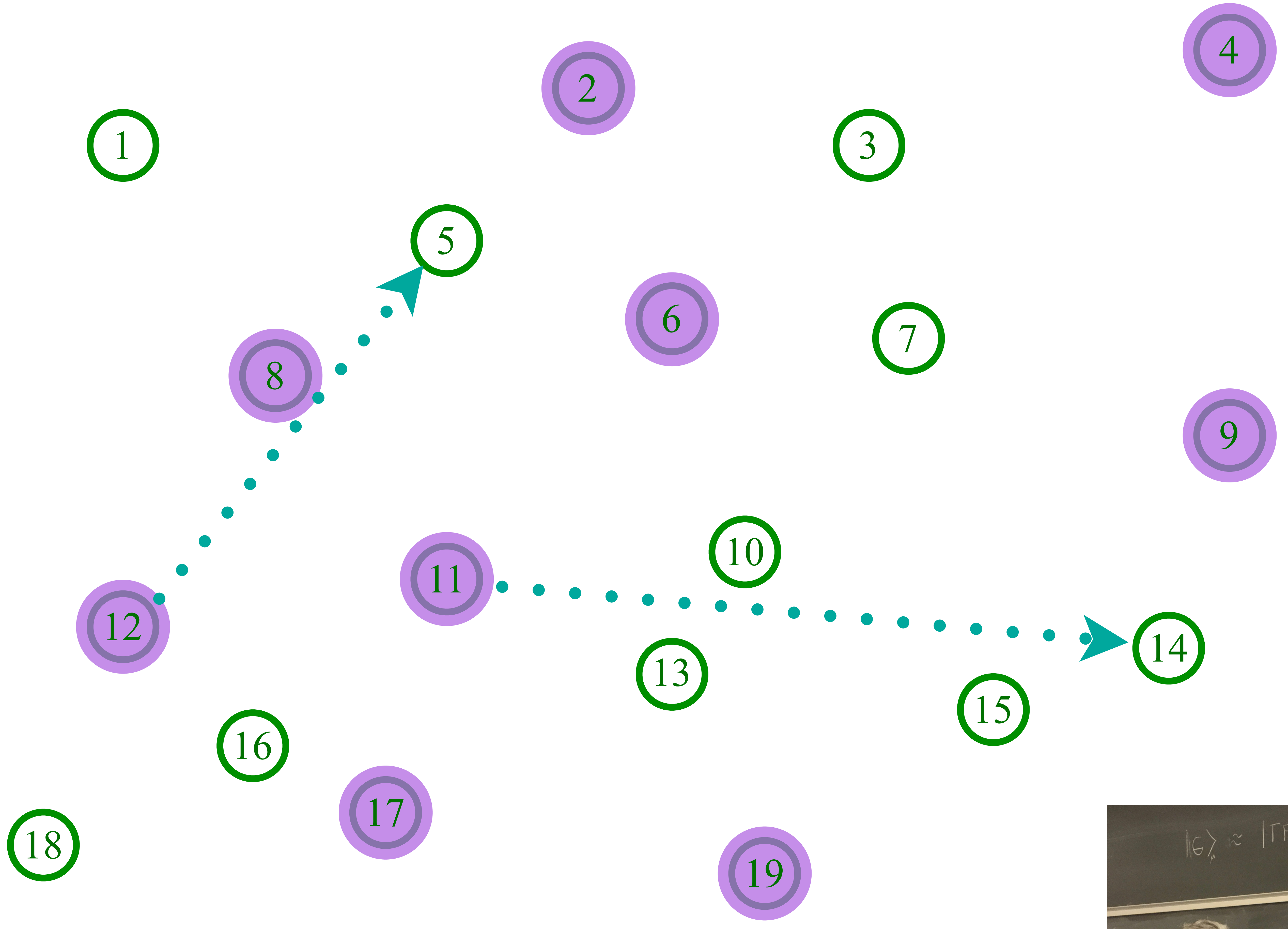
$$U_{11,12;5,14}$$

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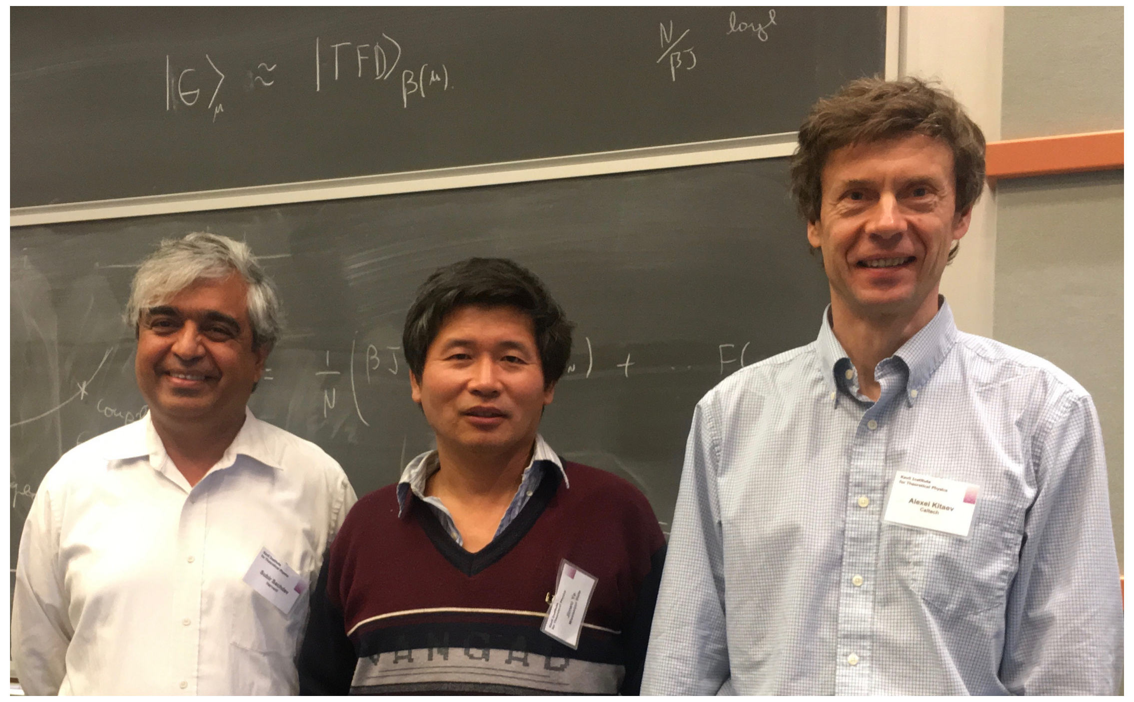

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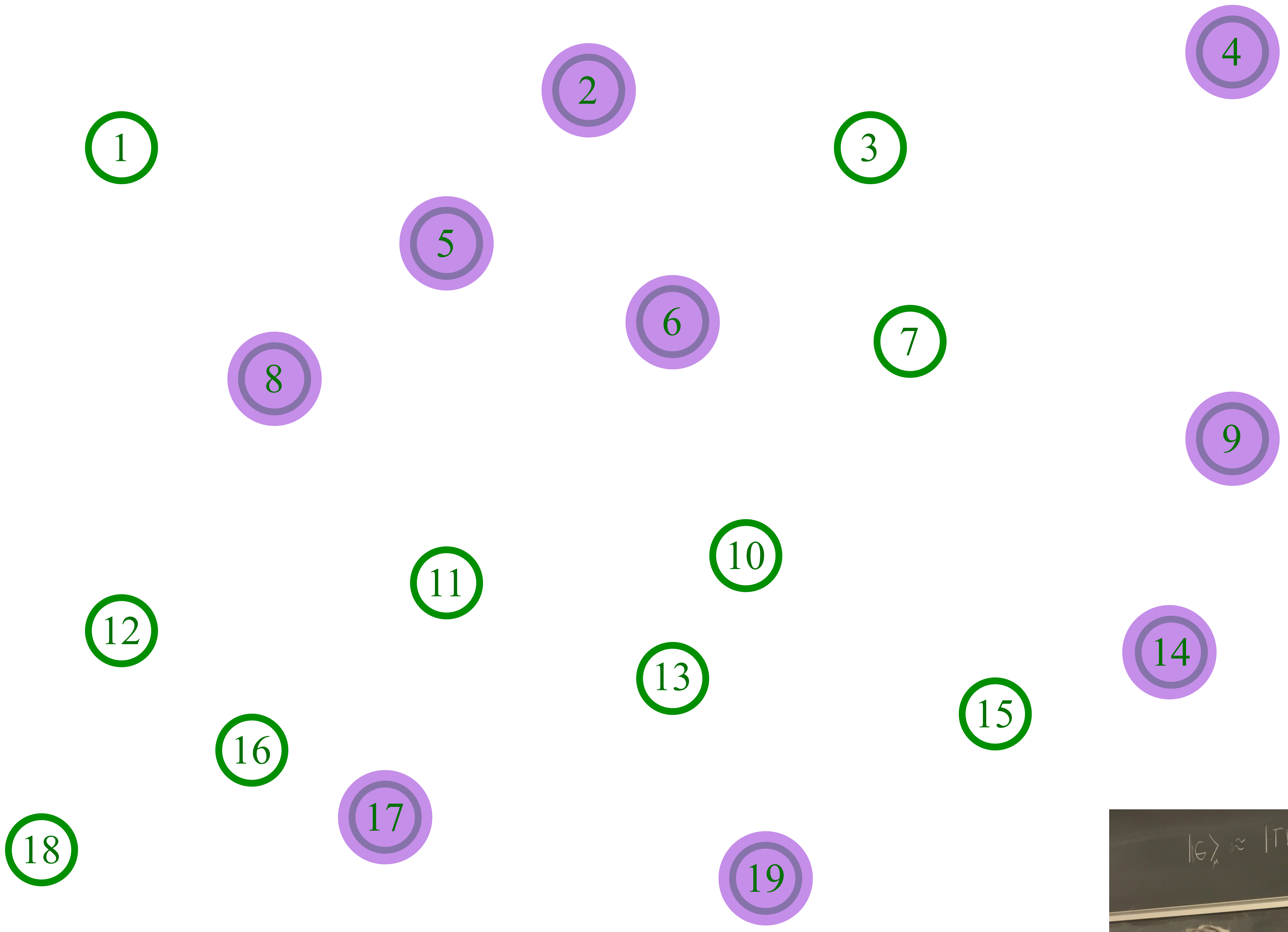
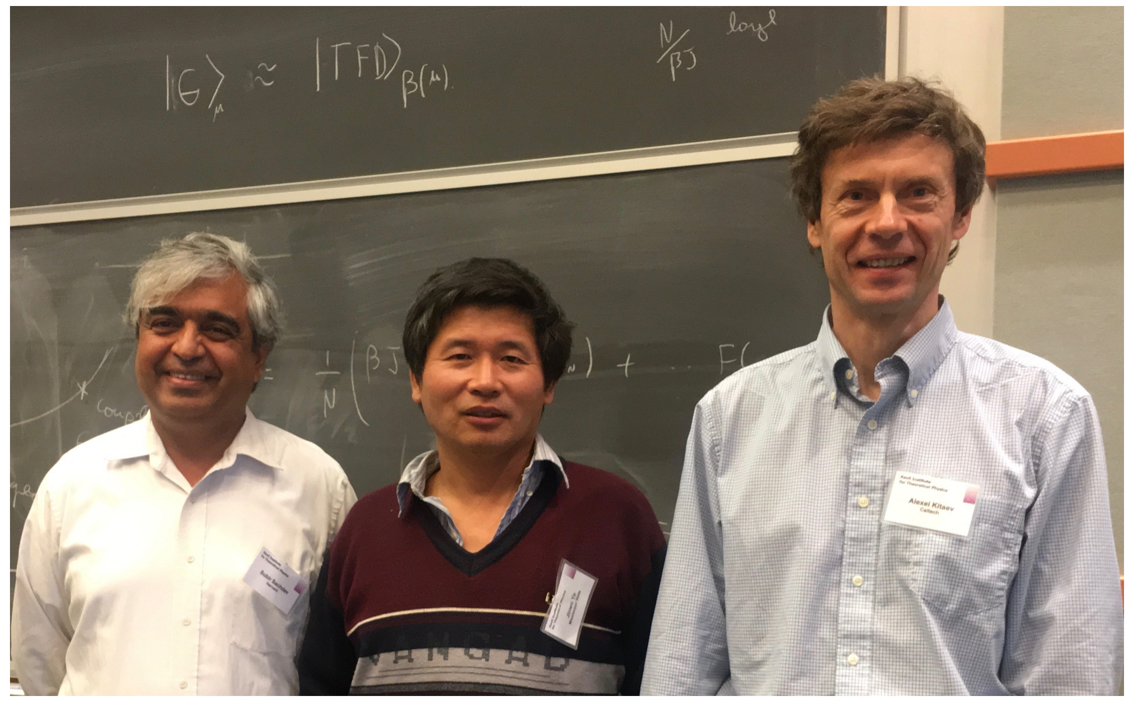



Entangle electrons pairwise randomly



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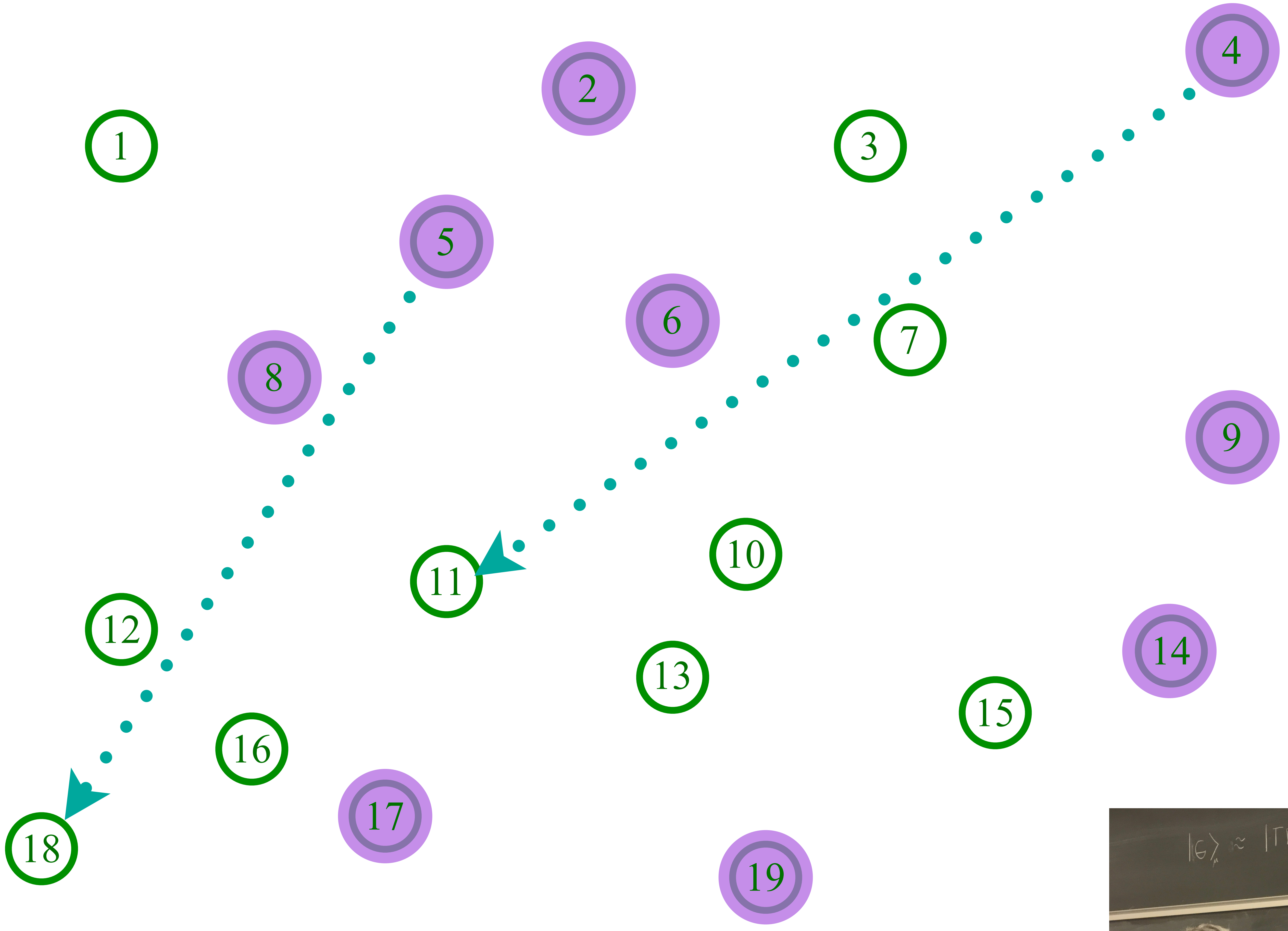
$$U_{4,5;11,18}$$

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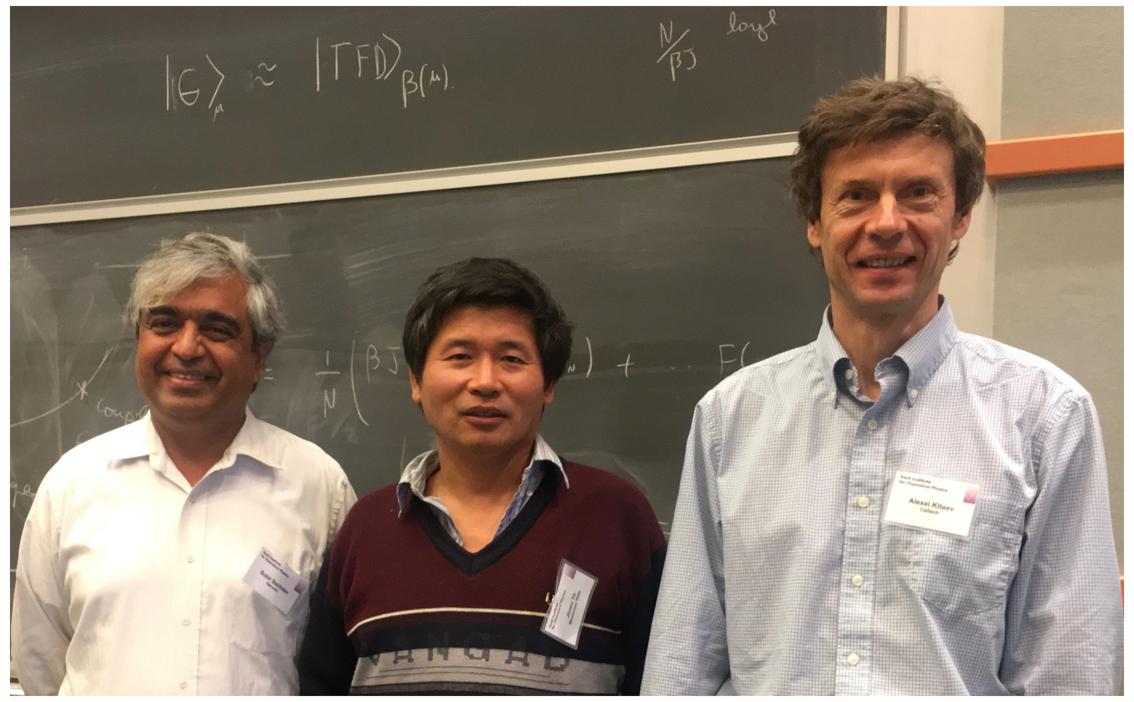

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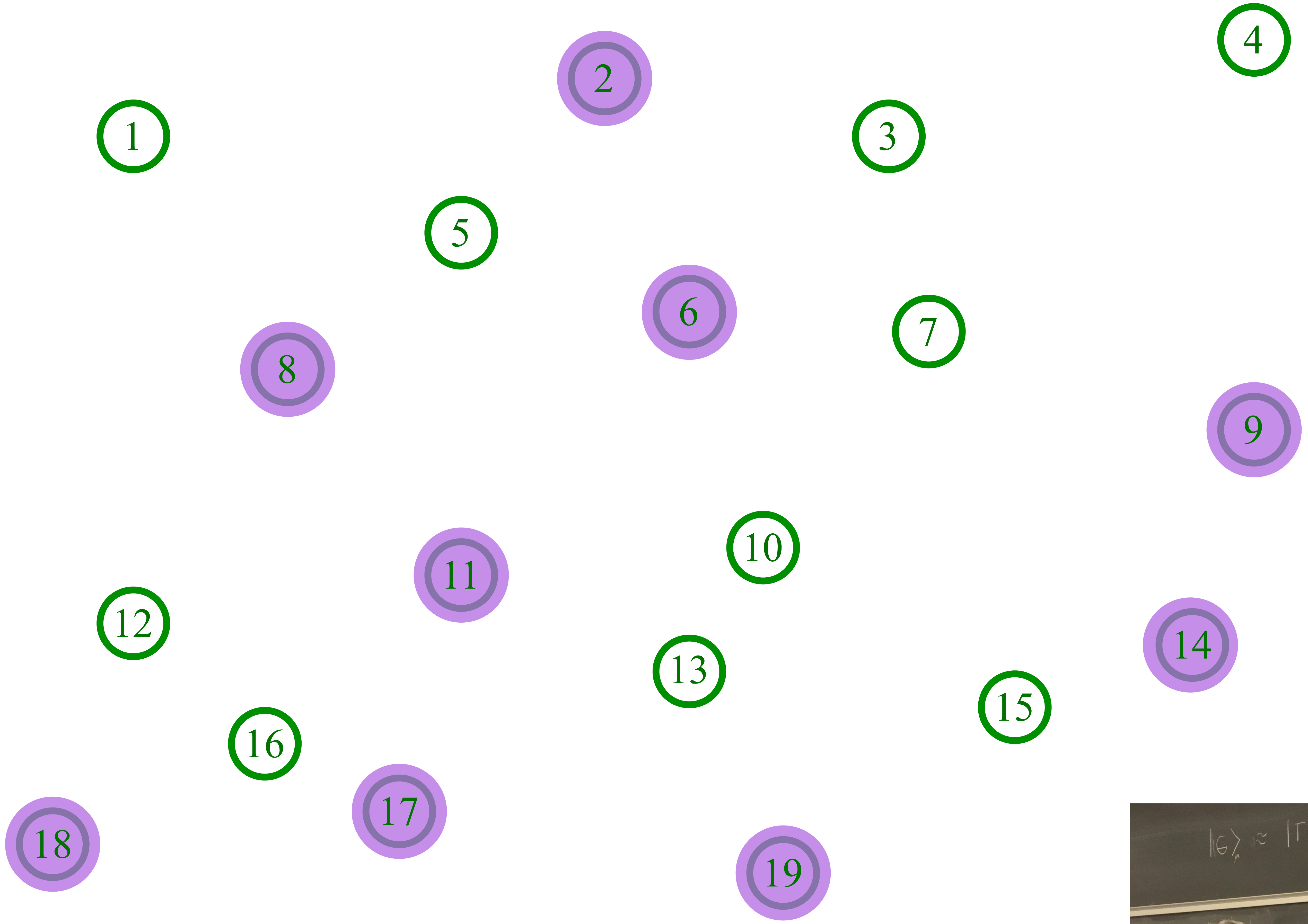


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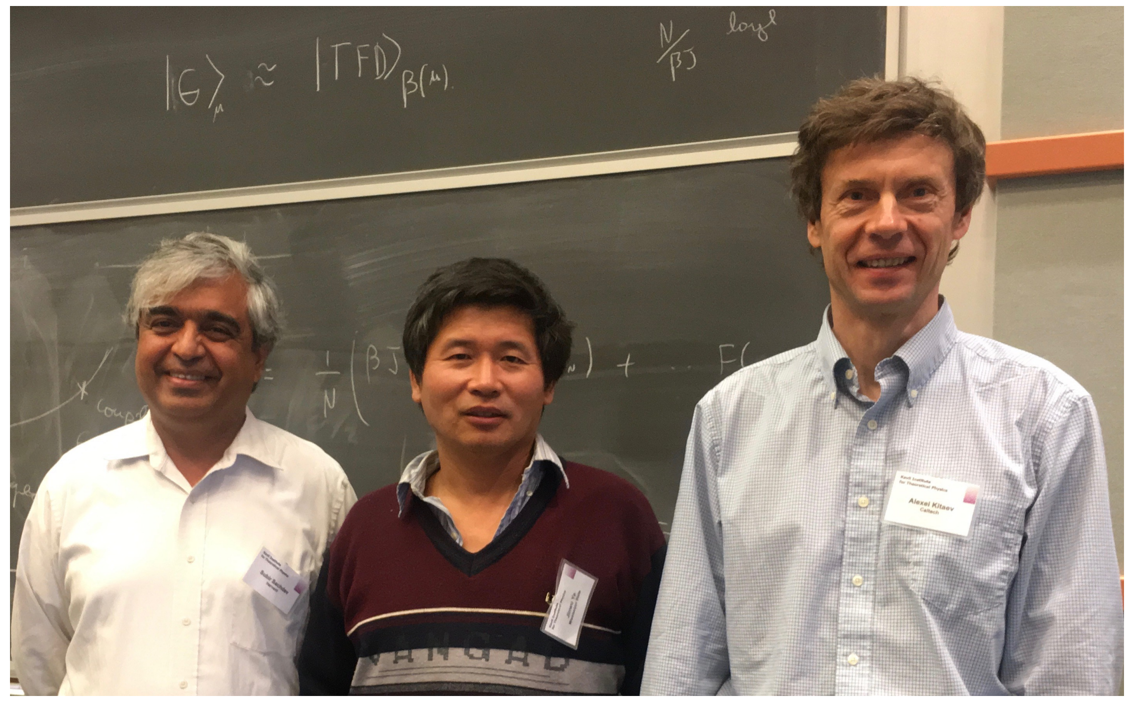

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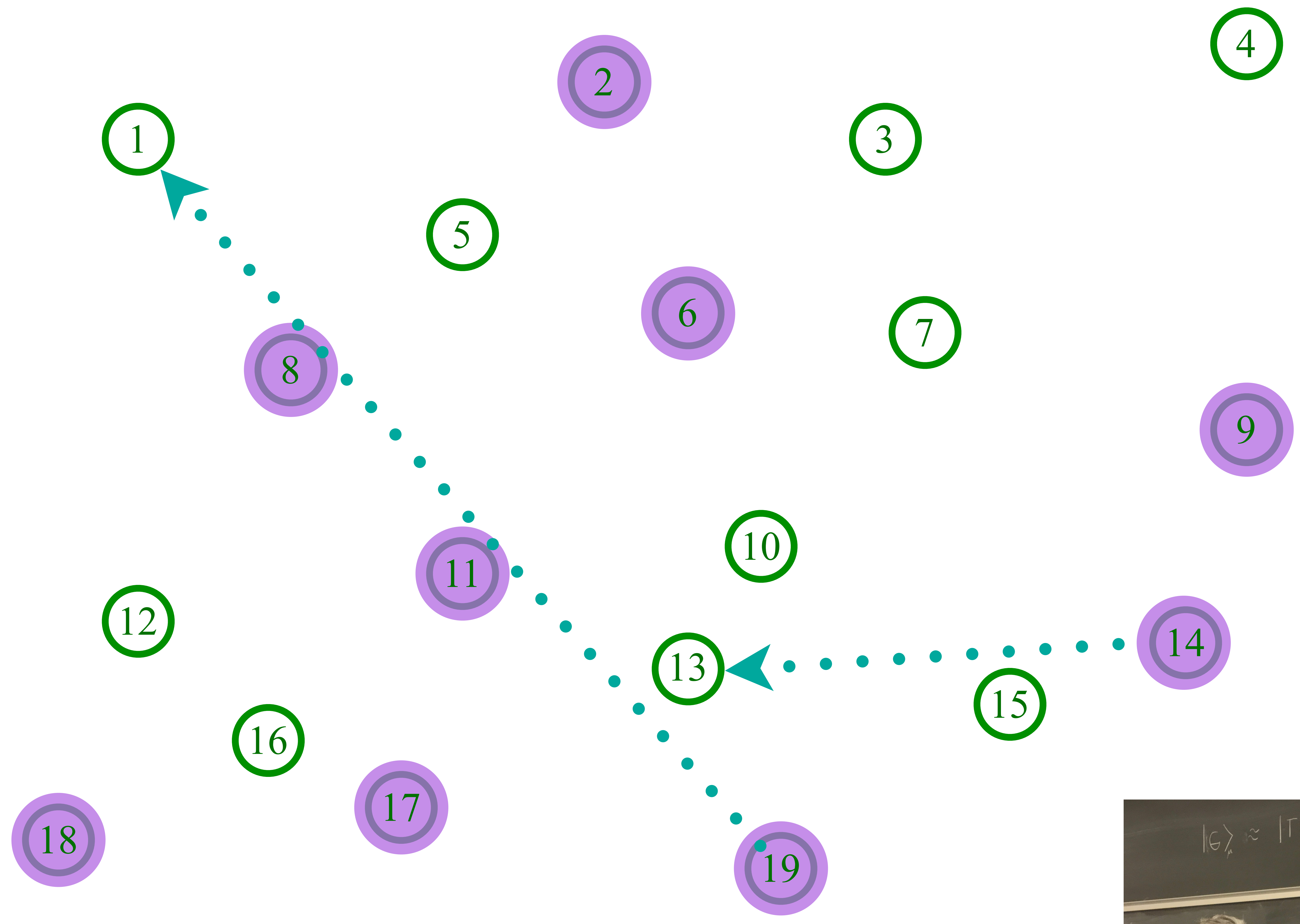


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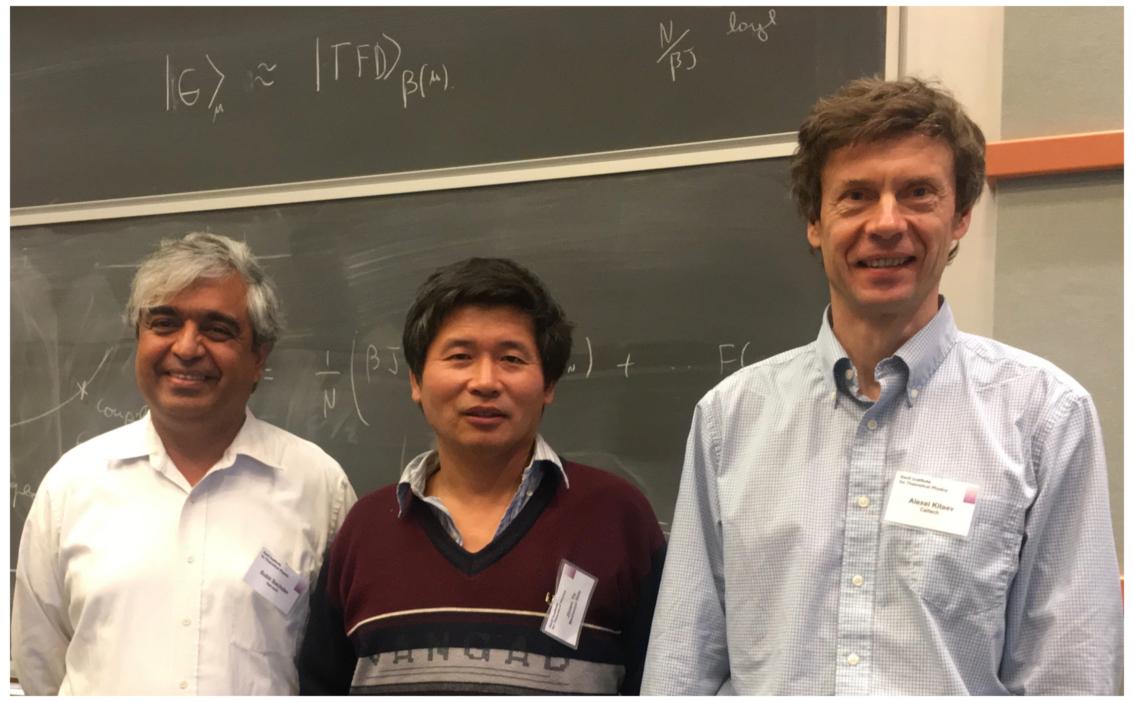

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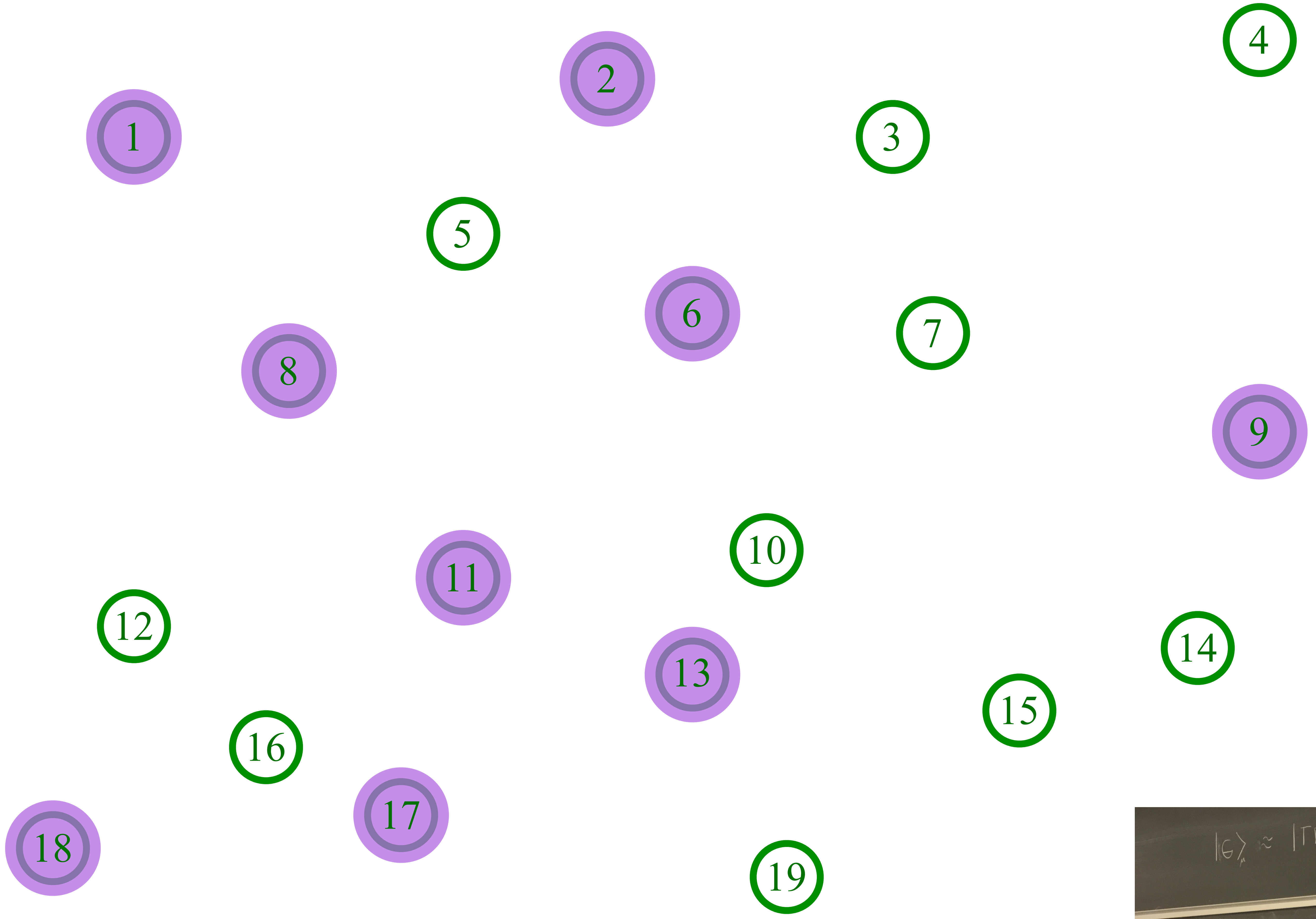


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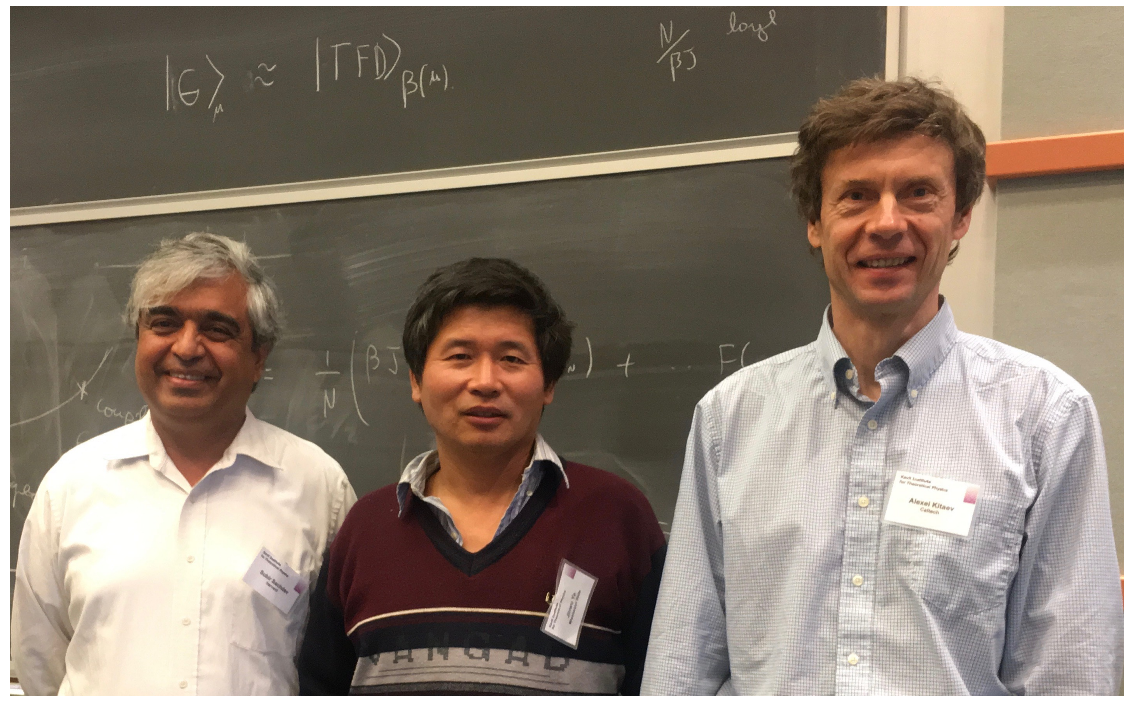

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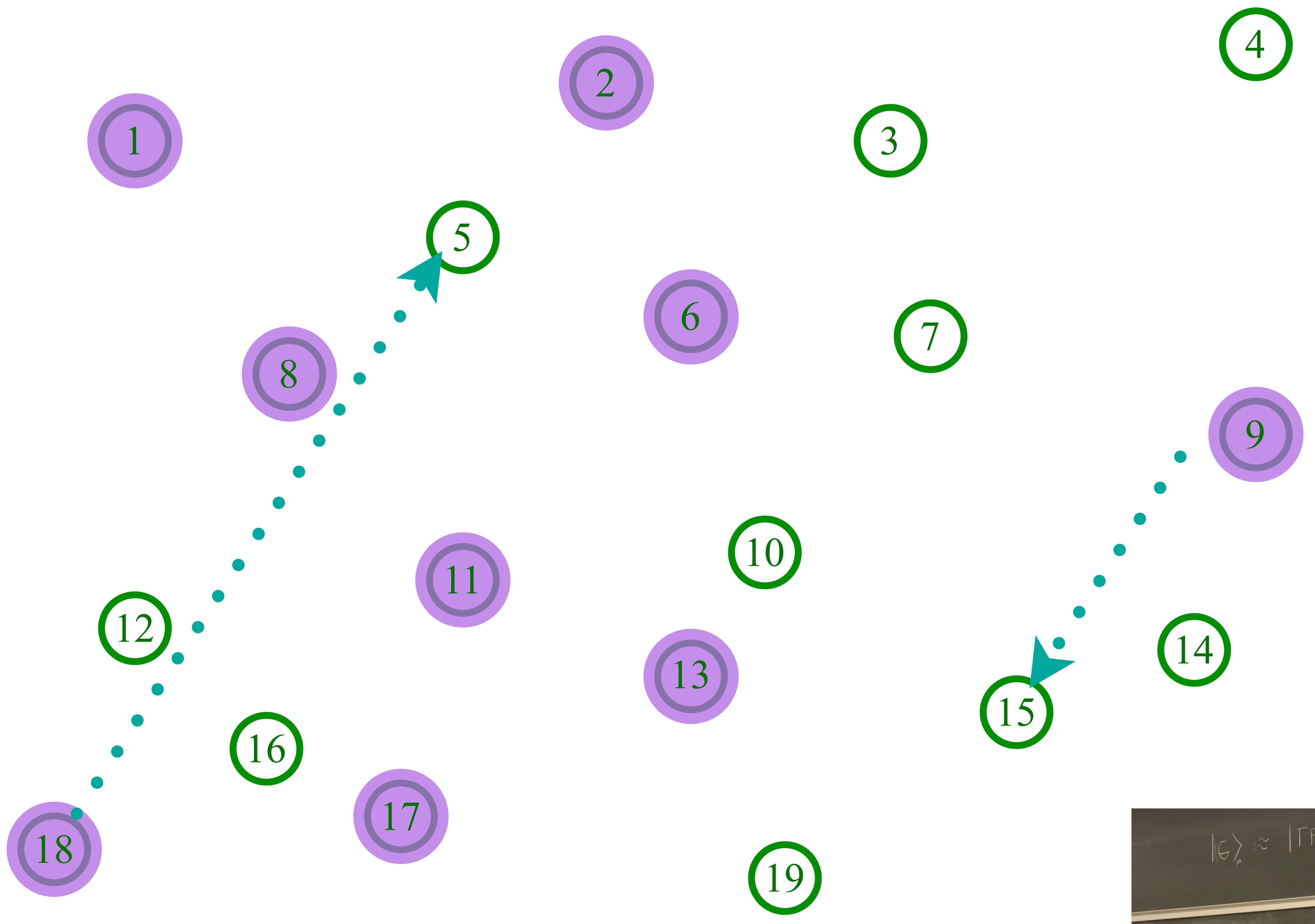


