Quantum spin liquids and the cuprate high temperature superconductors

Institute for Interdisciplinary Information Sciences Lecture Series
Tsinghua University, Beijing
July 18, 2024
Subir Sachdev

Video: https://www.koushare.com/live/details/36149

Maine Christos, Zhu-Xi Luo, Henry Shackleton, Ya-Hui Zhang, Mathias Scheurer, and S. Sachdev, *Proc. Nat. Acad. Sci.* **120**, e2302701120 (2023)
Maine Christos and S.Sachdev, *npj Quantum Materials* **9**, 4 (2024)
M. Christos, H. Shackleton, S. Sachdev, and Zhu-Xi Luo, arXiv:2402.09502
Pietro M. **B**onetti, Maine **C**hristos, S. **S**achdev (BCS), arXiv: 2405.08817
Jia-Xin Zhang, S. Sachdev, arXiv:2406.12964

PHYSICS
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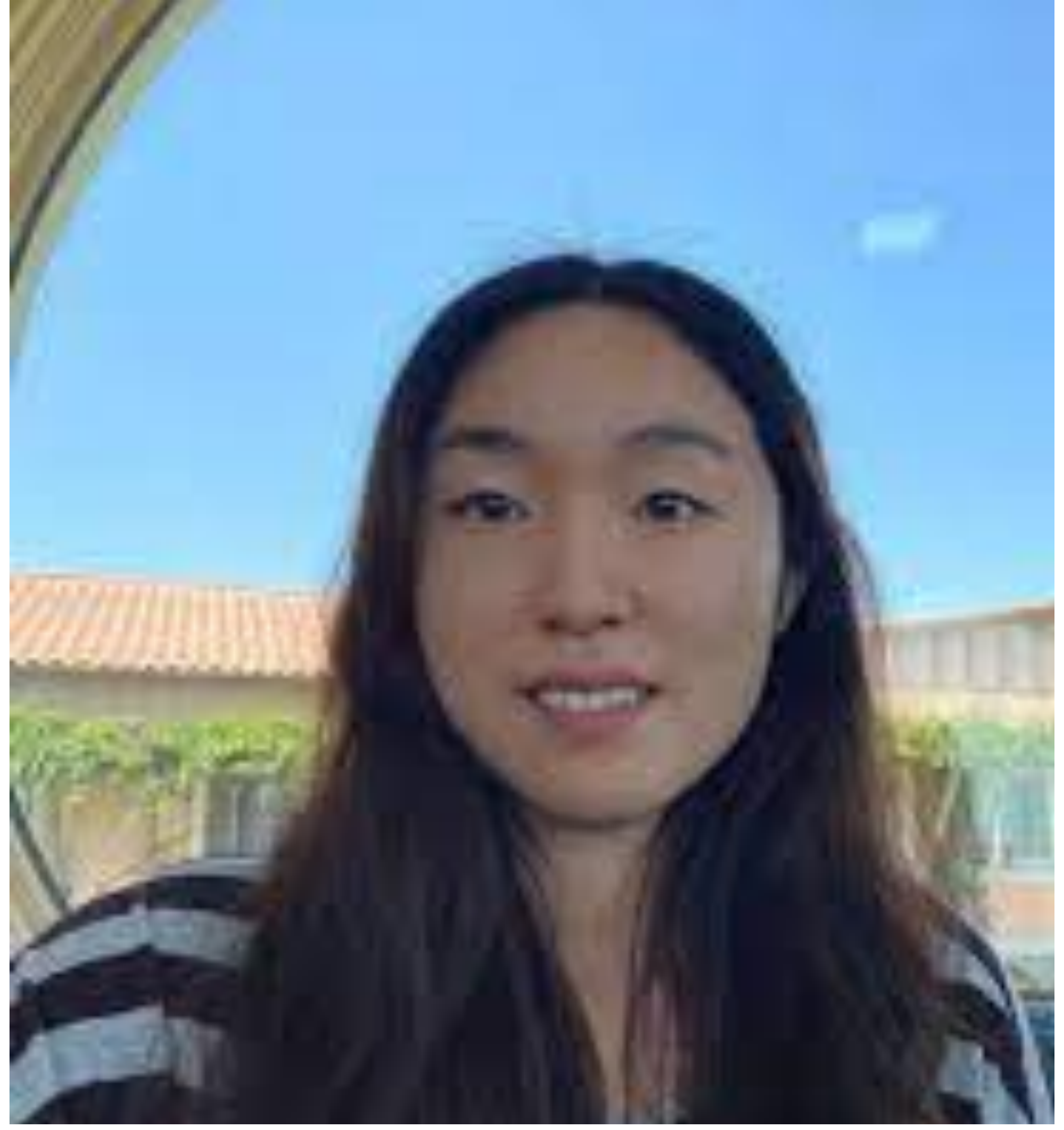
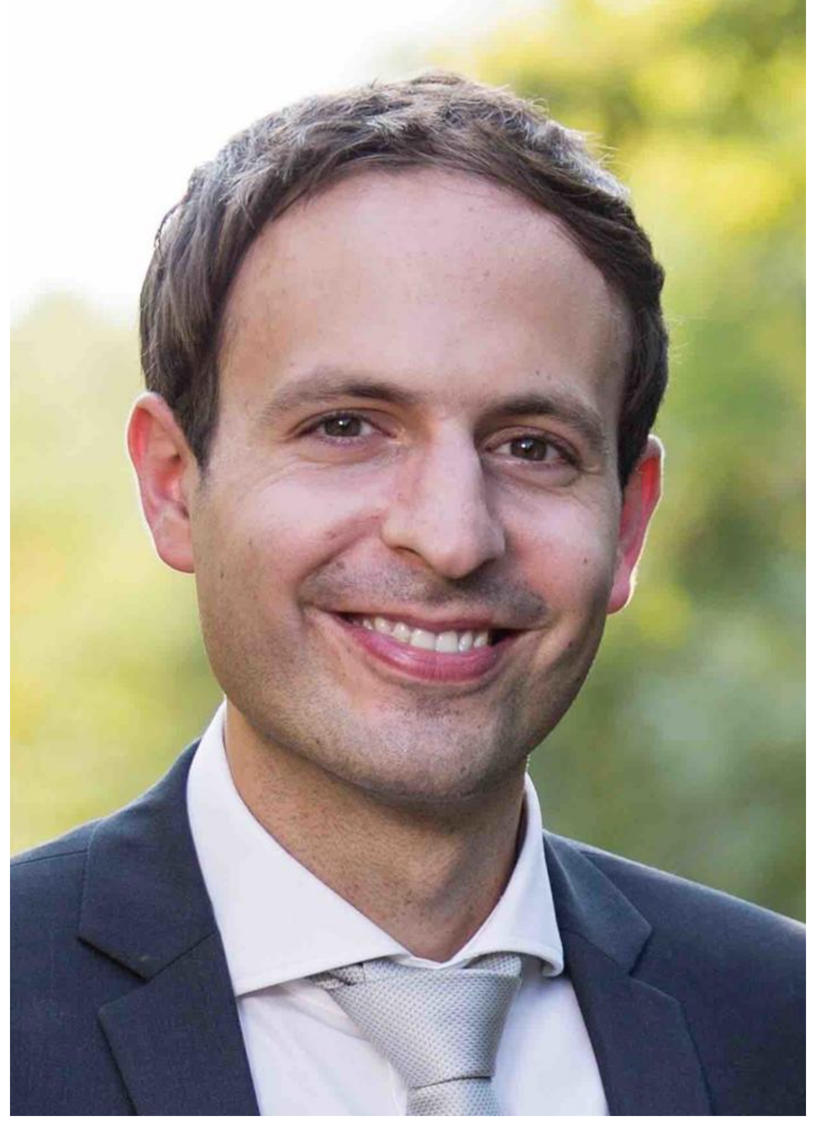
HARVARD

Maine Christos



Zhu-Xi Luo

Maria Tikhanovskaya



Mathias Scheurer

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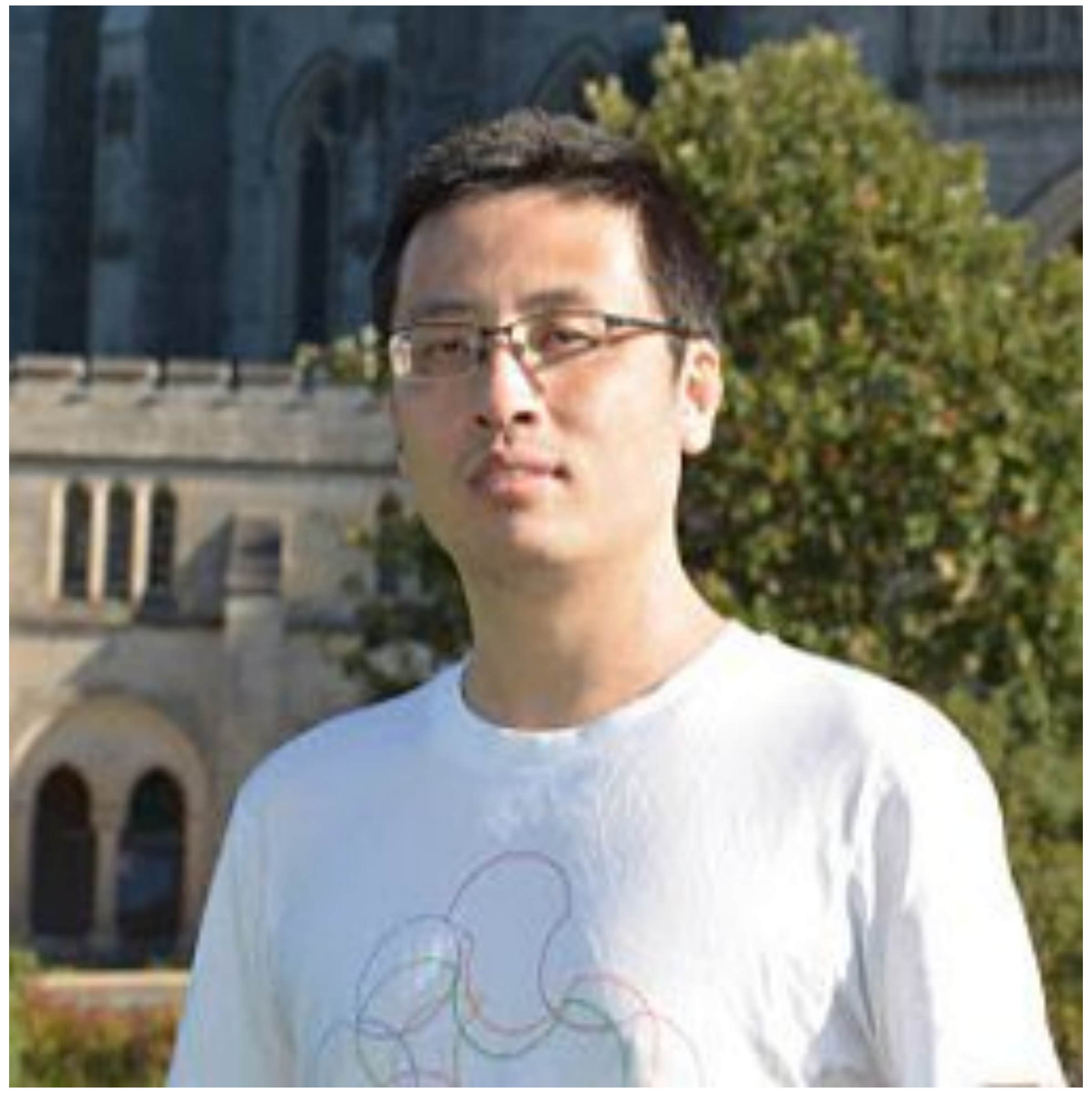
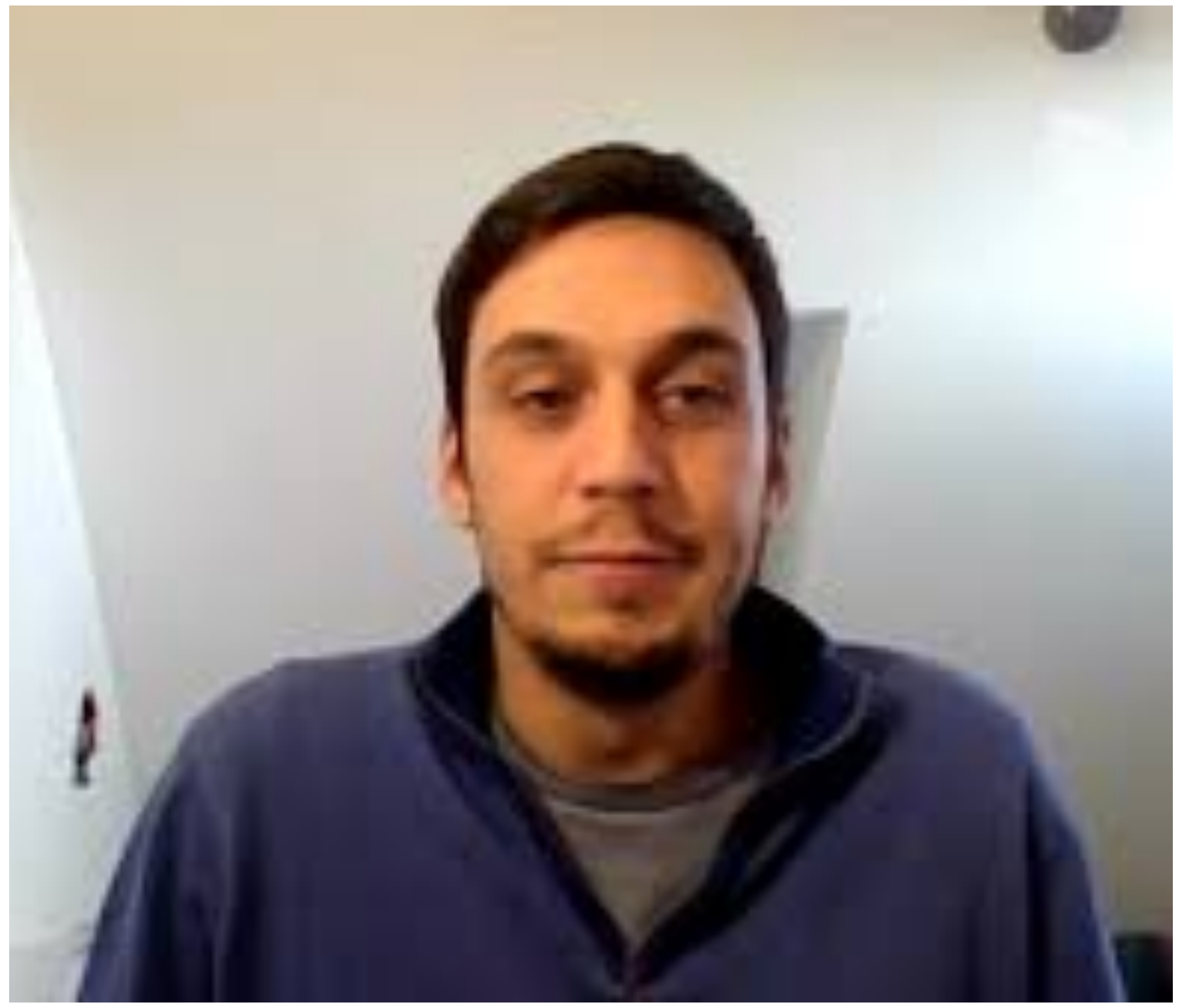