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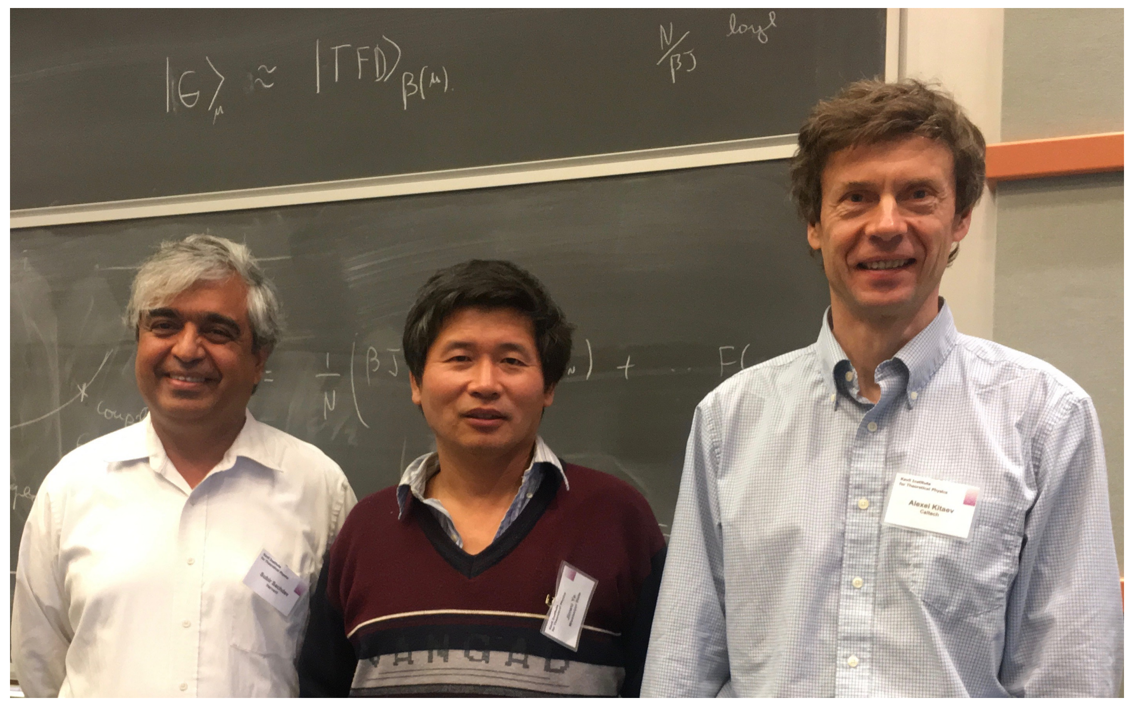

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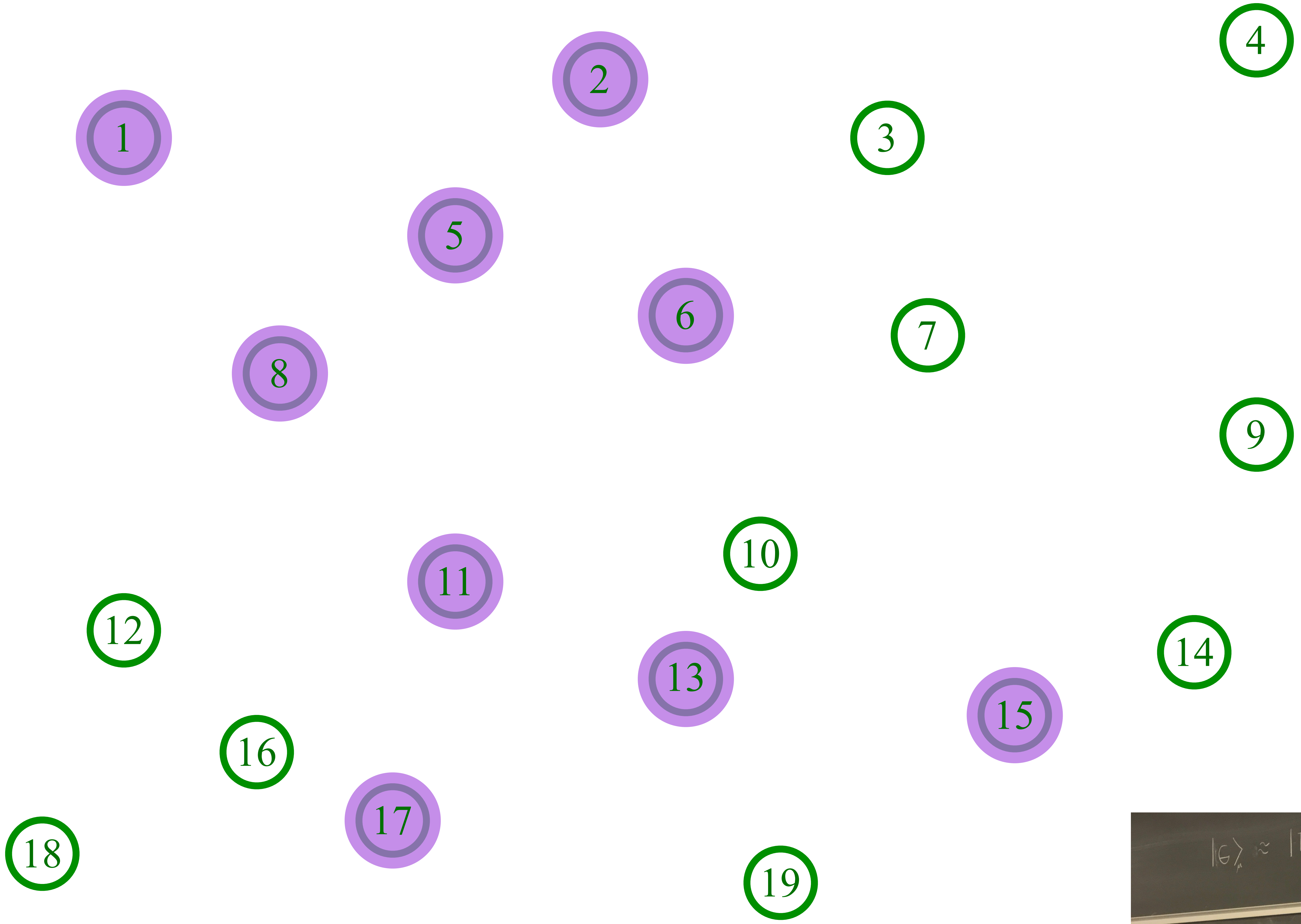


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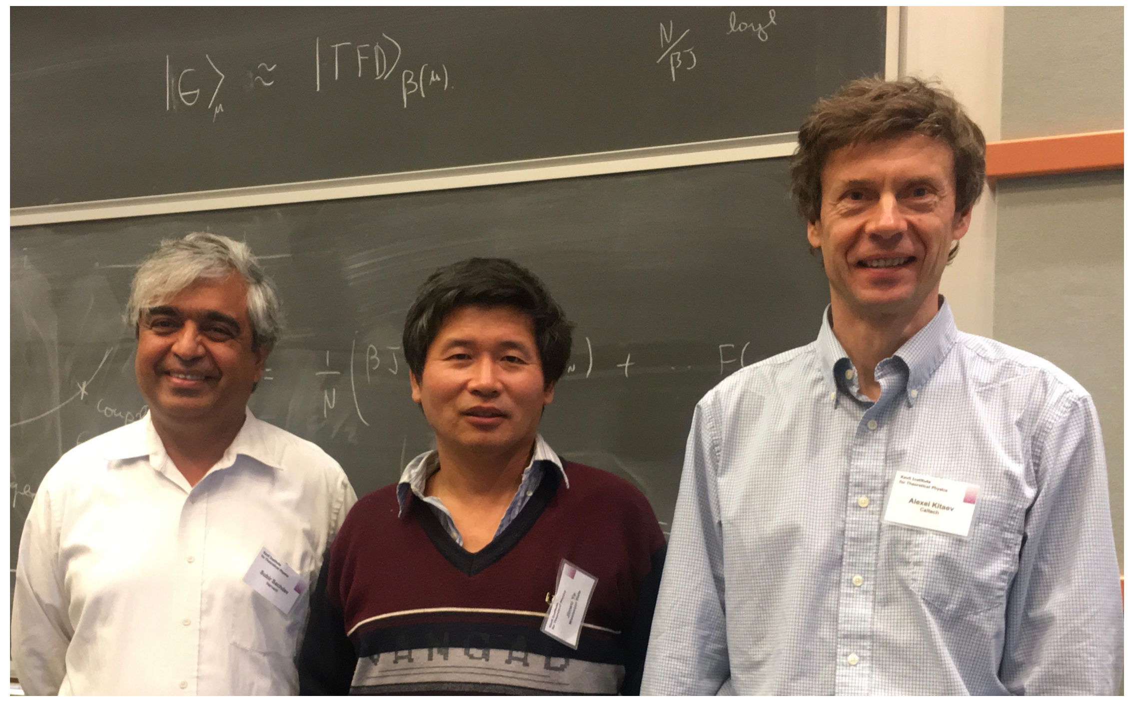
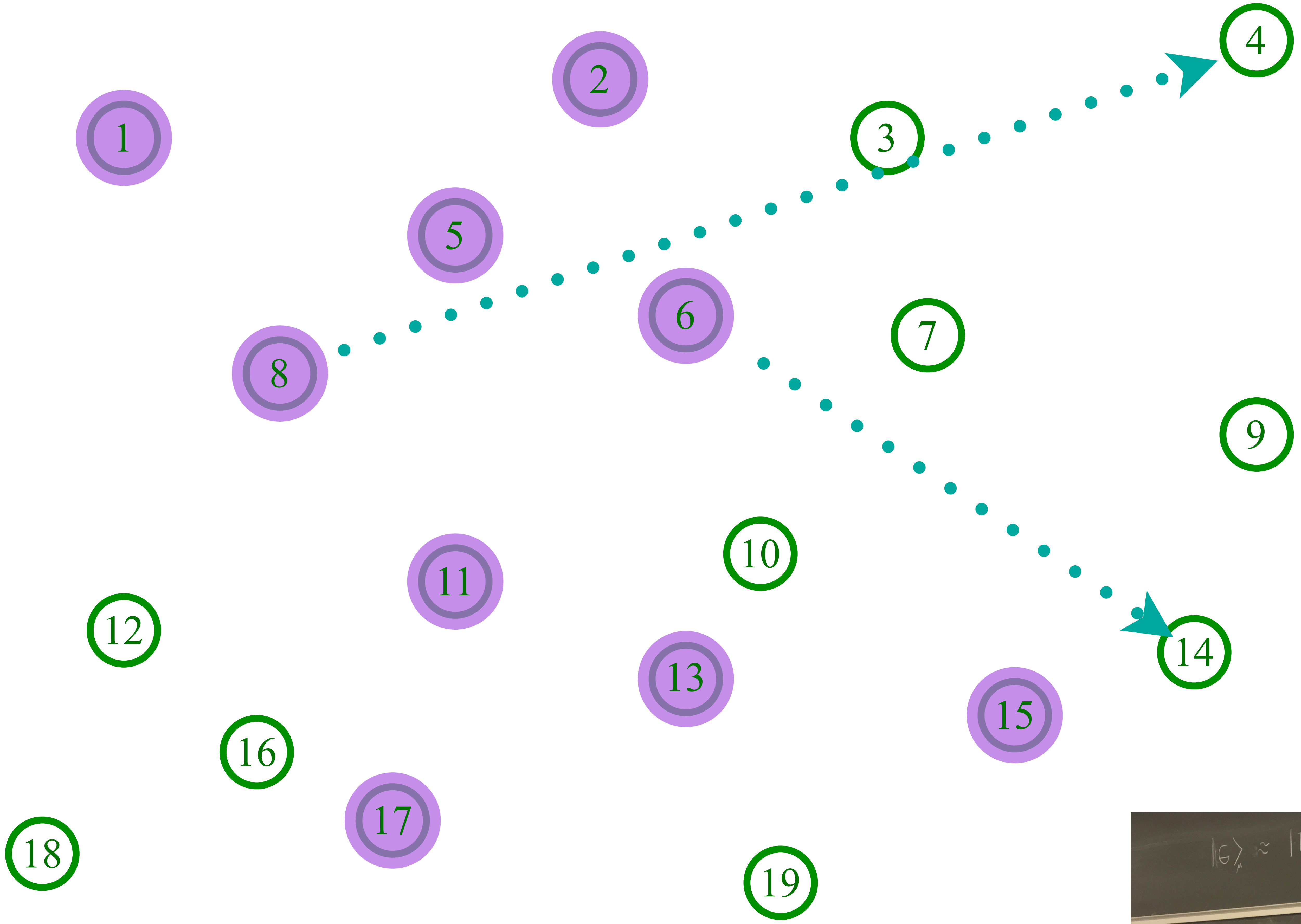


# The SYK model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{6,8;4,14}$$

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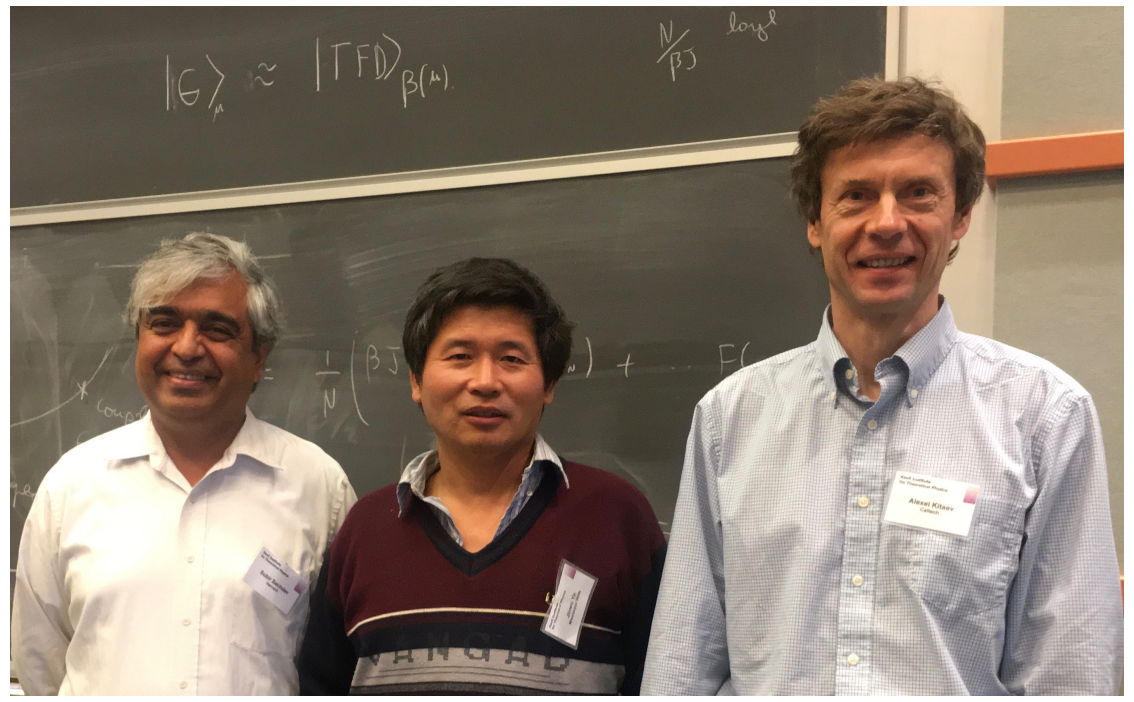

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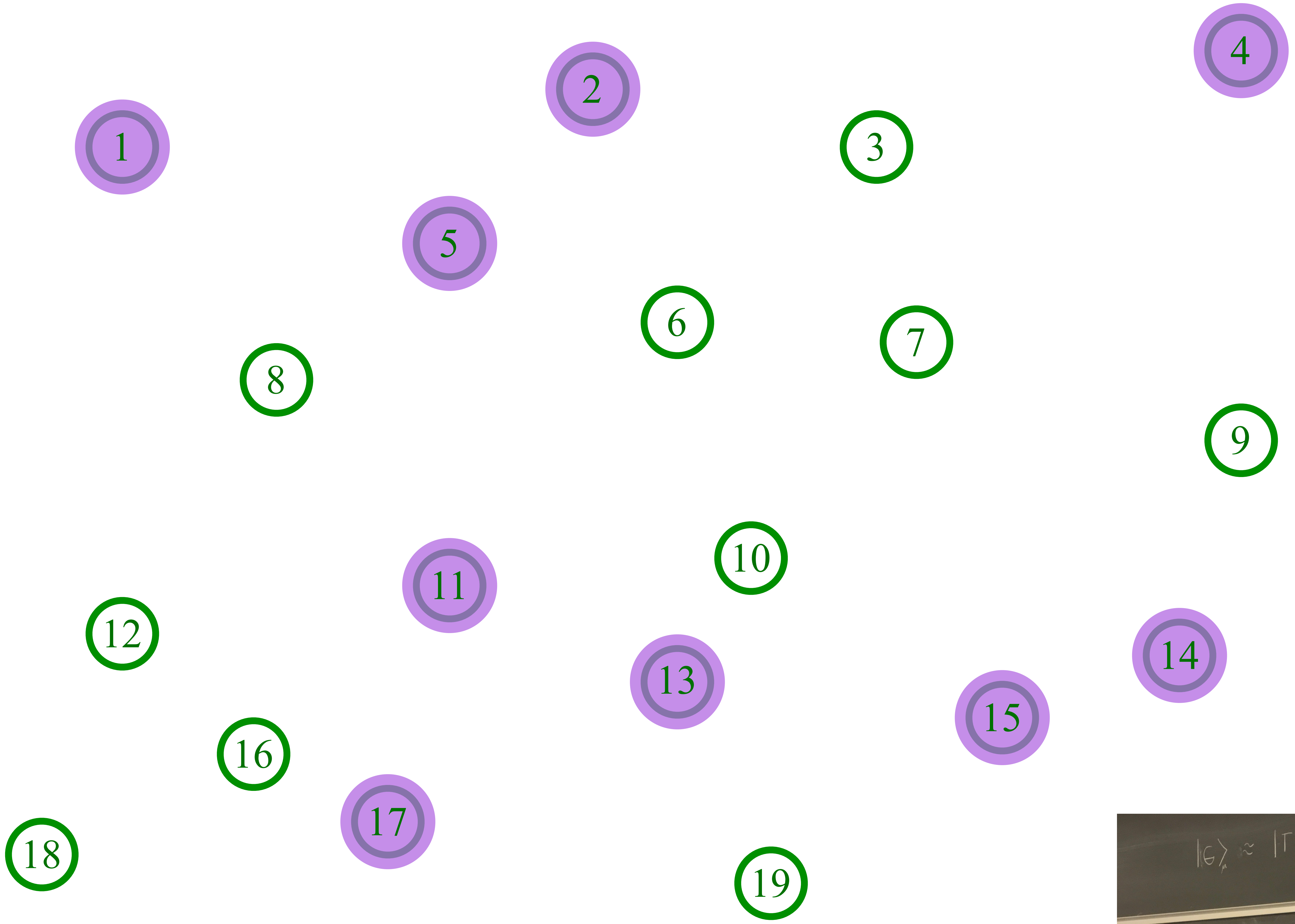


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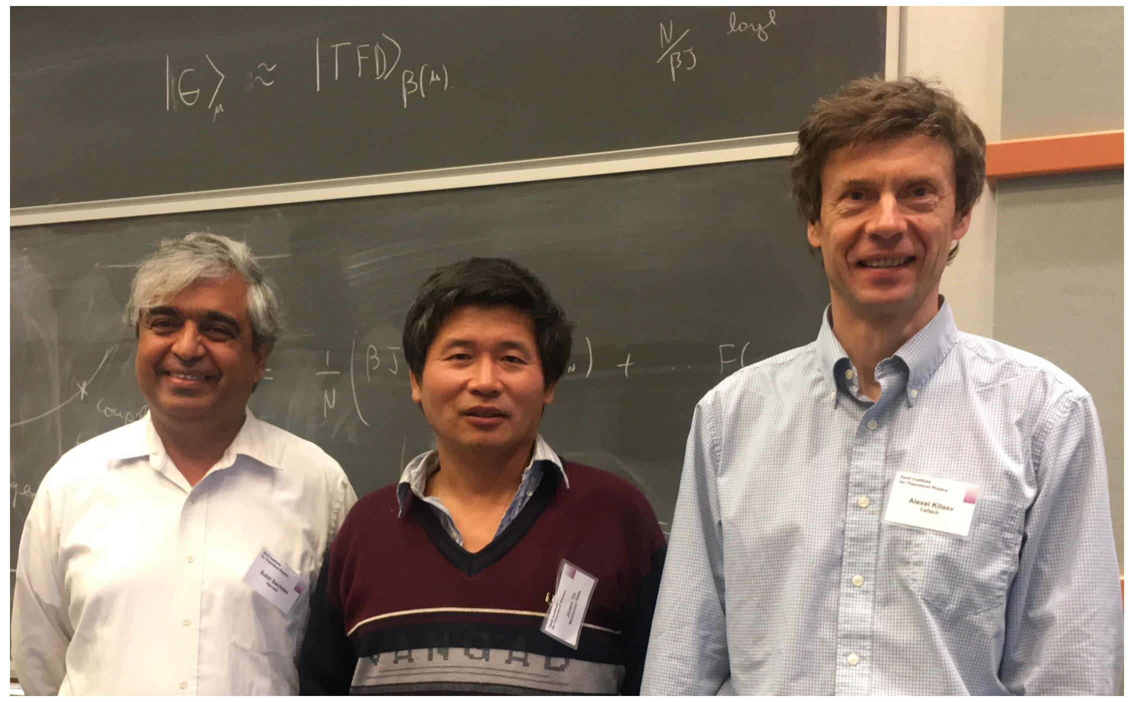


# The Sachdev-Ye-Kitaev (SYK) model

(See also: the "2-Body Random Ensemble" in nuclear physics; did not obtain the large *N* limit;
T.A. Brody, J. Flores, J.B. French, P.A. Mello, A. Pandey, and S.S.M. Wong, Rev. Mod. Phys. **53**, 385 (1981))

$$\mathcal{H} = \frac{1}{(2N)^{3/2}} \sum_{\alpha,\beta,\gamma,\delta=1}^{N} U_{\alpha\beta;\gamma\delta}\, c_\alpha^\dagger c_\beta^\dagger c_\gamma c_\delta - \mu \sum_\alpha c_\alpha^\dagger c_\alpha$$

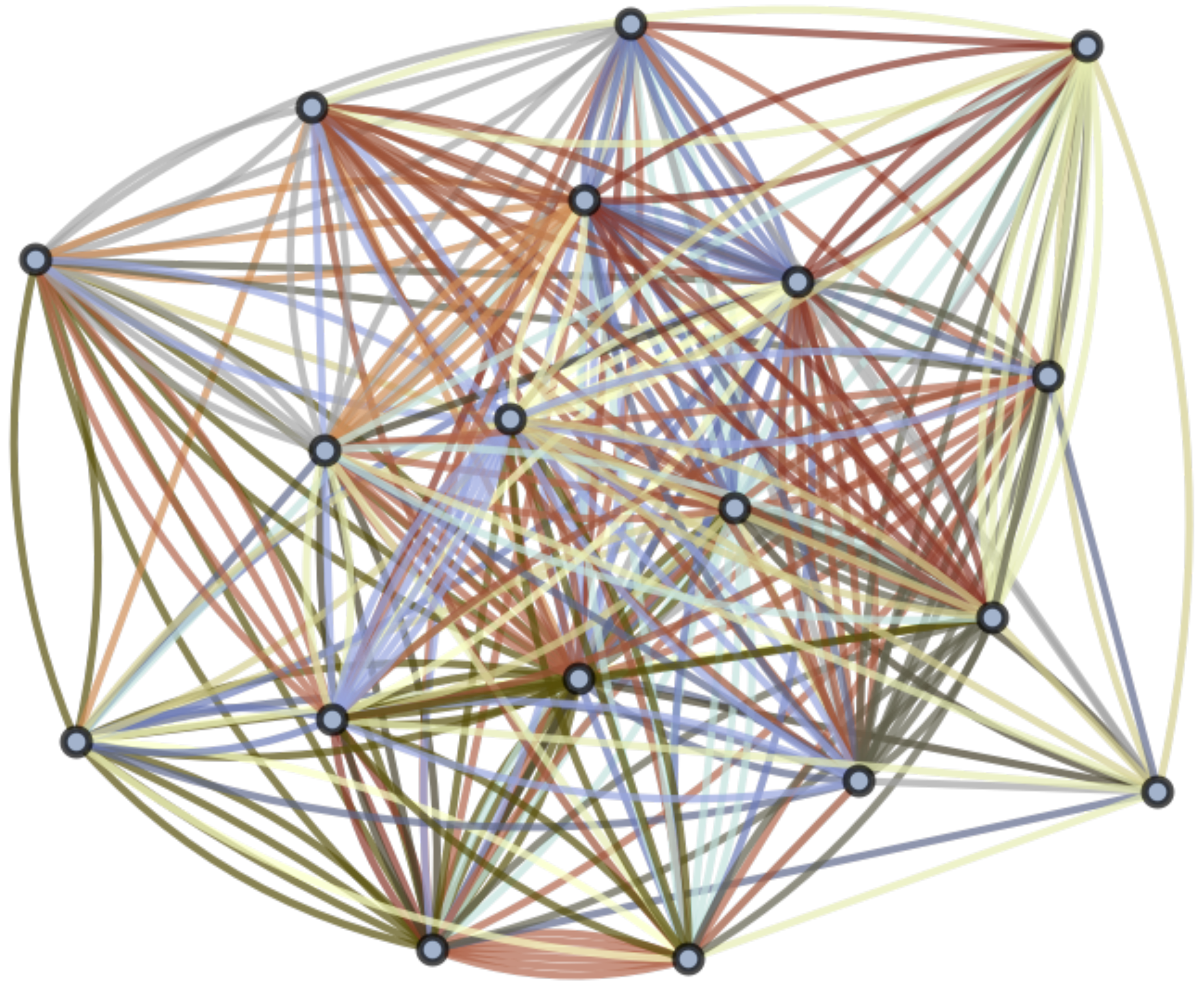
$$c_\alpha c_\beta + c_\beta c_\alpha = 0 \quad , \quad c_\alpha c_\beta^\dagger + c_\beta^\dagger c_\alpha = \delta_{\alpha\beta}$$

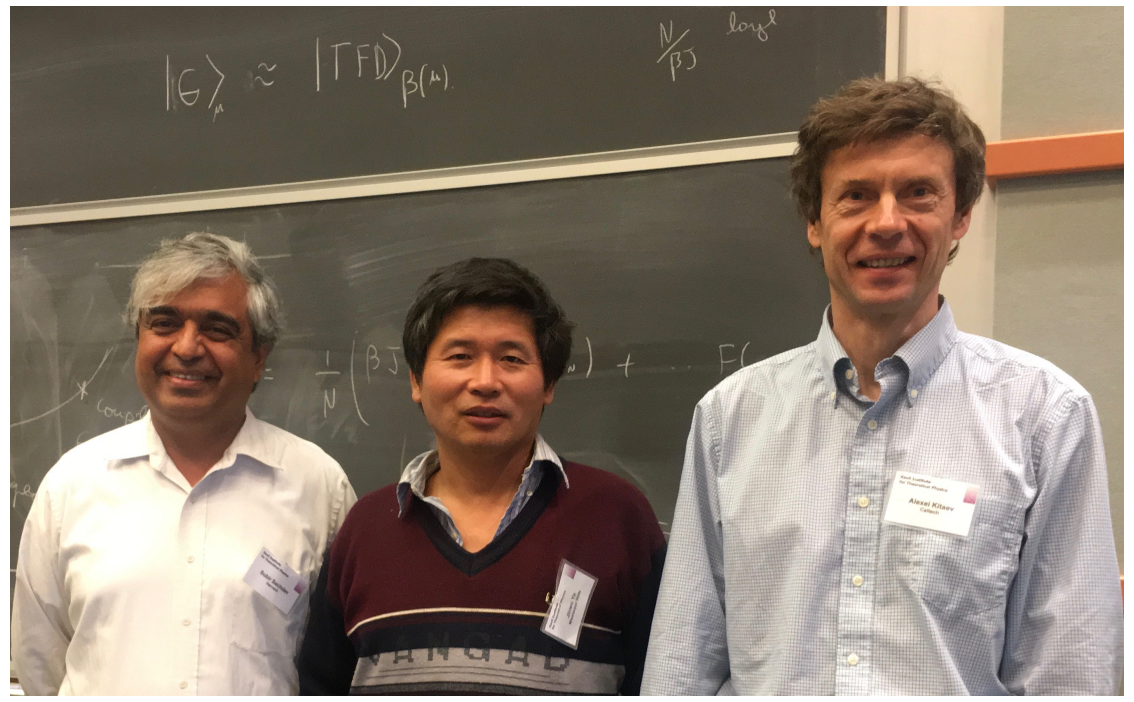
$$\mathcal{Q} = \frac{1}{N} \sum_\alpha c_\alpha^\dagger c_\alpha \, ; \quad [\mathcal{H}, \mathcal{Q}] = 0\, ; \quad 0 \leq \mathcal{Q} \leq 1$$

$U_{\alpha\beta;\gamma\delta}$ are independent random variables with $\overline{U_{\alpha\beta;\gamma\delta}} = 0$ and $\overline{|U_{\alpha\beta;\gamma\delta}|^2} = U^2$
$N \to \infty$ yields critical strange metal.

S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

A. Kitaev, unpublished; S. Sachdev, PRX **5**, 041025 (2015)



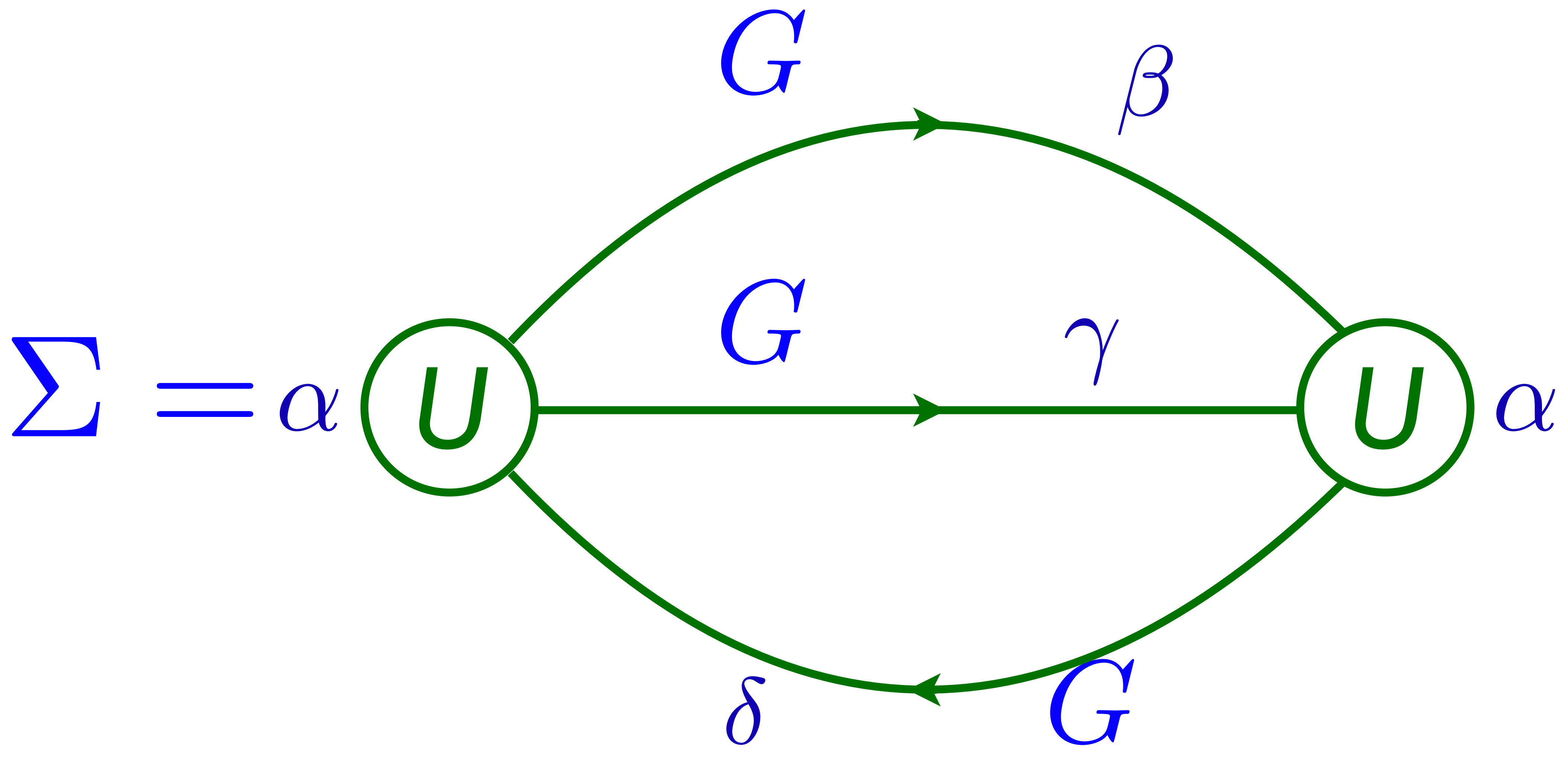


# The Sachdev-Ye-Kitaev (SYK) model

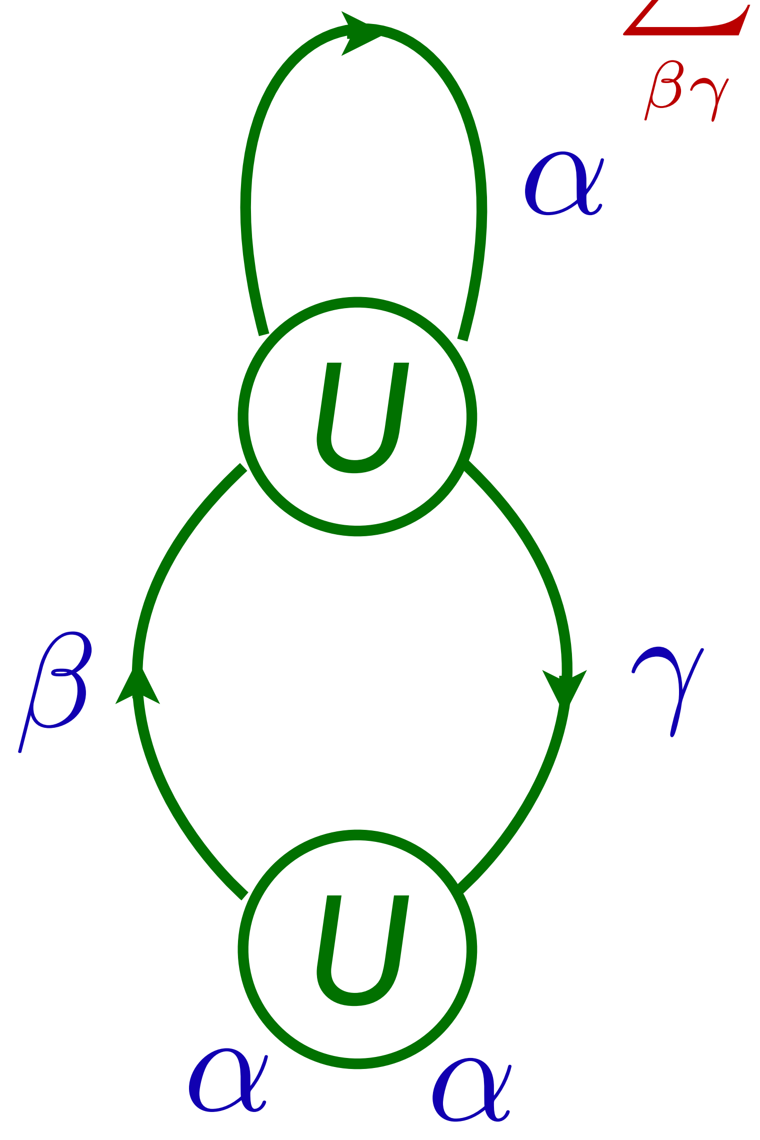
Feynman graph expansion in $U_{\alpha\beta;\gamma\delta}$, and graph-by-graph average, yields exact equations in the large $N$ limit:

$$G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = -U^2 G^2(\tau) G(-\tau)$$
$$G(\tau = 0^-) = \mathcal{Q}.$$

$$\sum_{\beta\gamma\delta} \frac{U^2}{N^3} = U^2$$




$$\sum_{\beta\gamma} \frac{U^2}{N^3} = \frac{U^2}{N}$$




S. Sachdev and J. Ye,
PRL **70**, 3339 (1993)

# The Sachdev-Ye-Kitaev (SYK) model

Sachdev, Ye (1993); Kitaev (2015)

A solvable model of multi-particle quantum entanglement.