Ya-Hui Zhang

Alexander Nikolaenko



Pietro Bonetti

Henry Shackleton

1. Identify spin liquid states of the insulator, describing the dynamics and symmetries of its anyons.
2. Work with metallic states in which the low energy anyons are essentially the same as those of the insulator, along with a ‘trivial’ fermion with the same quantum numbers as the electron.

1. Square lattice spin liquids

2. Spin liquids on the Kondo lattice: non-Luttinger volume Fermi surfaces (FL*)

3. Doping square lattice spin liquids for $t \gg J$: FL* in a single-band model

4. FL* theory of the pseudogap metal of the cuprates

5. Nodal fermionic quasiparticles in d-wave SC

6. Quantum oscillations in hole-doped cuprates

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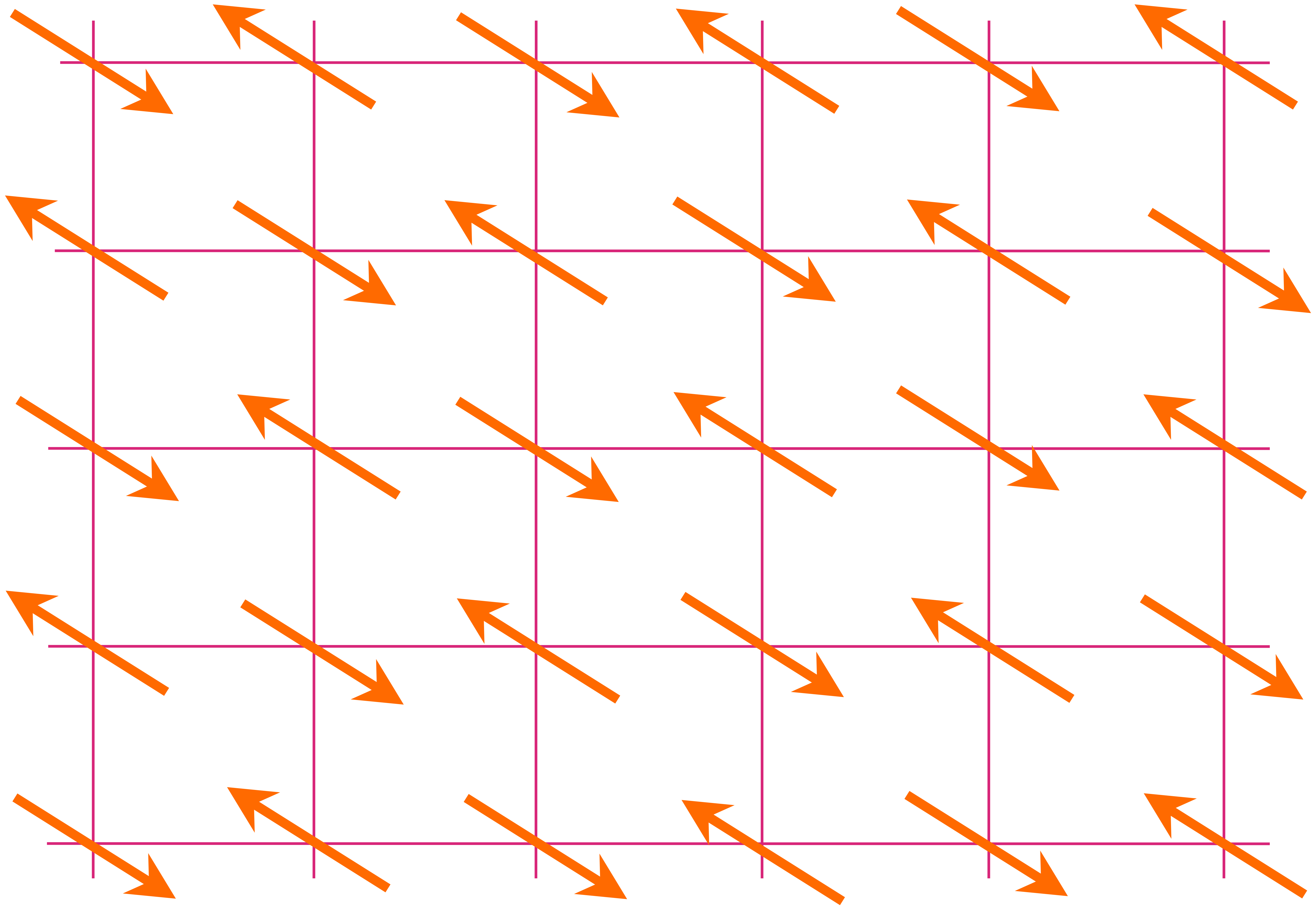

Insulating antiferromagnet

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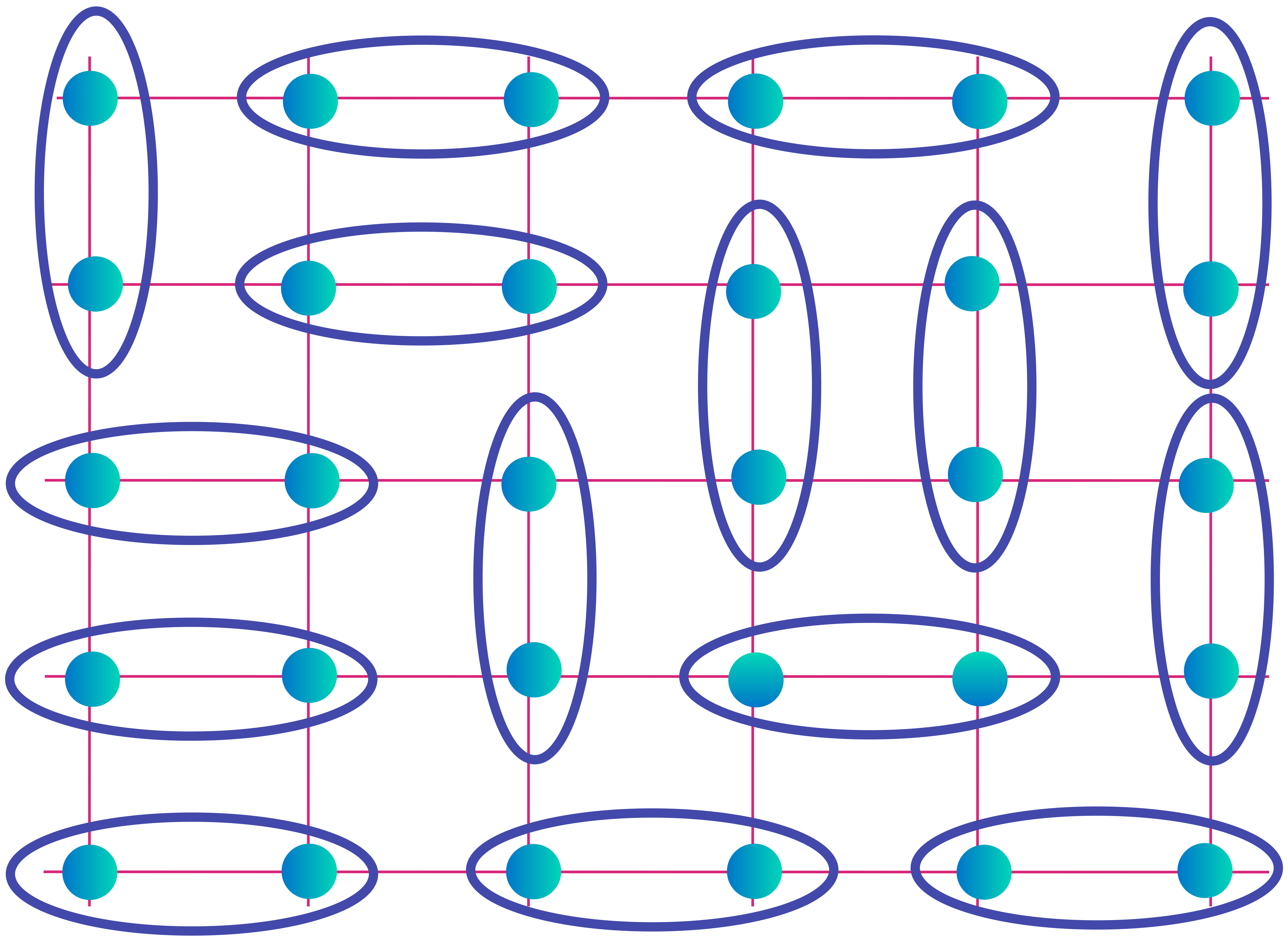

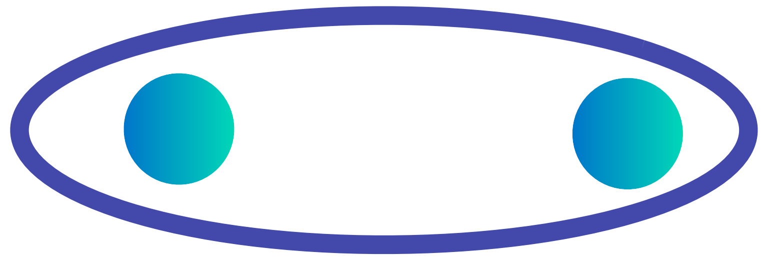 $= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

P.W. Anderson (1973)

Spin liquid

Electrons form entangled pairs, and the pairs entangle across the entire sample

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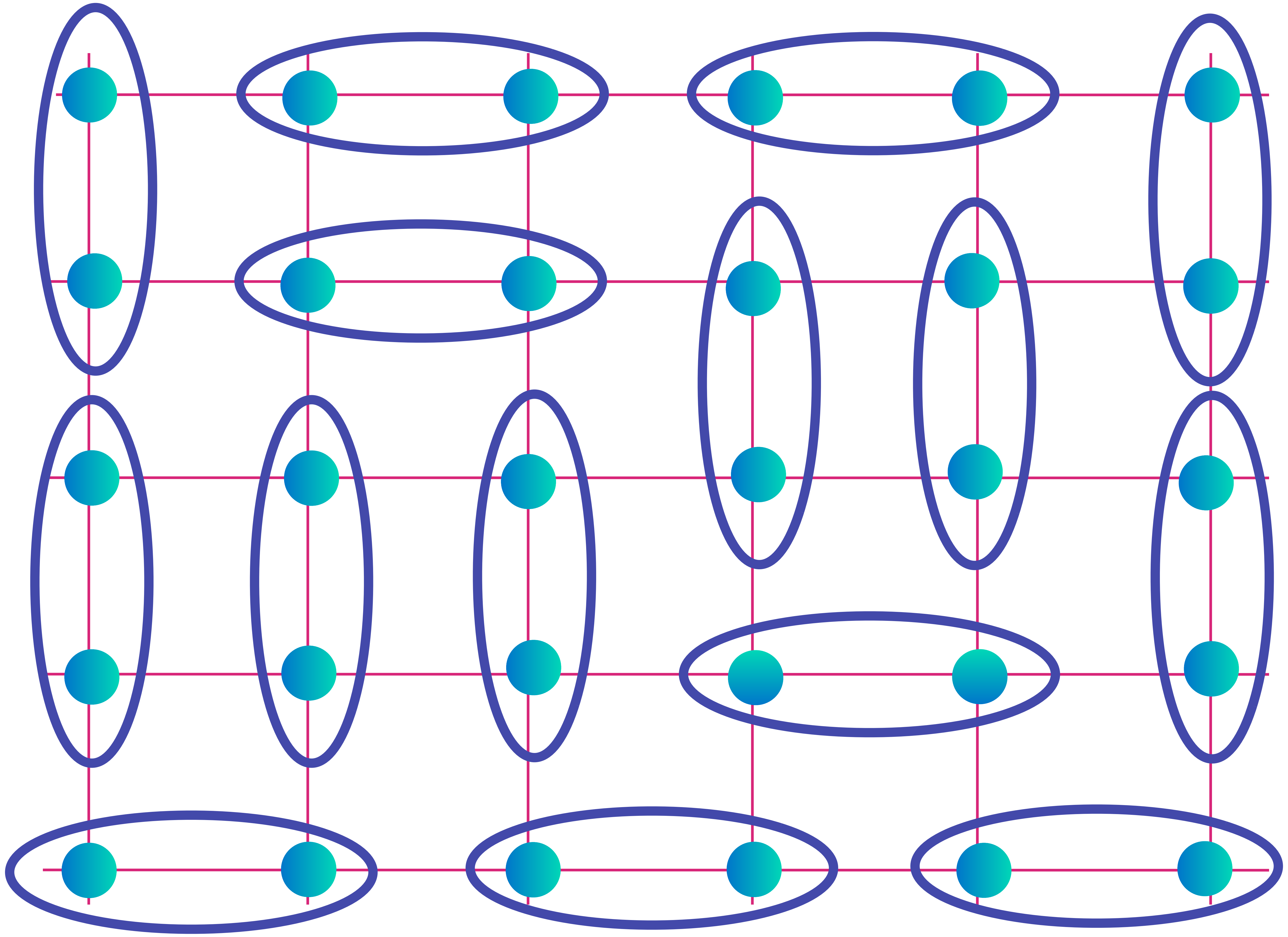

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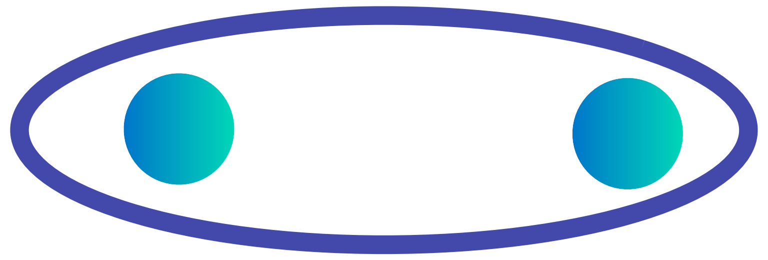 $= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

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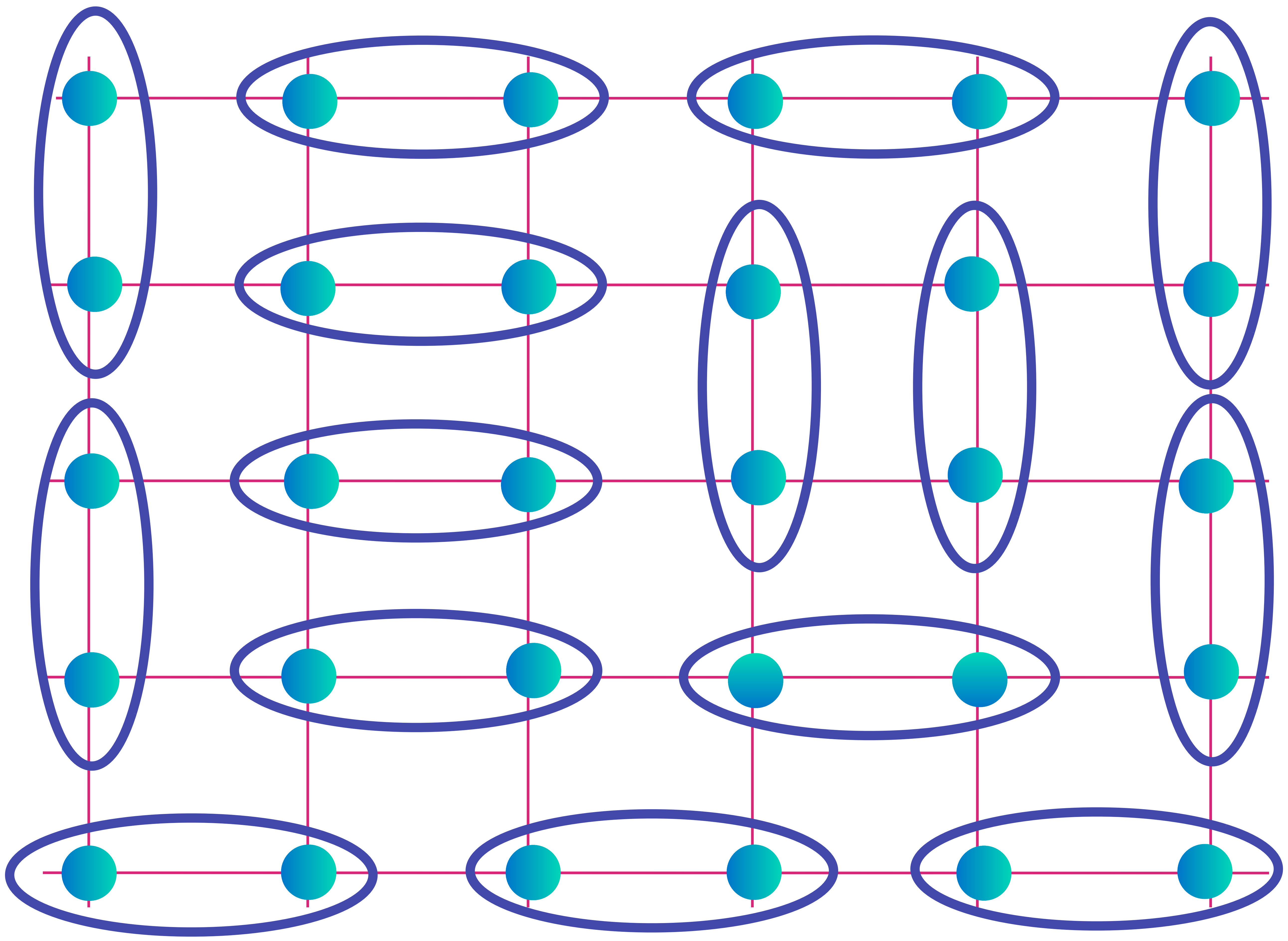

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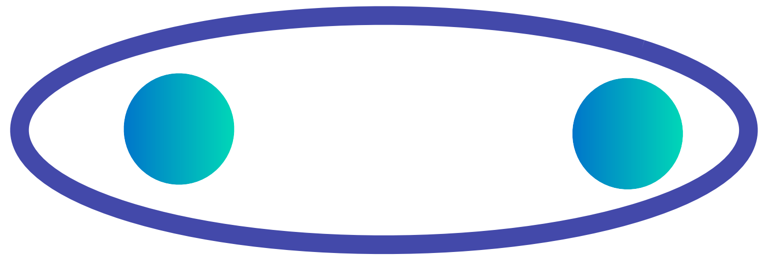

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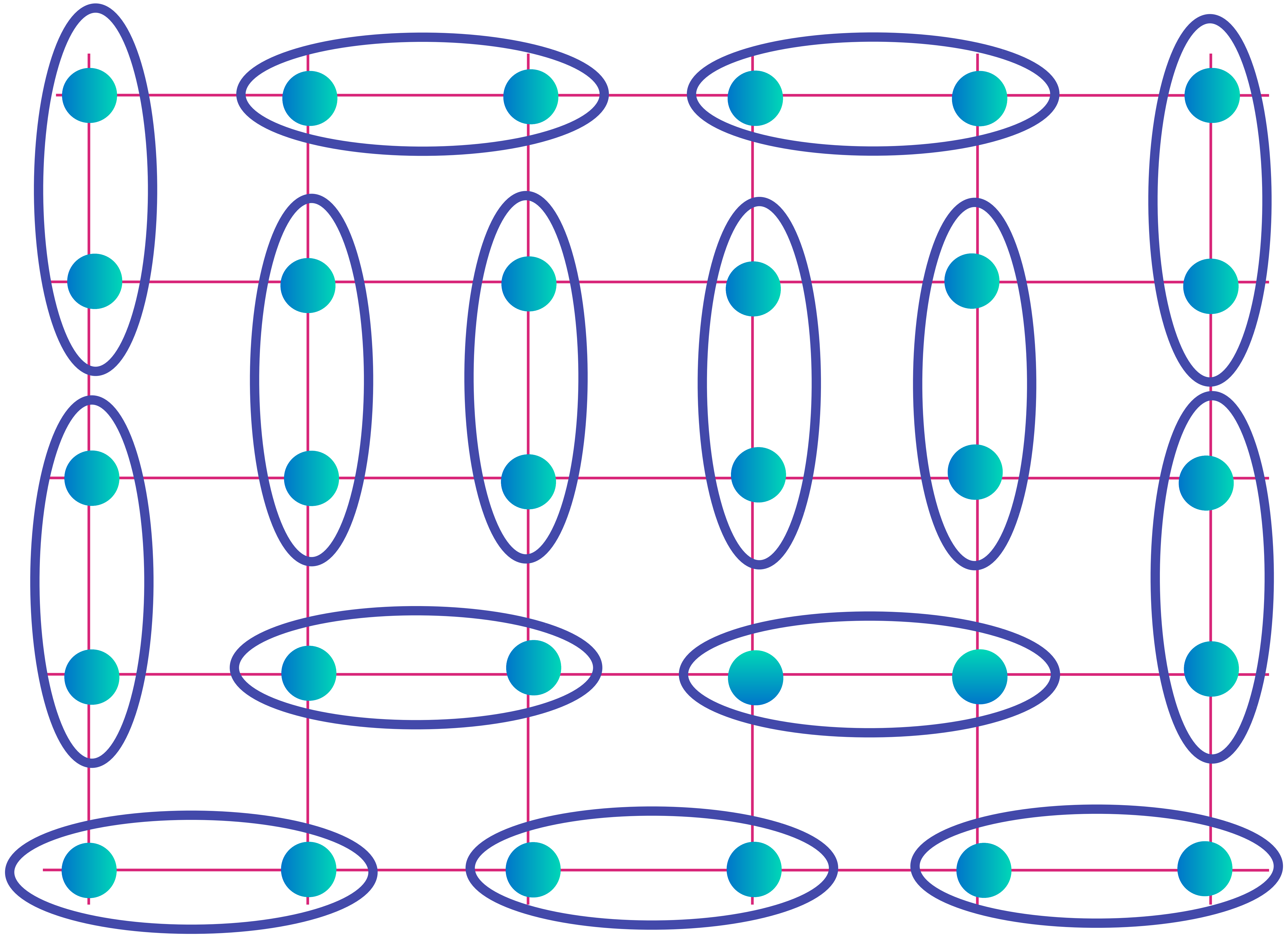