No quasiparticles: yields a metal in which current is carried not by individual electrons, but by an entangled "quantum soup"

# The SYK model

Consequences of emergent time-reparameterization and conformal symmetries in low-energy theory in 0+1 spacetime dimensions:

1. Planckian dynamics!

$$G(\omega) = \frac{A}{(k_B T)^{1/2}} F_1\left(\frac{\hbar\omega}{k_B T}\right) \; ; \qquad \tau(\omega) = \frac{\hbar}{k_B T} F_2\left(\frac{\hbar\omega}{k_B T}\right) \text{ independent of } U.$$

No bosons, fermions, anyons ...: $F_1 \neq (e^{\hbar\omega/(k_B T)} - 1)^{-1}$

S. Sachdev and J. Ye, PRL **70**, 3339 (1993); A. Georges and O. Parcollet PRB **59**, 5341 (1999)

# The complex SYK model

$$G_*(\tau) = -C \frac{e^{-2\pi\mathcal{E}T\tau}}{\sqrt{1+e^{-4\pi\mathcal{E}}}} \left(\frac{T}{\sin(\pi T \tau)}\right)^{1/2} .$$

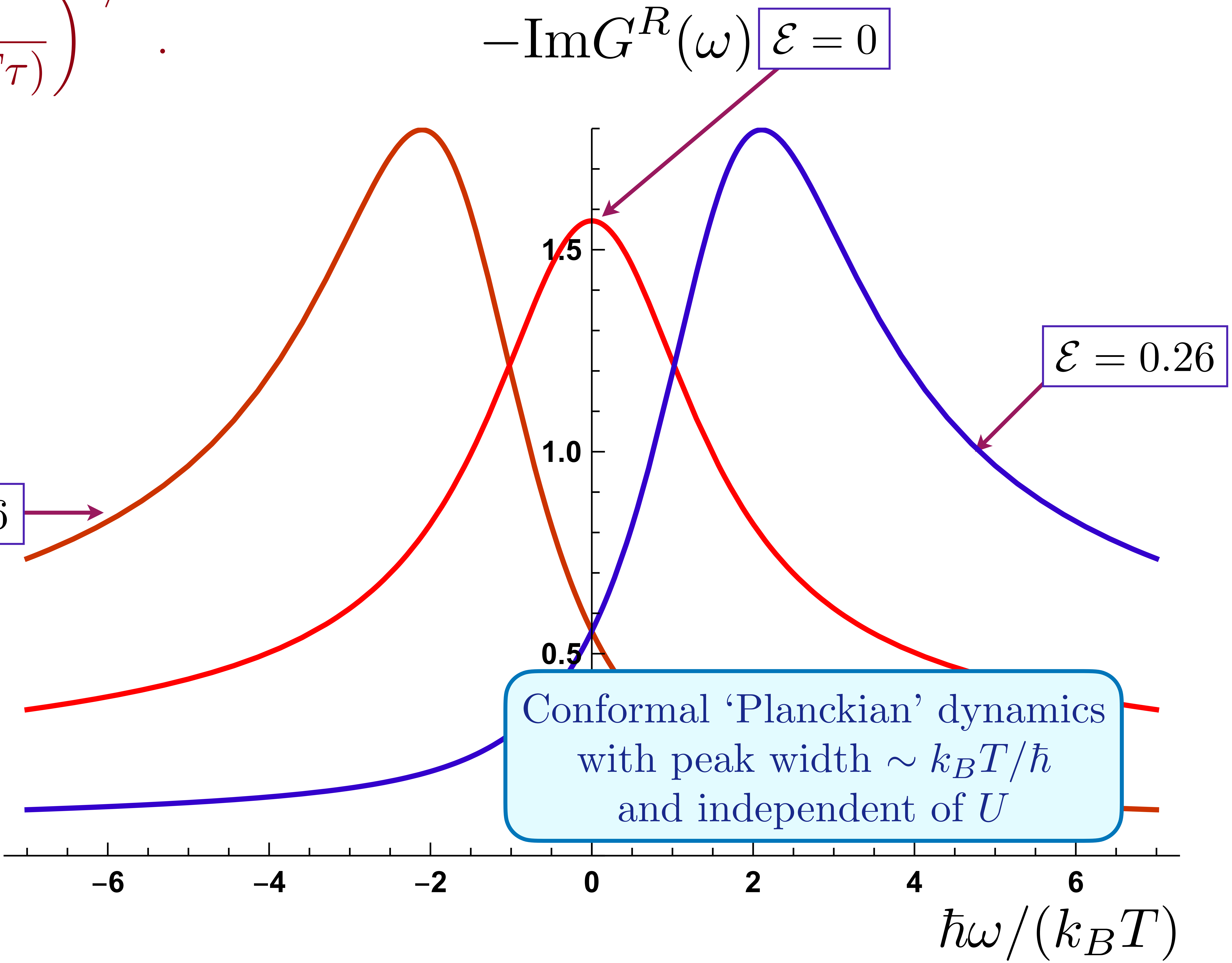
$$G_*^R(\omega) =$$

$$\frac{-iCe^{-i\theta}}{(2\pi T)^{1/2}} \frac{\Gamma\left(\frac{1}{4} - \frac{i\omega}{2\pi T} + i\mathcal{E}\right)}{\Gamma\left(\frac{3}{4} - \frac{i\omega}{2\pi T} + i\mathcal{E}\right)} .$$

$$e^{2\pi\mathcal{E}} = \frac{\sin(\pi/4+\theta)}{\sin(\pi/4-\theta)}$$

$$C = \left(\frac{\pi}{U^2\cos(2\theta)}\right)^{1/4}$$

$\mathcal{E}$ is a known function of $\mathcal{Q}$ (Luttinger relation)

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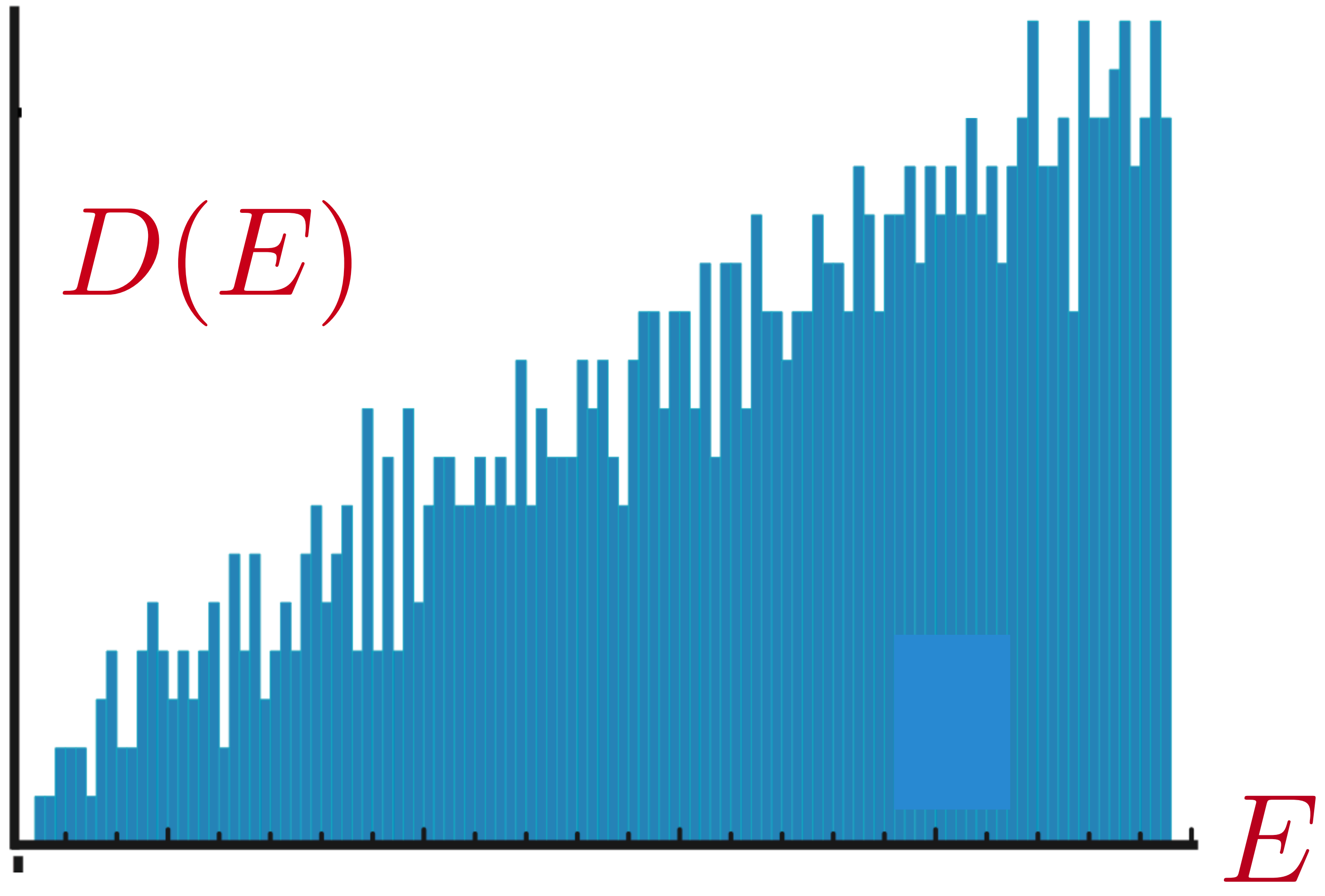
No bosons, fermions, anyons . . . : $F_1 \neq (e$

2. Zero temperature entropy without exponential ground state degeneracy!

$$\lim_{T\to 0} \lim_{N\to\infty} \frac{1}{N} S(T) = s_0 \quad , \quad D(E \to 0) = e^{N s_0} f_{\text{smooth}}(E)$$

$$s_0 = 0.4648476991708051074 9.... \text{ for } \mathcal{Q} = 1/2.$$




A. Georges, O. Parcollet, and S. Sachdev (**GPS**), Physical Review B **63**, 134406 (2001)

# Connections between the SYK model and black holes

- Black hole 'ring-down' or 'quasinormal mode damping' or 'chaos' times are Planckian $\sim \hbar/(k_B T)$
- Charged black holes have a non-zero Bekenstein-Hawking entropy in the limit $T \to 0$:

  $S_{BH} = A_0 c^3/(4\hbar G)$ where $A_0 = 2GQ^2/c^4$ is the area of the charged black hole horizon at $T = 0$.

S. Sachdev, Phys. Rev. Lett. **105**, 151602 (2010)

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  Also applies to rotating neutral black holes.

U. Moitra, S.K. Sake, S.P. Trivedi and V. Vishal, JHEP 11 (2019) 047.
D. Kapec, A. Sheta, A. Strominger and C. Toldo, PRL 133 (2024) 021601
M. Kolanowski, D. Marolf, I. Rakic, M. Rangamani and G.J. Turiaci, arXiv:2409.16248

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  Also applies to rotating neutral black holes.

- The example of the SYK model implies that $S_{BH}$ is *not* realized by an exponentially large ground state degeneracy (as is the case in all earlier string-theoretic computations).

S. Sachdev, Phys. Rev. Lett. **105**, 151602 (2010)

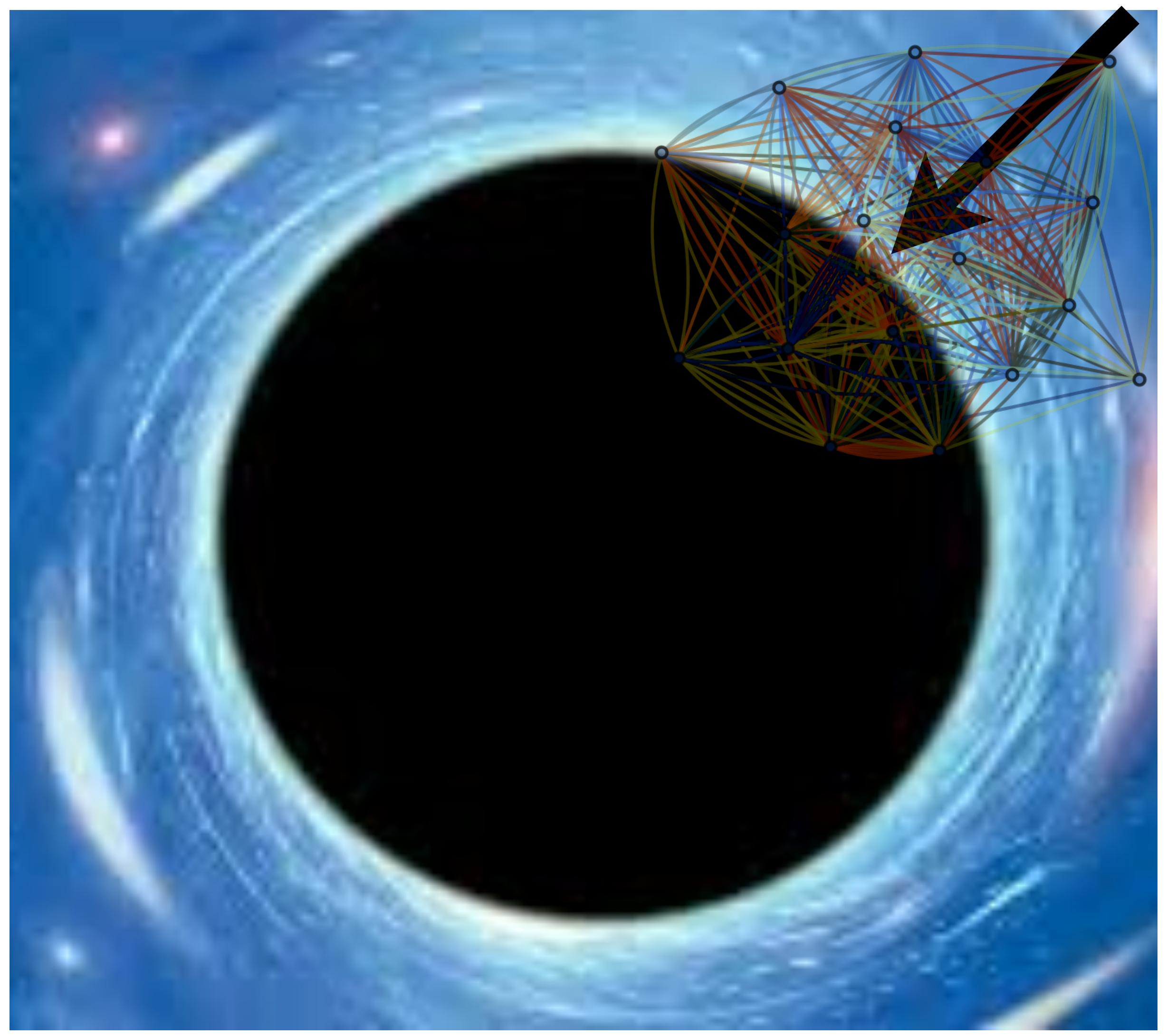
Thermodynamics of quantum black holes with charge $\mathcal{Q}$:

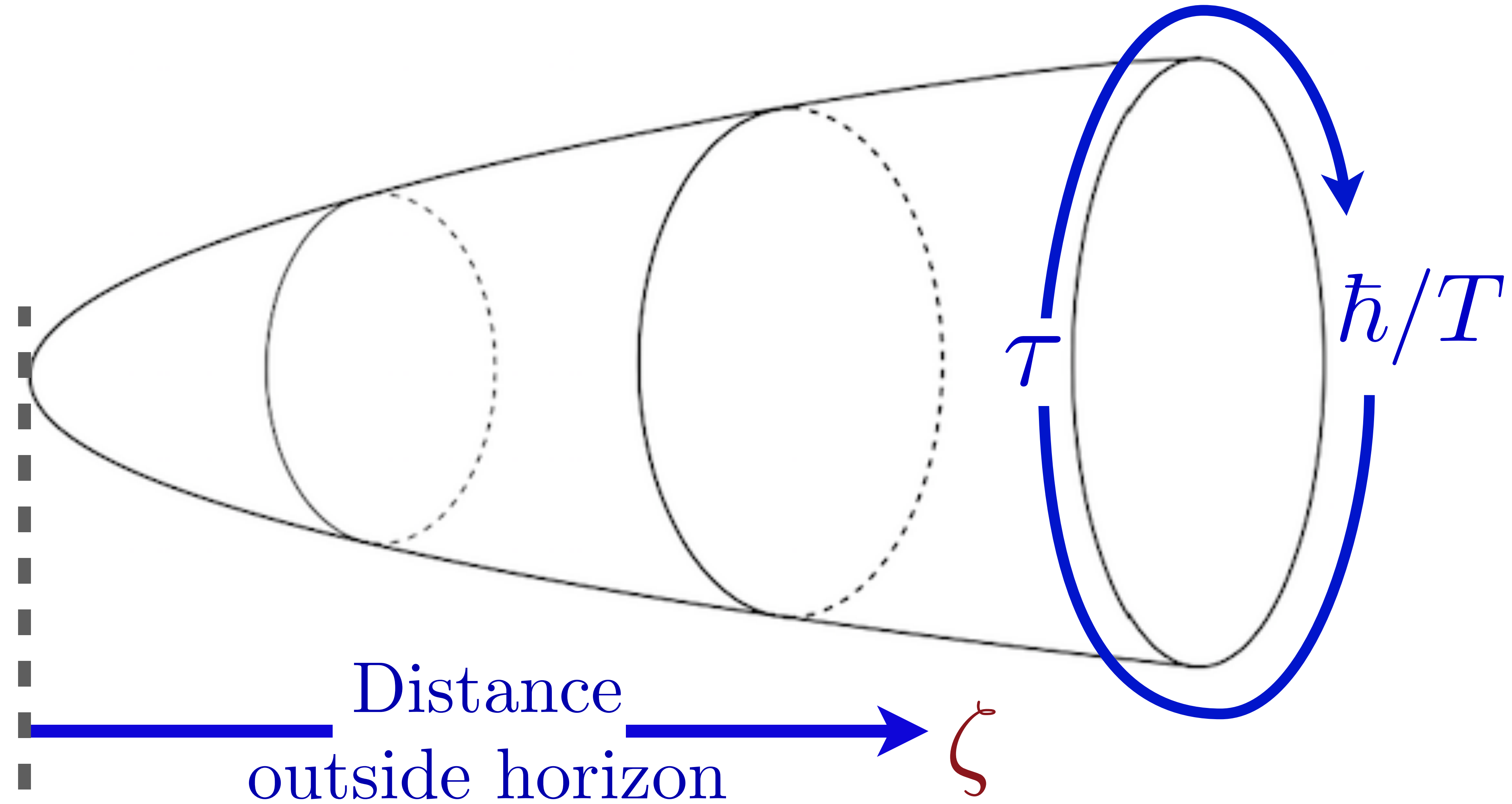
$$\mathcal{Z}(\mathcal{Q},T) = \int \mathcal{D}g_{\mu\nu}\mathcal{D}A_\mu \exp\left(-\frac{1}{\hbar}\, I^{(3+1)}_{\text{Einstein gravity+Maxwell EM}}[g_{\mu\nu},A_\mu]\right)$$

$$\approx \exp\left(\frac{A_0 c^3}{4\hbar G}\right)\int \mathcal{D}g_{\mu\nu}\mathcal{D}A_\mu \exp\left(-\frac{1}{\hbar}\, I^{(1+1)}_{\text{JT gravity of AdS}_2\text{+boundary graviton}}[g_{\mu\nu},A_\mu]\right)$$

Holography: quantum entanglement on the surface

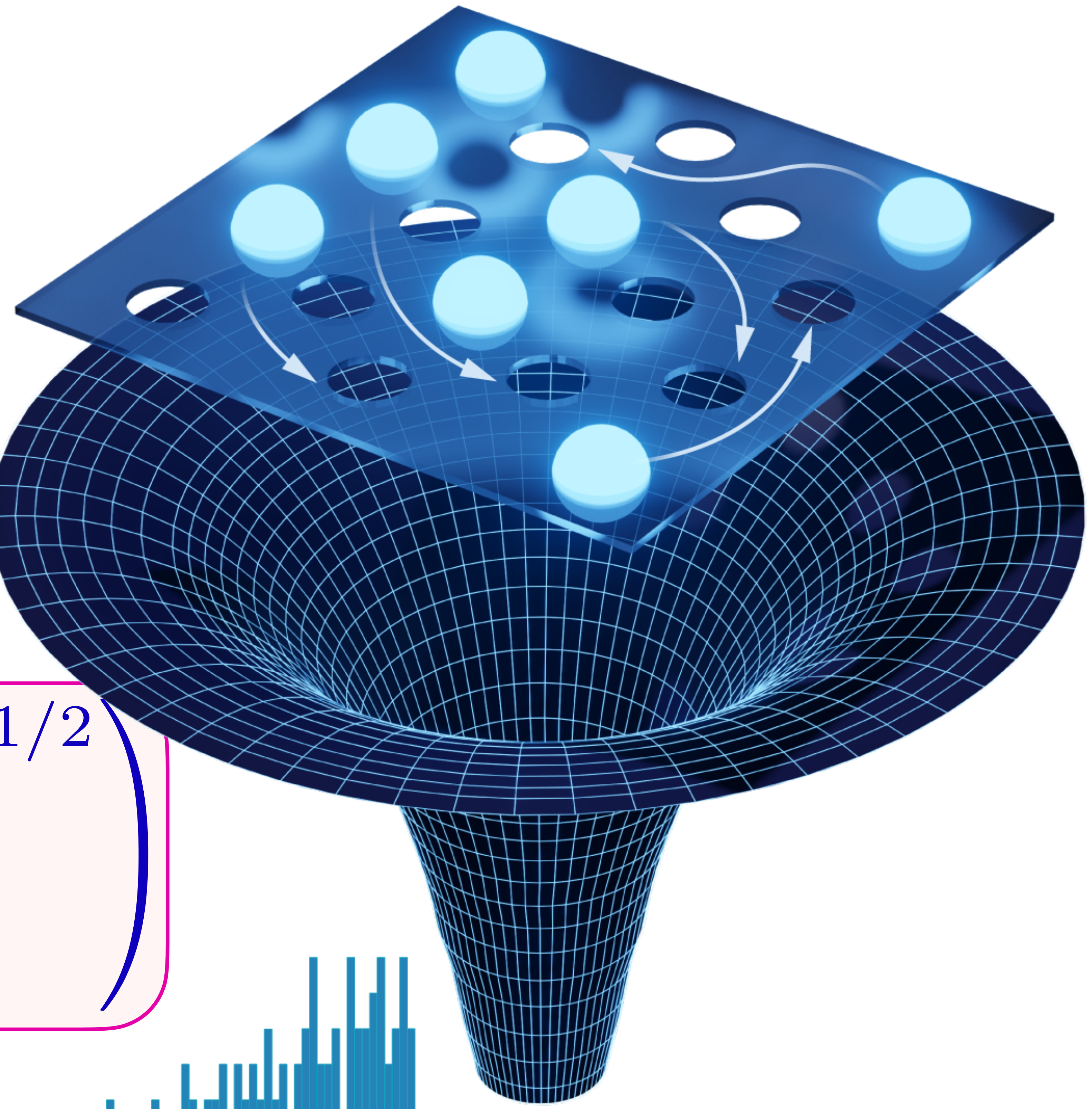
$$= \int \mathcal{D}f(\tau)\mathcal{D}\phi(\tau)\exp\left(-\frac{1}{\hbar} I^{(0+1)}_{\text{SYK}}[\text{time reparameterizations } f(\tau),\ \text{phase rotations } \phi(\tau)]\right)$$






Kitaev (2015); Maldacena, Stanford, Yang (2016); Cotler *et al.* (2017)

# D(E) of charged black holes from the SYK model



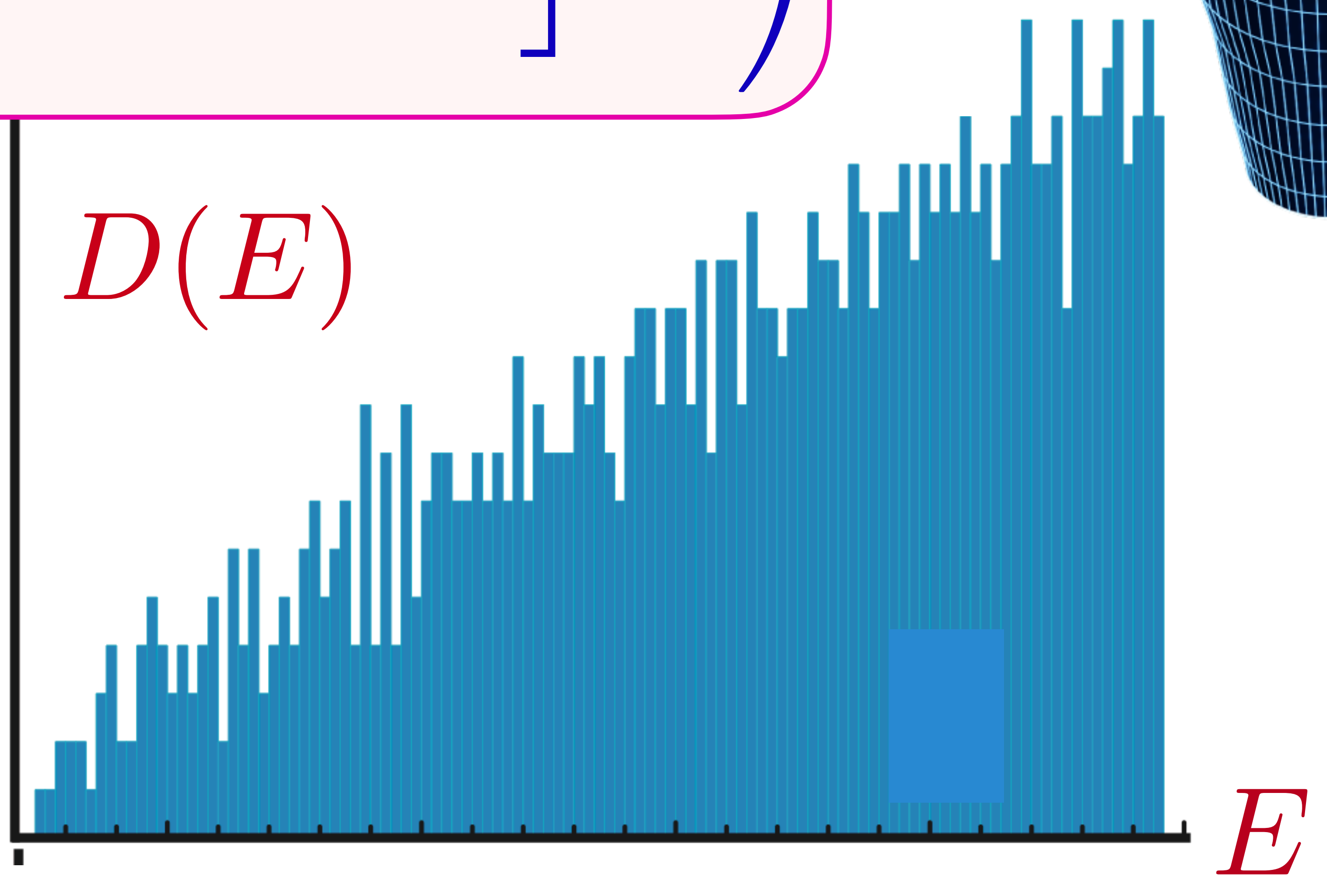
- For generic charged black holes in 3+1 dimensions with horizon area $A_0$ at $T = 0$ and fixed charge $\mathcal{Q}$ ($A_0 = 2G\mathcal{Q}^2/c^4$), the density of quantum states at small energy $E$ is

Bekenstein-Hawking

$$D(E) \sim \left(\frac{A_0 c^3}{\hbar G}\right)^{-347/90} \exp\left(\frac{A_0 c^3}{4\hbar G}\right) \sinh\left(\left[\frac{\sqrt{\pi}{A_0}^{3/2} c^2}{\hbar^2 G} E\right]^{1/2}\right)$$

Iliesiu, Murthy, Turiaci (2022)

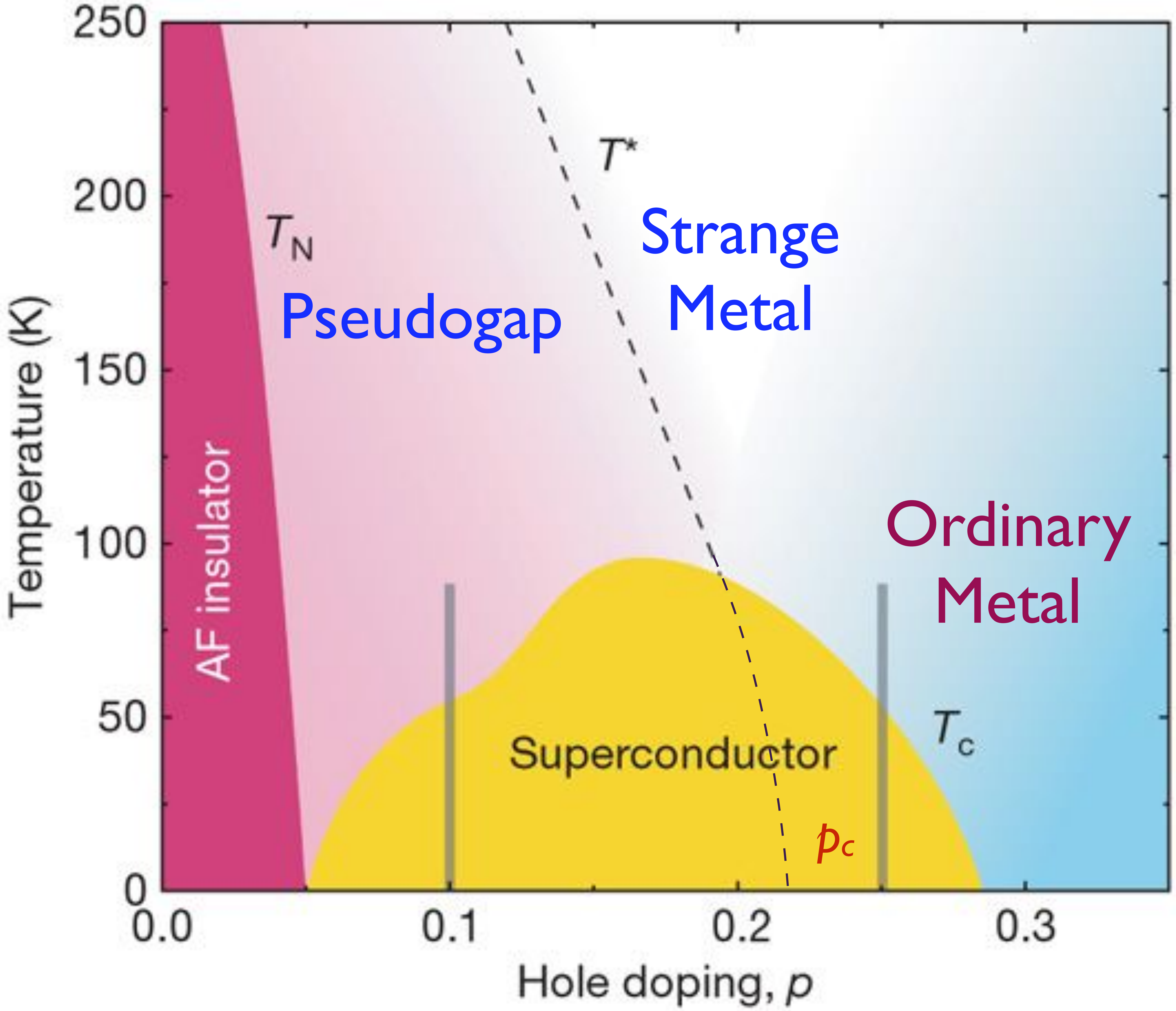
$f_{\rm smooth}(E)$ : developments from the SYK model

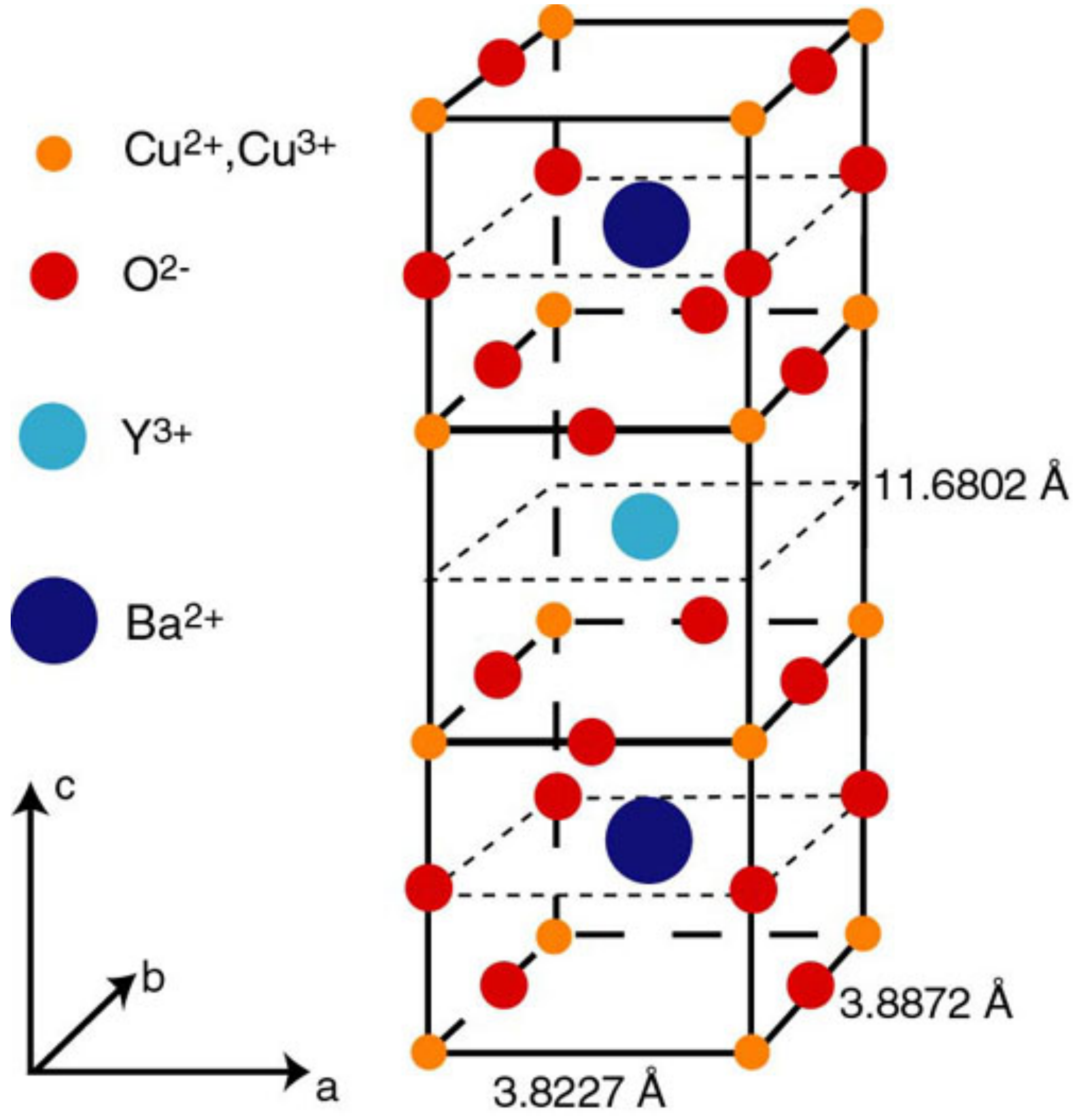



Similar remarks apply to rotating neutral black holes.

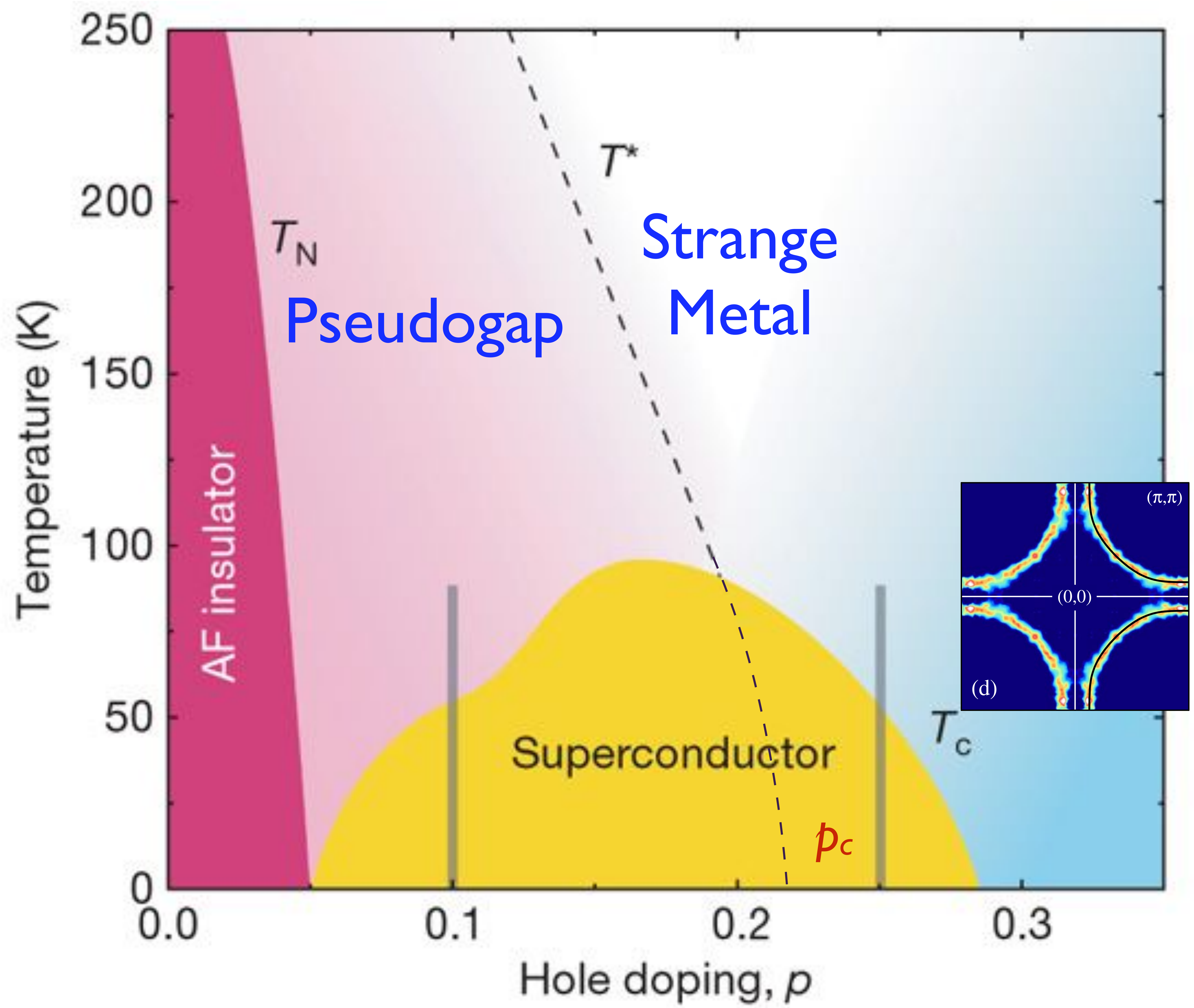
D. Chowdhury, A. Georges, O. Parcollet, and S. S., Rev. Mod. Phys. **94**, 035004 (2022)

# Quantum phase transitions of metals

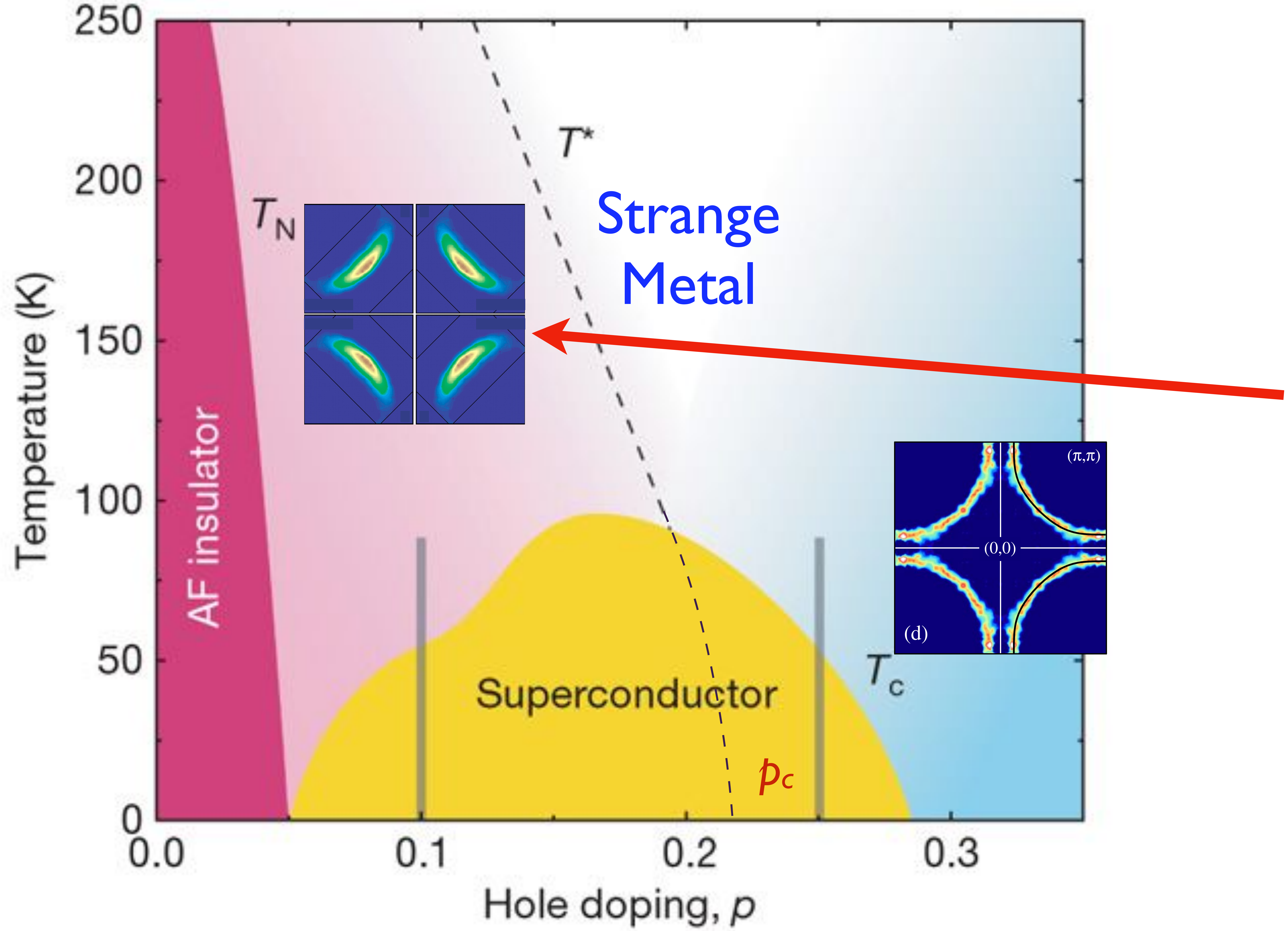

250
200
150
100
50
0
Temperature (K)
0.0
0.1
0.2
0.3
Hole doping, p
AF insulator
T_N
Pseudogap
T*
Strange Metal
Ordinary Metal
Superconductor
T_c
p_c


Cu2+,Cu3+
O2-
Y3+
Ba2+
c
b
a
11.6802 Å
3.8872 Å
3.8227 Å

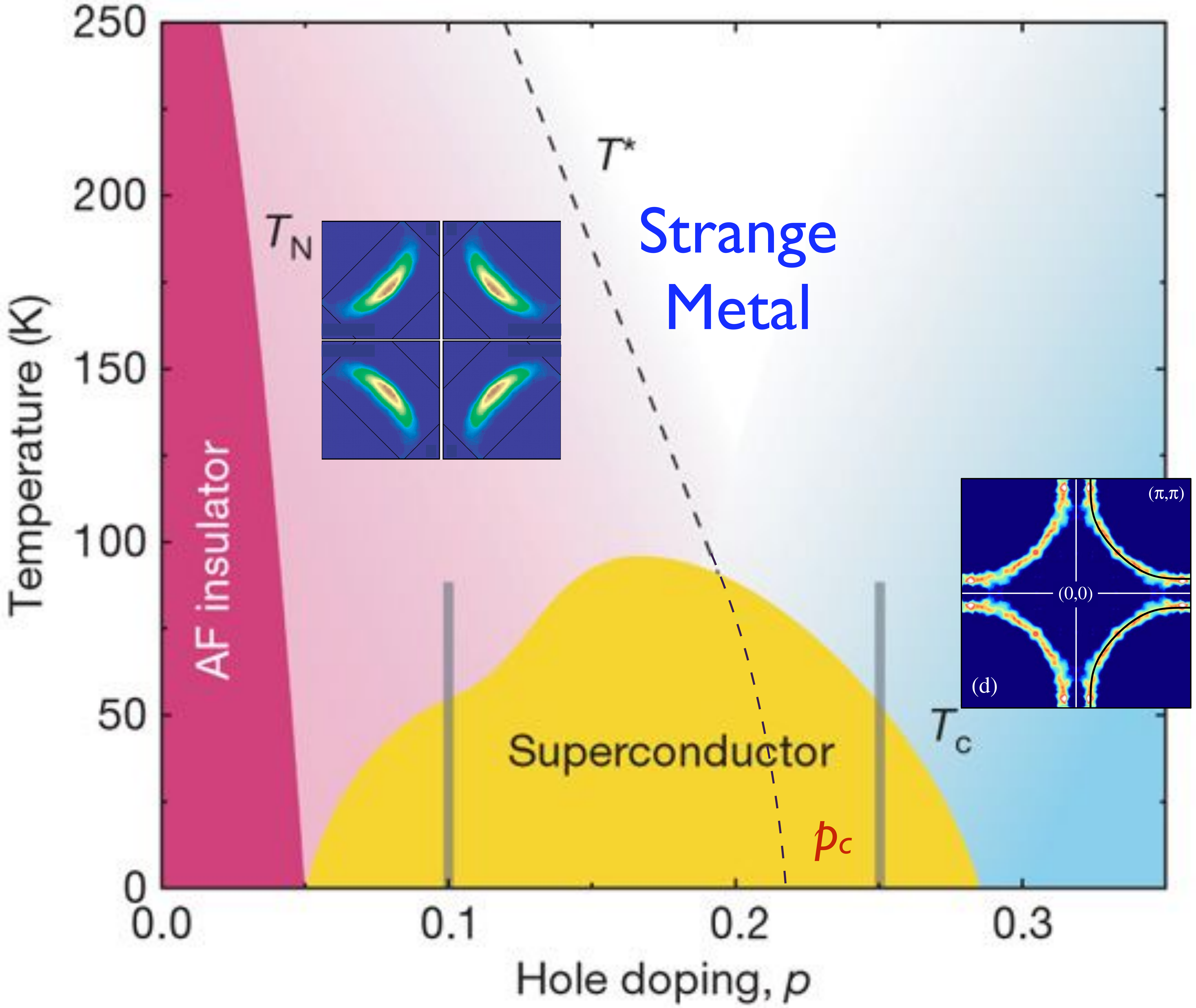
$YBa_2Cu_3O_{6+x}$




Fermi surface
as expected
in a model
of free electrons



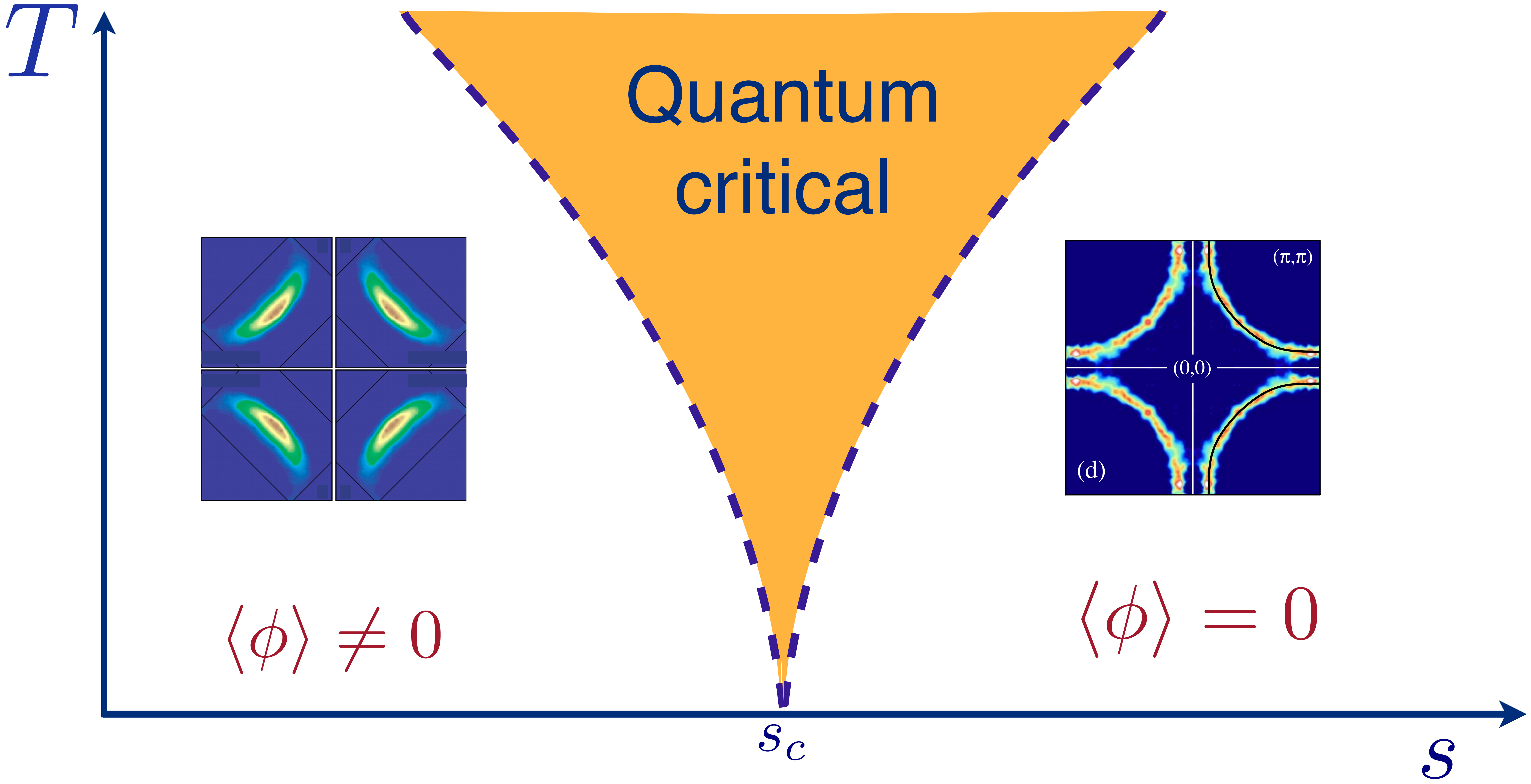

"Pseudogap metal"
Fermi surface
modified by
electron-electron
interactions




View the strange metal as a property of a $T = 0$ quantum phase transition involving change in the Fermi surface.