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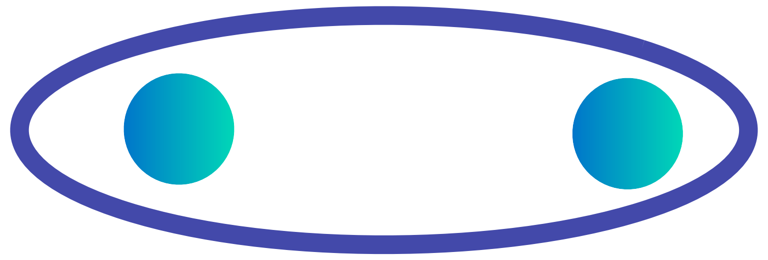 $= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

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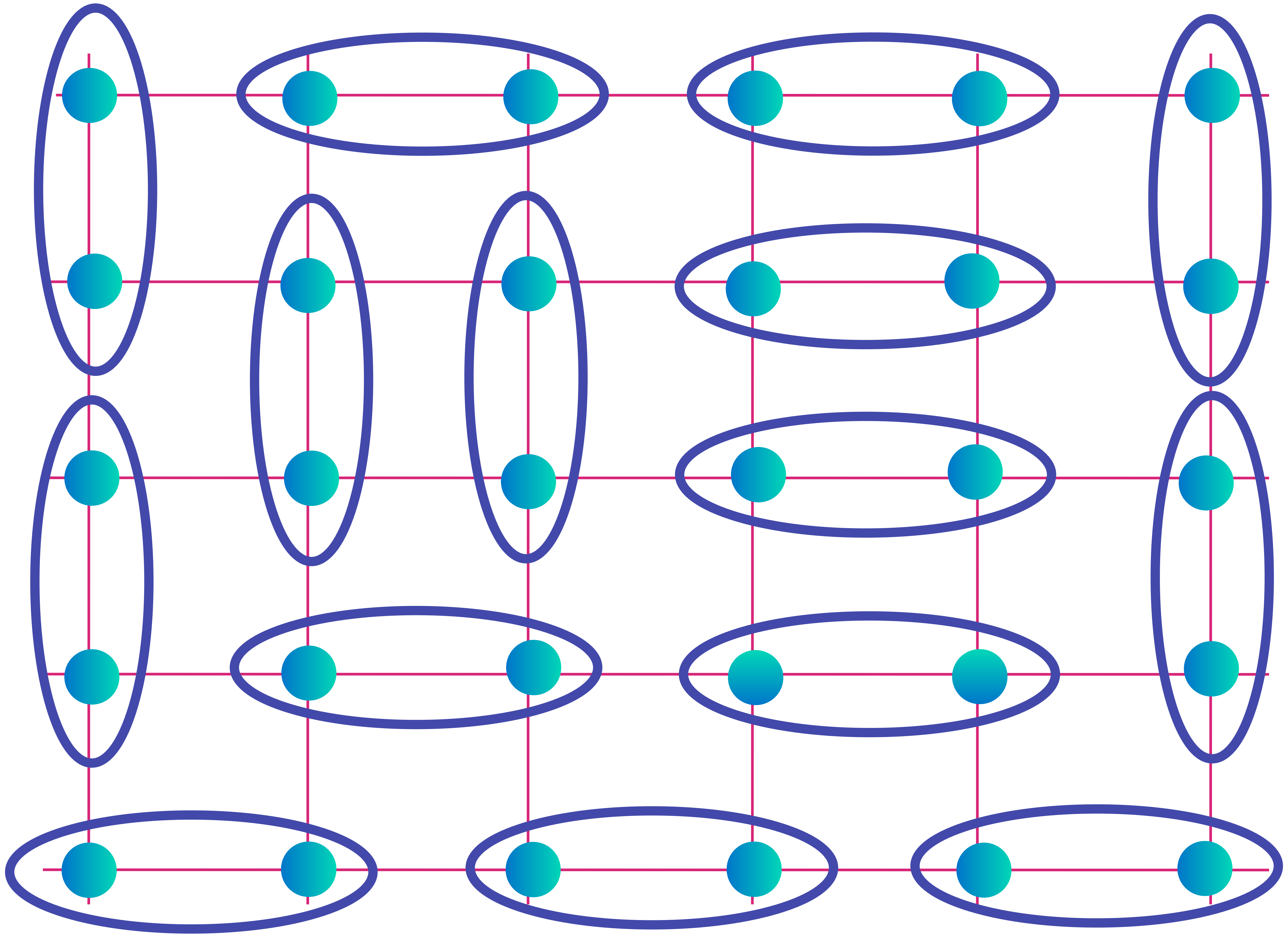

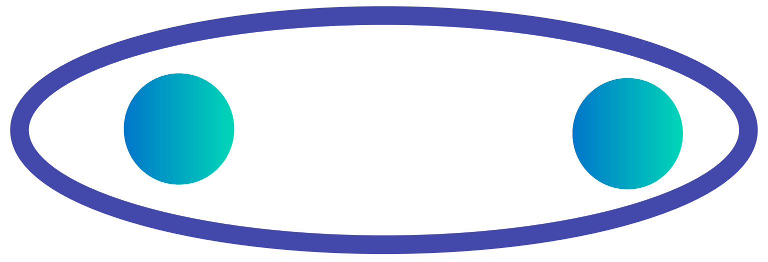 $= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

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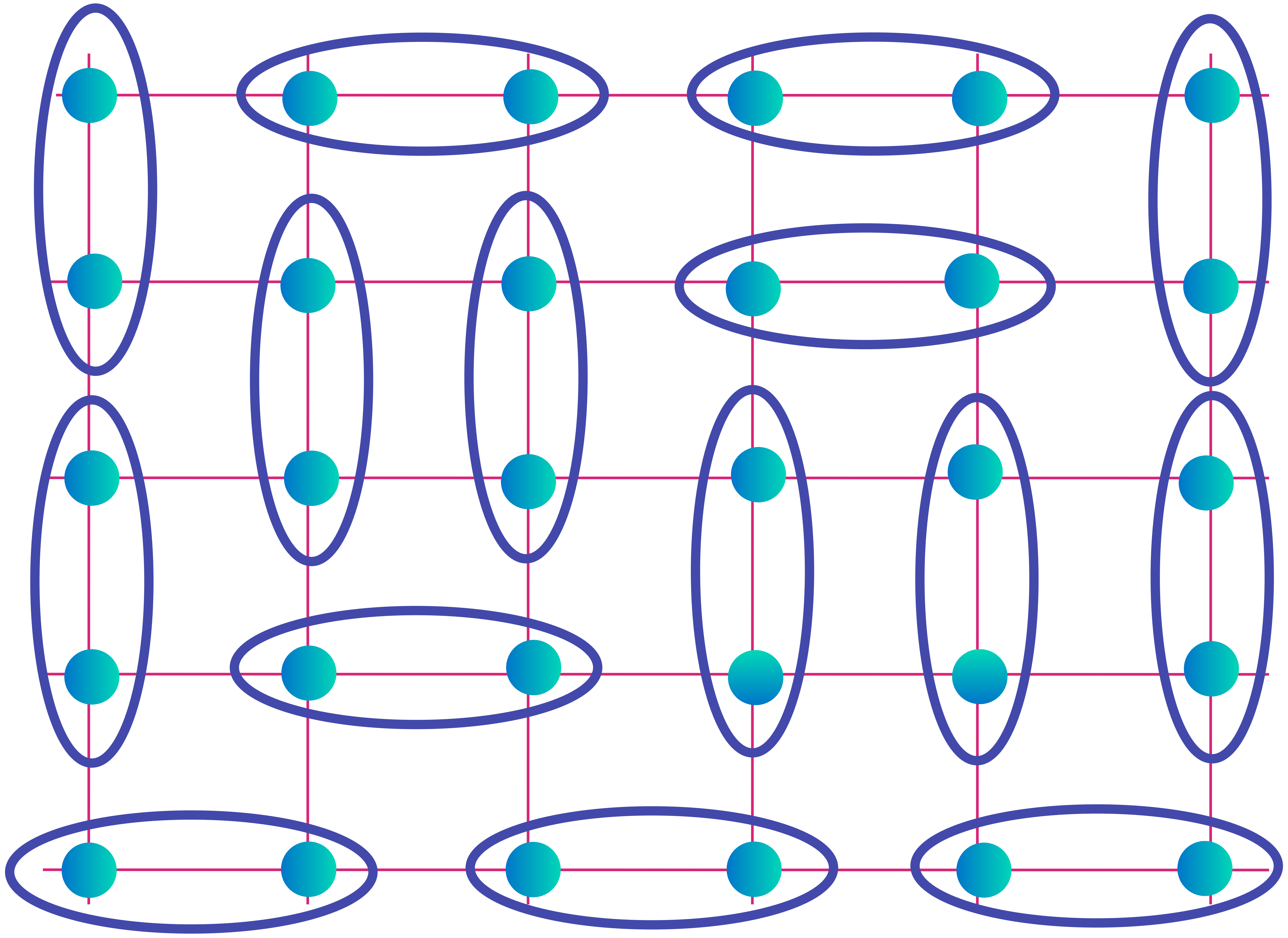

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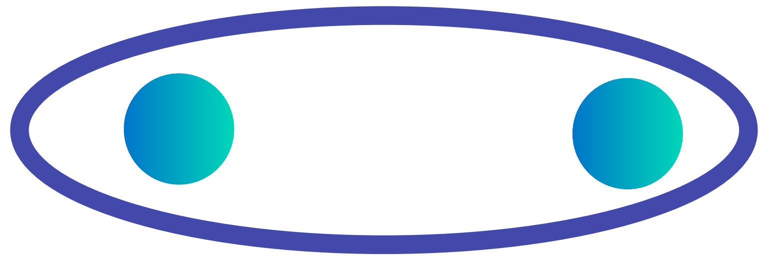

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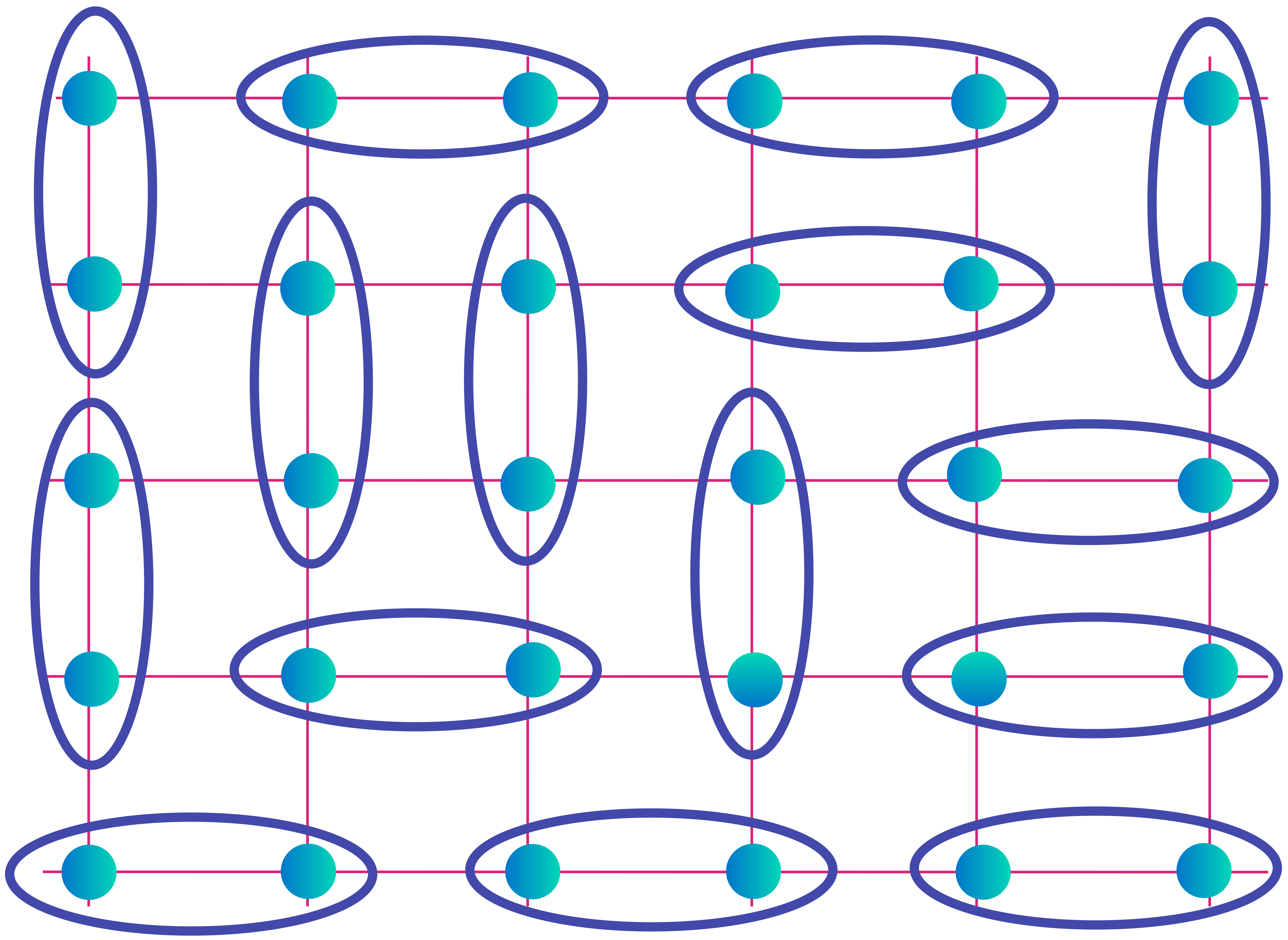