The onset of superconductivity may "hide" this quantum transition.

# Quantum phase transitions of Fermi surface change




Fermi surface

$+$

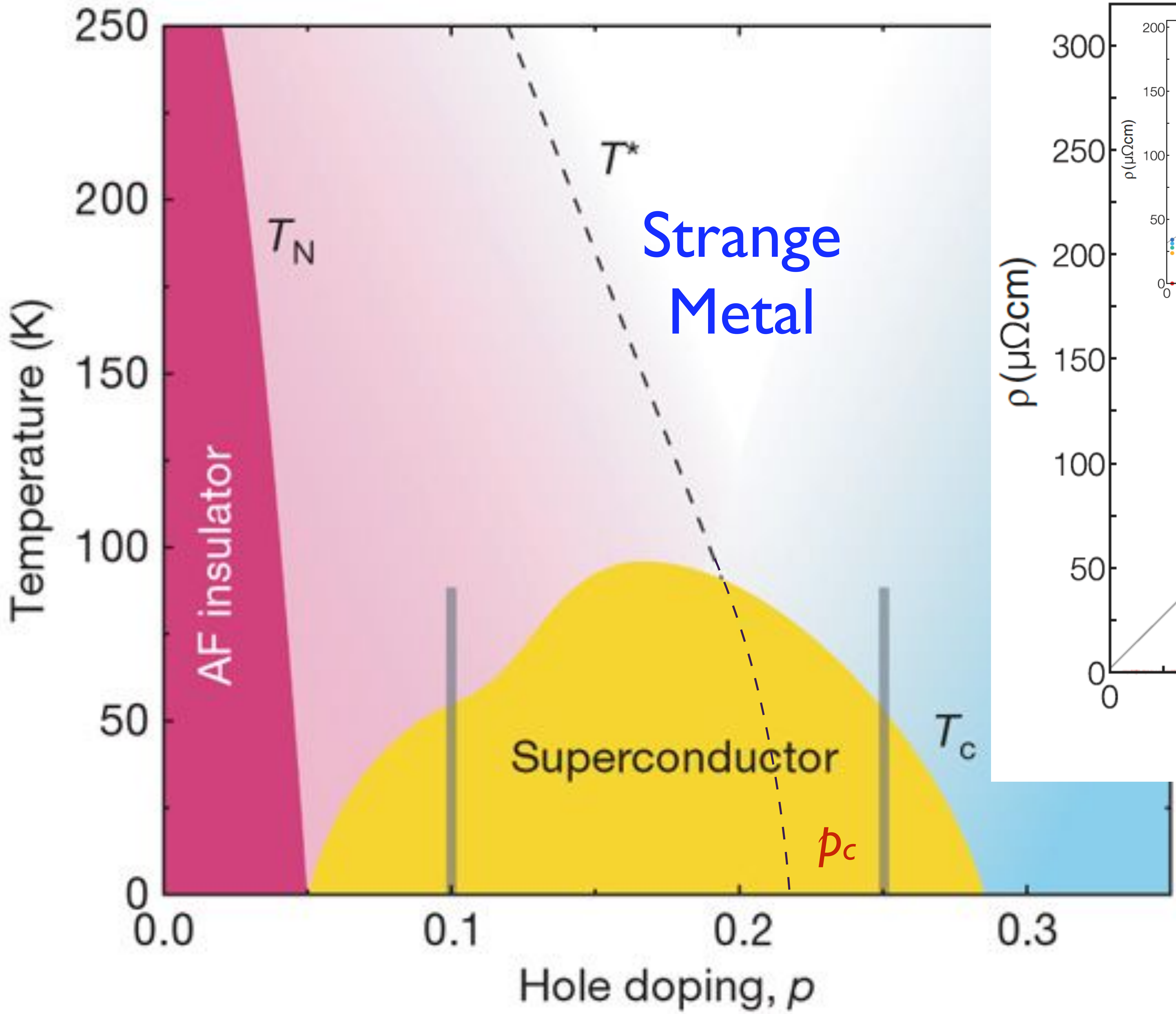
boson $\phi$

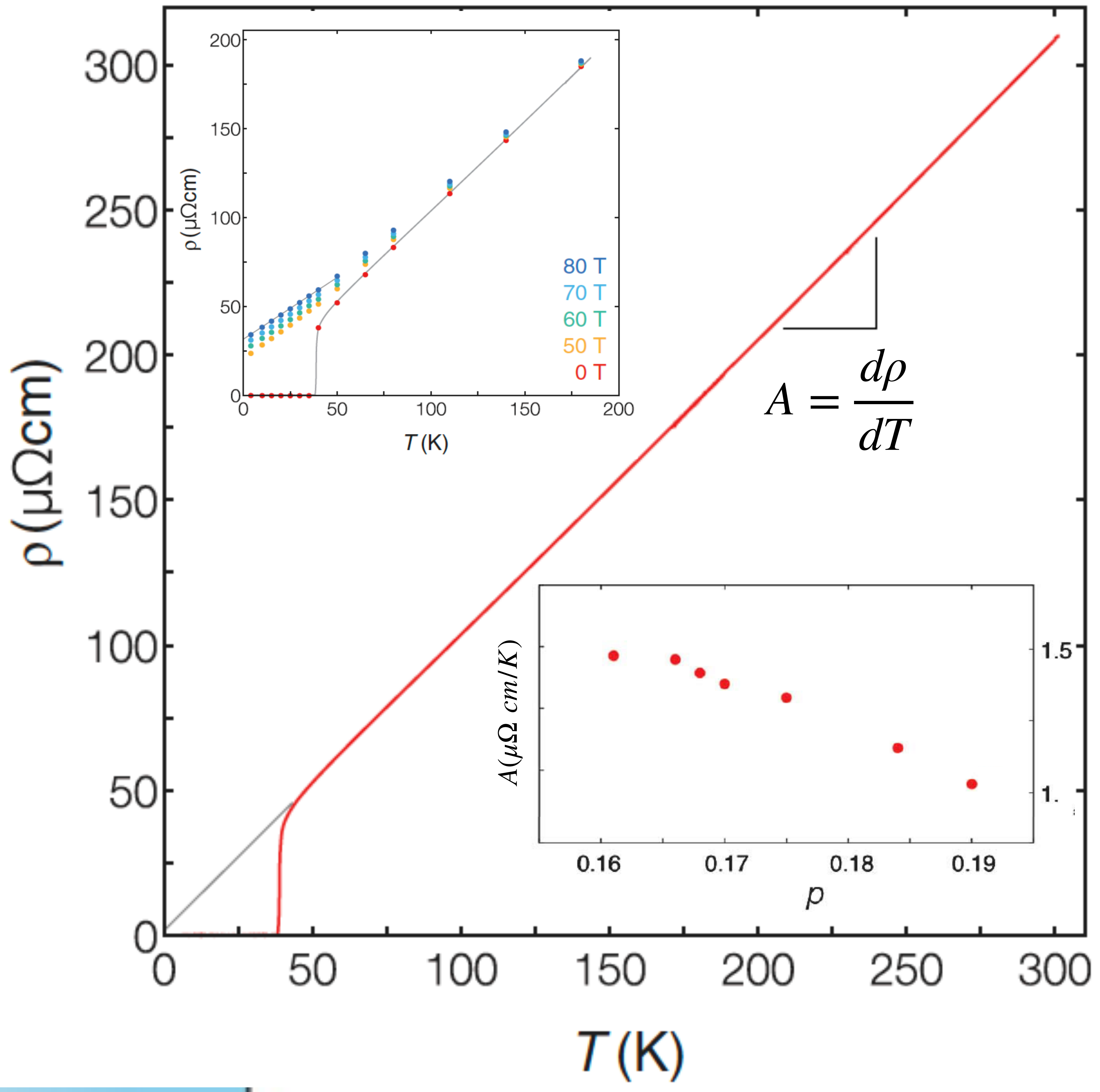
with a 'mass' $s$

and

a boson-fermion

Yukawa coupling $g$.



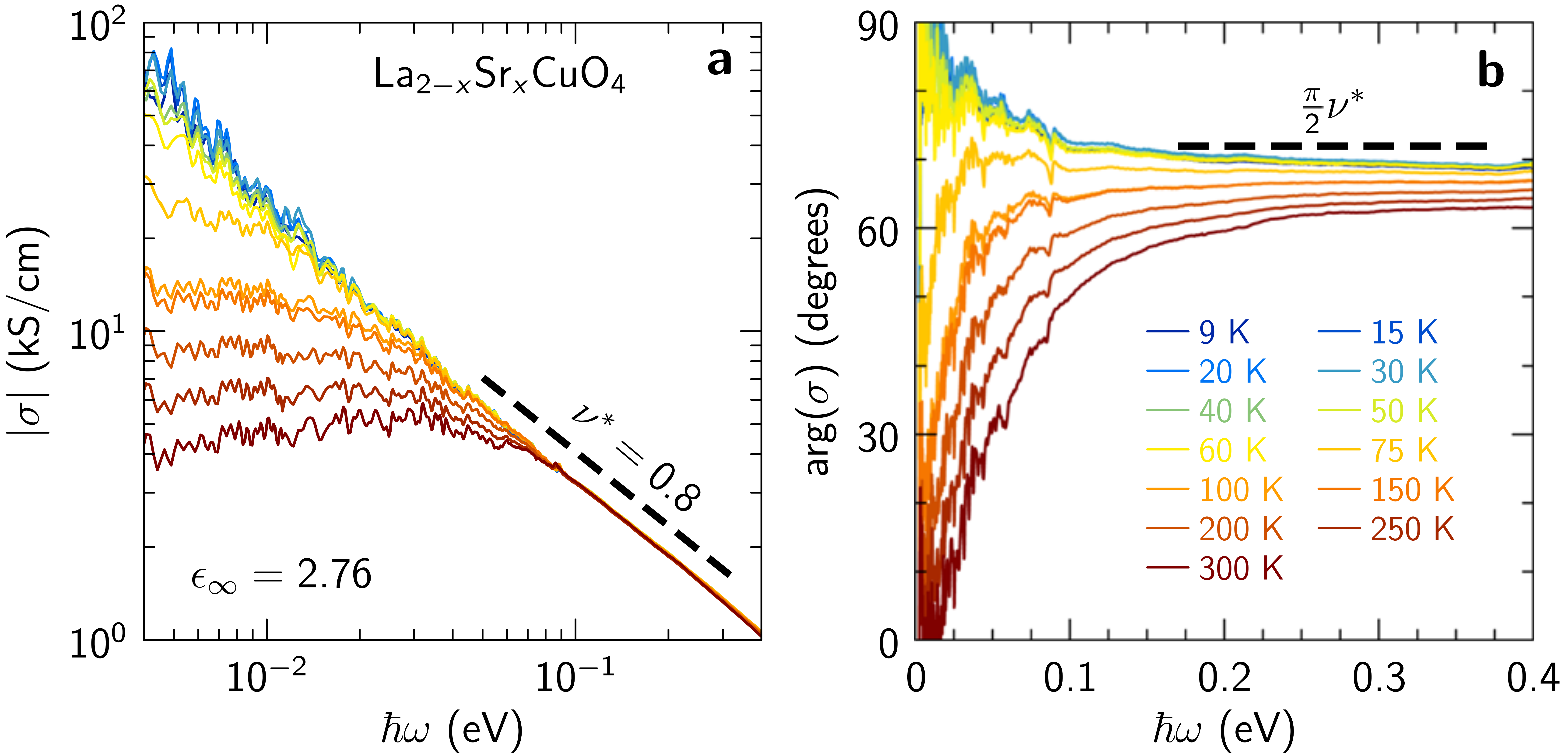

LSCO: Giraldo-Gallo et al. 2018

# Reconciling scaling of the optical conductivity of cuprate superconductors with Planckian resistivity and specific heat

B. Michon, C. Berthod, C. W. Rischau, A. Ataei, L. Chen, S. Komiya, S. Ono, L. Taillefer, D. van der Marel, A. Georges

*Nature Communications* **14**, Article number: 3033 (2023)

$$\sigma(\omega) = i\frac{e^2K/(\hbar d_c)}{\hbar\omega\frac{m^*(\omega)}{m} + i\frac{\hbar}{\tau(\omega)}}$$




$La_{2-x}Sr_xCuO_4$
$p = 0.24$
$T_c = 19$ K

# Reconciling scaling of the optical conductivity of cuprate superconductors with Planckian resistivity and specific heat

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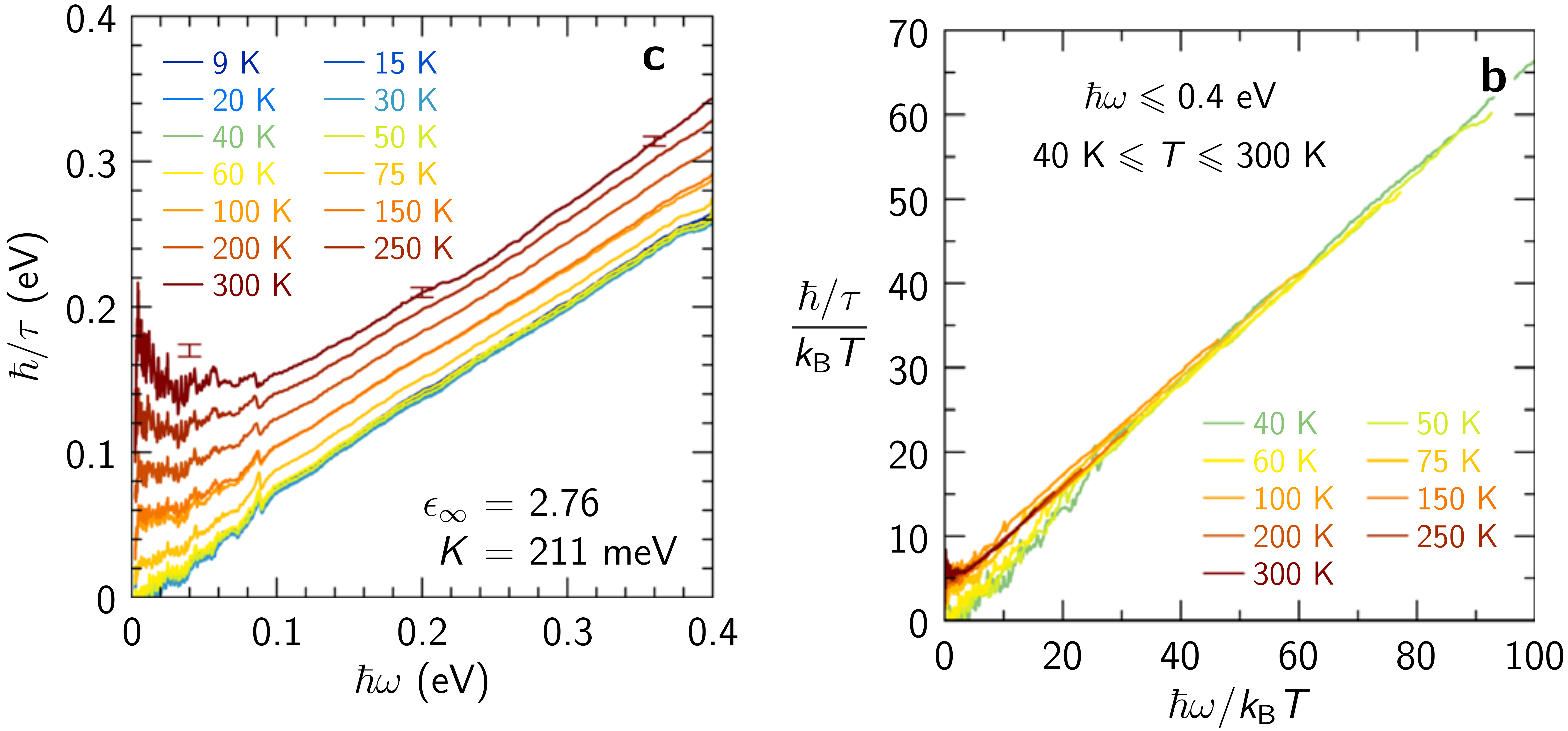



Planckian dynamics!

$$\tau(\omega) = \frac{\hbar}{k_B T} F\left(\frac{\hbar\omega}{k_B T}\right)$$

and entropy

$S(T \to 0) \sim T \ln(1/T)$.

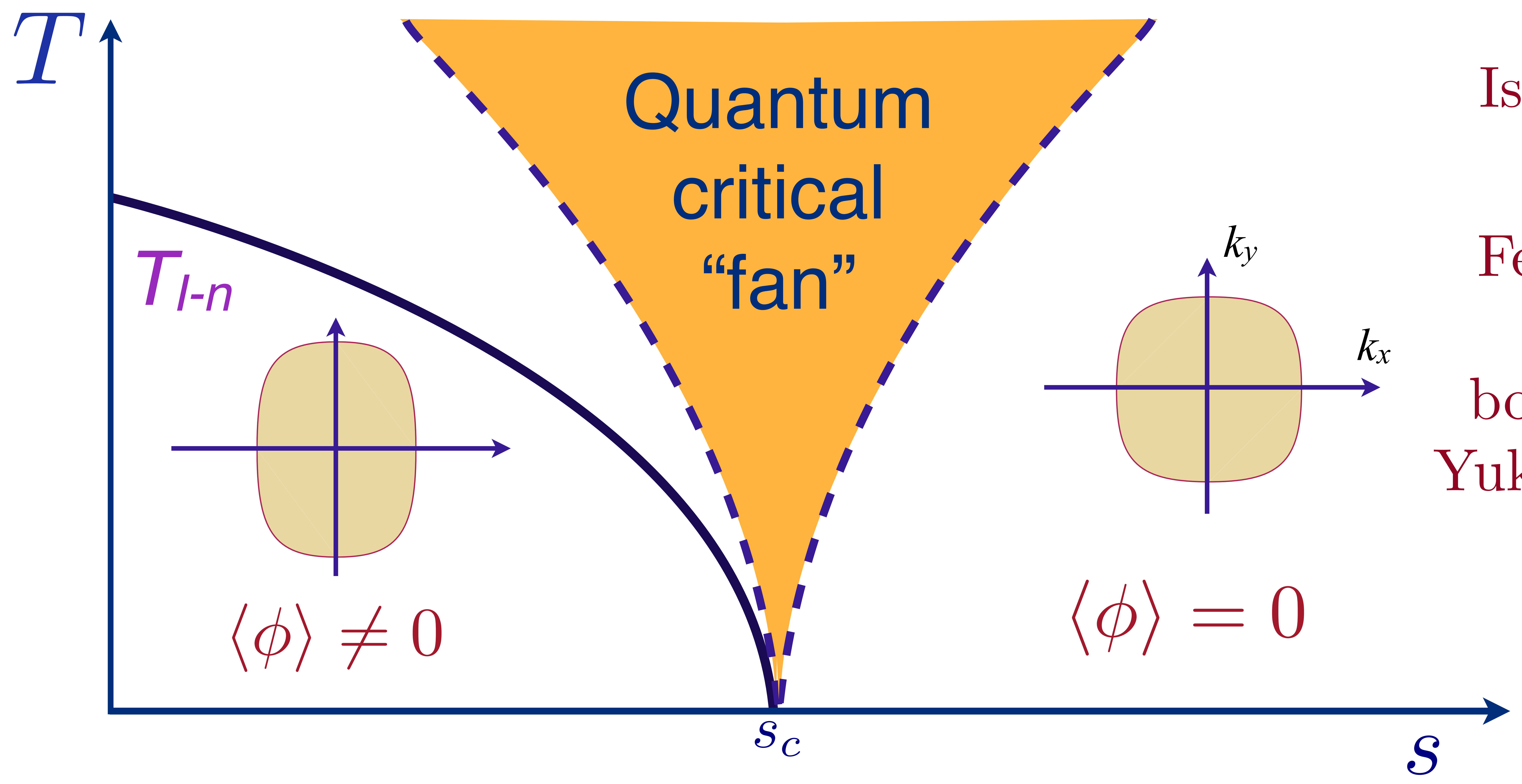
$La_{2-x}Sr_xCuO_4$

$p = 0.24$

$T_c = 19$ K

# Quantum phase transitions in two-dimensional metals

## Type I: Fermi surface deformation



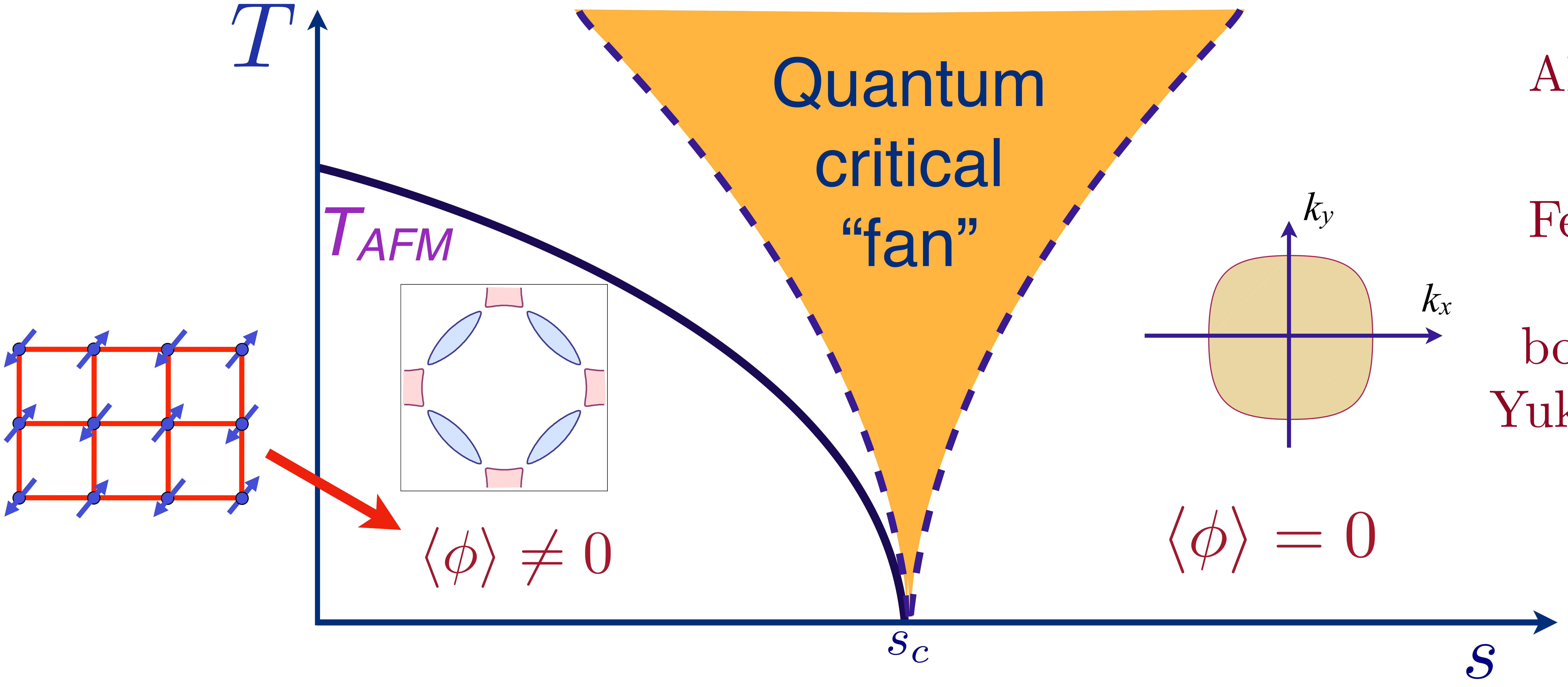

Ising boson $\phi$

$+$

Fermi surface
with a
boson-fermion
Yukawa coupling

# Quantum phase transitions in two-dimensional metals

## Type II: Fermi surface reconstruction



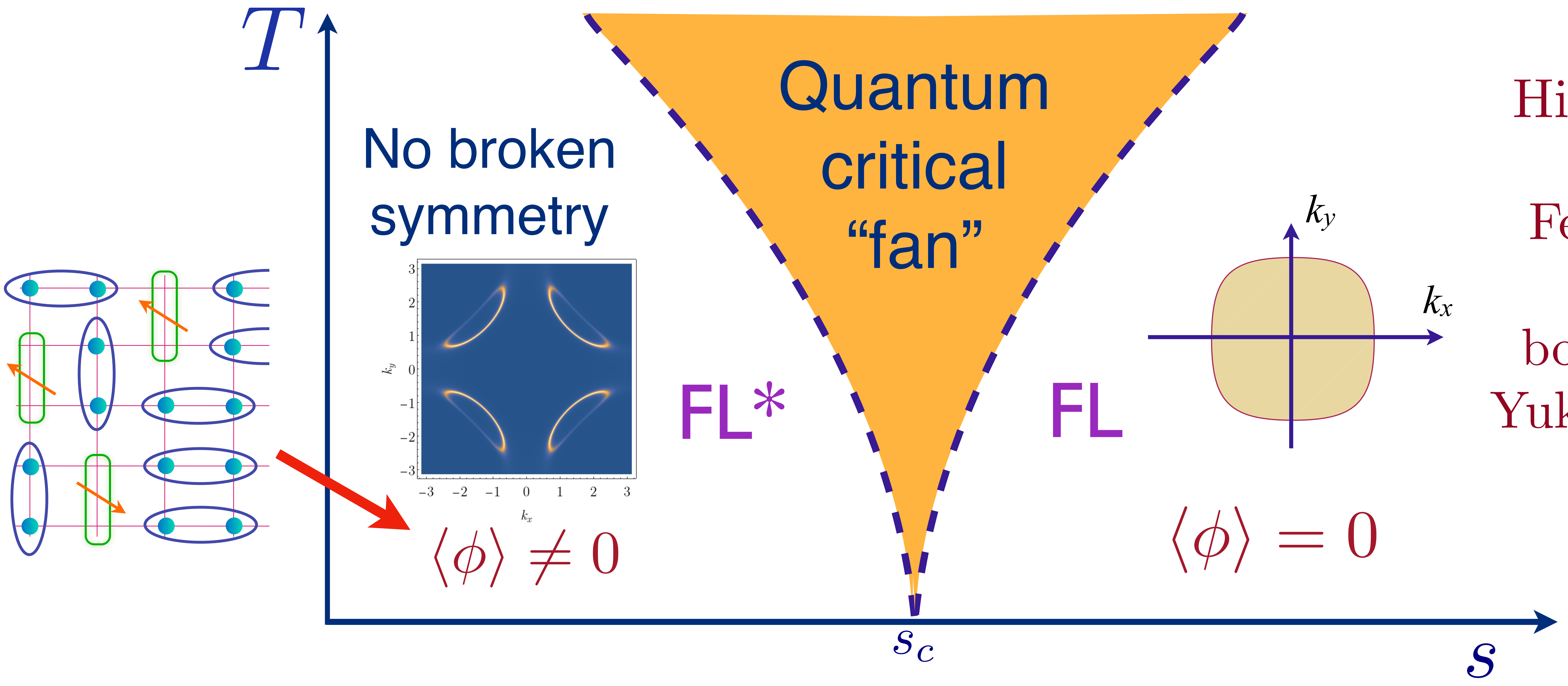

AFM boson $\phi$

$+$

Fermi surface with a boson-fermion Yukawa coupling

# Quantum phase transitions in two-dimensional metals

## Type III: Fermi surface jump




Spin liquid
$+$
Higgs boson $\phi$
$+$
Fermi surface with a boson-fermion Yukawa coupling

Applies to hole-doped cuprates

# Yukawa-SYK model

# Yukawa-SYK model

$$\mathcal{H} = -\mu \sum_i \psi_i^\dagger \psi_i + \sum_\ell \frac{1}{2}\left(\pi_\ell^2 + \omega_0^2 \phi_\ell^2\right) + \frac{1}{N} \sum_{ij\ell} g_{ij\ell} \psi_i^\dagger \psi_j \phi_\ell$$

with $g_{ij\ell}$ independent random numbers with zero mean.

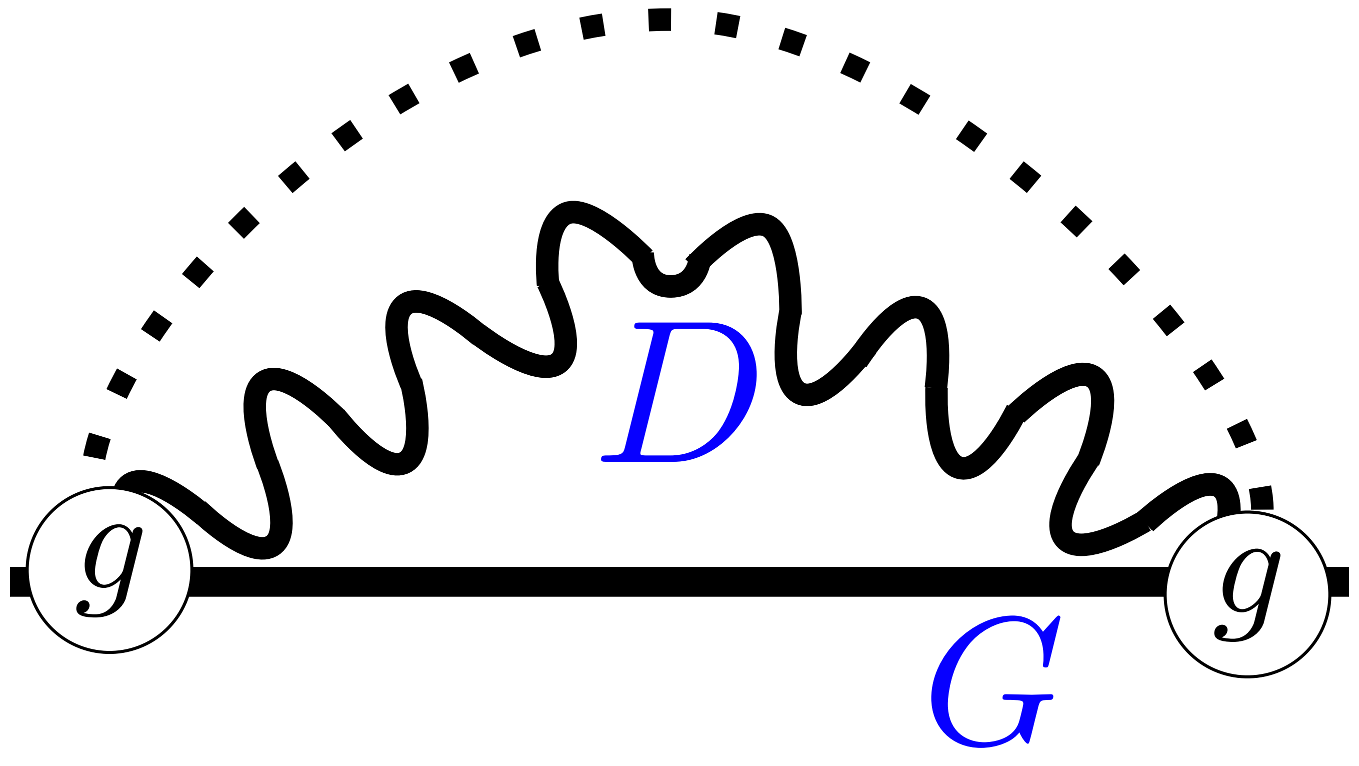

I. Esterlis and J. Schmalian,
PRB **100**, 115132 (2019)
See also Yuxuan Wang,
PRL **124**, 017002 (2020)

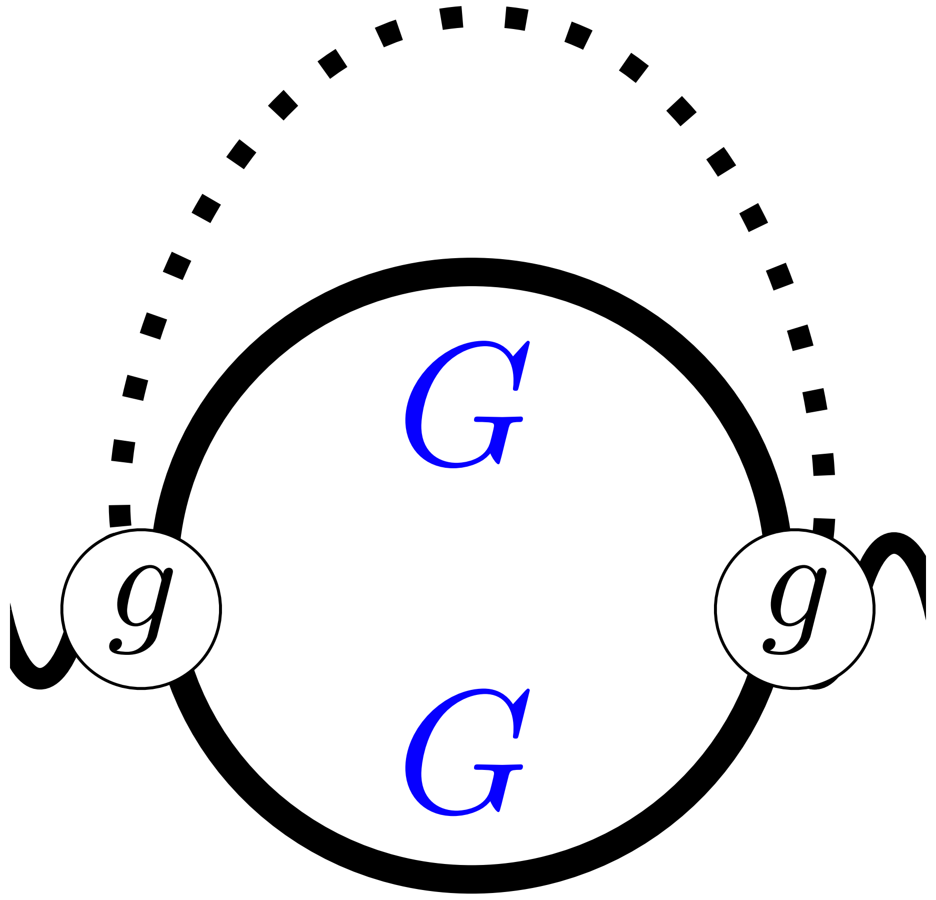
# Yukawa-SYK model

$$\mathcal{H} = -\mu \sum_i \psi_i^\dagger \psi_i + \sum_\ell \frac{1}{2}\left(\pi_\ell^2 + \omega_0^2 \phi_\ell^2\right) + \frac{1}{N}\sum_{ij\ell} g_{ij\ell}\psi_i^\dagger \psi_j \phi_\ell$$

with $g_{ij\ell}$ independent random numbers with zero mean. The large $N$ equations for the Green's functions and self energies of the fermions $(G, \Sigma)$ and bosons $(D, \Pi)$ are

$$\boxed{\begin{array}{ccc} G(i\omega_n) = \dfrac{1}{i\omega_n + \mu - \Sigma(i\omega_n)} & , & D(i\omega_n) = \dfrac{1}{\omega_n^2 + \omega_0^2 - \Pi(i\omega_n)} \\ \Sigma(\tau) = g^2 G(\tau) D(\tau) & , & \Pi(\tau) = -g^2 G(\tau) G(-\tau) \end{array}}$$

$\Sigma =$ 


$\Pi =$ 


I. Esterlis and J. Schmalian, PRB **100**, 115132 (2019)
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Make the low frequency ansatz