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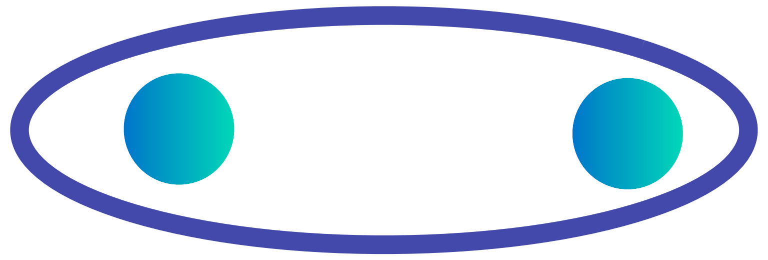

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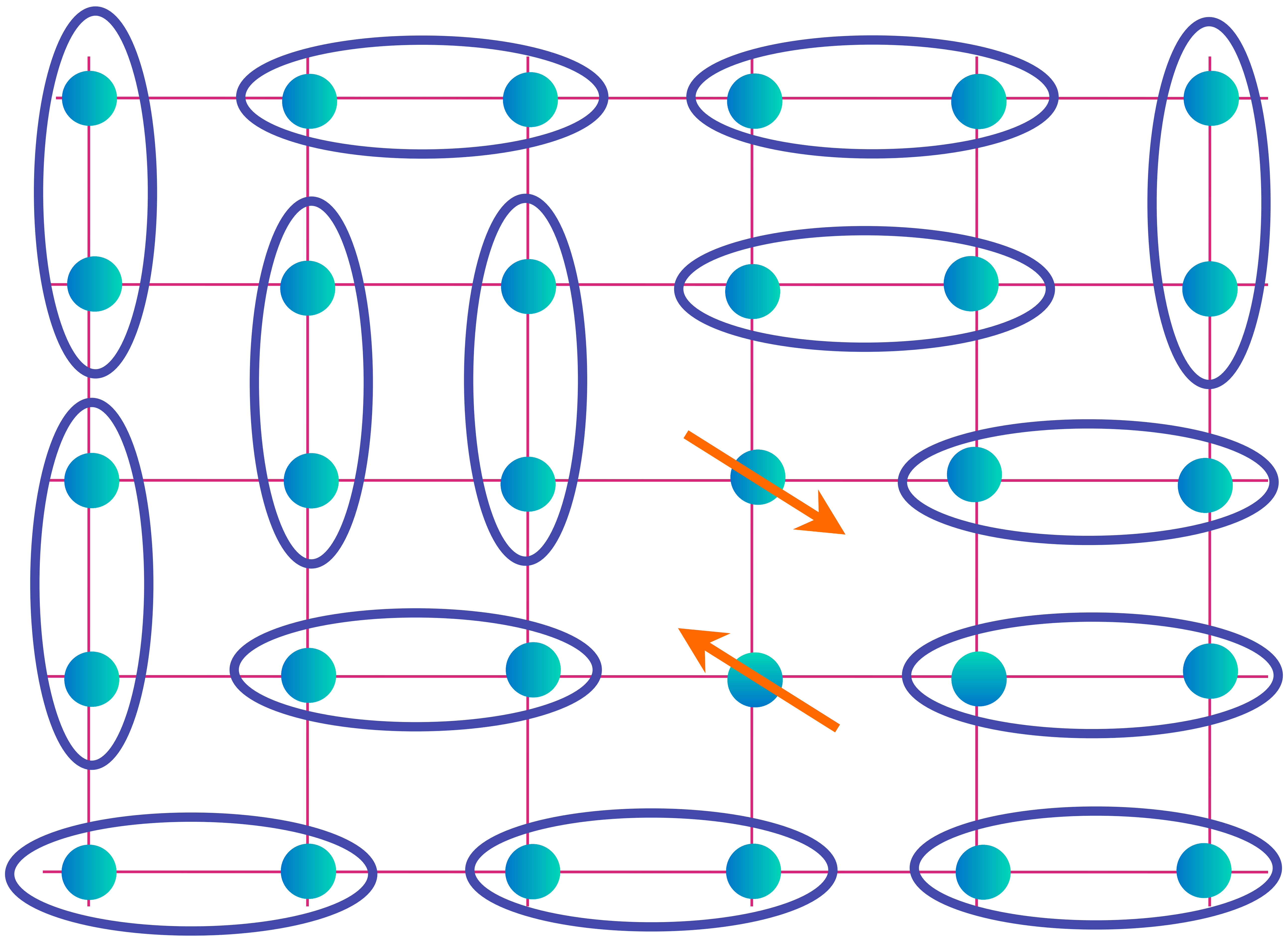

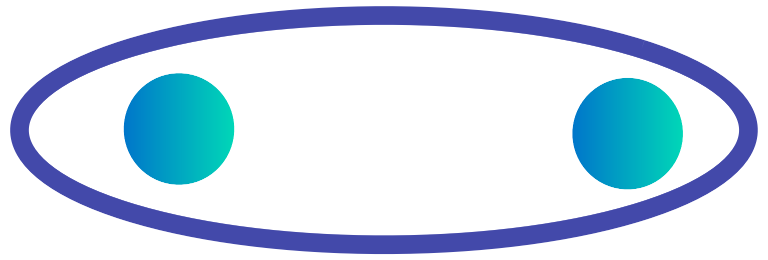 $= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

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Fractionalized spinon excitations with spin S=1/2 and charge 0.

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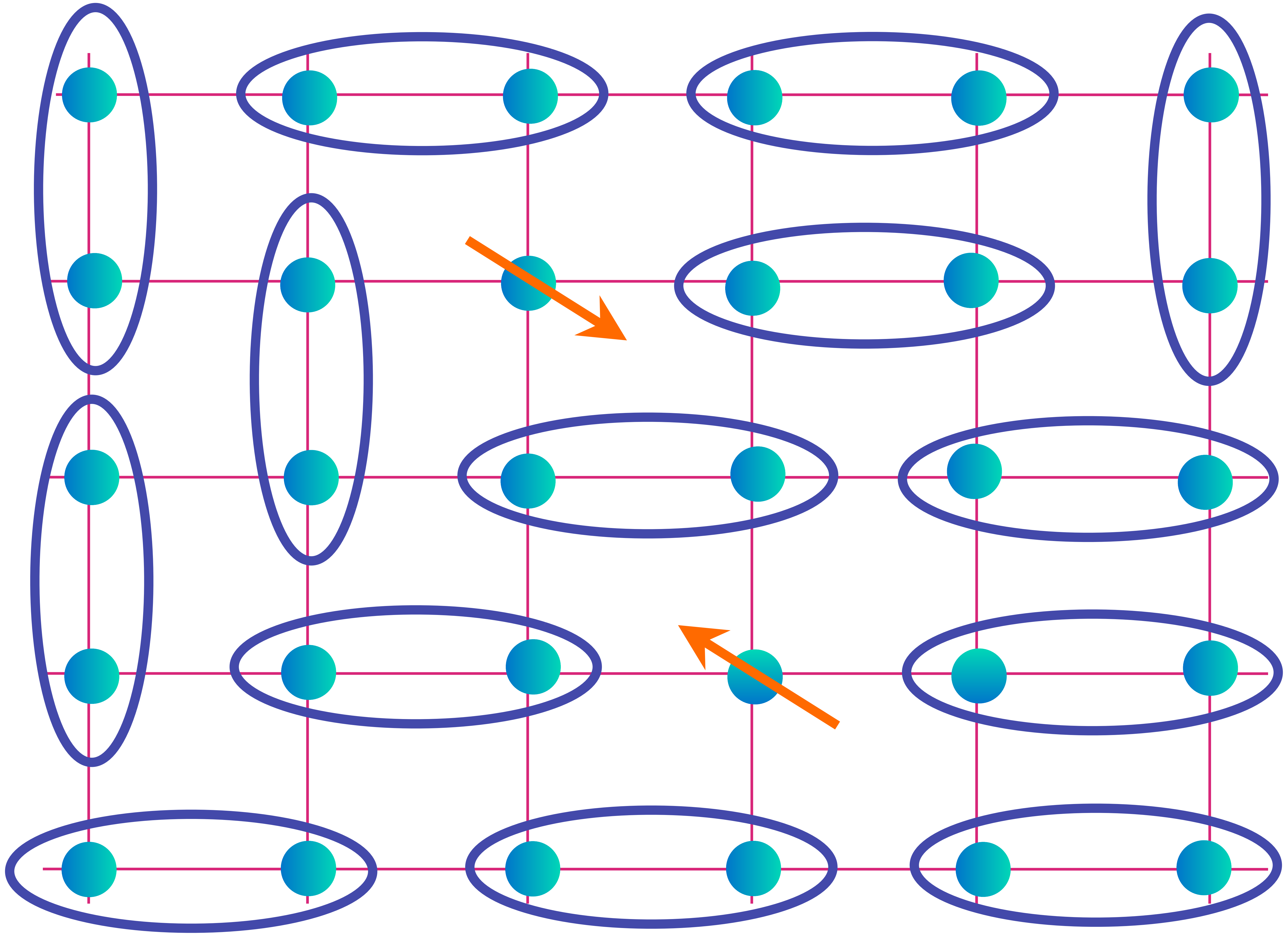

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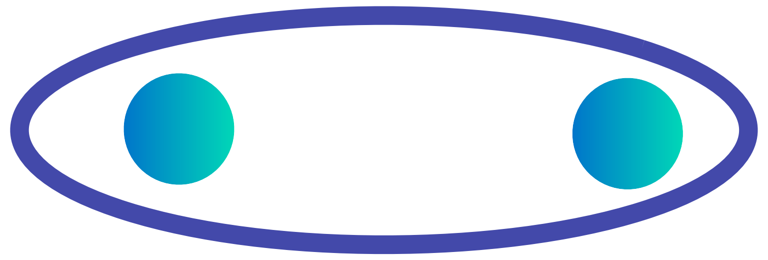