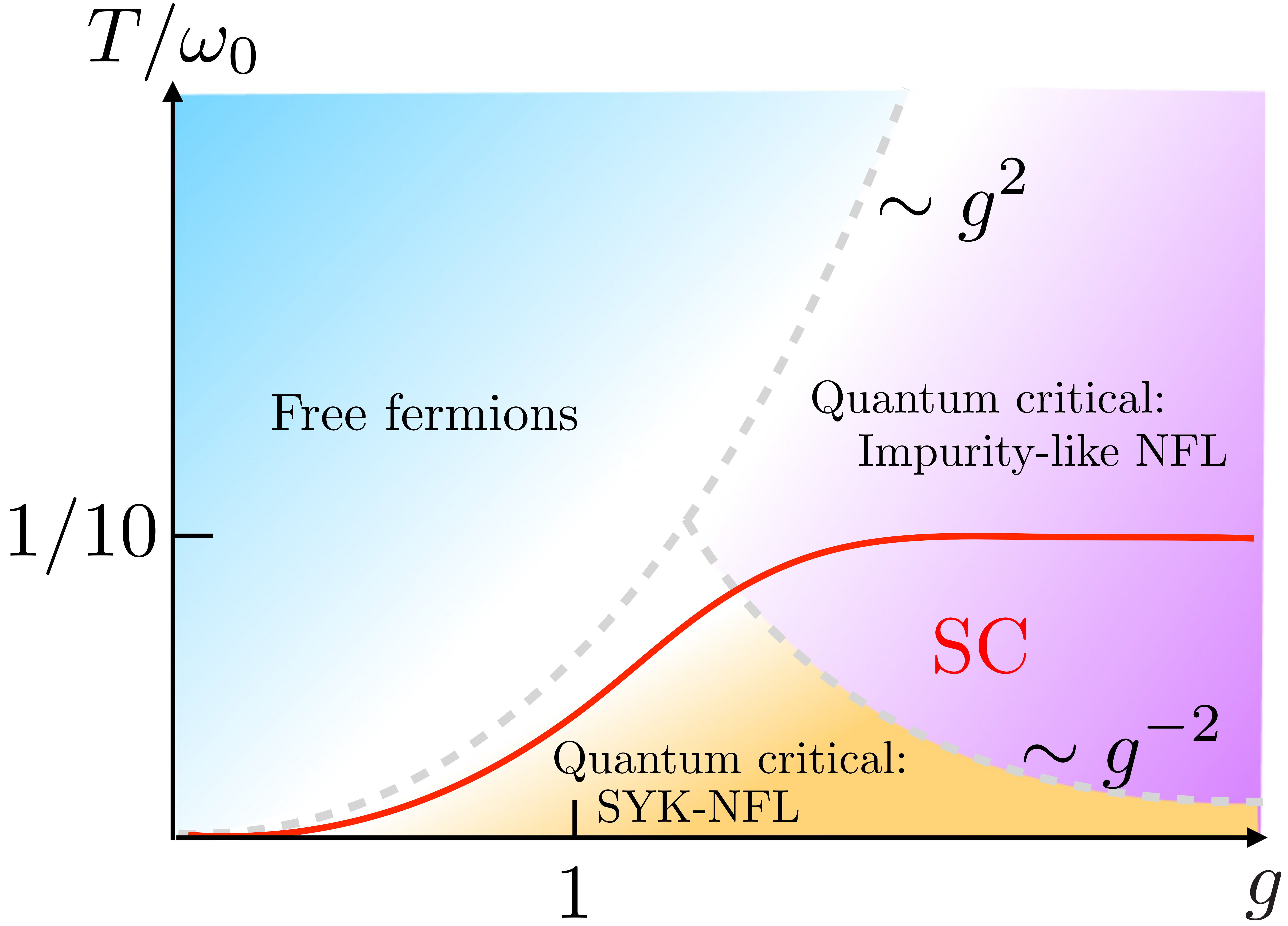
$$G(i\omega) \sim -i\,\mathrm{sgn}(\omega)|\omega|^{-(1-2\Delta)} \quad , \quad D(i\omega) \sim |\omega|^{1-4\Delta} \quad , \quad \frac{1}{4} < \Delta < \frac{1}{2}$$

A consistent solution exists for

$$\frac{4\Delta - 1}{2(2\Delta-1)[\sec(2\pi\Delta) - 1]} = 1 \quad , \quad \Delta = 0.42037\ldots$$

I. Esterlis and J. Schmalian,
PRB **100**, 115132 (2019)
See also Yuxuan Wang,
PRL **124**, 017002 (2020)

# Yukawa-SYK model



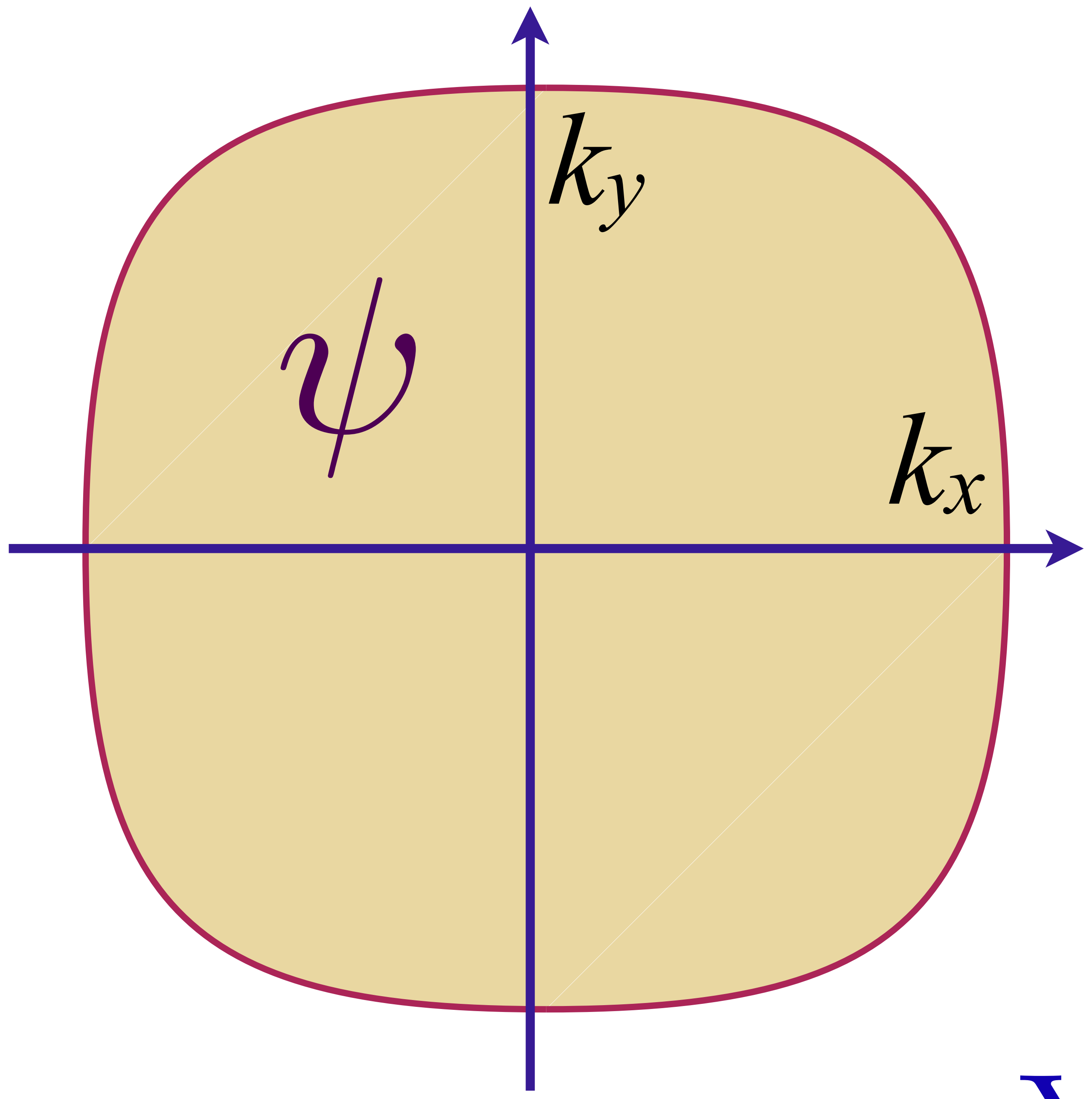

I. Esterlis and J. Schmalian, PRB **100**, 115132 (2019)
See also Yuxuan Wang, PRL **124**, 017002 (2020)

# Quantum criticality in clean metals

# Fermi surface + critical boson with no spatial disorder

$$\mathcal{L}_\psi = \psi^\dagger_{\boldsymbol{k}} \left( \frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$

Type I: a critical boson $\phi$
*e.g.* Ising ferromagnetism




$$+s\,[\phi(\boldsymbol{r})]^2 \qquad +g\,\psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\,\phi(\boldsymbol{r})$$

$$+K\,[\nabla_{\boldsymbol{r}}\phi(\boldsymbol{r})]^2 + u\,[\phi(\boldsymbol{r})]^4$$

Yukawa coupling $g$ between fermions and bosons

# Fermi surface + critical boson with no spatial disorder

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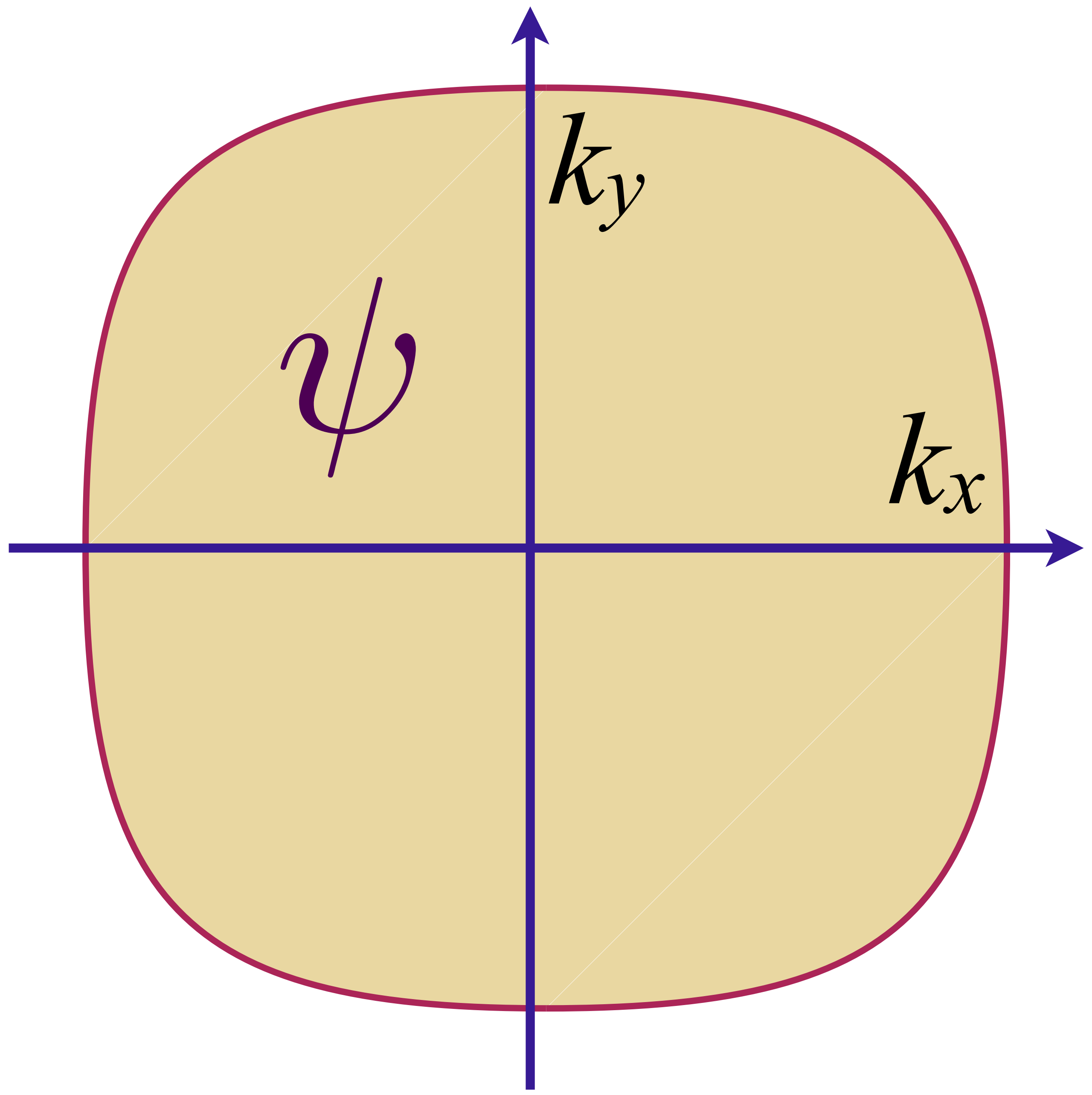


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$$+K\,[\nabla_{\boldsymbol{r}}\phi(\boldsymbol{r})]^2 + u\,[\phi(\boldsymbol{r})]^4$$

Large $N$ limit of:
$$\frac{g_{\alpha\beta\gamma}}{N}\,\psi^\dagger_\alpha(\boldsymbol{r})\psi_\beta(\boldsymbol{r})\,\phi_\gamma(\boldsymbol{r})$$
with $\alpha, \beta, \gamma = 1 \ldots N$
and $g_{\alpha\beta\gamma}$ random in flavor space,
as in *Yukawa-SYK* models
of fermions and bosons

# Fermi surface + critical boson with no spatial disorder

$$\mathcal{L}_\psi = \psi^\dagger_{\boldsymbol{k}} \left( \frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$

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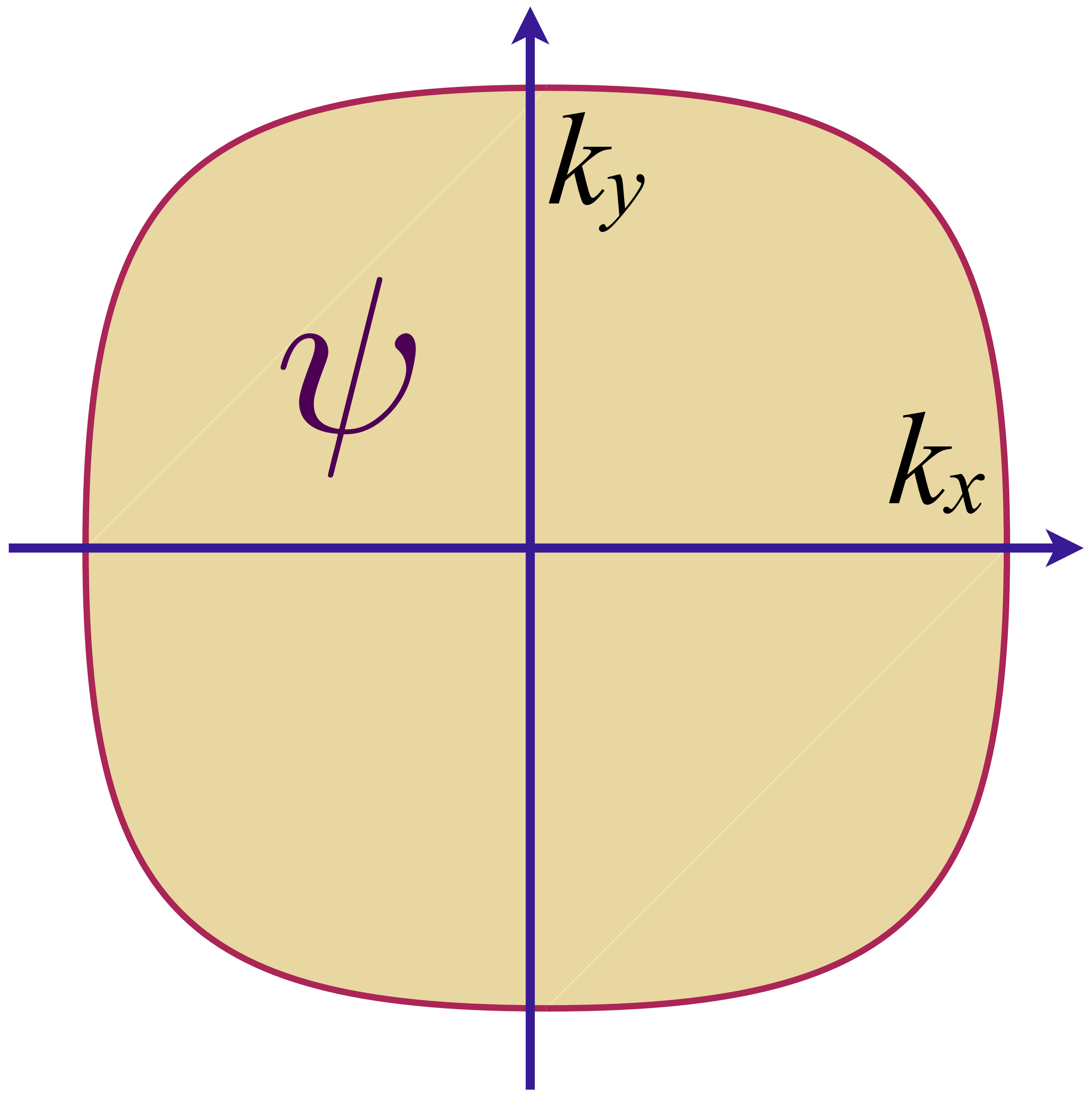
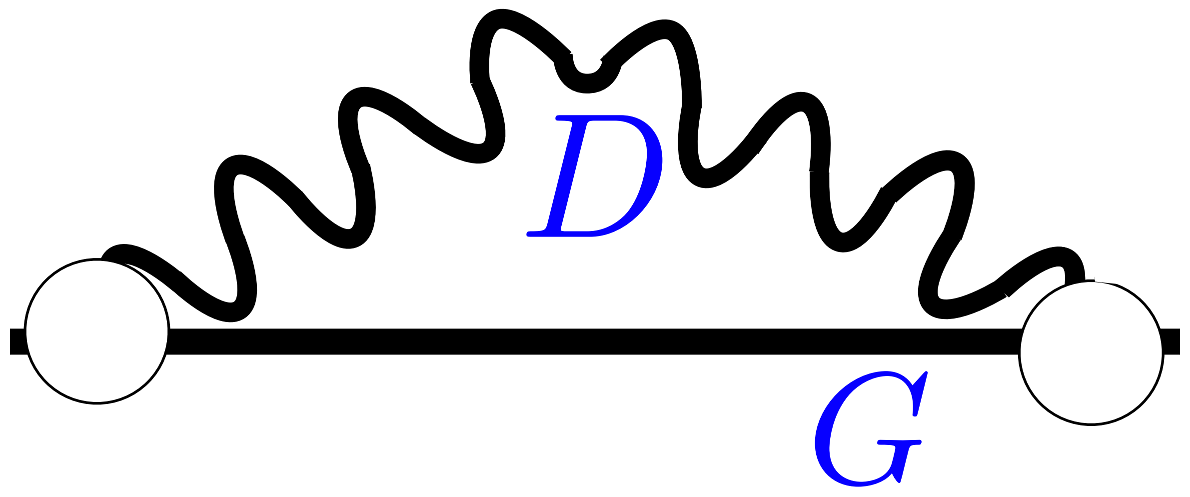
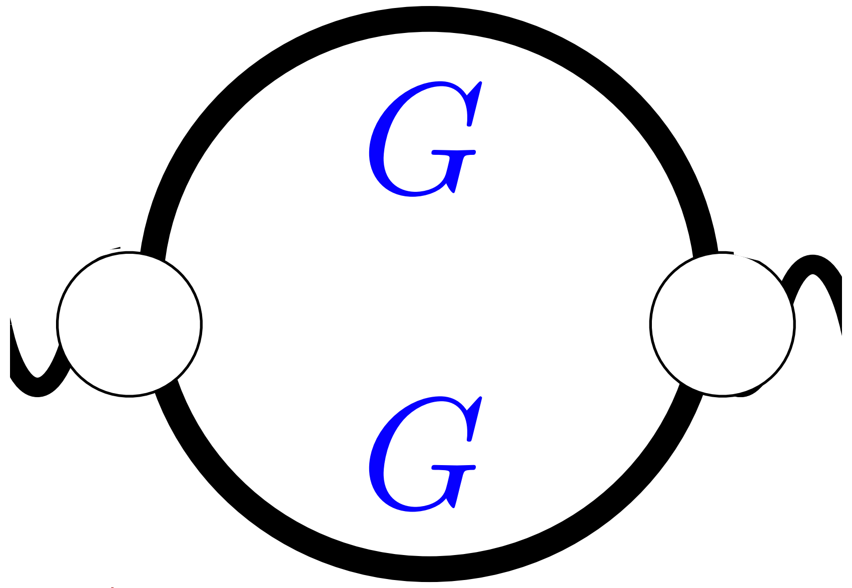



$$+s\,[\phi(\boldsymbol{r})]^2 \qquad +g\,\psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\,\phi(\boldsymbol{r})$$

$$+K\,[\nabla_{\boldsymbol{r}}\phi(\boldsymbol{r})]^2 + u\,[\phi(\boldsymbol{r})]^4$$

$\Sigma =$ 


$\Pi =$ 

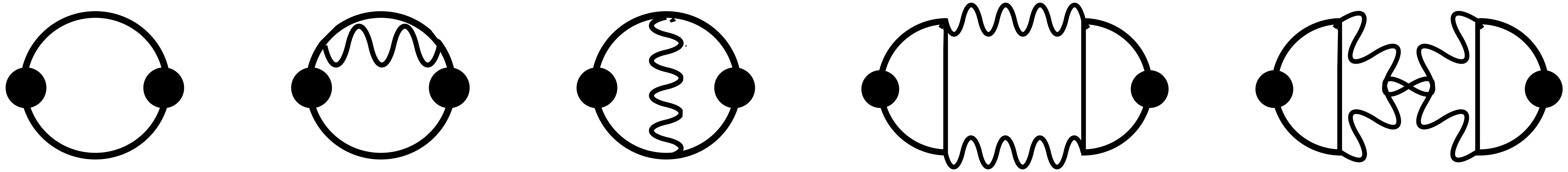

$$\Sigma(\hat{\boldsymbol{k}}, i\omega) \sim -i\,\mathrm{sgn}(\omega)|\omega|^{2/3}\,, \quad G(\boldsymbol{k}, i\omega) = \frac{1}{i\omega - \varepsilon(\boldsymbol{k}) - \Sigma(\hat{\boldsymbol{k}}, i\omega)}$$

P.A. Lee (1989)

**Sharp Fermi surface but no quasiparticles**

# Fermi surface + critical boson with no spatial disorder

## Optical conductivity—Diagrams



$$\mathrm{Re}\left[\sigma(\omega)\right] = C\,|\omega|^{-2/3}$$
$$\rho(T) \sim T^{4/3}$$

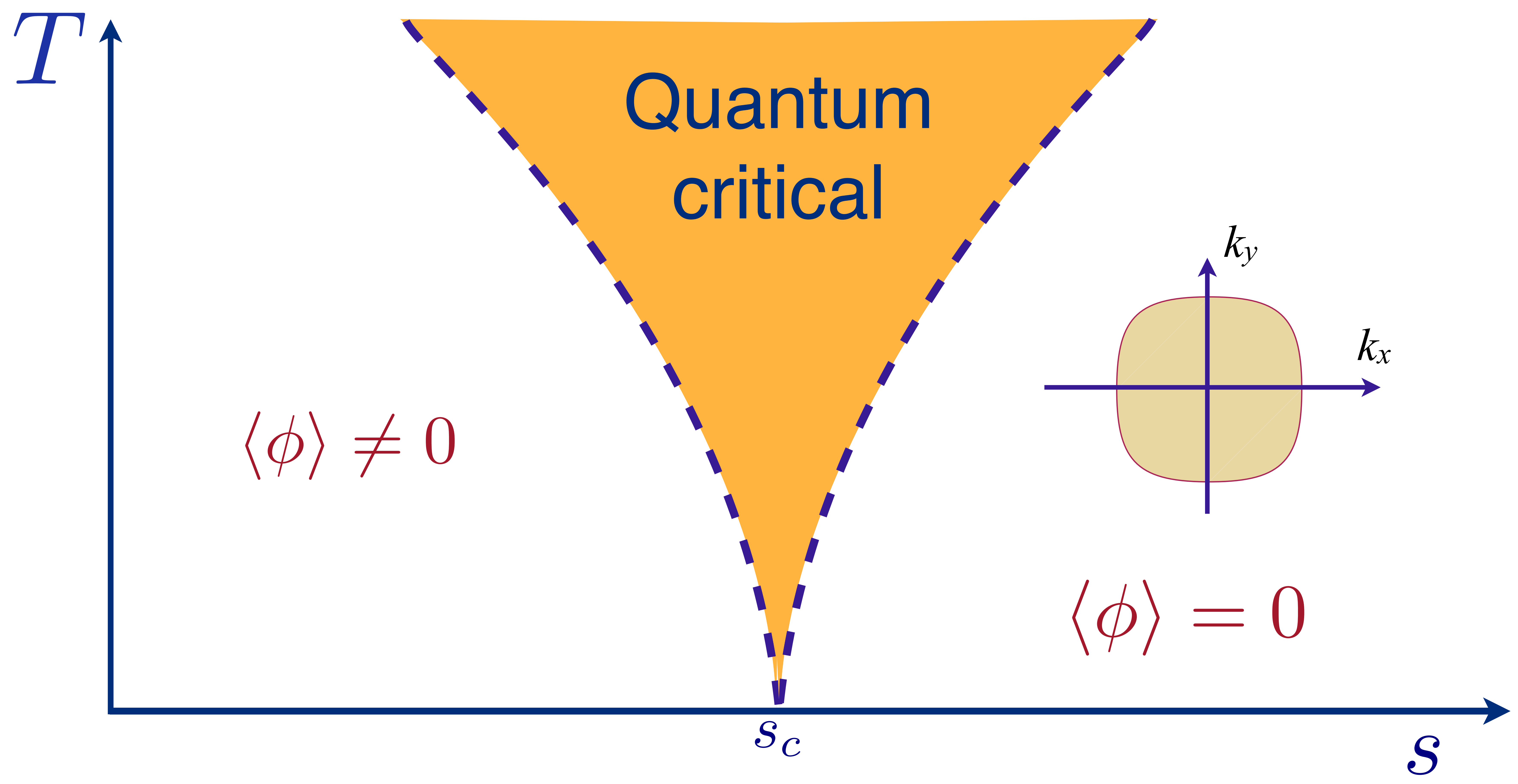
Yong Baek Kim, A. Furusaki, Xiao-Gang Wen, and P. A. Lee, PRB **50**, 17917 (1994).

$$C = 0\,; \quad \sigma(\omega) = iD/(\omega - \omega_c) + \omega^0 + \ldots$$

Haoyu Guo, Aavishkar Patel, Ilya Esterlis, S.S., PRB **106**, 115151 (2022);
Haoyu Guo, Davide Valentinis, J. Schmalian, S.S., Aavishkar Patel, PRB **109**, 075162 (2024);
D.L. Maslov and A.V. Chubukov, Rep. Prog. Phys. **80**, 026503 (2017);
Zhengyan Darius Sh Dominic V. Else, Hart Goldman, T. Senthil, SciPost Phys. **14**, 113 (2023);
Haoyu Guo, arXiv: 2406.12967

# Fermi surface + critical boson with no spatial disorder

## Type I, II, or III



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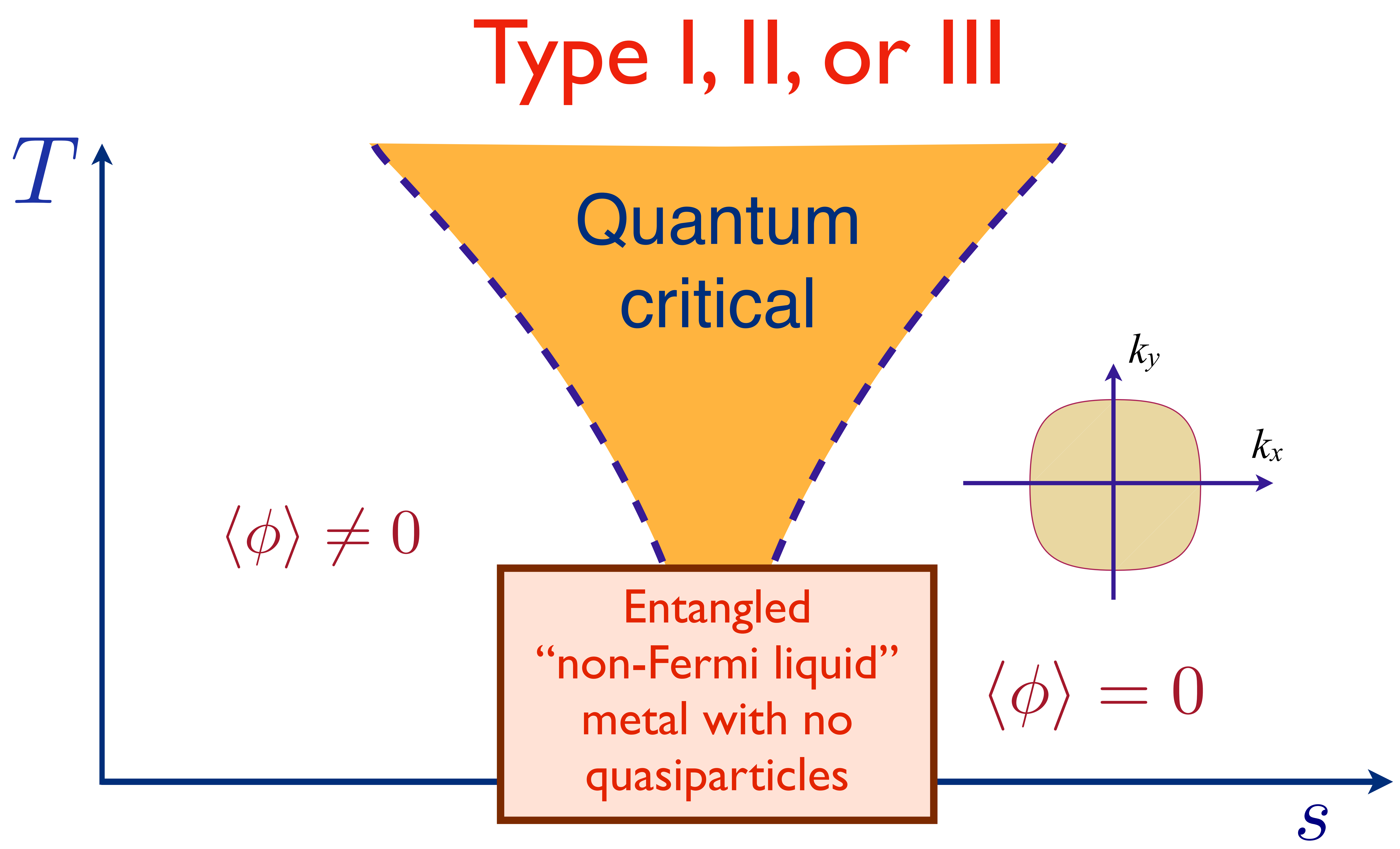

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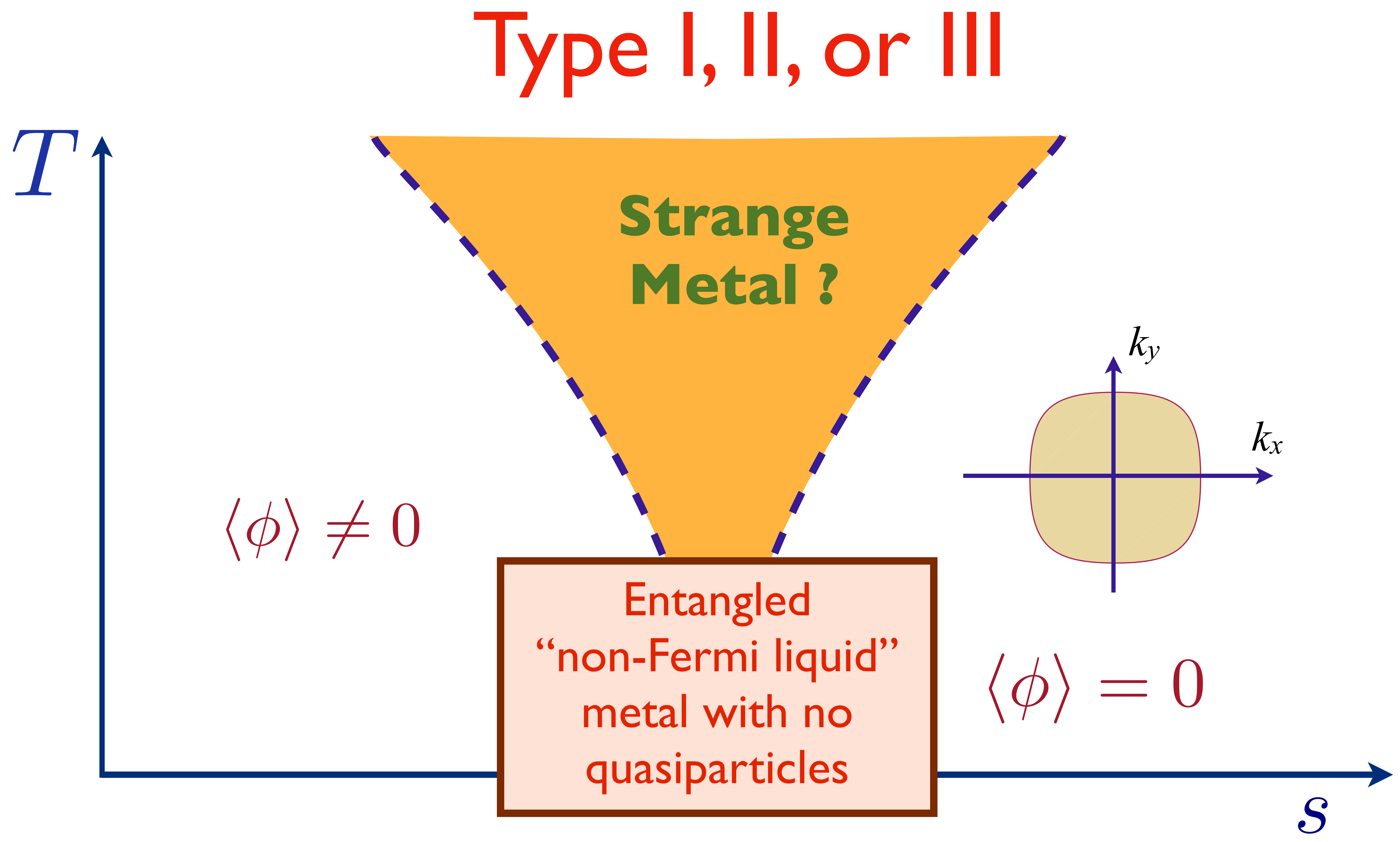

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S.A. Hartnoll, P. K. Kovtun, M. Muller, and S.S., PRB **76**, 144502 (2007); S.A. Hartnoll, R. Mahajan, M. Punk, and S. S., PRB **89**, 155130 (2014); Aavishkar Patel and S.S., PRB **90**, 165146 (2014); Haoyu Guo, Aavishkar Patel, Ilya Esterlis, S.S., PRB **106**, 115151 (2022); Haoyu Guo, Davide Valentinis, J. Schmalian, S.S., Aavishkar Patel, PRB **109**, 075162 (2024); D.L. Maslov and A.V. Chubukov, Rep. Prog. Phys. **80**, 026503 (2017); Zhengyan Darius Sh Dominic V. Else, Hart Goldman, T. Senthil, SciPost Phys. **14**, 113 (2023); Haoyu Guo, arXiv: 2406.12967

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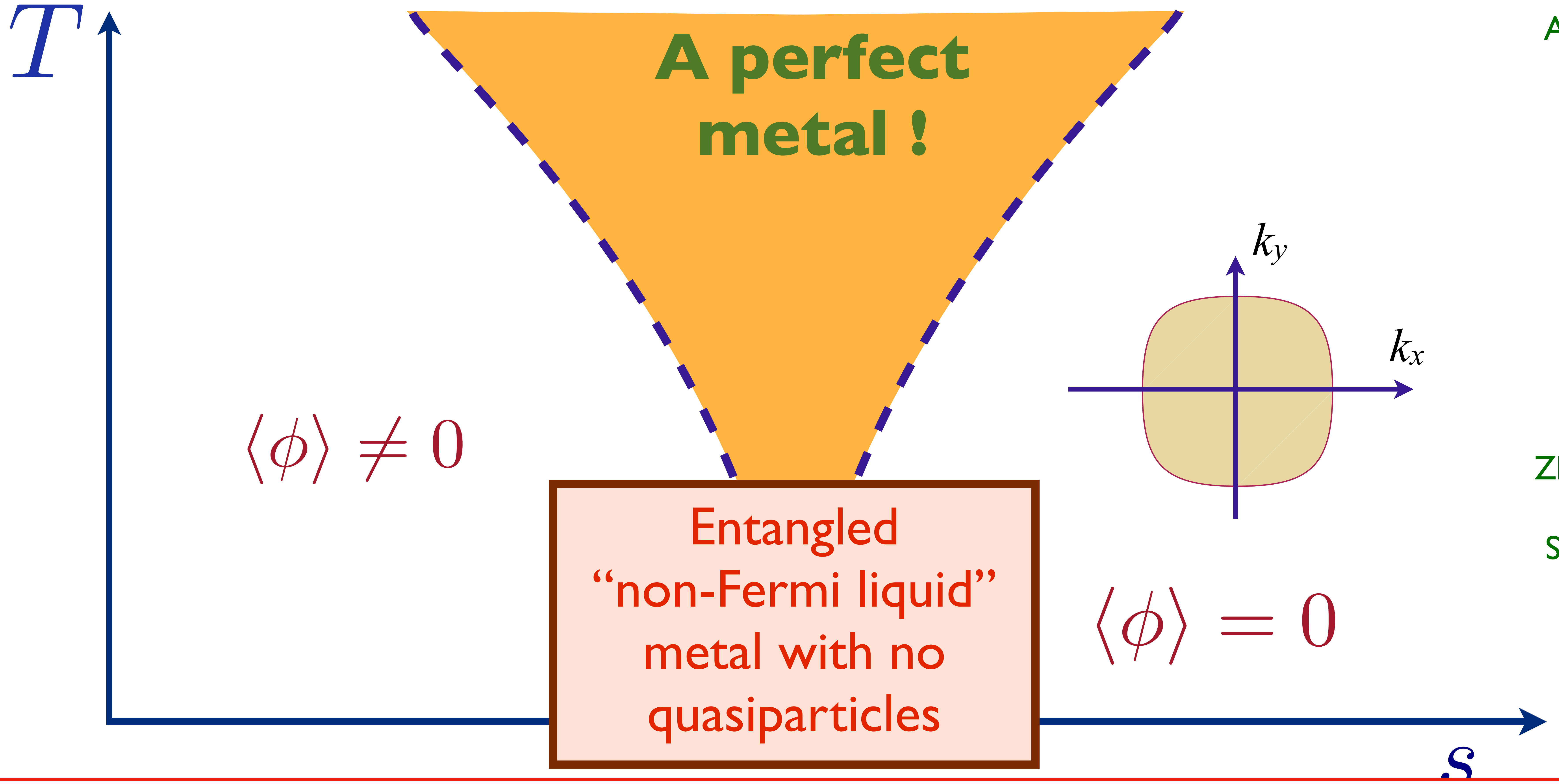
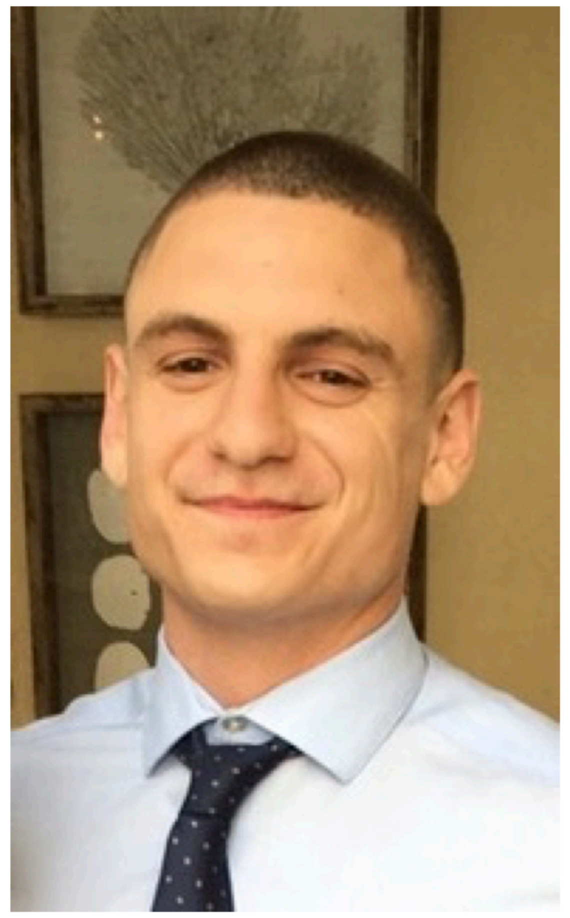



Extreme drag: the fermions $\psi$ "drag" the bosons $\phi$ as they move, and so electrical current does not relax, even though strong $\psi$-$\phi$ scattering leads to absence of $\psi$ quasiparticles.

# Universal theory of strange metals:

**Quantum phase transitions
in inhomogeneous metals
described by the
two-dimensional Yukawa-SYK model**

Theory applies for types I, II, III, with only minor differences.