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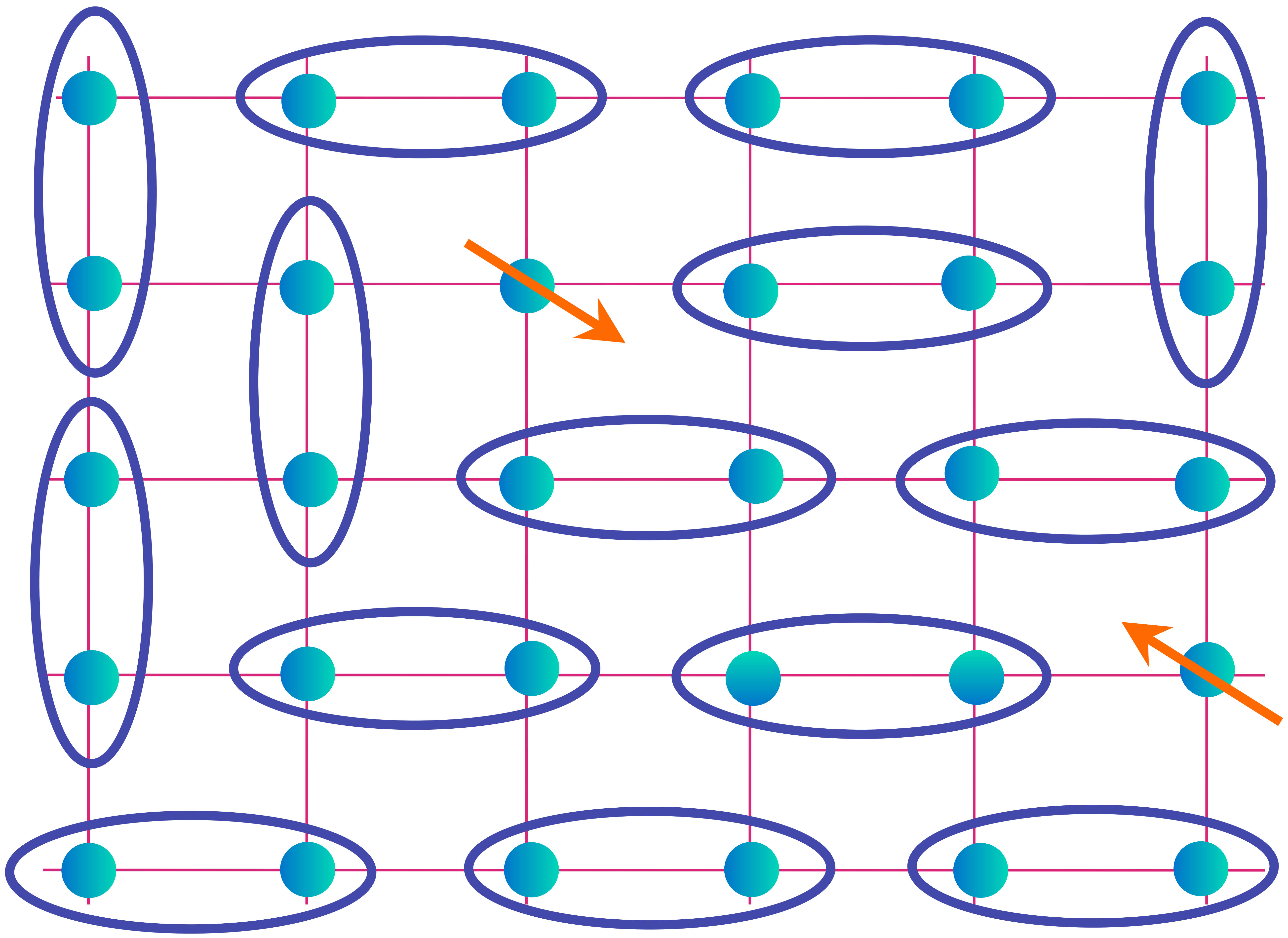

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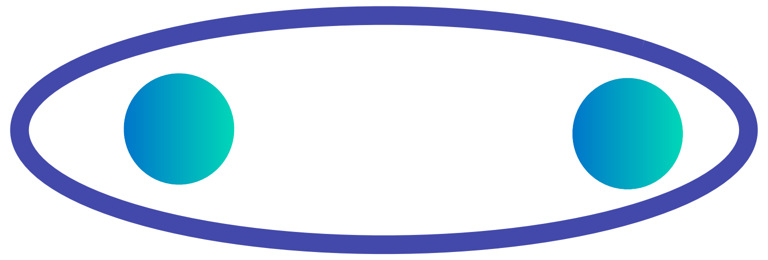

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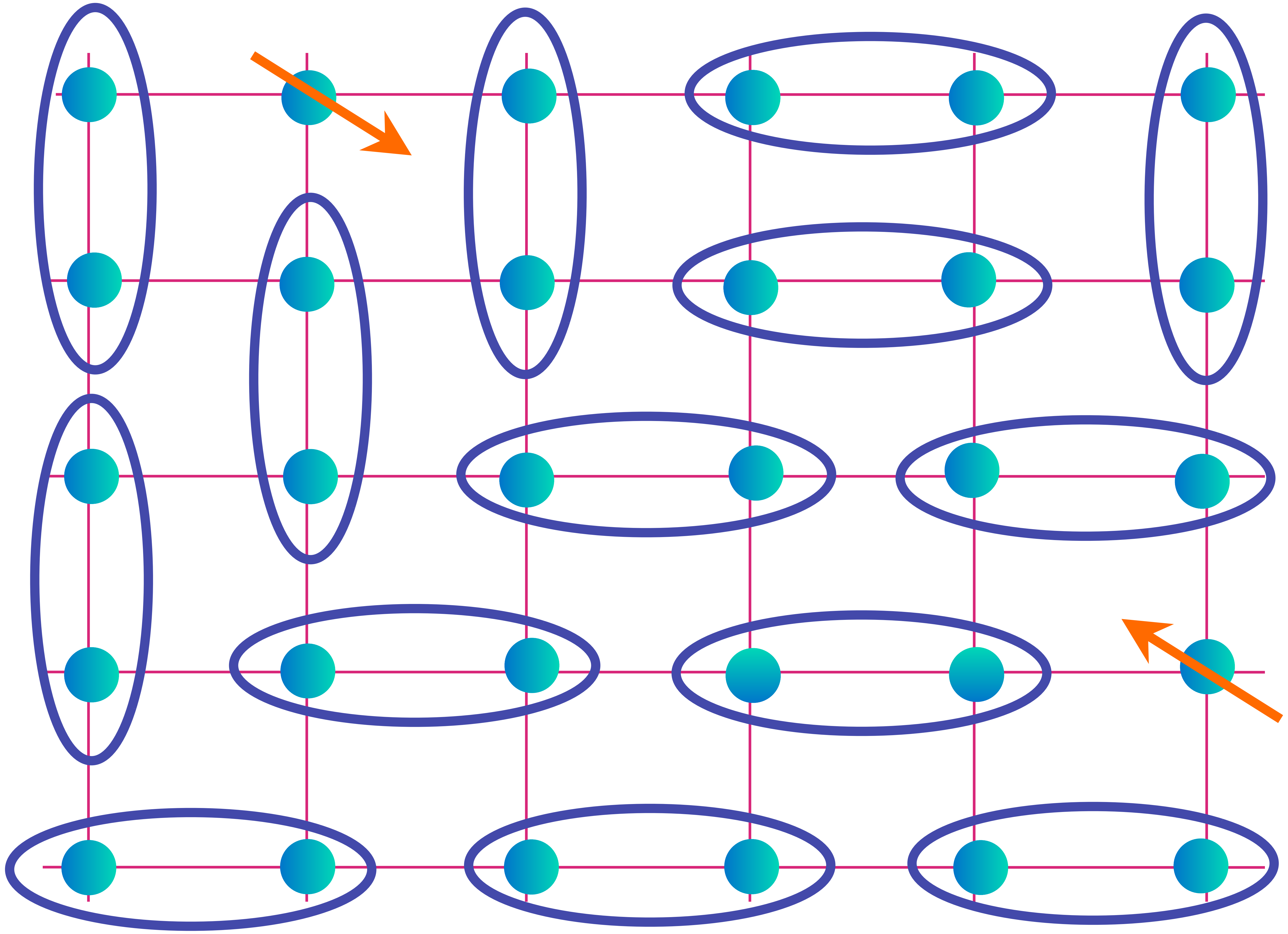

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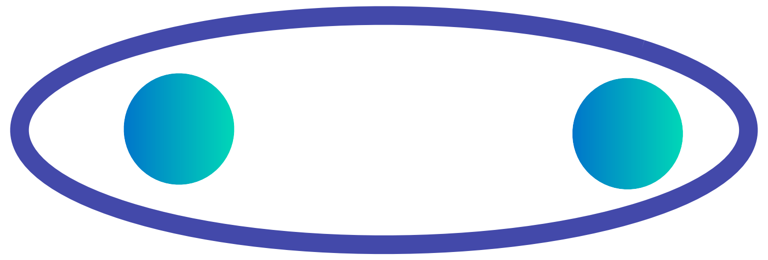

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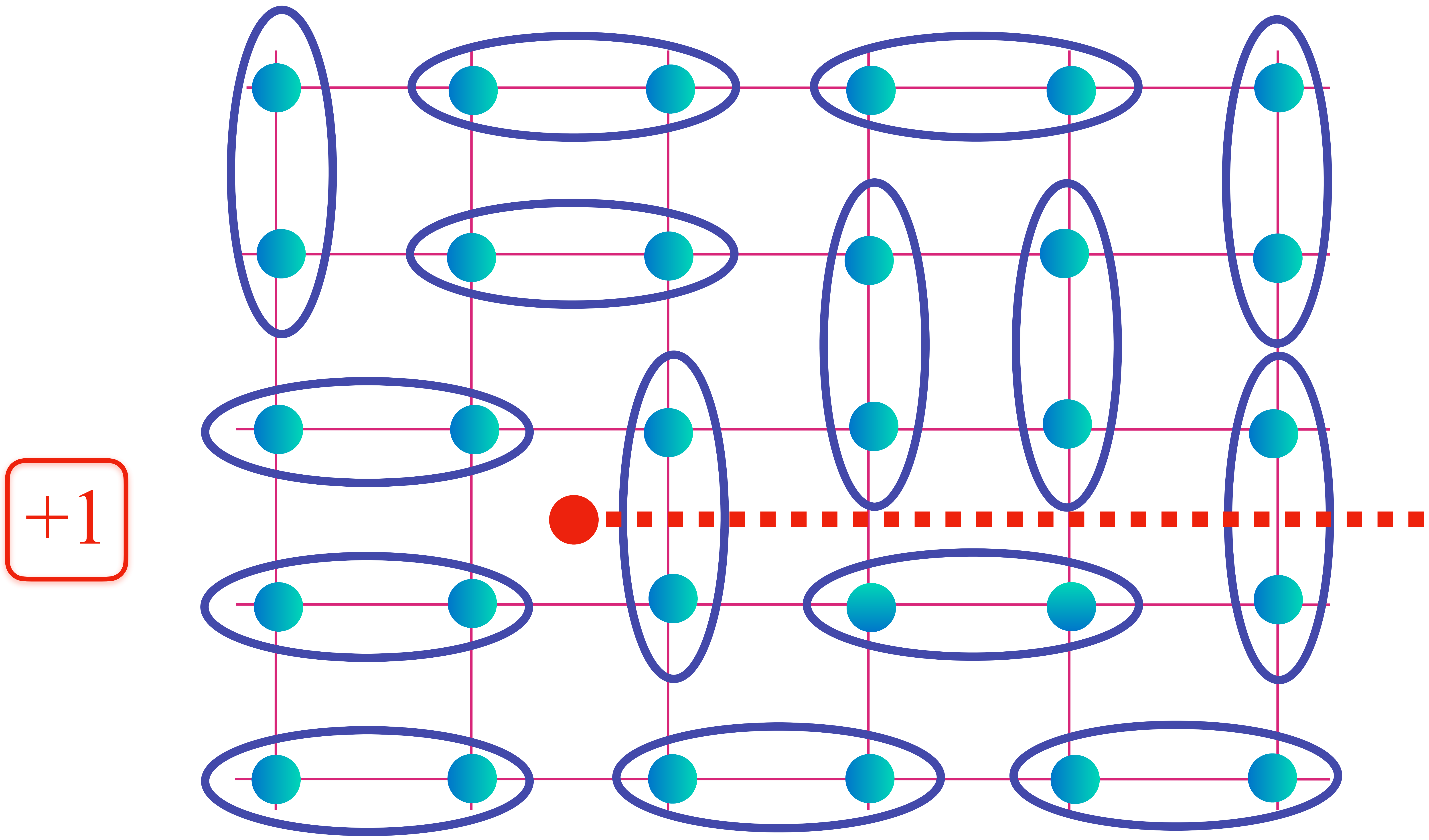

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$\mathbb{Z}_2$ spin liquid

Vison excitation with $S = 0$.