Ilya Esterlis
Wisconsin

Haoyu Guo
Cornell

Aavishkar Patel
Flatiron

Chenyuan Li
Rice

Davide
Valentinis
KIT

Joerg
Schmalian
KIT

Peter Lunts
Harvard

Aavishkar A. Patel, Haoyu Guo, Ilya Esterlis, S. S., *Science* **381**, 790 (2023)
*Universal theory of strange metals from spatially random interactions*

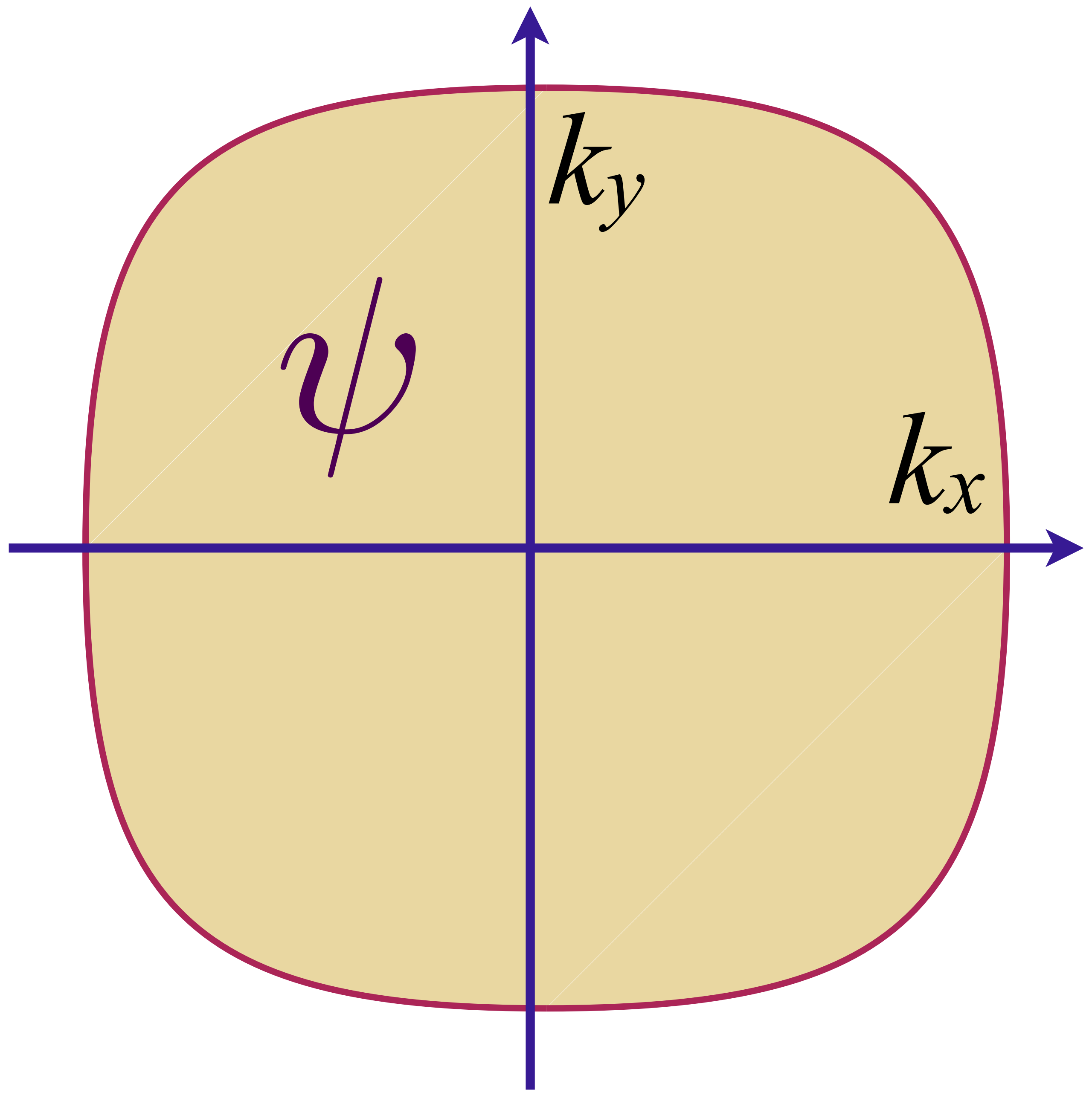
Aavishkar A. Patel, Peter Lunts, S.S., *PNAS* **121**, e2402052121 (2024)
*Localization of overdamped bosonic modes and transport in strange metals*

Chenyuan Li, Aavishkar A. Patel, Haoyu Guo, Davide Valentinis, Jorg Schmalian, S.S., Ilya Esterlis, *PRL* **133**, 186502 (2024)
*Strange metal and superconductor in the two-dimensional Yukawa-Sachdev-Ye-Kitaev model*

Aavishkar A. Patel, Peter Lunts, and Michael Albergo, arXiv:2410.05365
*Strange metals and planckian transport in a gapless phase from spatially random interactions*

# Fermi surface + critical boson with no spatial disorder

$$\mathcal{L}_\psi = \psi_{\boldsymbol{k}}^\dagger \left( \frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$




Type I, II, III: A critical boson $\phi$
*e.g.* Ising ferromagnetism,
spin-density wave order,
Higgs boson for Fermi-volume changing transition

$$+s\,[\phi(\boldsymbol{r})]^2 \qquad +g\,\psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\,\phi(\boldsymbol{r})$$

$$+K\,[\nabla_{\boldsymbol{r}}\phi(\boldsymbol{r})]^2 + u\,[\phi(\boldsymbol{r})]^4$$

# Fermi surface + critical boson with potential disorder

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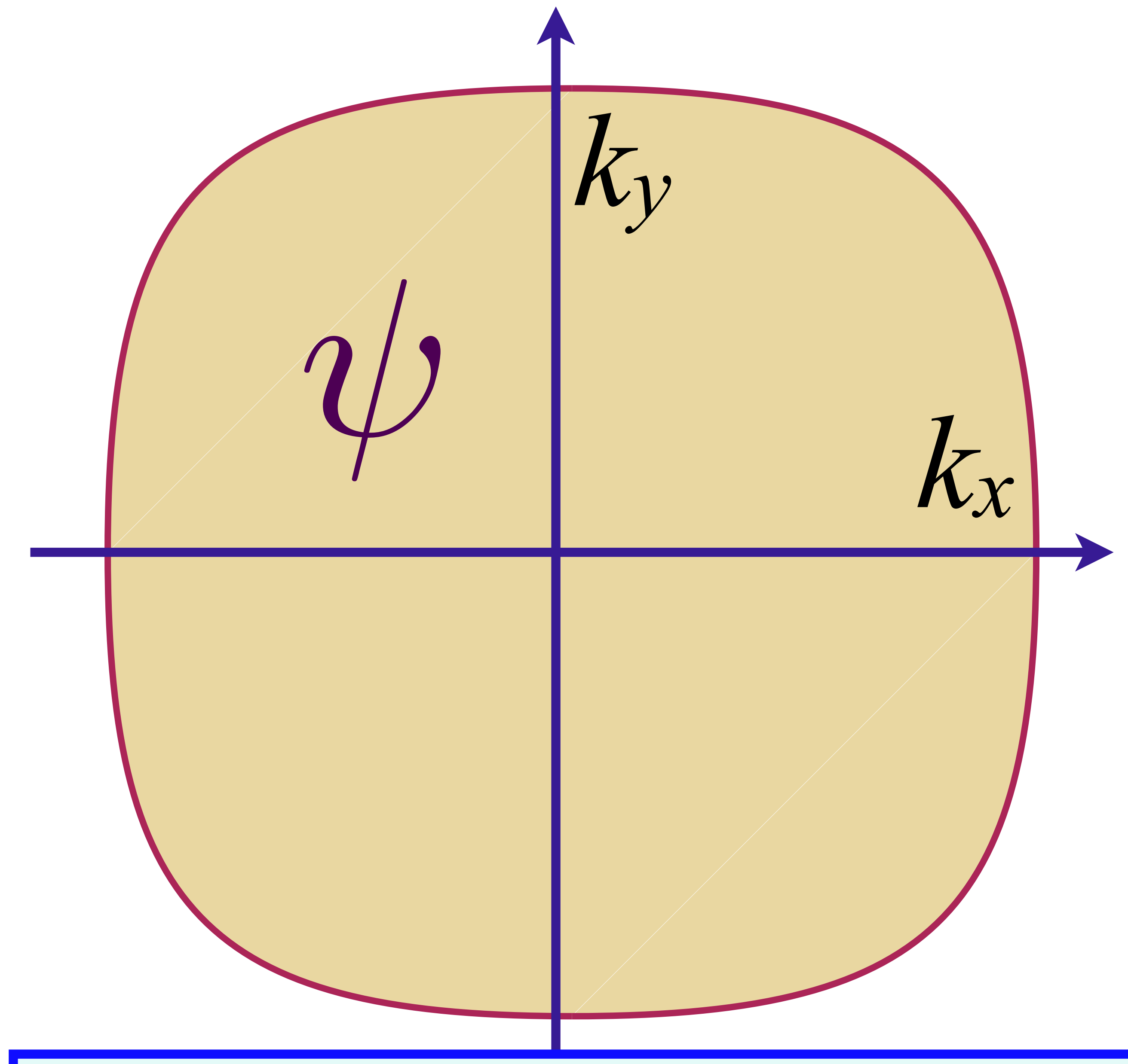
Type I, II, III: A critical boson $\phi$
*e.g.* Ising ferromagnetism,
spin-density wave order,
Higgs boson for Fermi-volume changing transition




$$+s\,[\phi(\boldsymbol{r})]^2 \qquad +g\,\psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\,\phi(\boldsymbol{r})$$

$$+K\,[\nabla_{\boldsymbol{r}}\phi(\boldsymbol{r})]^2 + u\,[\phi(\boldsymbol{r})]^4 \; +v(\boldsymbol{r})\psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})$$

Spatially random potential $v(\boldsymbol{r})$ with $\overline{v(\boldsymbol{r})} = 0$, $\overline{v(\boldsymbol{r})v(\boldsymbol{r}')} = v^2\delta(\boldsymbol{r} - \boldsymbol{r}')$

$v(\boldsymbol{r})$ leads to elastic scattering of $\psi$ and 'Altshuler-Aronov' corrections; localization of $\psi$ only at long length scales, not relevant for experiments

# 2d-YSYK model: Fermi surface + critical boson with interaction disorder

$$\mathcal{L}_\psi = \psi_{\boldsymbol{k}}^\dagger \left( \frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$

Type I, II, III: A critical boson $\phi$
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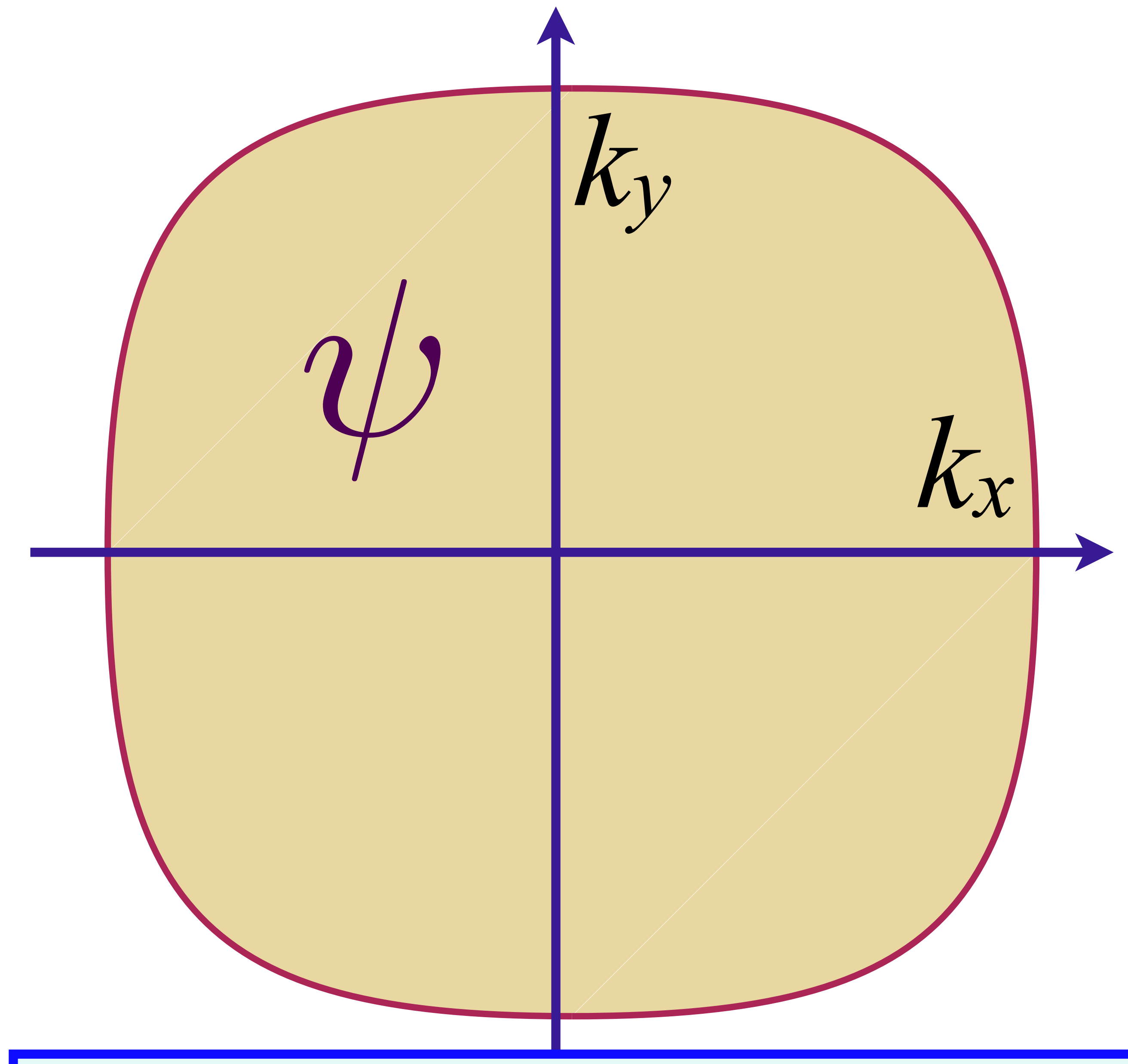



$$+s\,[\phi(\boldsymbol{r})]^2 + [g + g'(\boldsymbol{r})]\,\psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\,\phi(\boldsymbol{r})$$

$$+K\,[\nabla_{\boldsymbol{r}}\phi(\boldsymbol{r})]^2 + u\,[\phi(\boldsymbol{r})]^4 + v(\boldsymbol{r})\psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})$$

Aavishkar A. Patel, Haoyu Guo, Ilya Esterlis, S. Sachdev, *Science* **381**, 790 (2023)

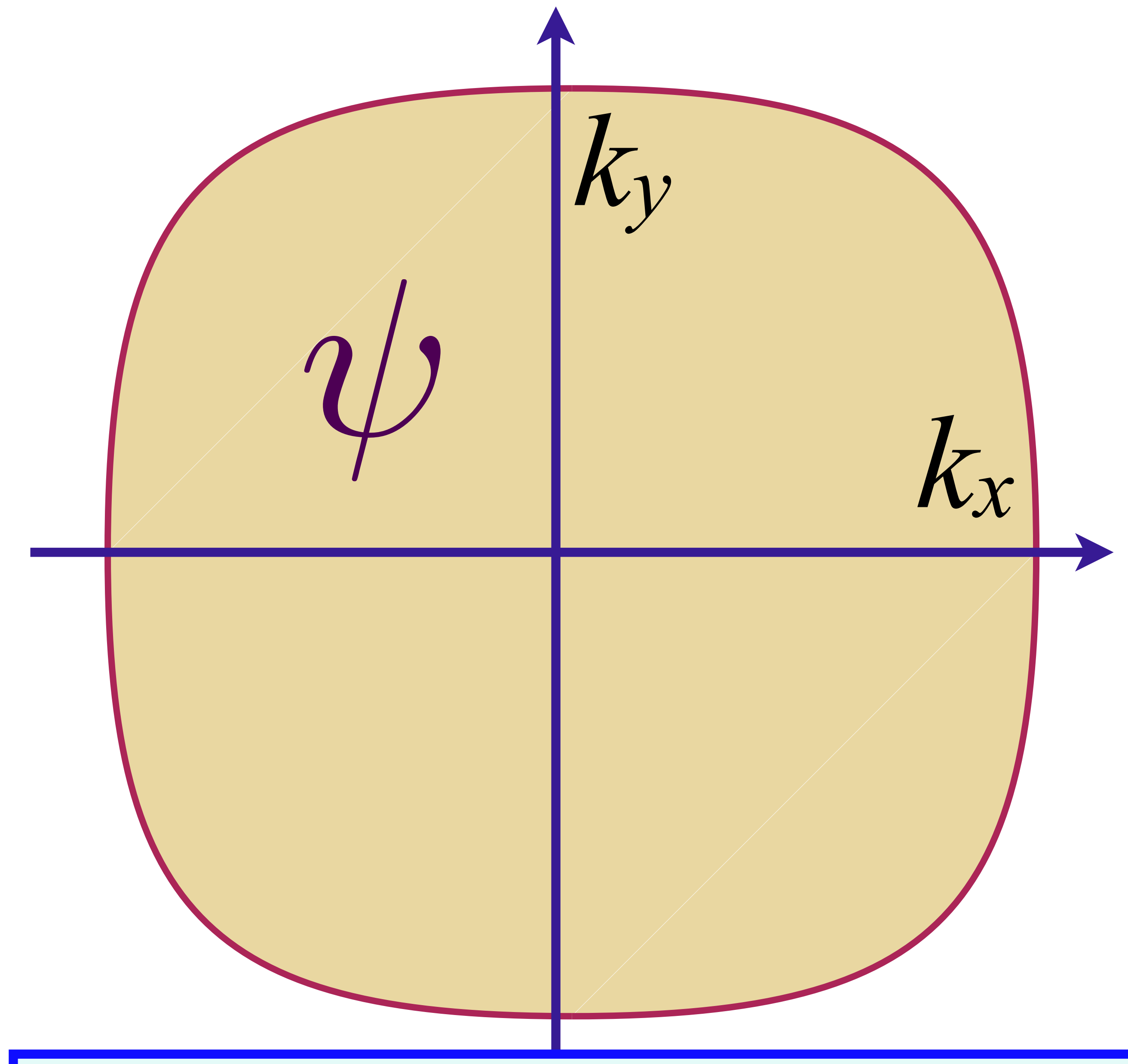
Spatially random potential $v(\boldsymbol{r})$ with $\overline{v(\boldsymbol{r})} = 0$, $\overline{v(\boldsymbol{r})v(\boldsymbol{r}')} = v^2\delta(\boldsymbol{r} - \boldsymbol{r}')$

Spatially random Yukawa coupling $g'(\boldsymbol{r})$ with $\overline{g'(\boldsymbol{r})} = 0$, $\overline{g'(\boldsymbol{r})g'(\boldsymbol{r}')} = g'^2\delta(\boldsymbol{r} - \boldsymbol{r}')$

$g'(\boldsymbol{r})$ creates inhomogeneity in the position of QCP (Harris disorder):
the two-dimensional Yukawa-Sachdev-Ye-Kitaev model.

# 2d-YSYK model: Fermi surface + critical boson with interaction disorder

$$\mathcal{L}_\psi = \psi^\dagger_{\boldsymbol{k}} \left( \frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$

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*e.g.* Ising ferromagnetism,
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Aavishkar A. Patel, Haoyu Guo, Ilya Esterlis, S. Sachdev, *Science* **381**, 790 (2023)

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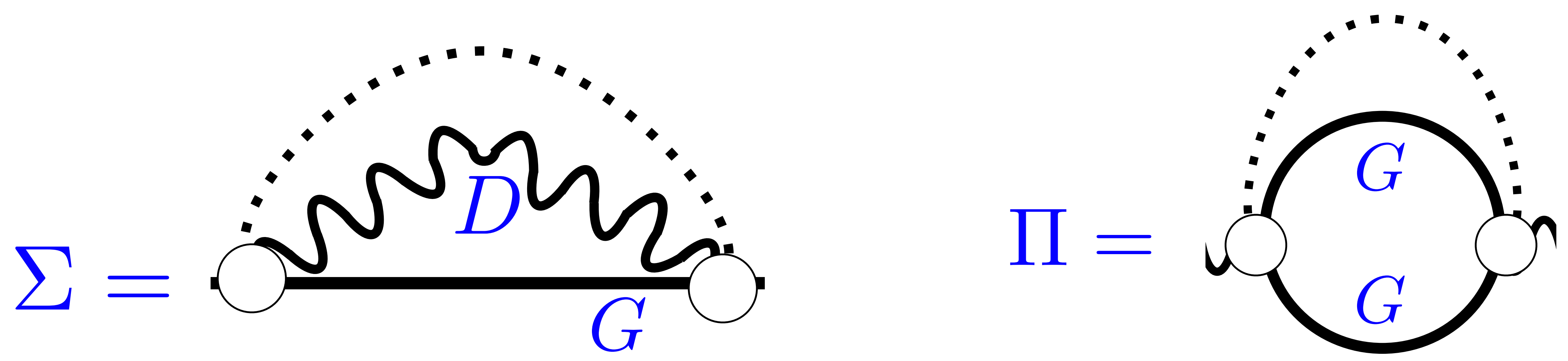
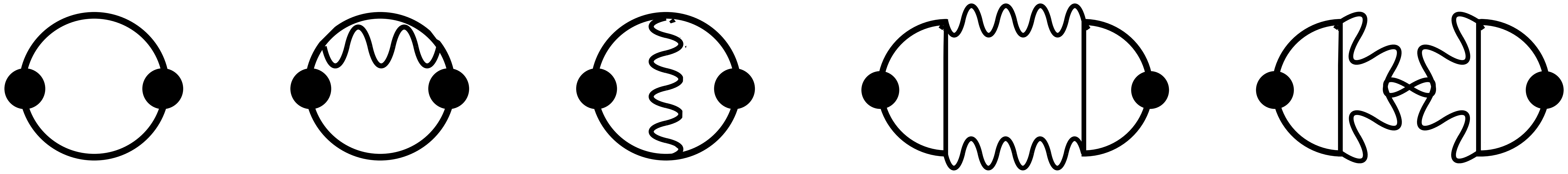
Spatially random Yukawa coupling $g'(\boldsymbol{r})$ with $\overline{g'(\boldsymbol{r})} = 0$, $\overline{g'(\boldsymbol{r})g'(\boldsymbol{r}')} = g'^2\delta(\boldsymbol{r} - \boldsymbol{r}')$

Analyze 2d-YSYK model in a self-averaging manner as in the SYK model.
Should be applicable as long as eigenmodes of $\phi(\boldsymbol{r})$ are extended.

# 2d-YSYK model: Fermi surface + critical boson with interaction disorder



Residual resistivity is determined by $v^2$

Linear-in-$T$ resistivity determined by $g'^2$

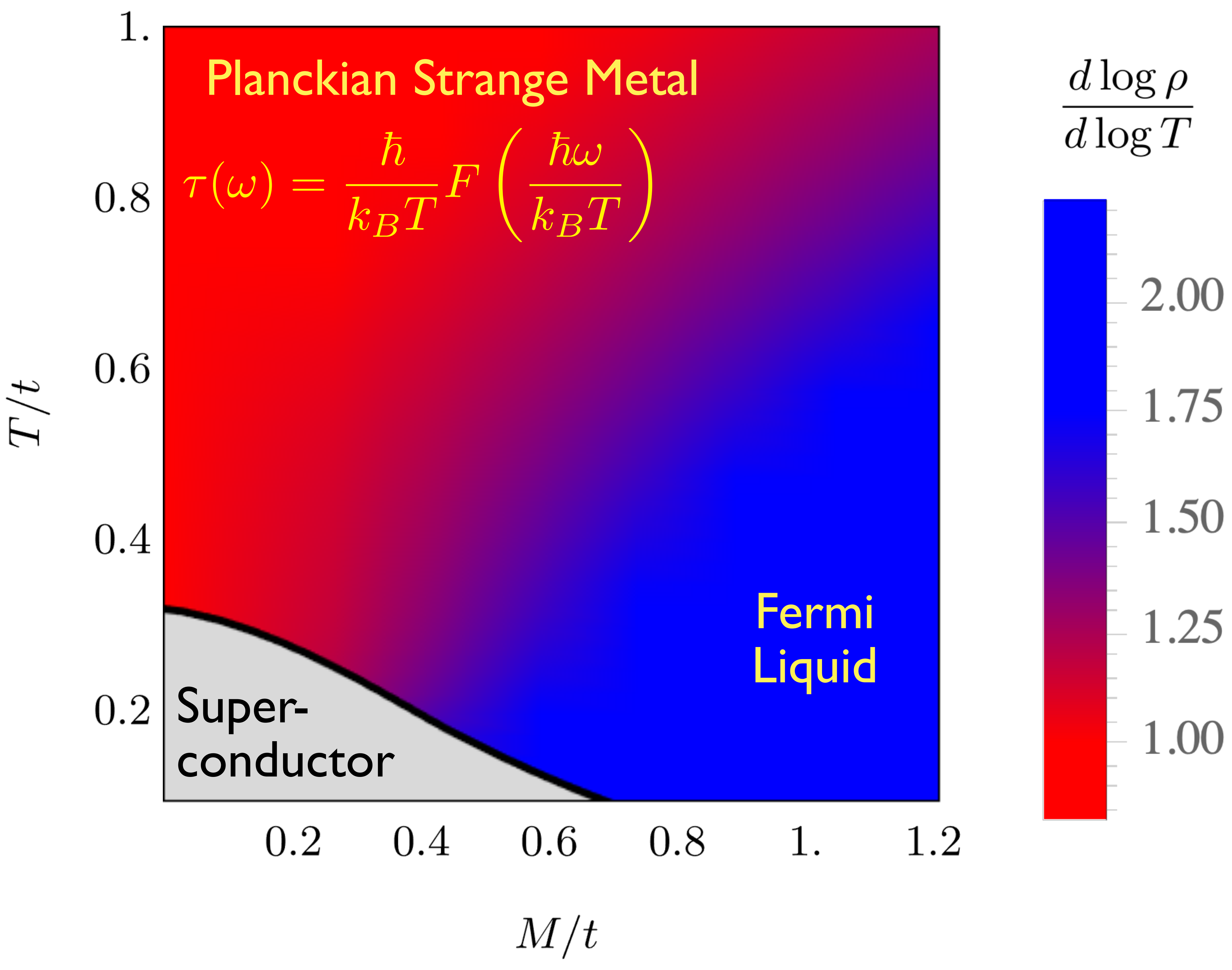
Transport insensitive to $g$

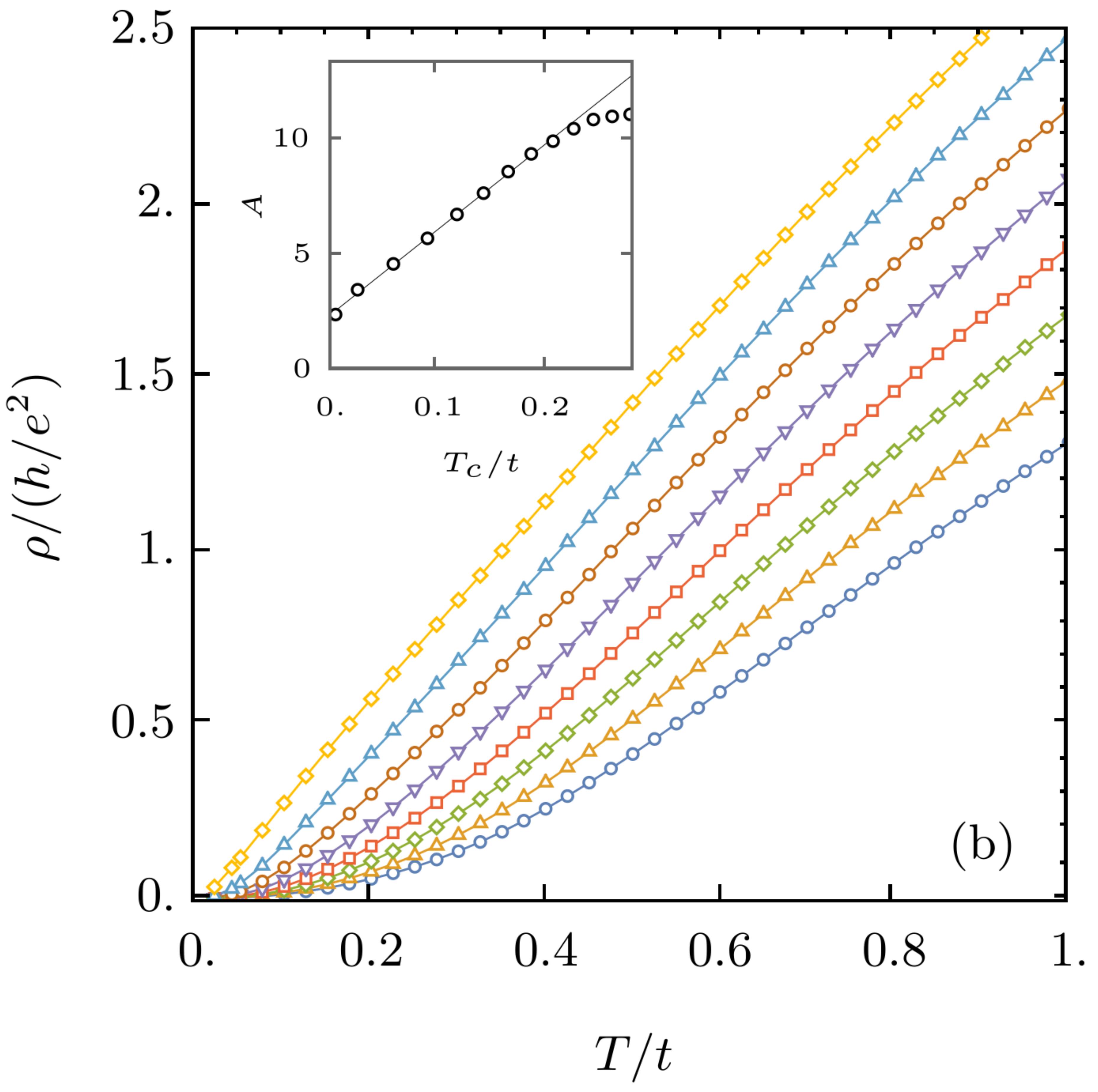
Marginal Fermi liquid self energy $\Sigma \sim \omega \ln \omega$

$T \ln(1/T)$ specific heat

# Strange metal and superconductor in the two-dimensional Yukawa-Sachdev-Ye-Kitaev model

Chenyuan Li, Aavishkar A. Patel, Haoyu Guo, Davide Valentinis, Jorg Schmalian, S.S., Ilya Esterlis, PRL in press; arXiv:2406.07608



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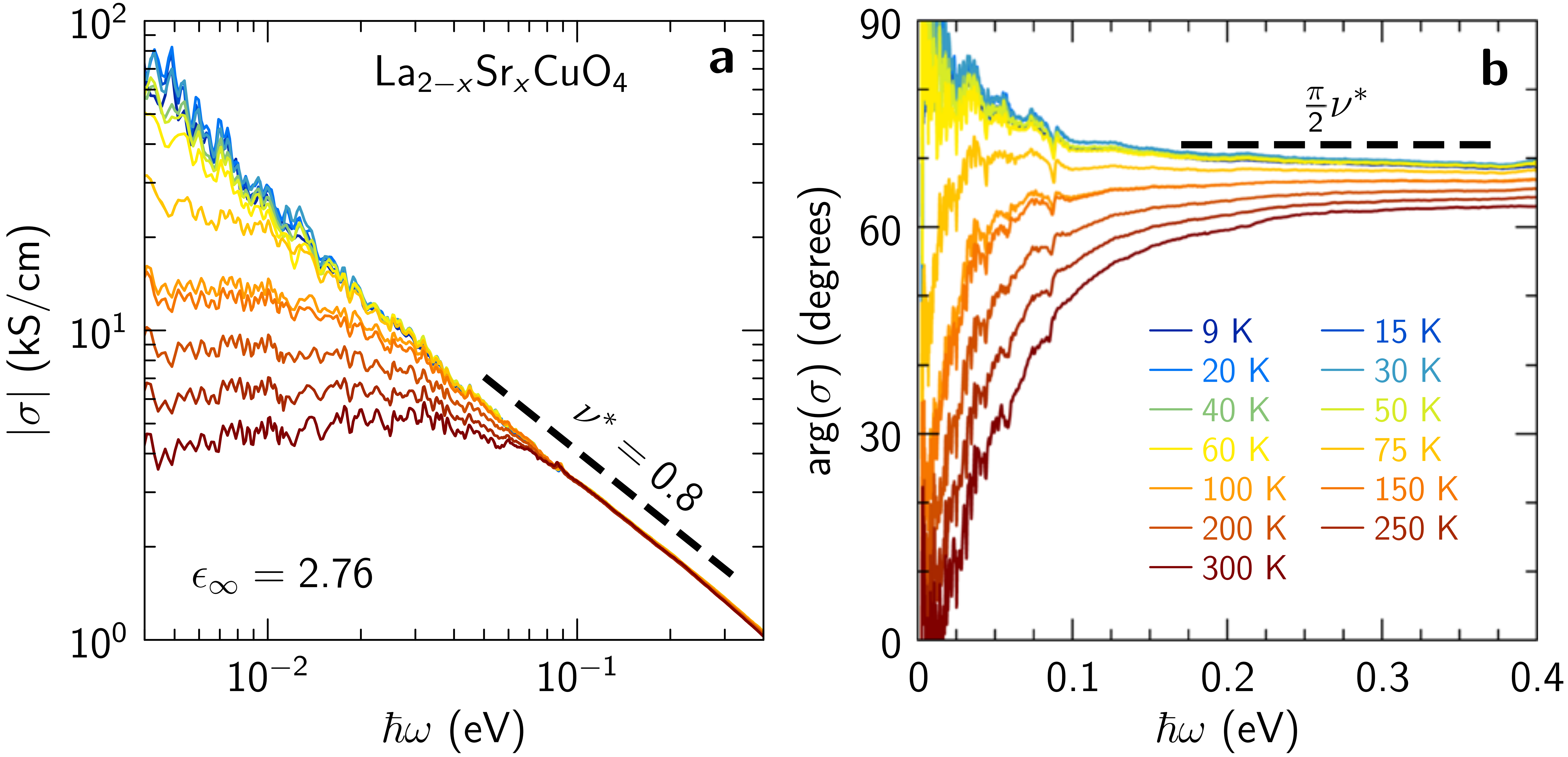
*Nature Communications* **14**, Article number: 3033 (2023)

$$\sigma(\omega) = i\frac{e^2K/(\hbar d_c)}{\hbar\omega\frac{m^*(\omega)}{m} + i\frac{\hbar}{\tau(\omega)}}$$

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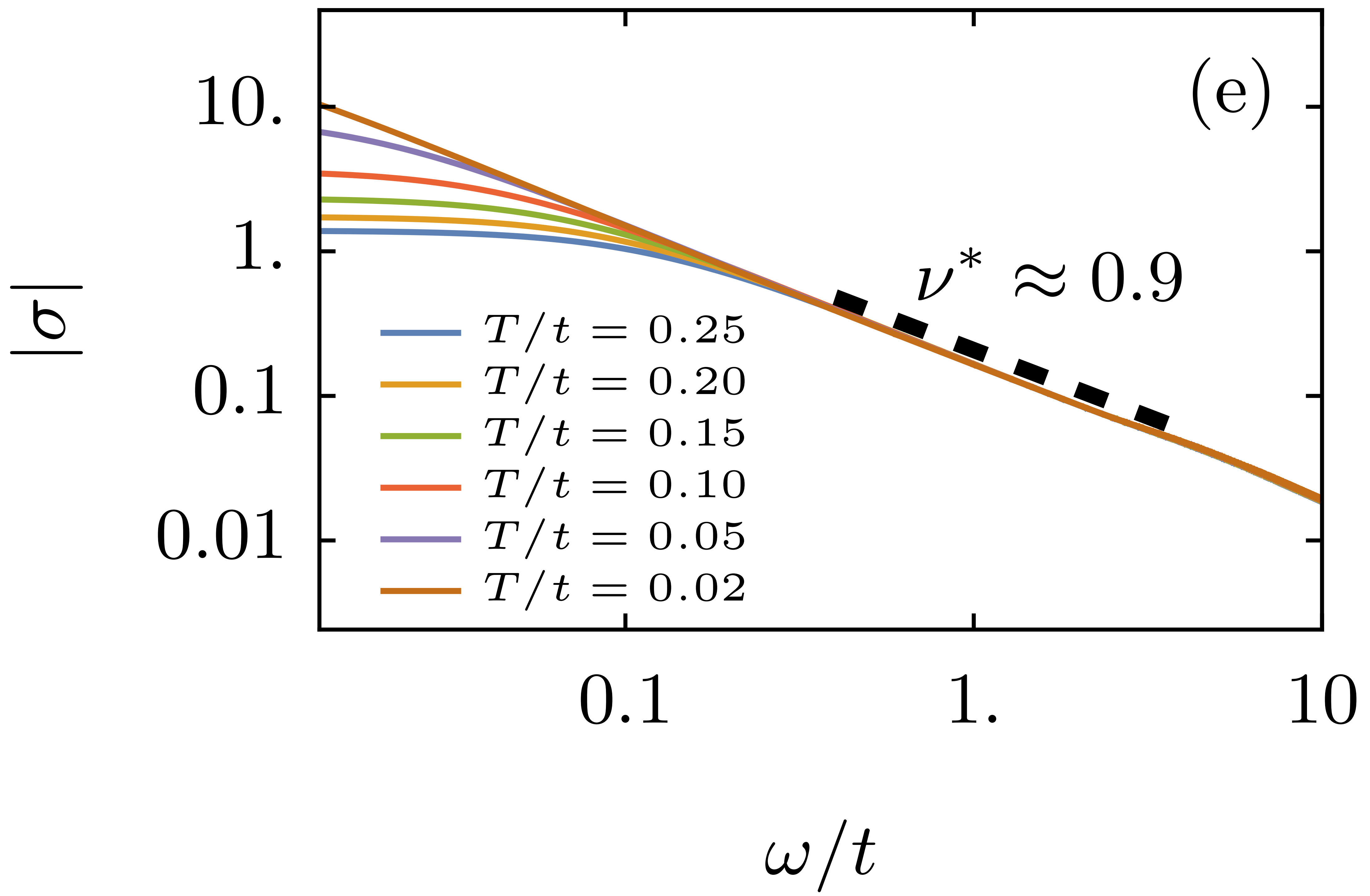
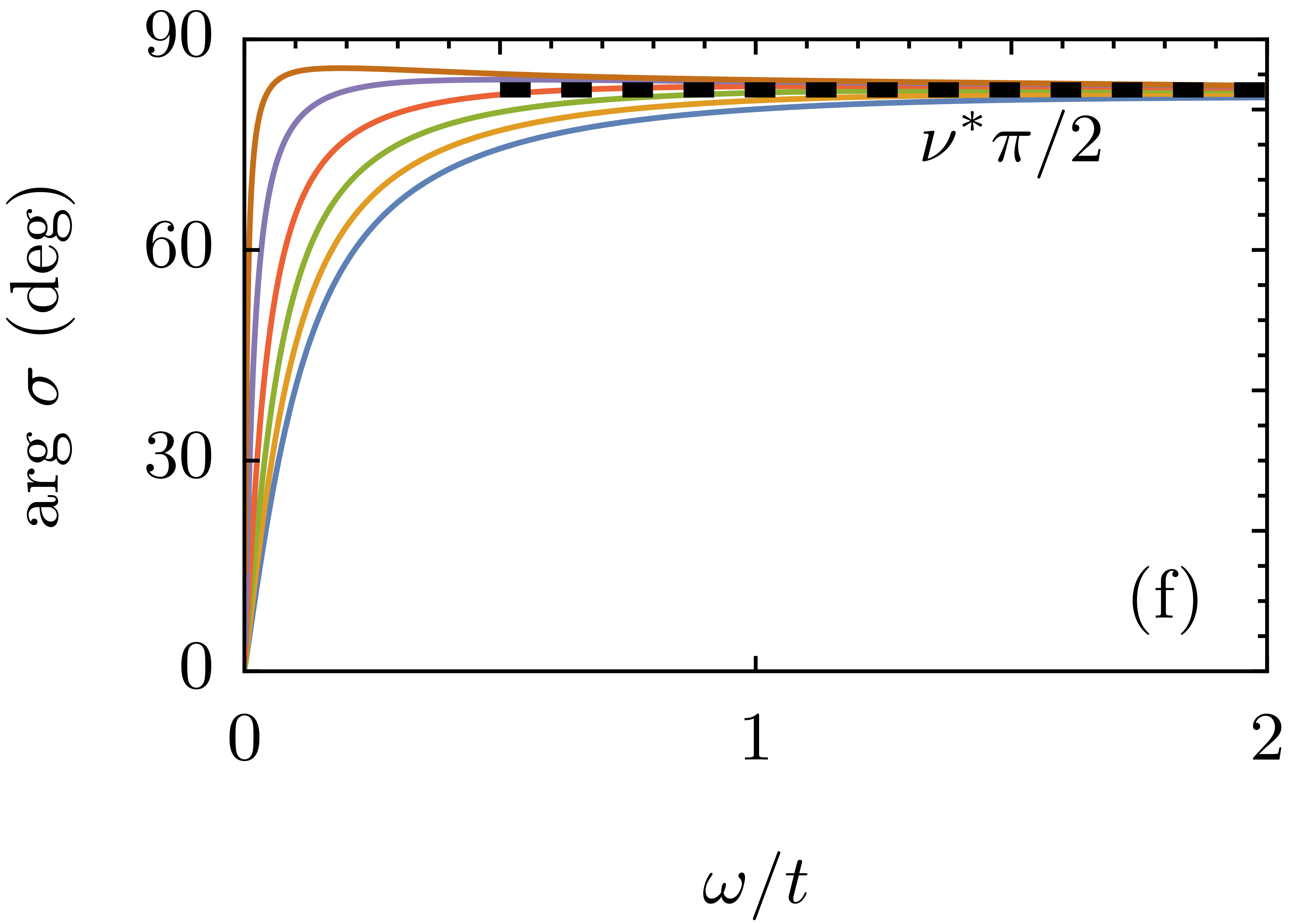
Chenyuan Li, Aavishkar A. Patel, Haoyu Guo, Davide Valentinis, Jorg Schmalian, S.S., Ilya Esterlis, PRL in press; arXiv:2406.07608

$$\sigma(\omega) = i\frac{e^2K/(\hbar d_c)}{\hbar\omega\frac{m^*(\omega)}{m} + i\frac{\hbar}{\tau(\omega)}}$$



# Reconciling scaling of the optical conductivity of cuprate superconductors with Planckian resistivity and specific heat