Spinons and visons are mutual semions.

The topological structure of the $\mathbb{Z}_2$ spin liquid is identical to Kitaev's toric code, discovered later (spinons and visons correspond to the e and m particles).

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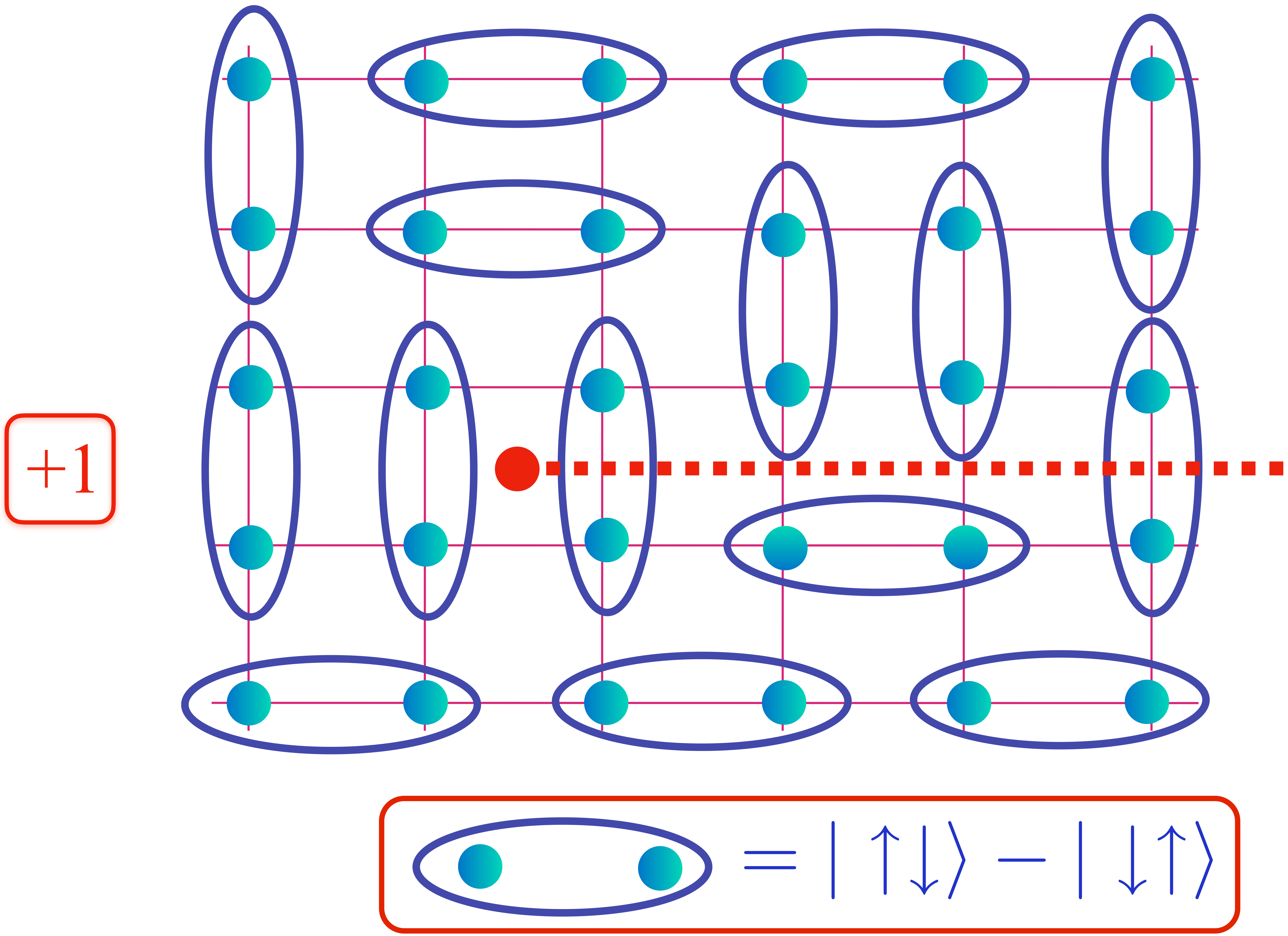

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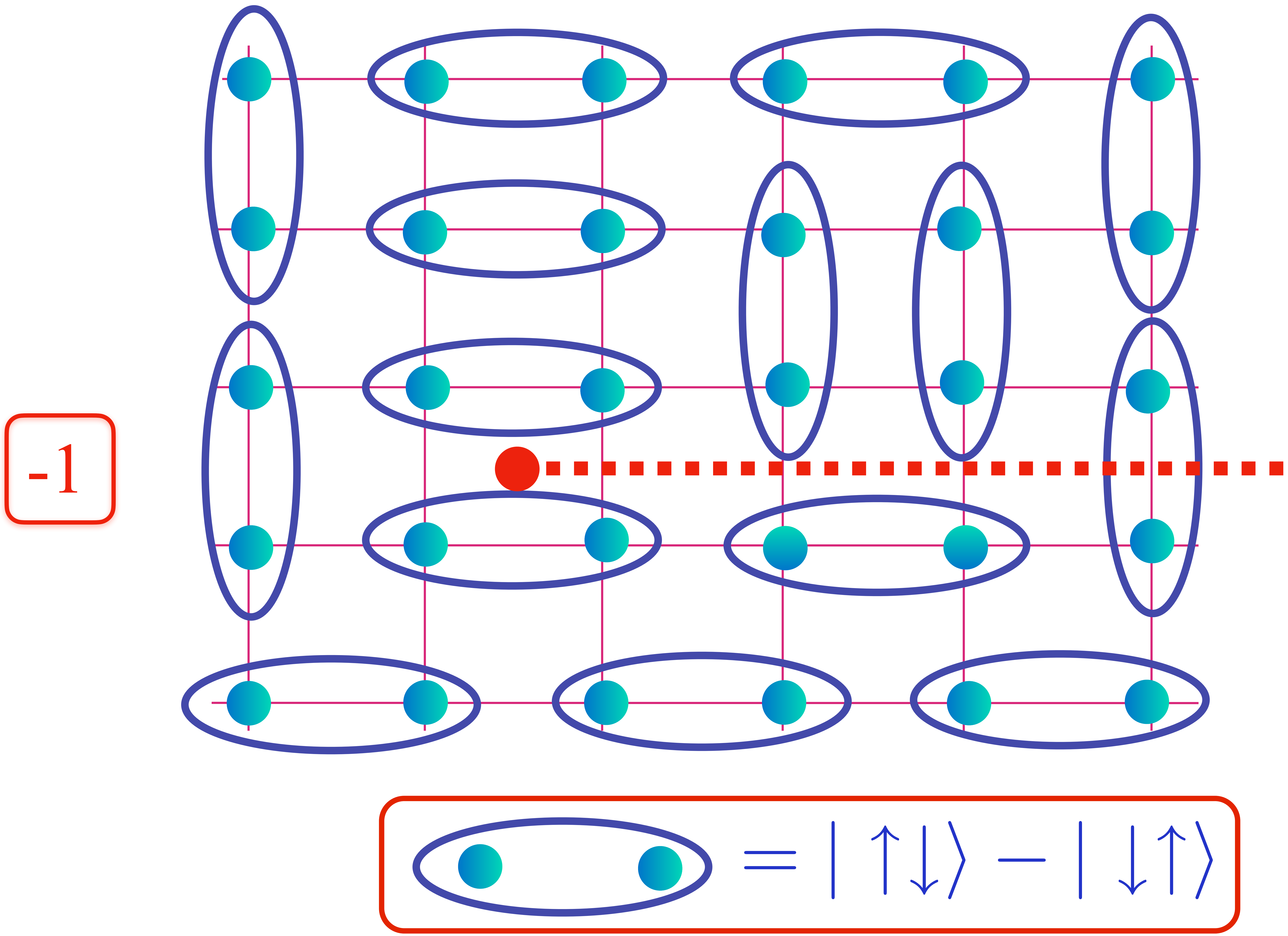

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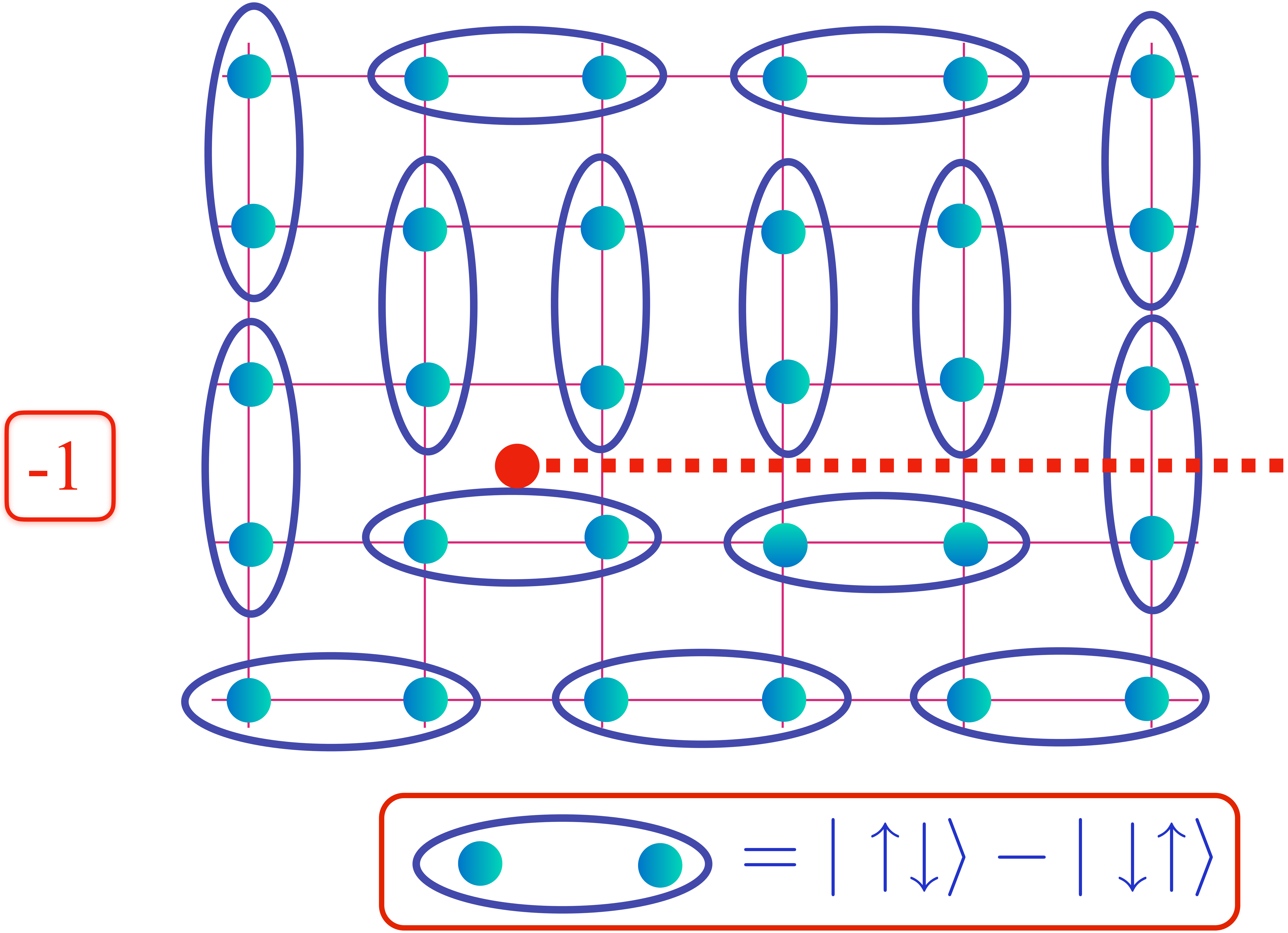