B. Michon, C. Berthod, C. W. Rischau, A. Ataei, L. Chen, S. Komiya, S. Ono, L. Taillefer, D. van der Marel, A. Georges

*Nature Communications* **14**, Article number: 3033 (2023)

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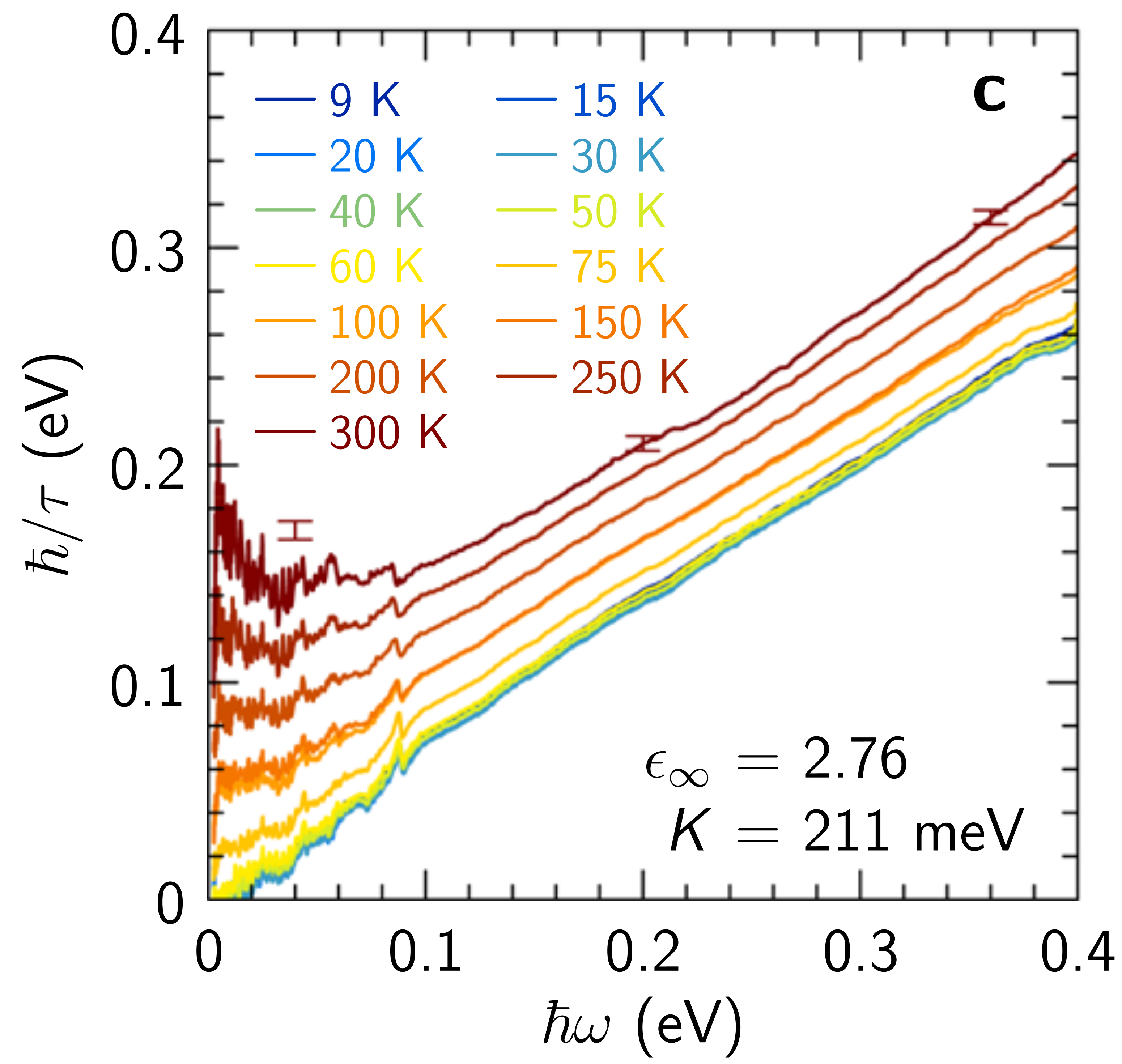


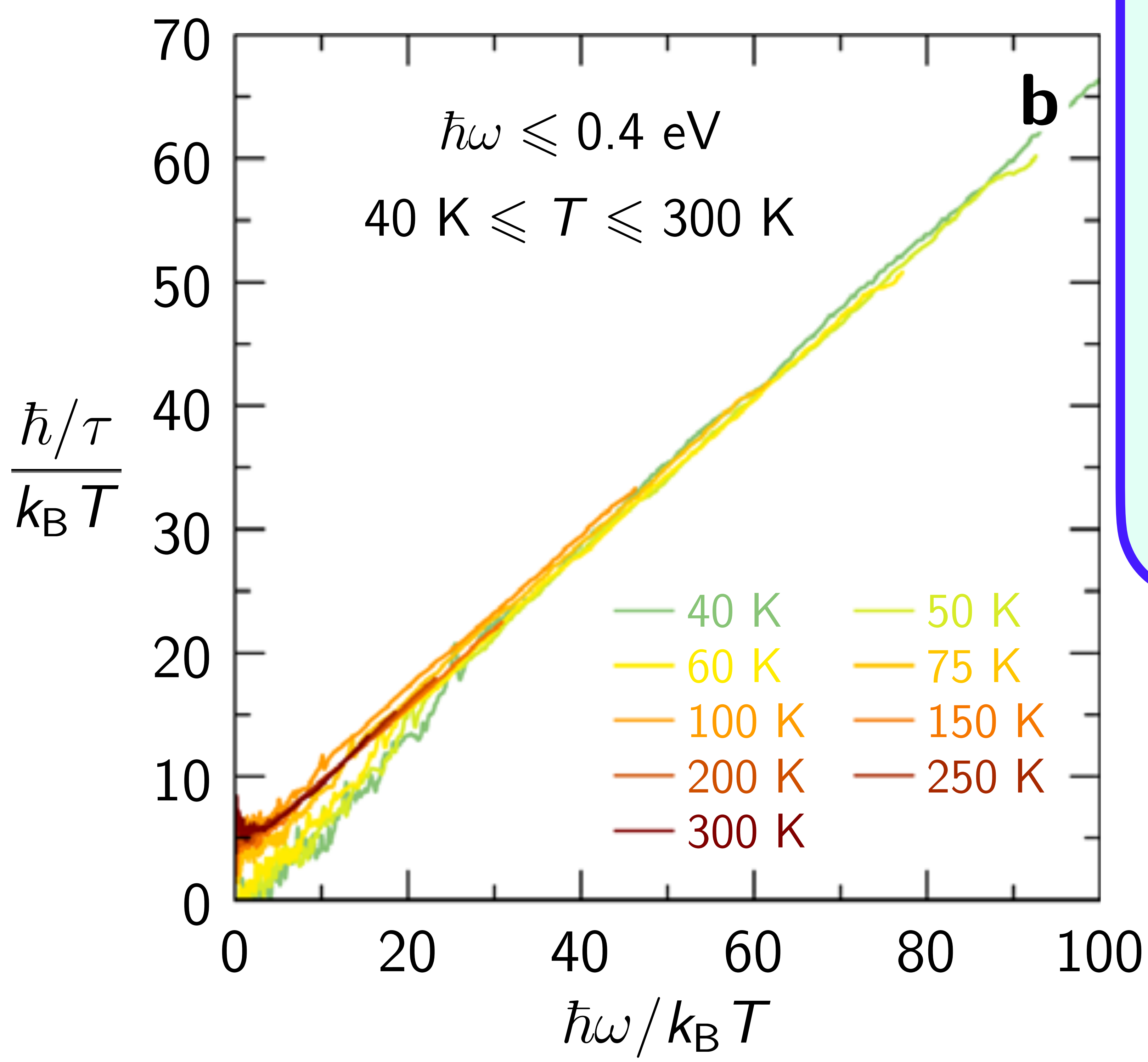


Planckian dynamics!

$$\tau(\omega) = \frac{\hbar}{k_B T} F\left(\frac{\hbar\omega}{k_B T}\right)$$

and entropy

$S(T \to 0) \sim T\ln(1/T)$.

$La_{2-x}Sr_xCuO_4$

$p = 0.24$

$T_c = 19$ K

# Strange metal and superconductor in the two-dimensional Yukawa-Sachdev-Ye-Kitaev model

Chenyuan Li, Aavishkar A. Patel, Haoyu Guo, Davide Valentinis, Jorg Schmalian, S.S., Ilya Esterlis, PRL in press; arXiv:2406.07608

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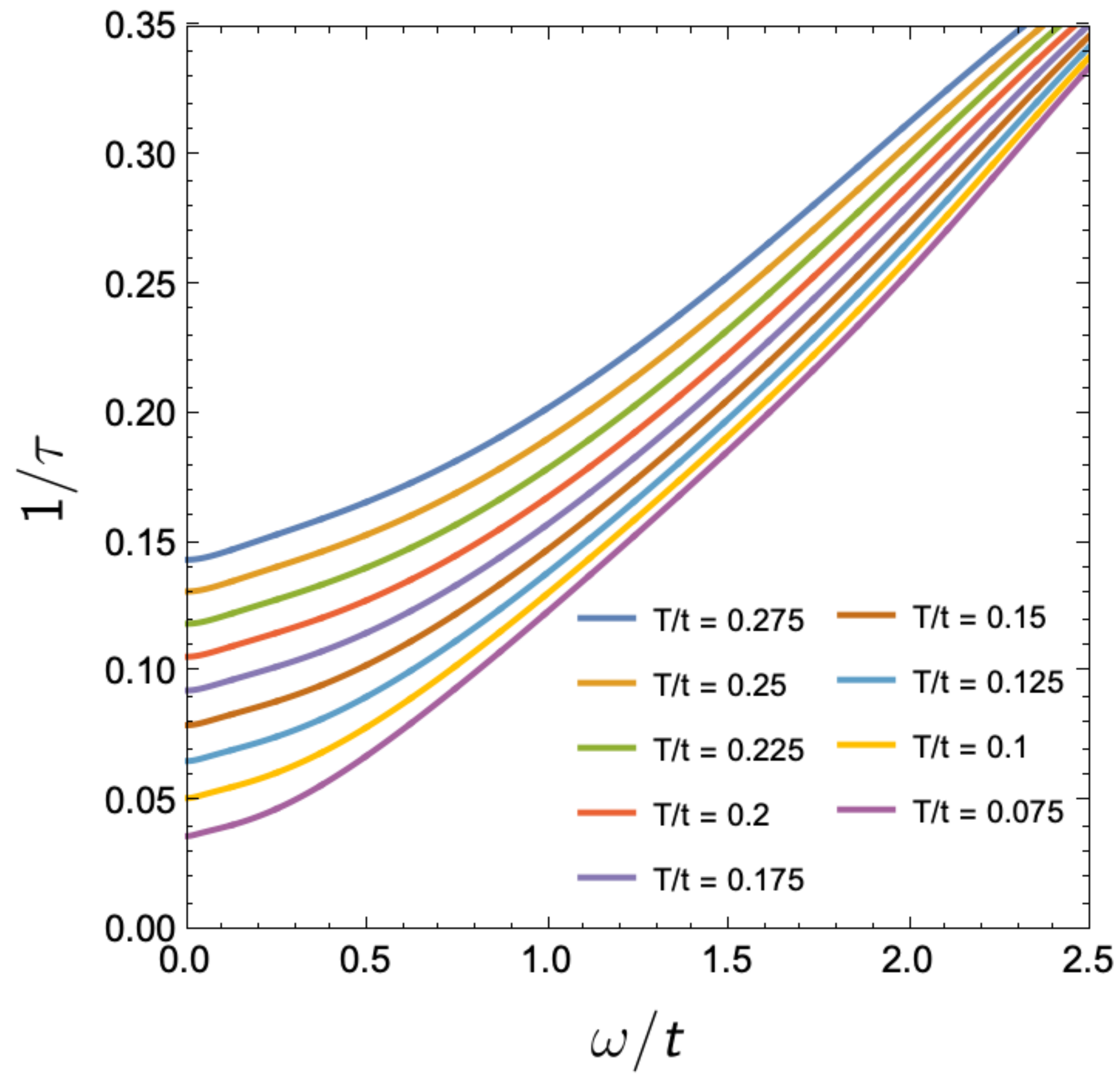
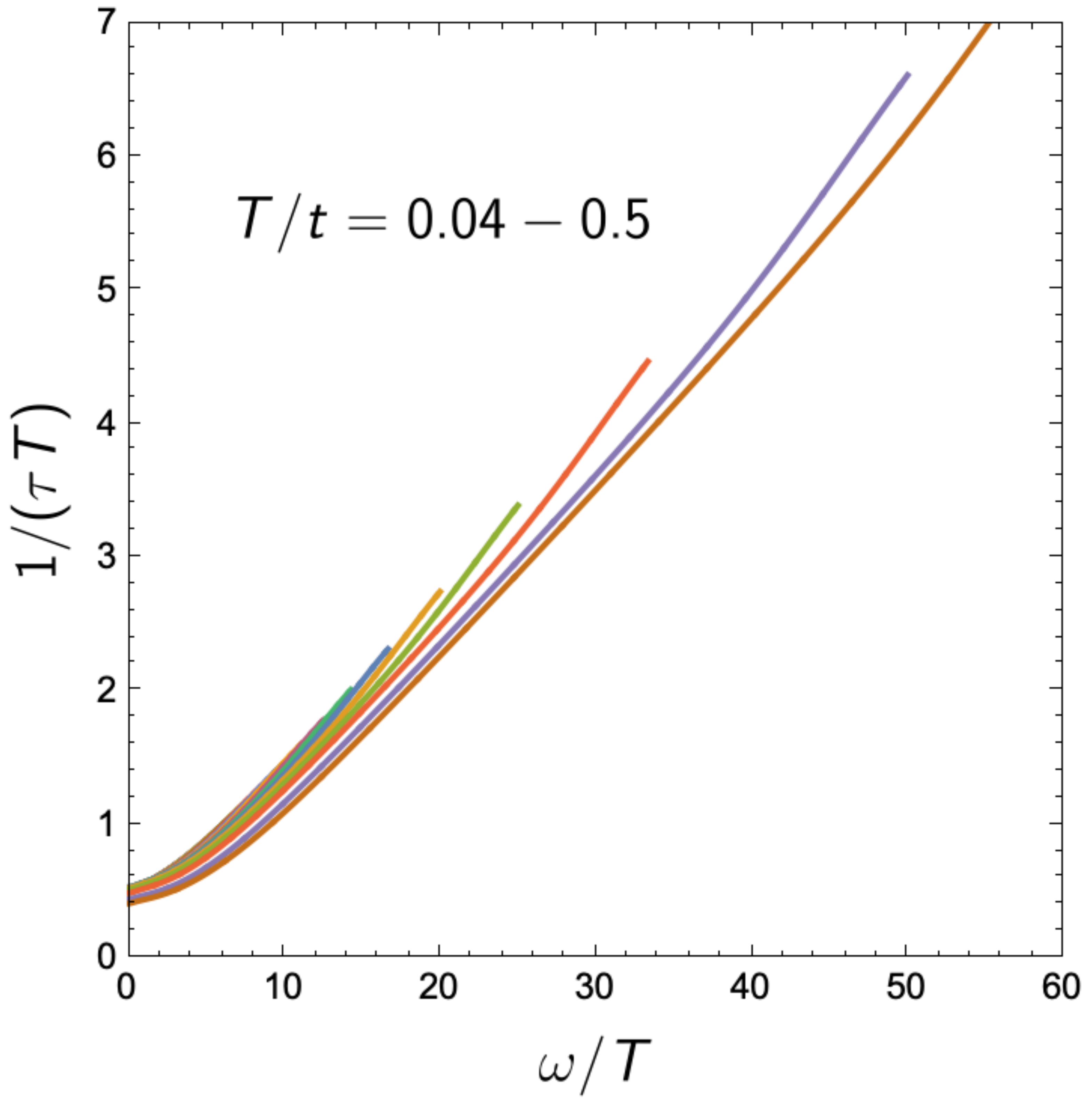



Planckian dynamics!

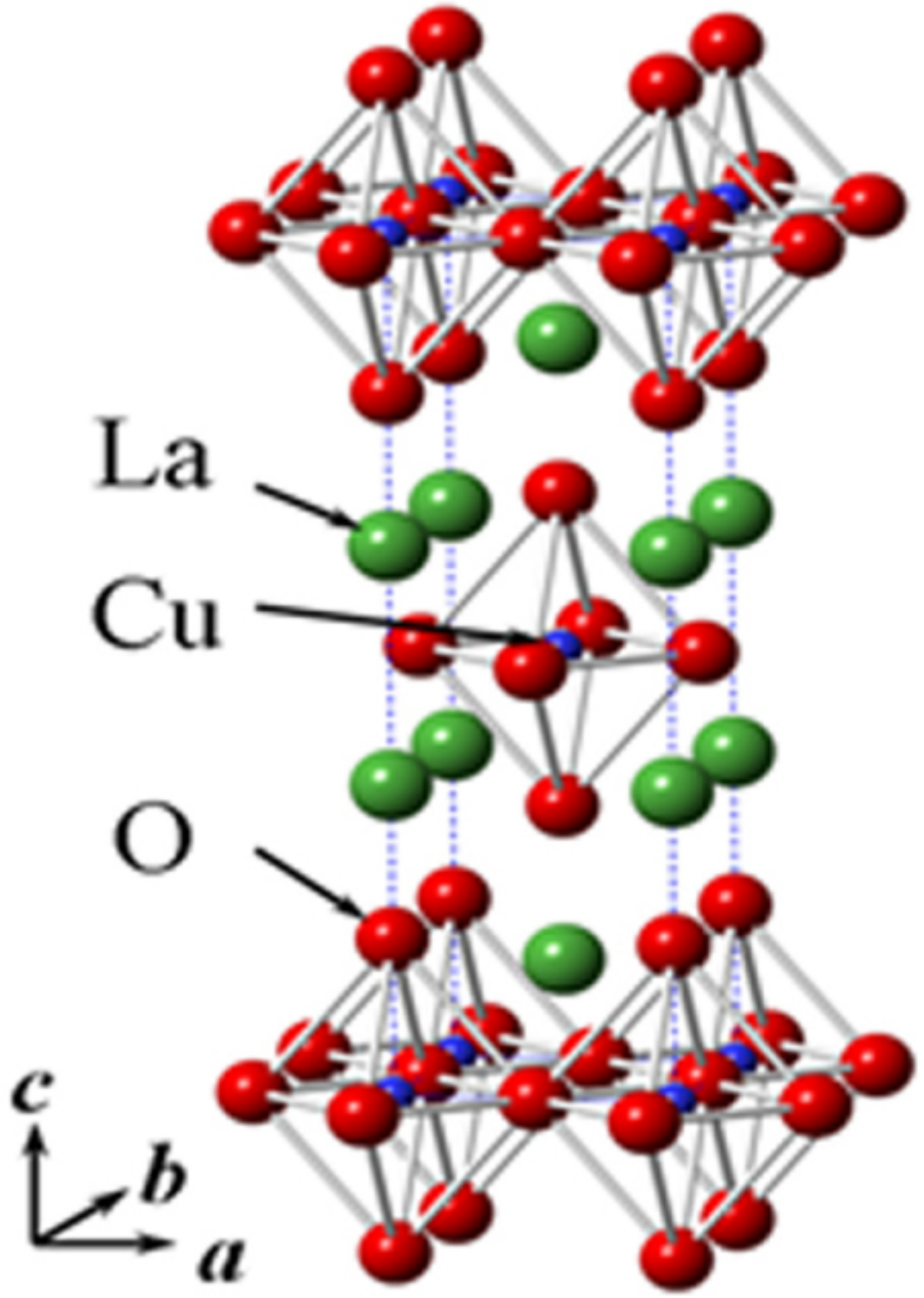
$$\tau(\omega) = \frac{\hbar}{k_B T} F\left(\frac{\hbar\omega}{k_B T}\right)$$

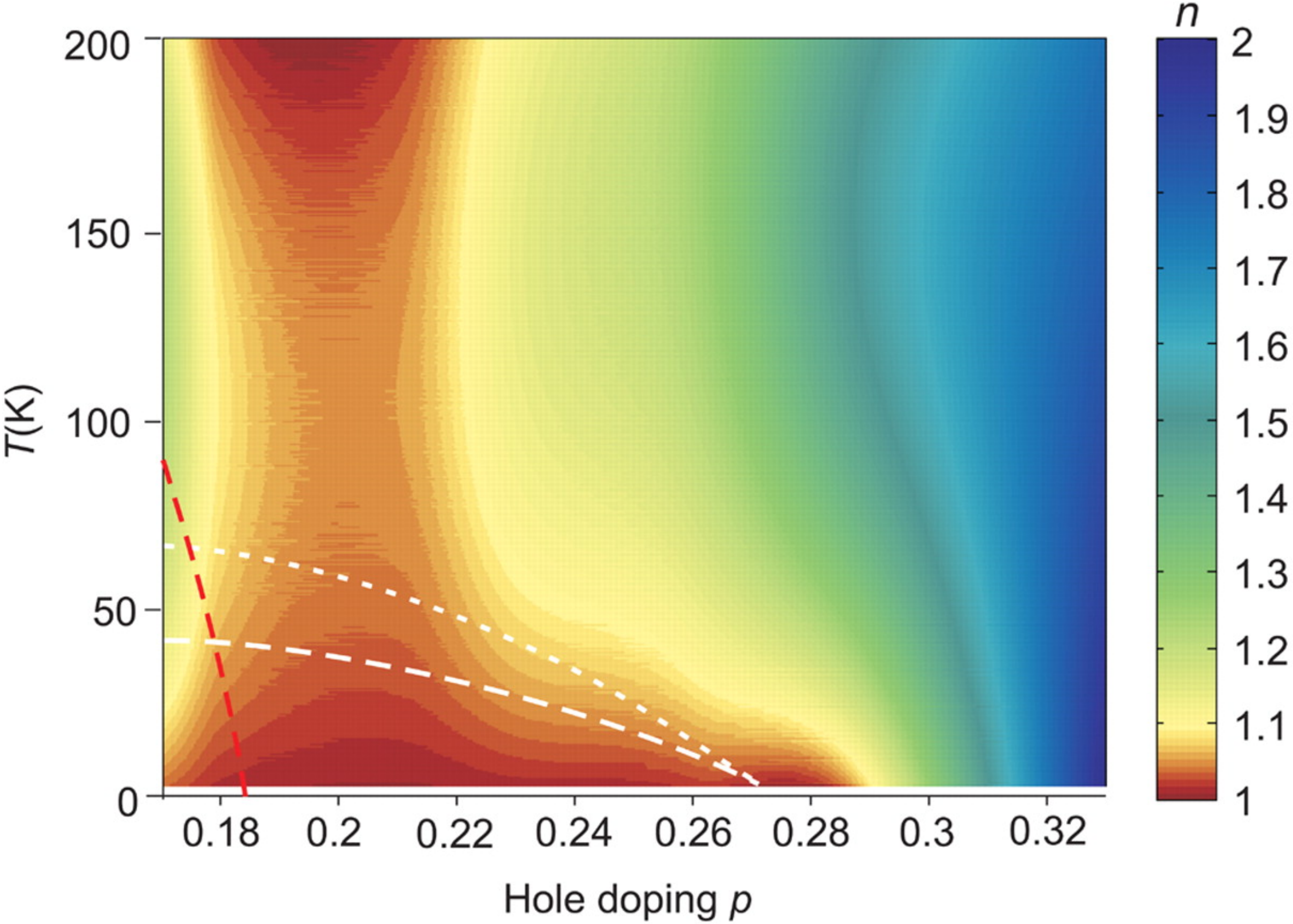
and entropy $S(T \to 0) \sim T\ln(1/T)$ in 2d-YSYK model (unlike zero temperature entropy in SYK model).

# Anomalous Criticality in the Electrical Resistivity of $La_{2-x}Sr_xCuO_4$

R. A. Cooper,[1] Y. Wang,[1] B. Vignolle,[2] O. J. Lipscombe,[1] S. M. Hayden,[1] Y. Tanabe,[3] T. Adachi,[3] Y. Koike,[3] M. Nohara,[4]* H. Takagi,[4] Cyril Proust,[2] N. E. Hussey[1]†

SCIENCE VOL 323 603 2009



# Two-dimensional metals with Harris disorder

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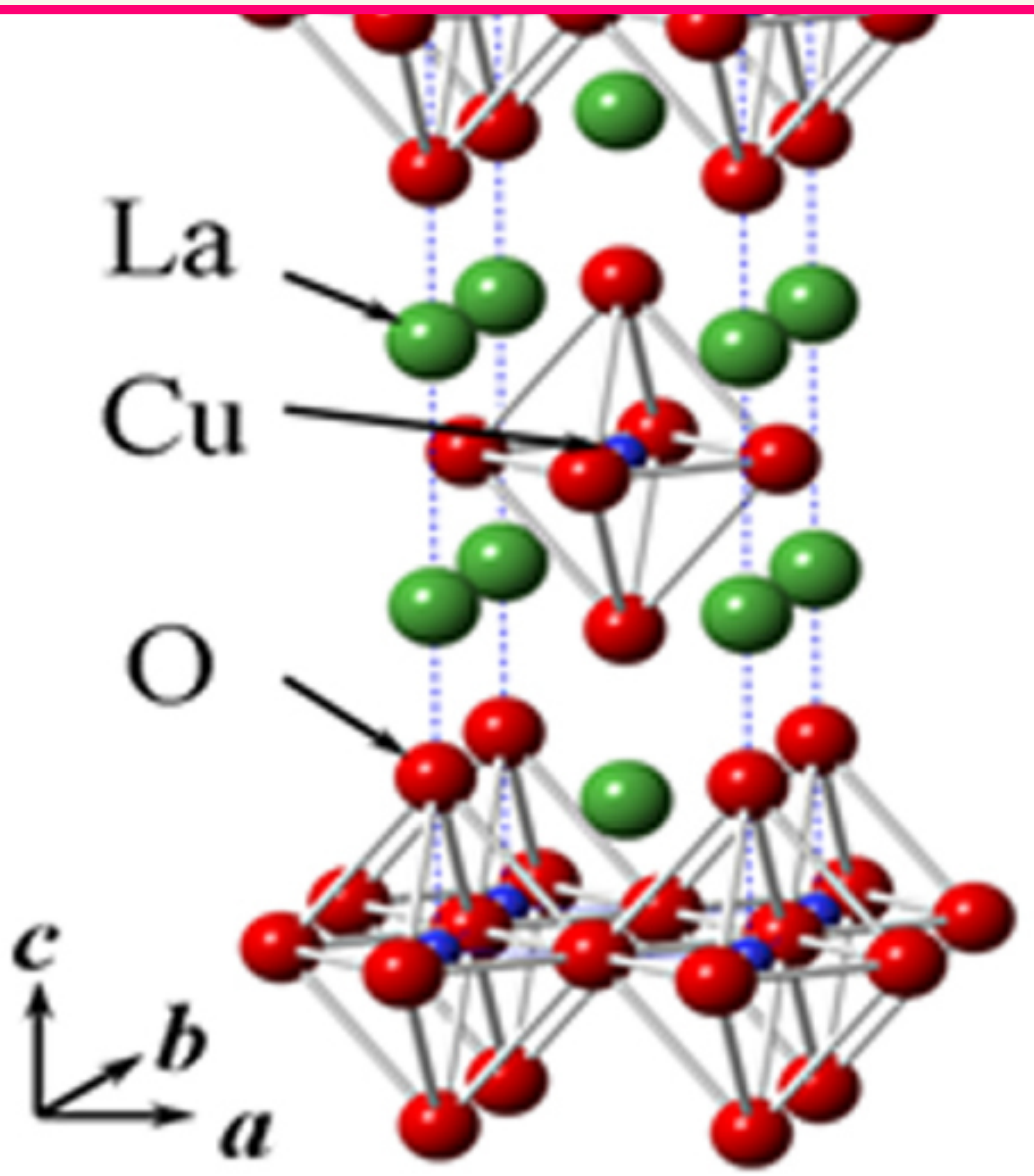
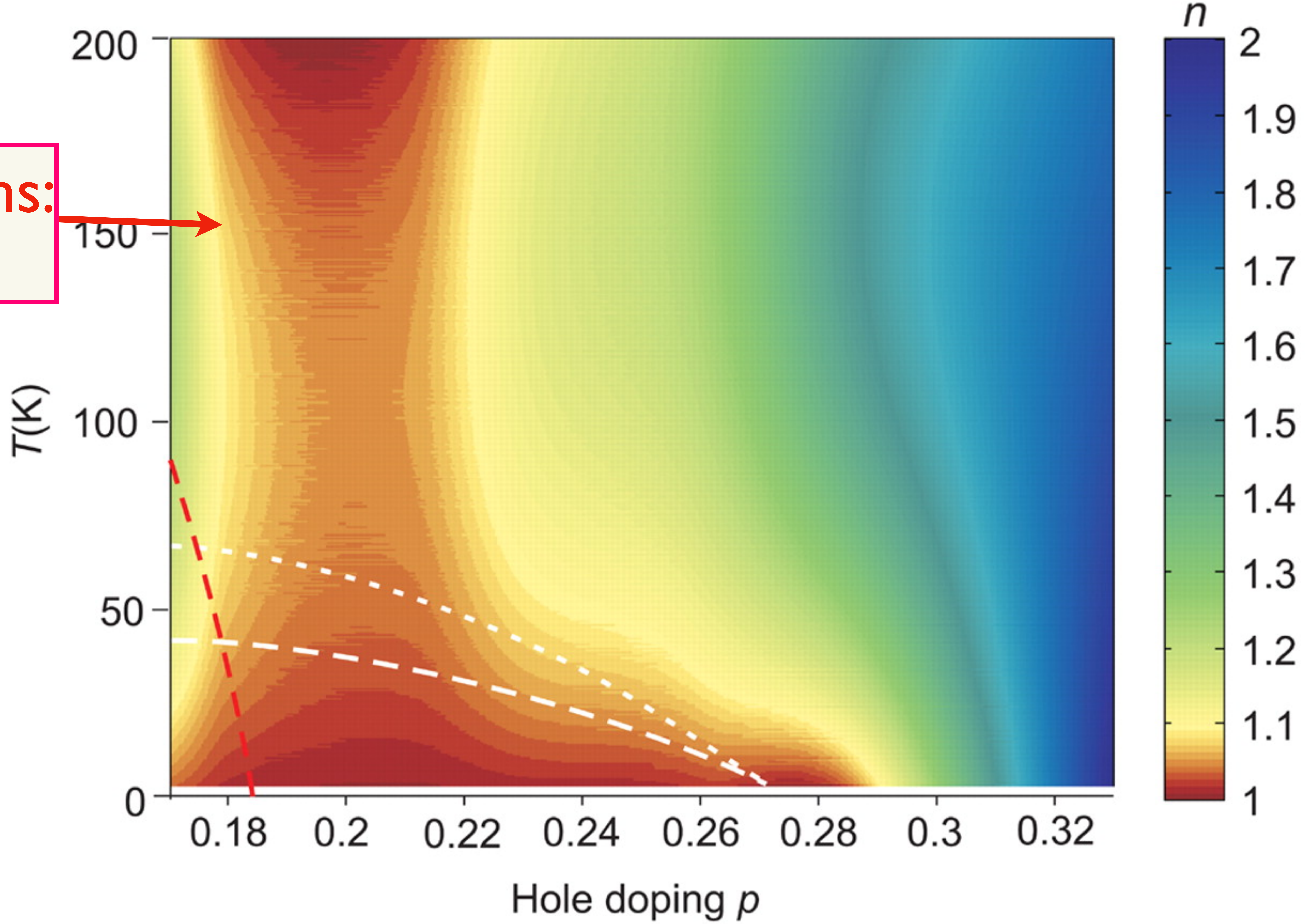
SCIENCE VOL 323 603 2009

Extended bosons and fermions: physics of 2d-YSYK

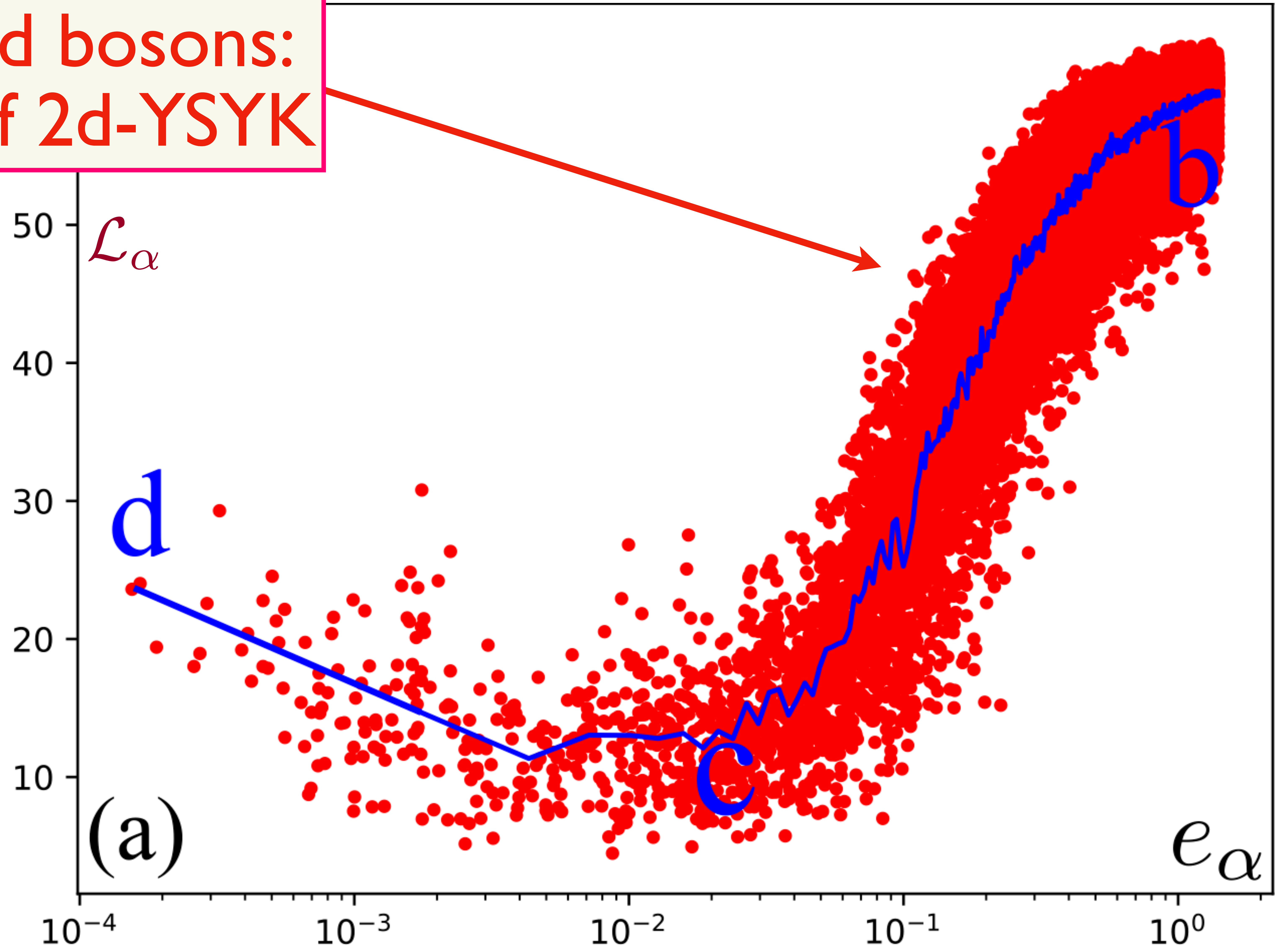


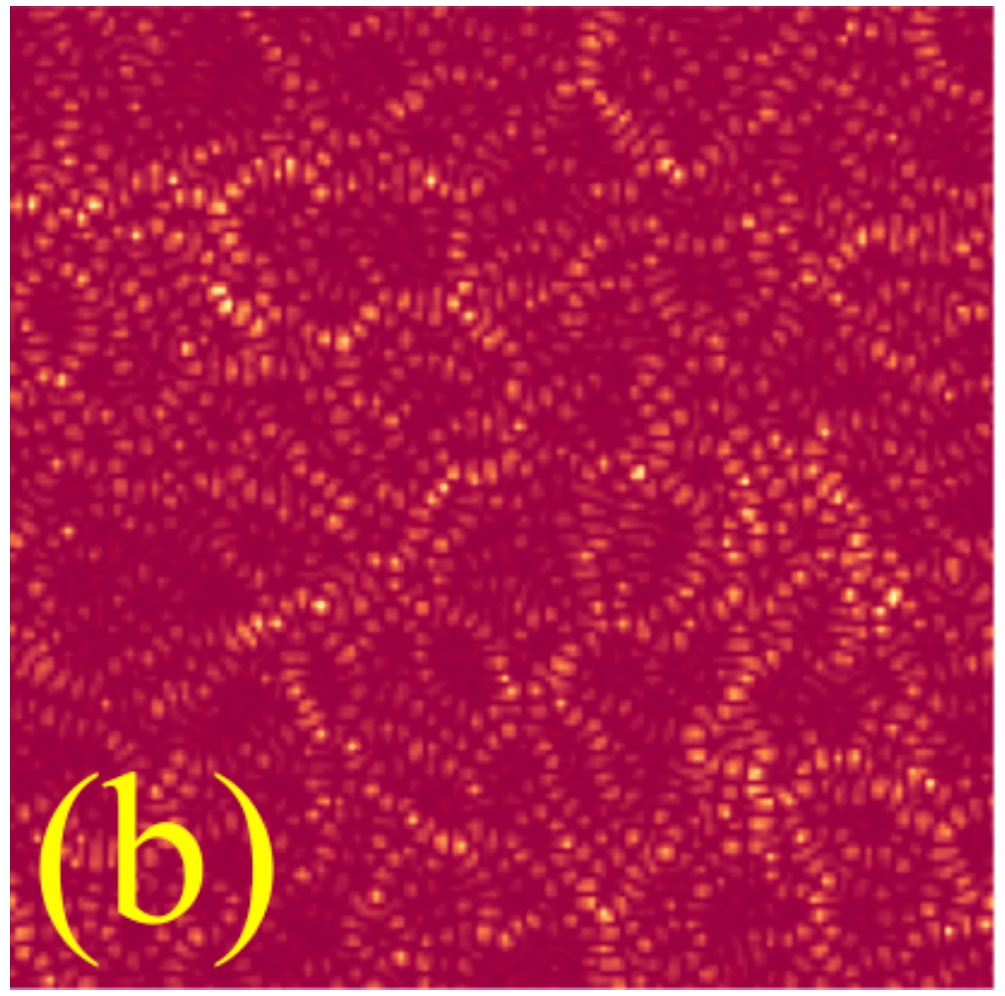



# Bosonic eigenmodes in Harris-disordered metals

$\phi$ eigenmodes localization length $\mathcal{L}_\alpha$

Extended bosons: physics of 2d-YSYK




QMC:
Aavishkar A. Patel,
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Michael Albergo,
arXiv:2410.05365

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*PNAS* **121**,
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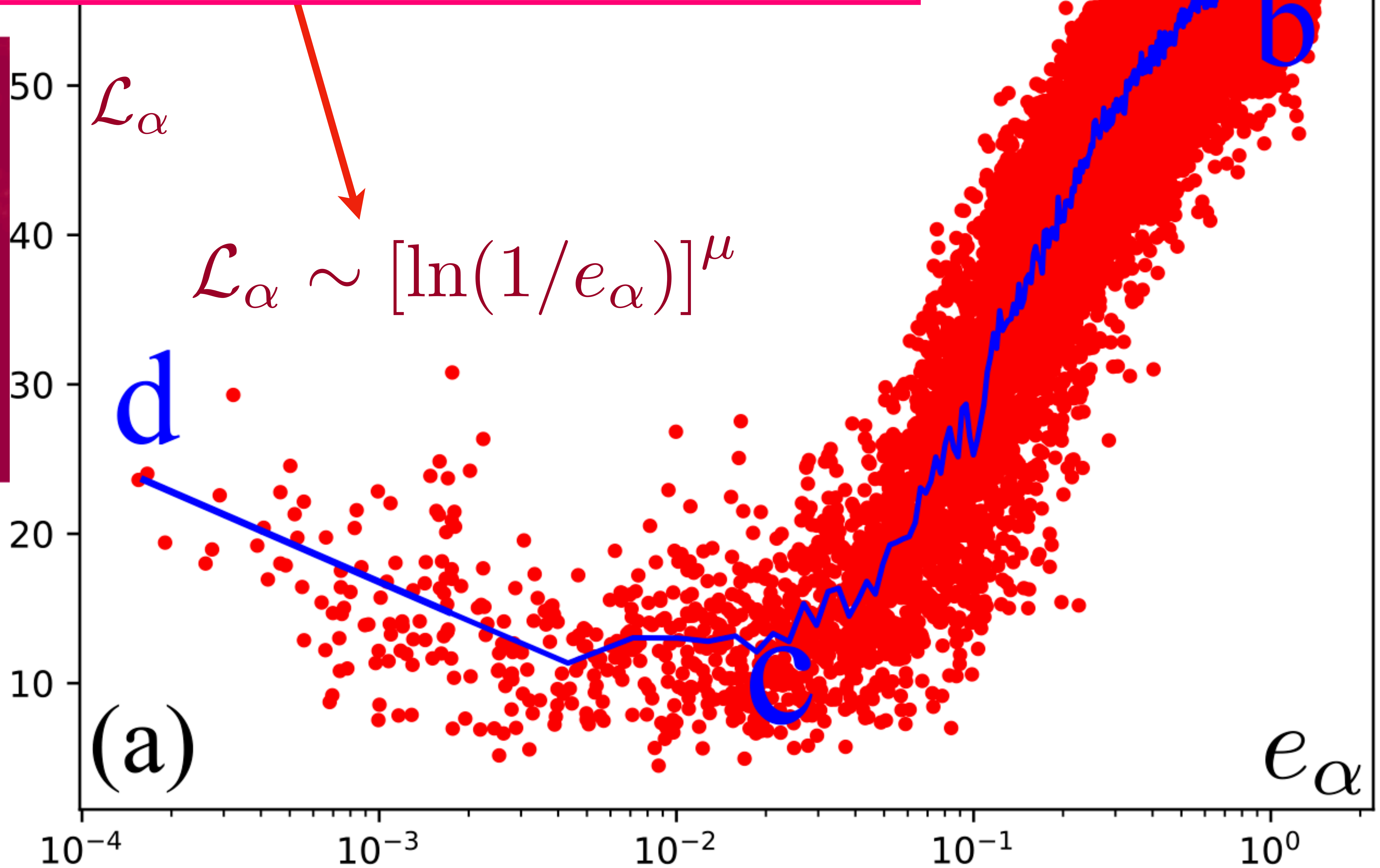
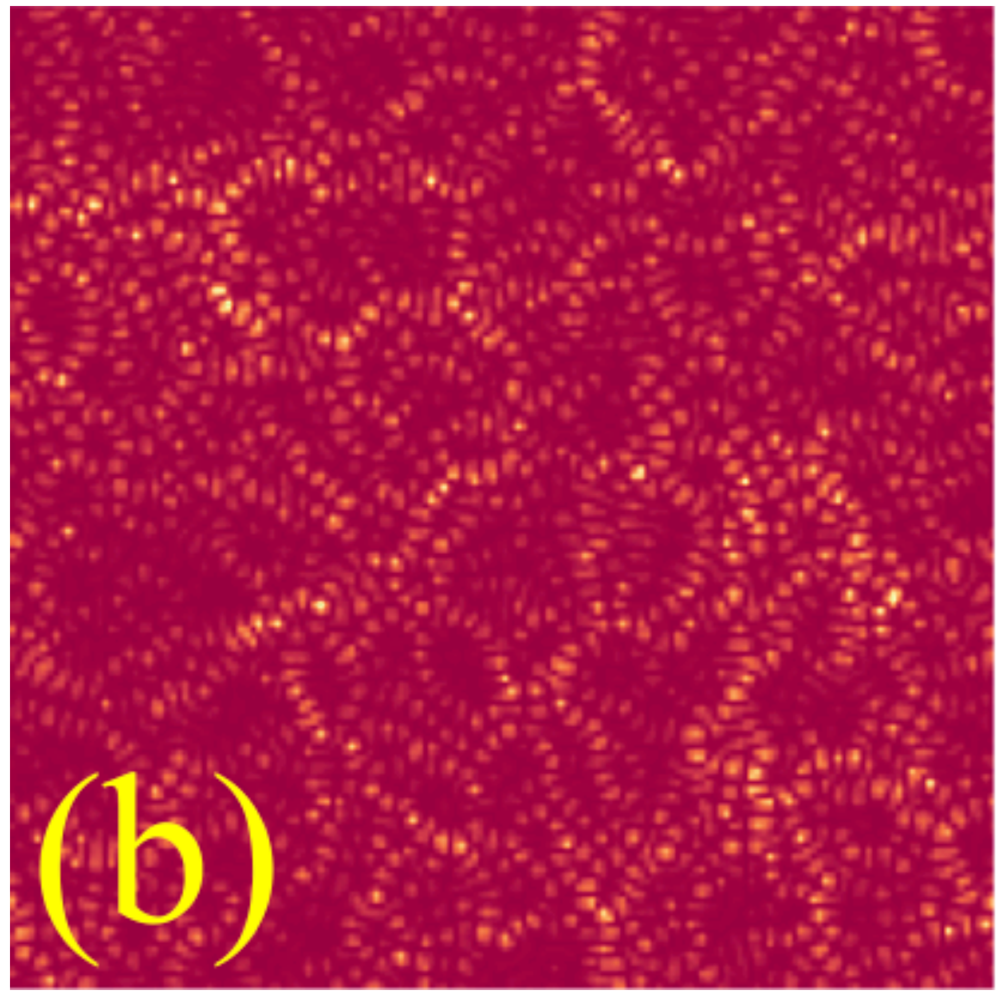
# Bosonic eigenmodes in Harris-disordered metals

$\phi$ eigenmodes localization length $\mathcal{L}_\alpha$

Physics of RTFIM, with logarithmically slow growth of localization length with decreasing energy

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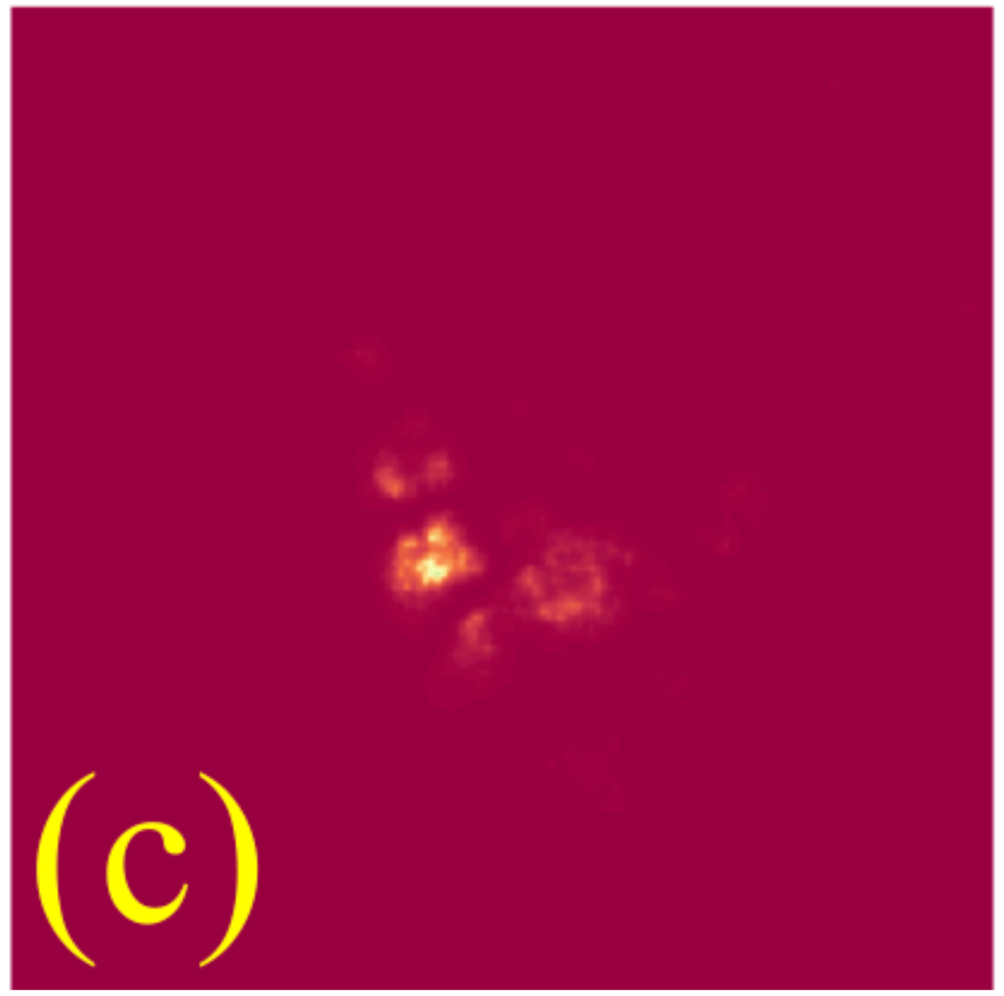


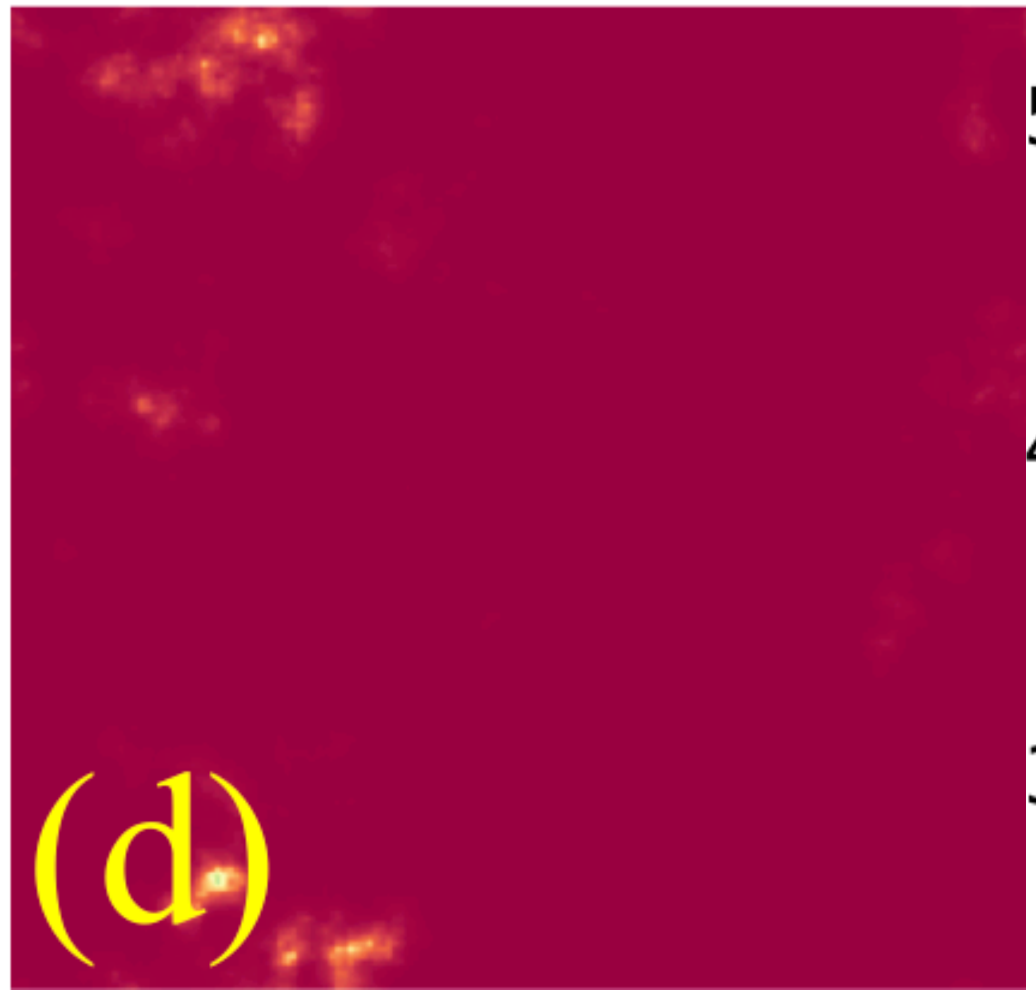


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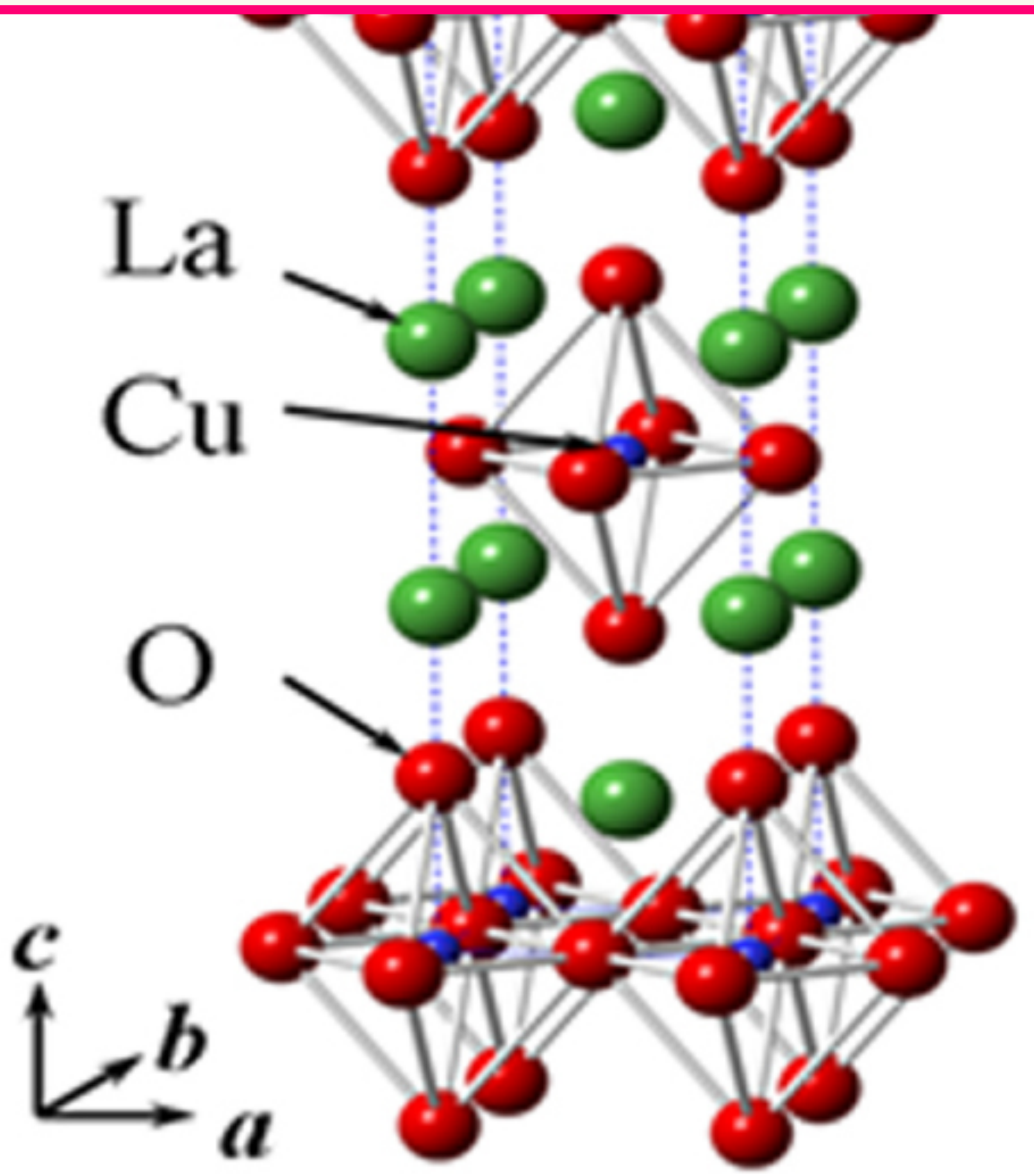
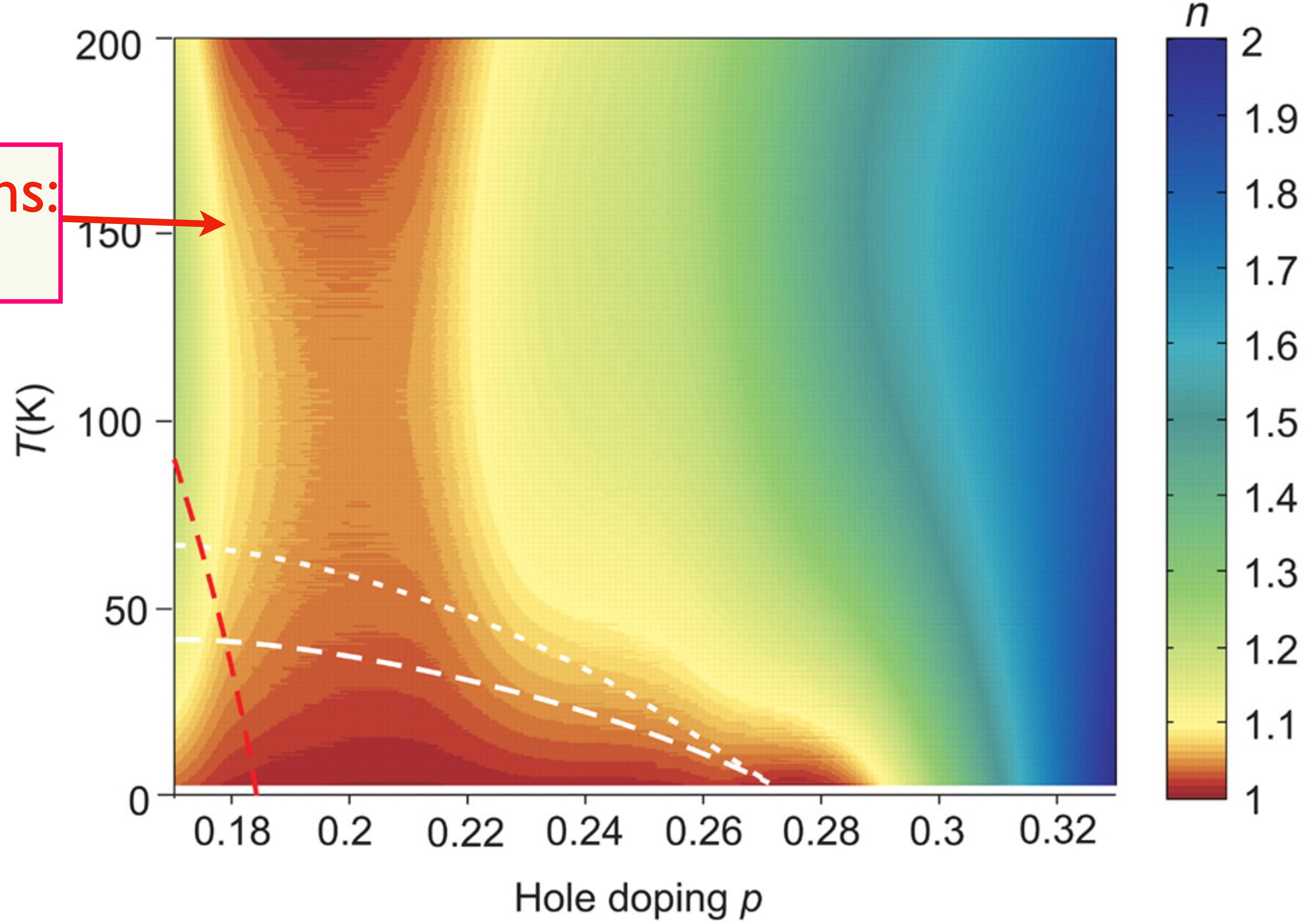
SCIENCE VOL 323 603 2009

# Two-dimensional metals with Harris disorder

Extended bosons and fermions:
physics of 2d-YSYK



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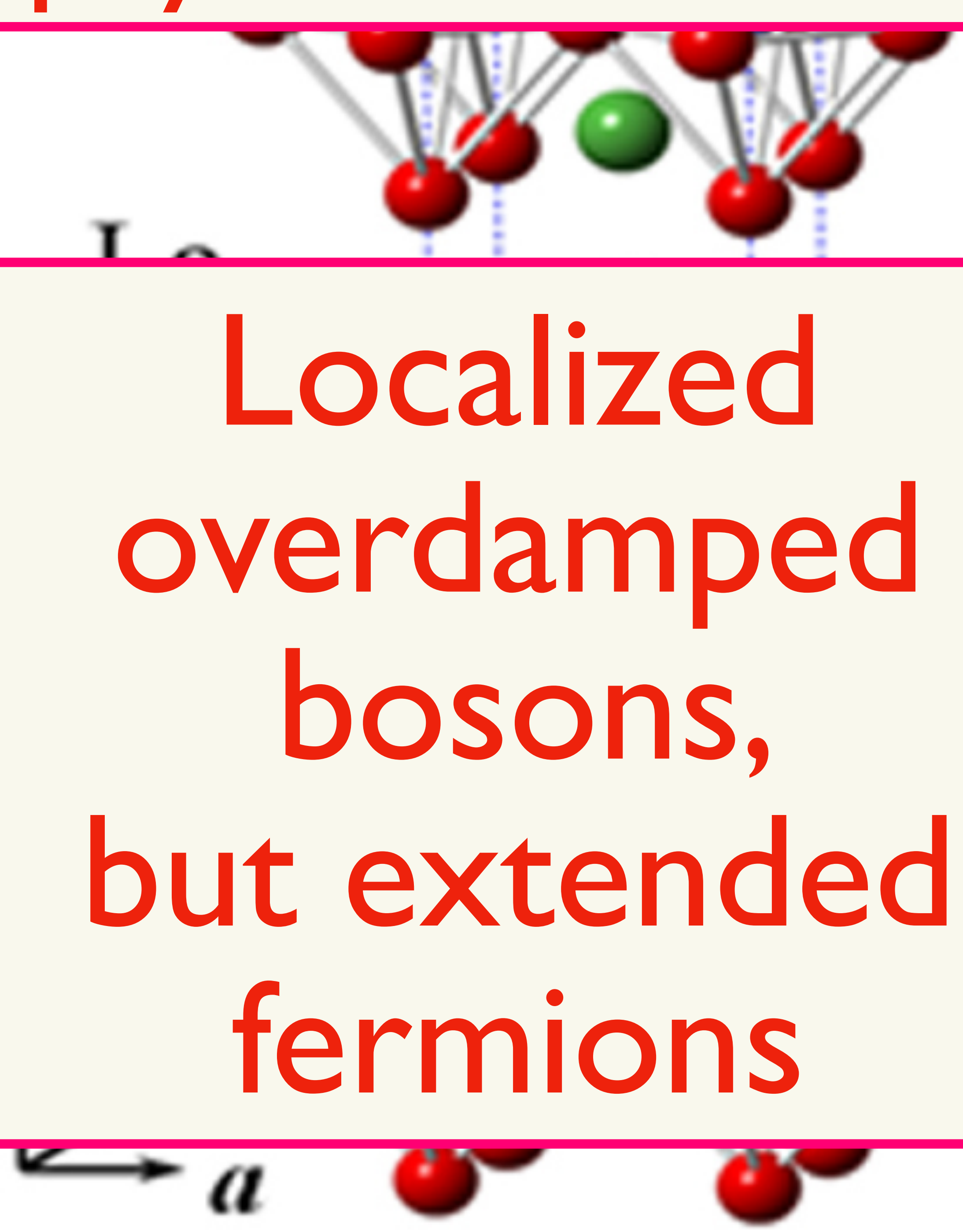
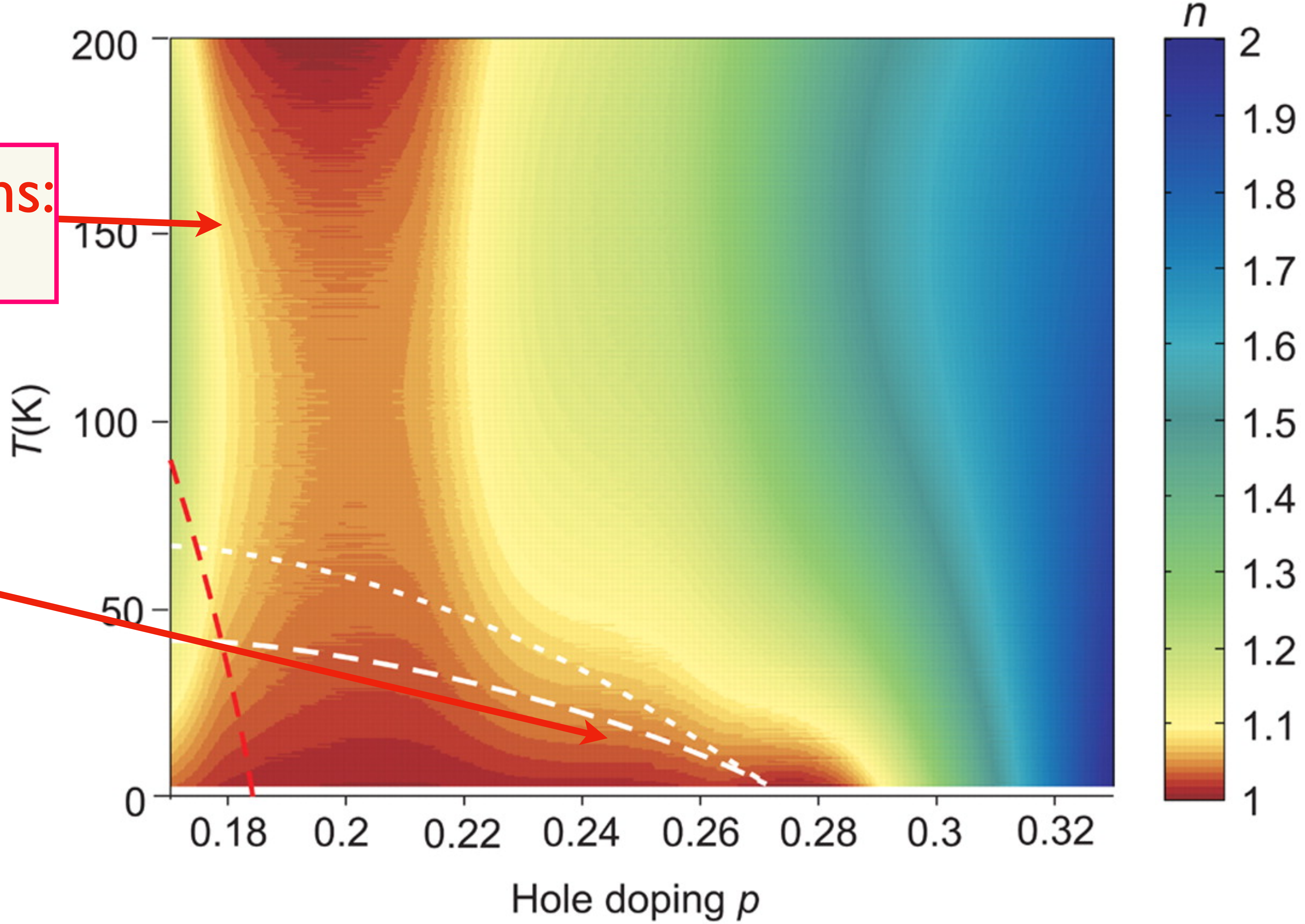
Extended bosons and fermions: physics of 2*d*-YSYK

Localized overdamped bosons, but extended fermions

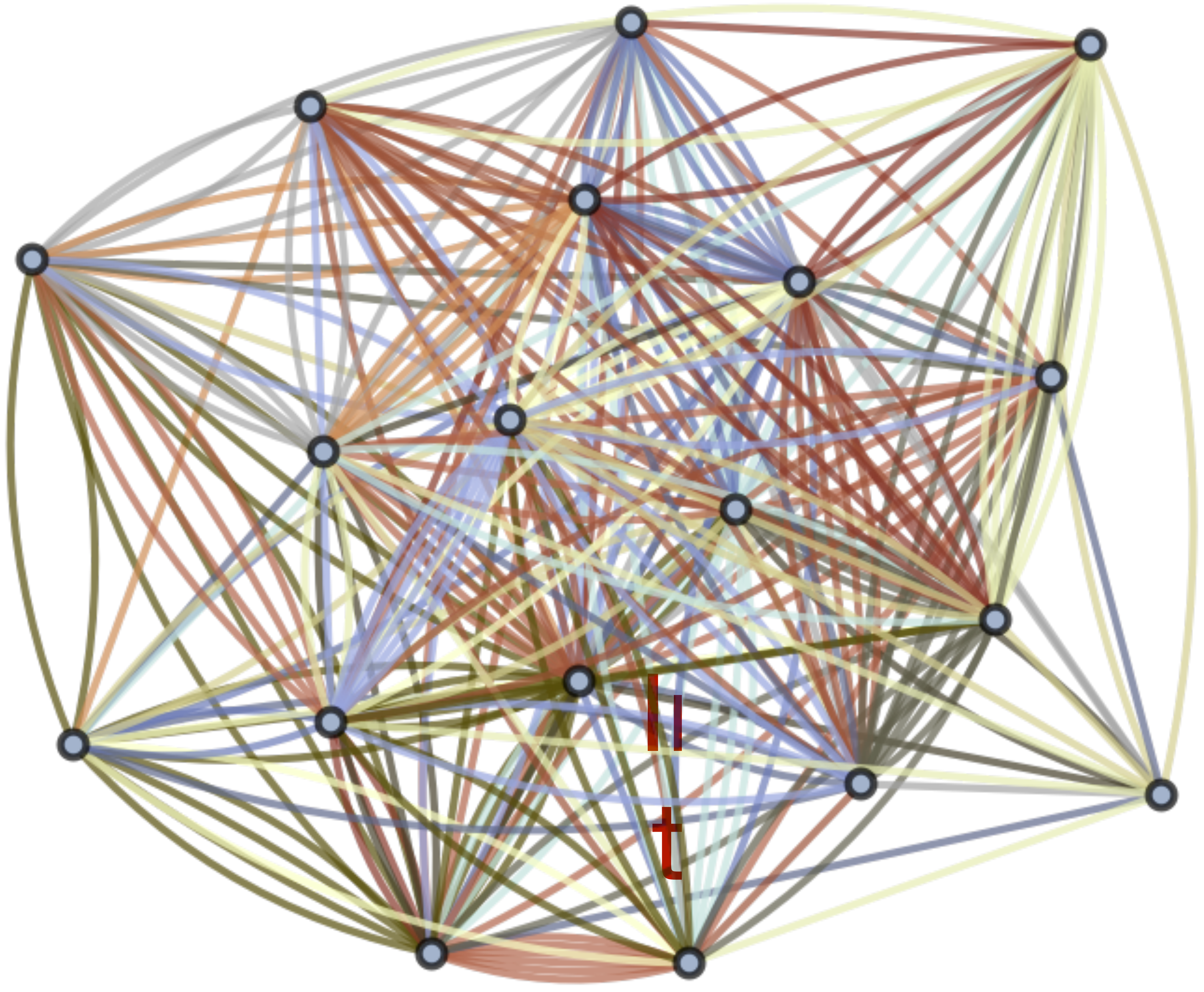




# Recap

# The Sachdev-Ye-Kitaev (SYK) model

The SYK model describes multi-particle quantum entanglement resulting in the loss of identity of the particles



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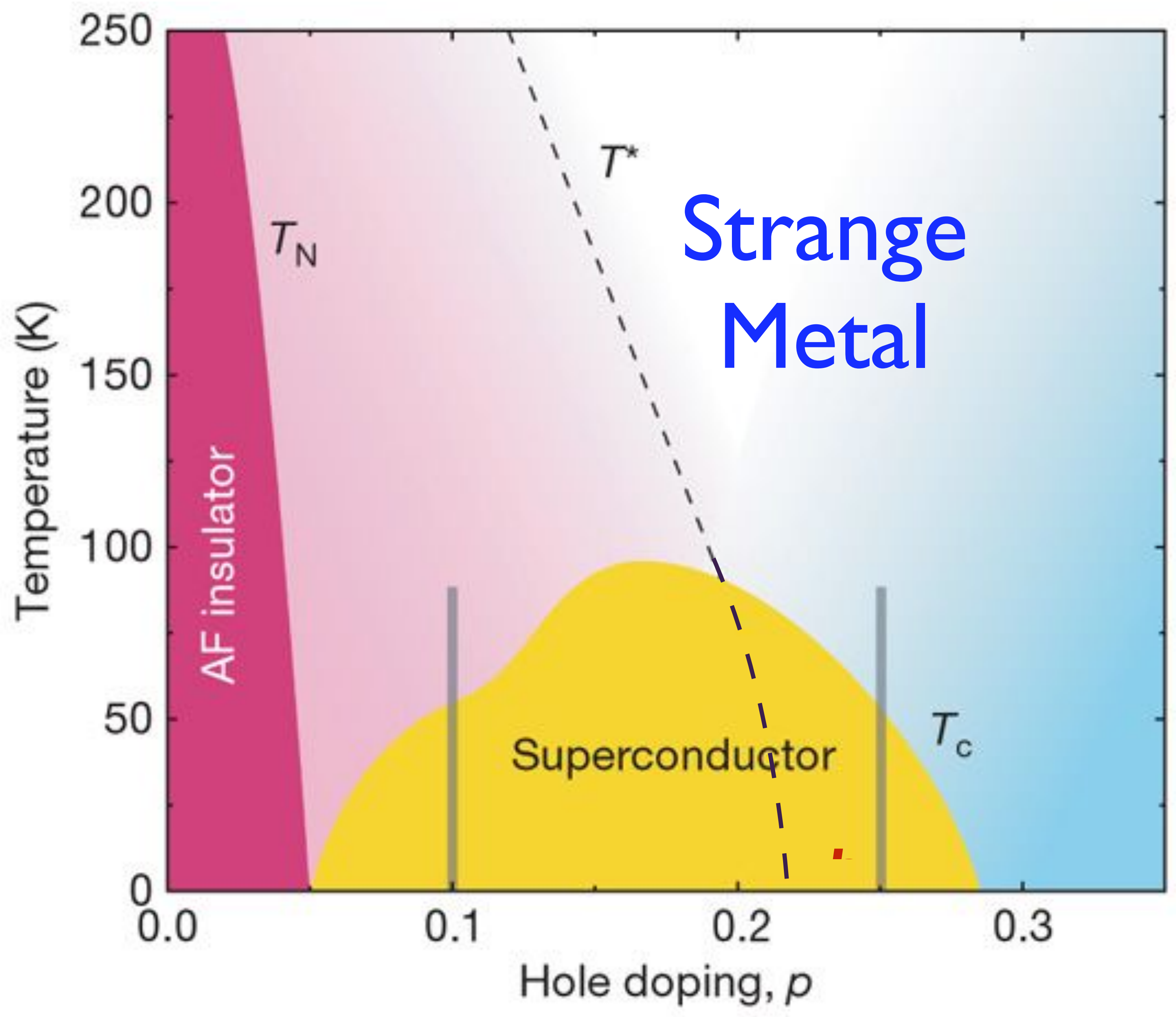
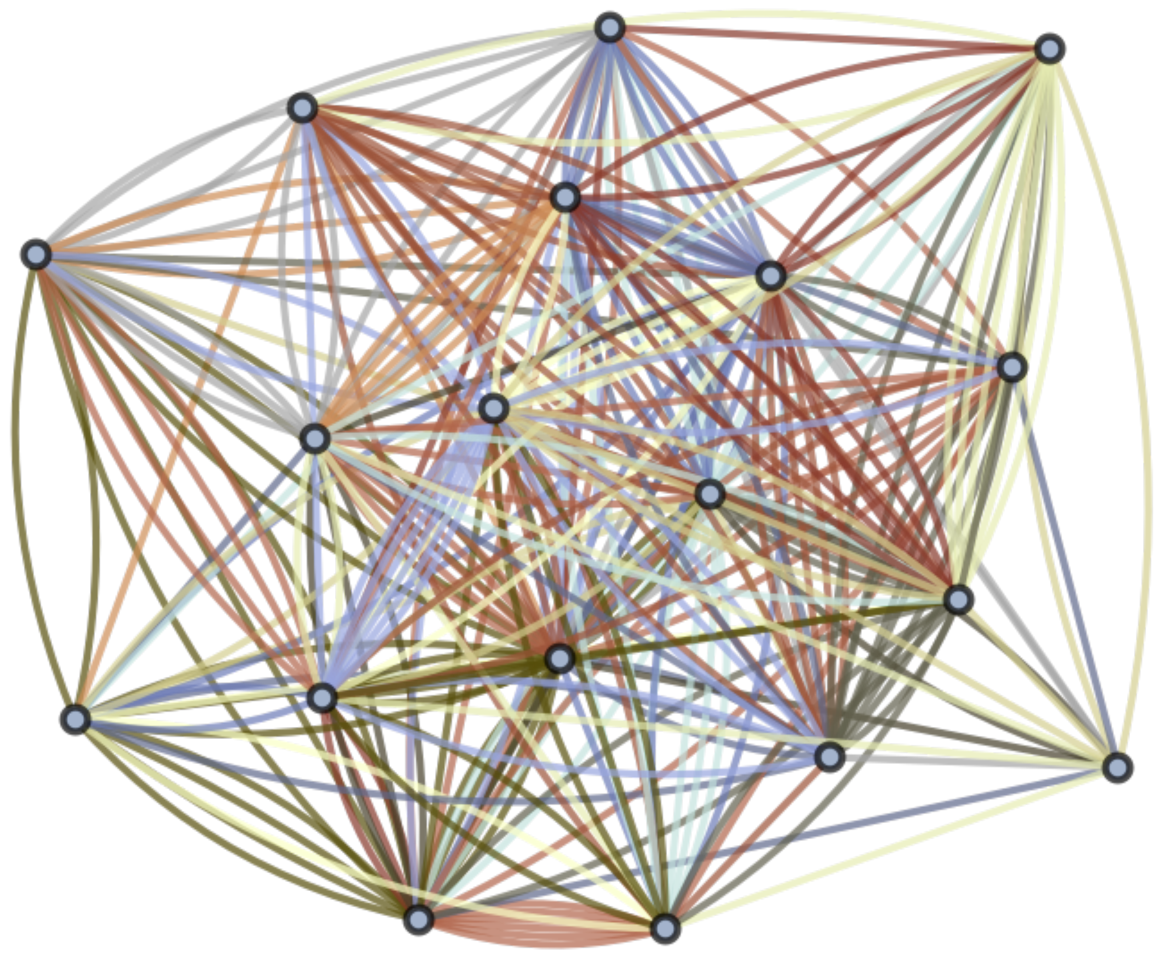



A 2d-YSYK theory describes the **strange metal** behavior of numerous quantum materials



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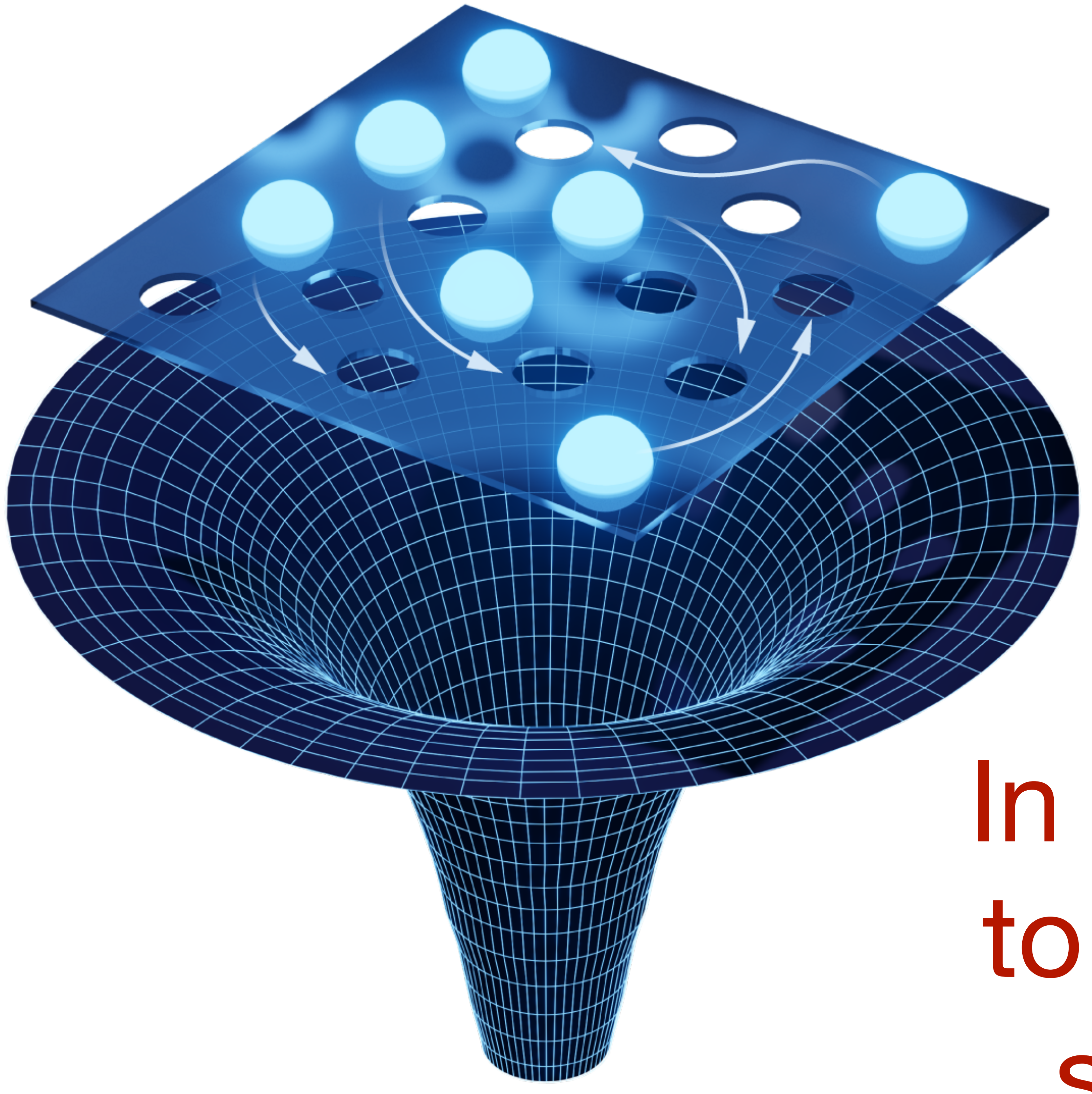

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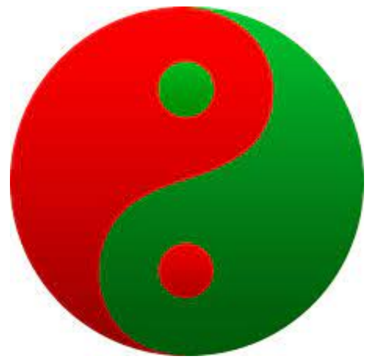

In a *dual* set of variables the SYK model has led to the computation of the low energy density of states of ***charged and rotating black holes***