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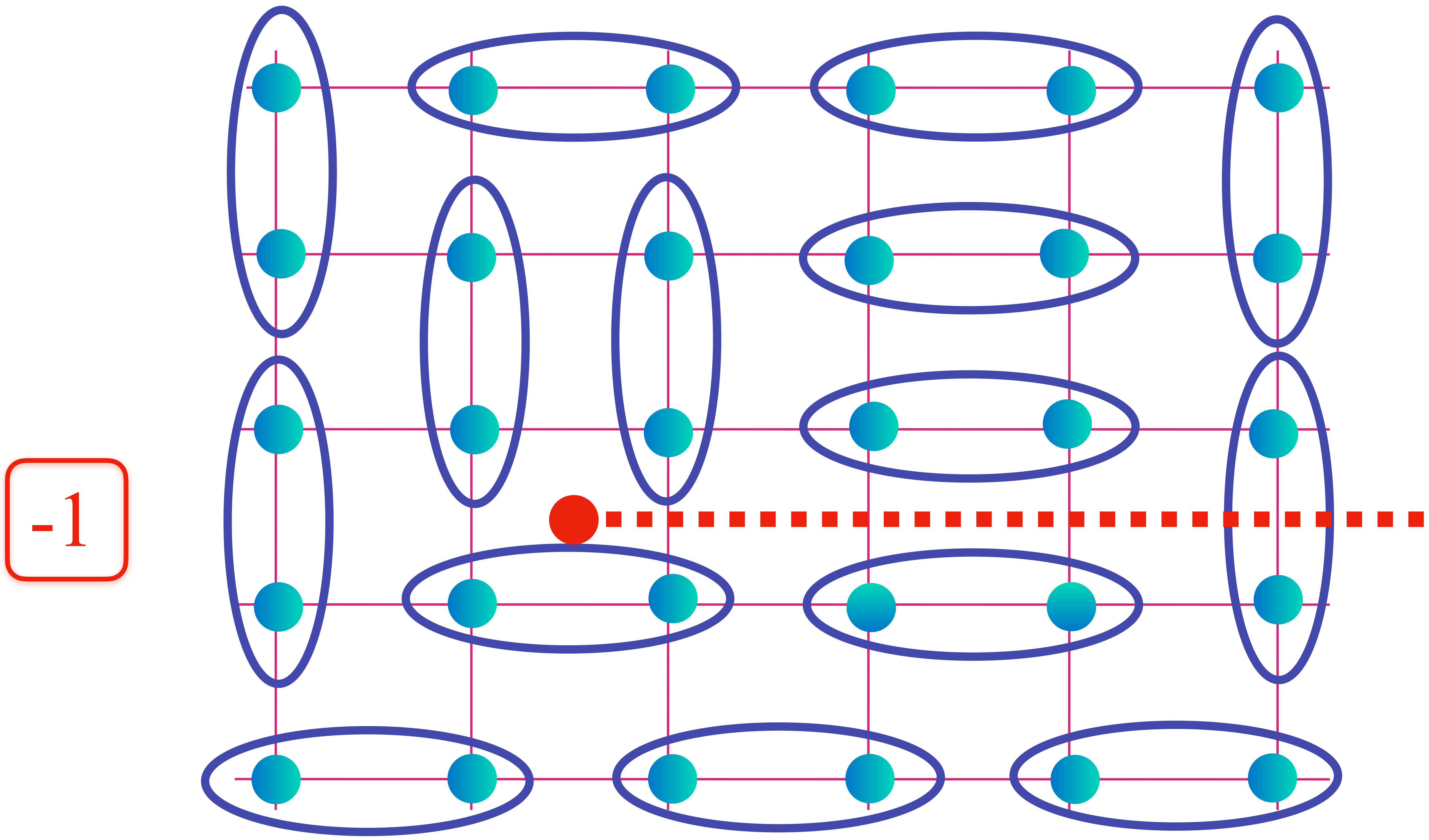

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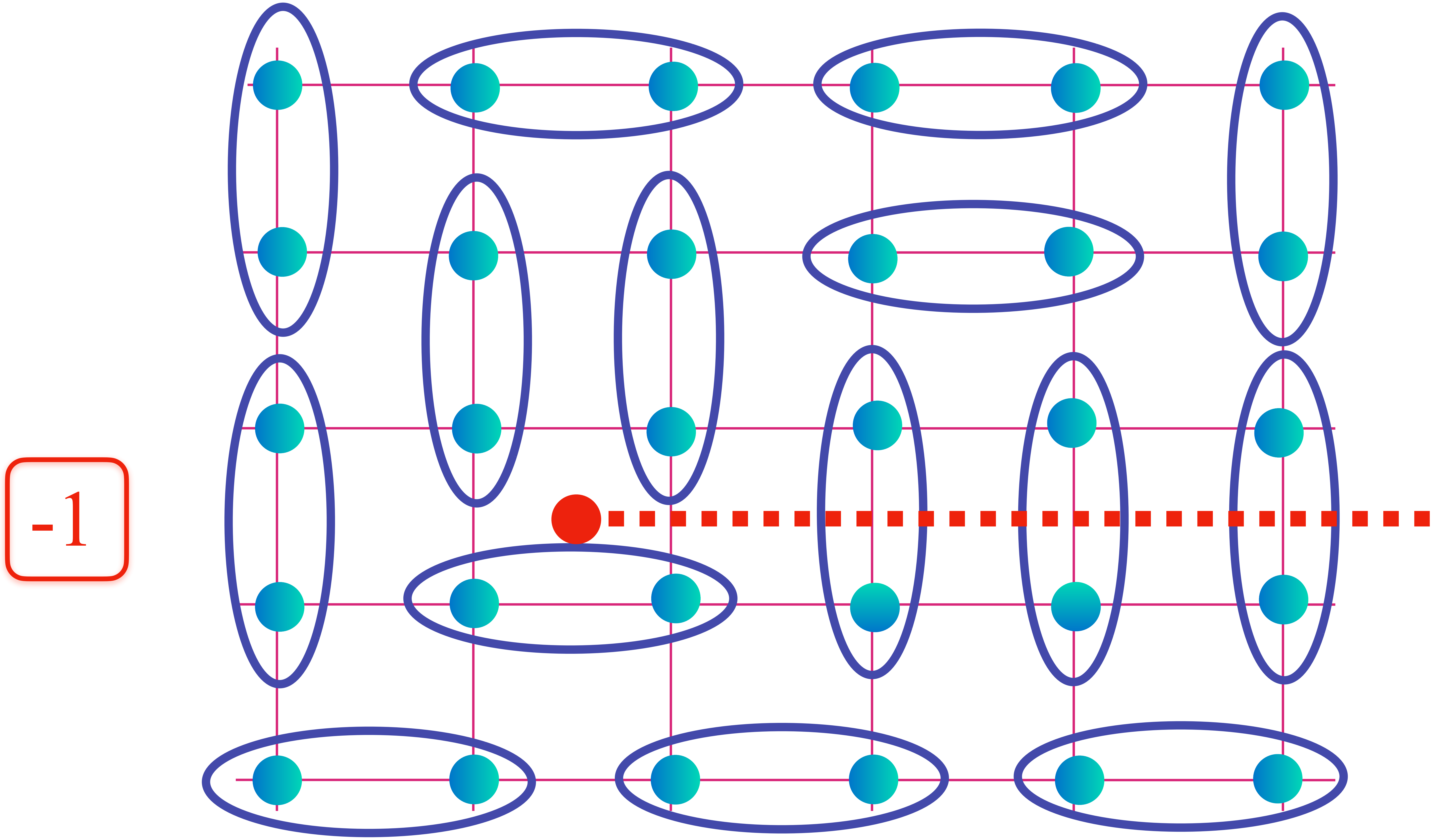

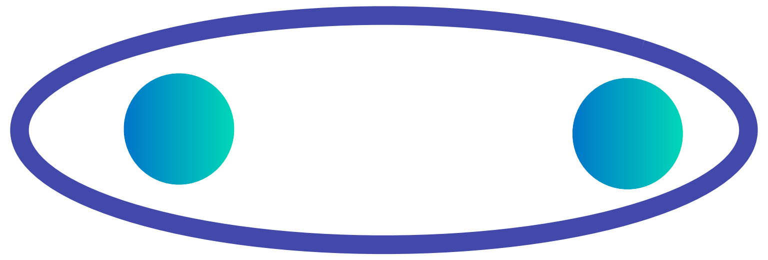

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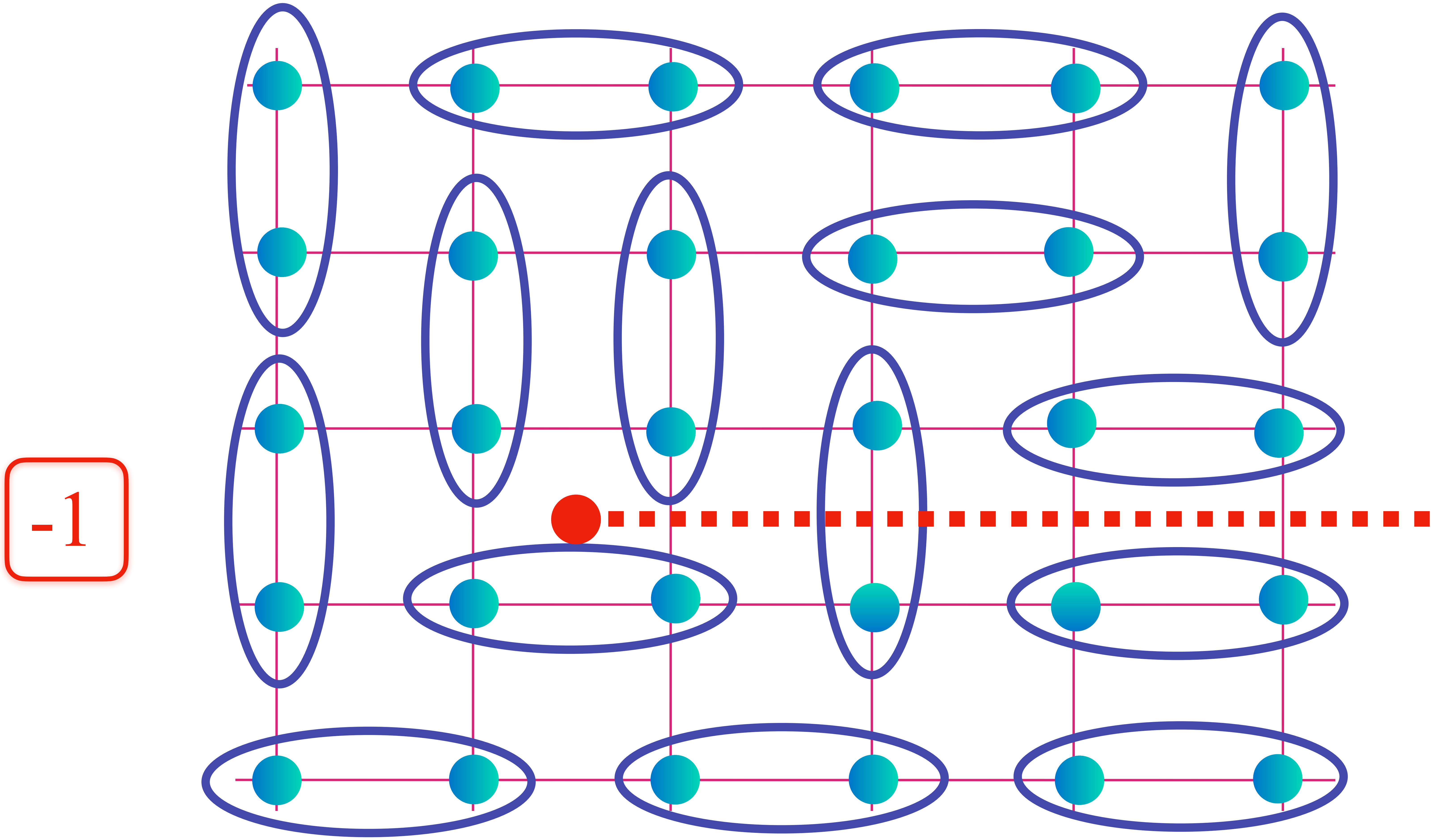

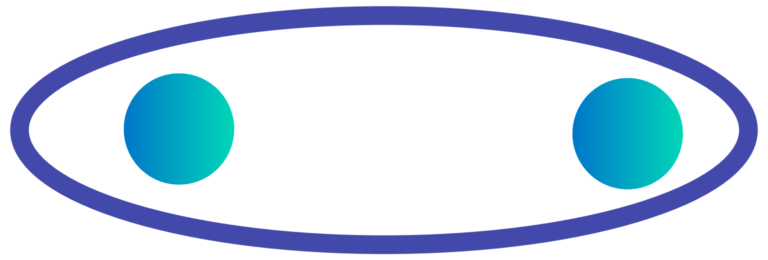 $= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

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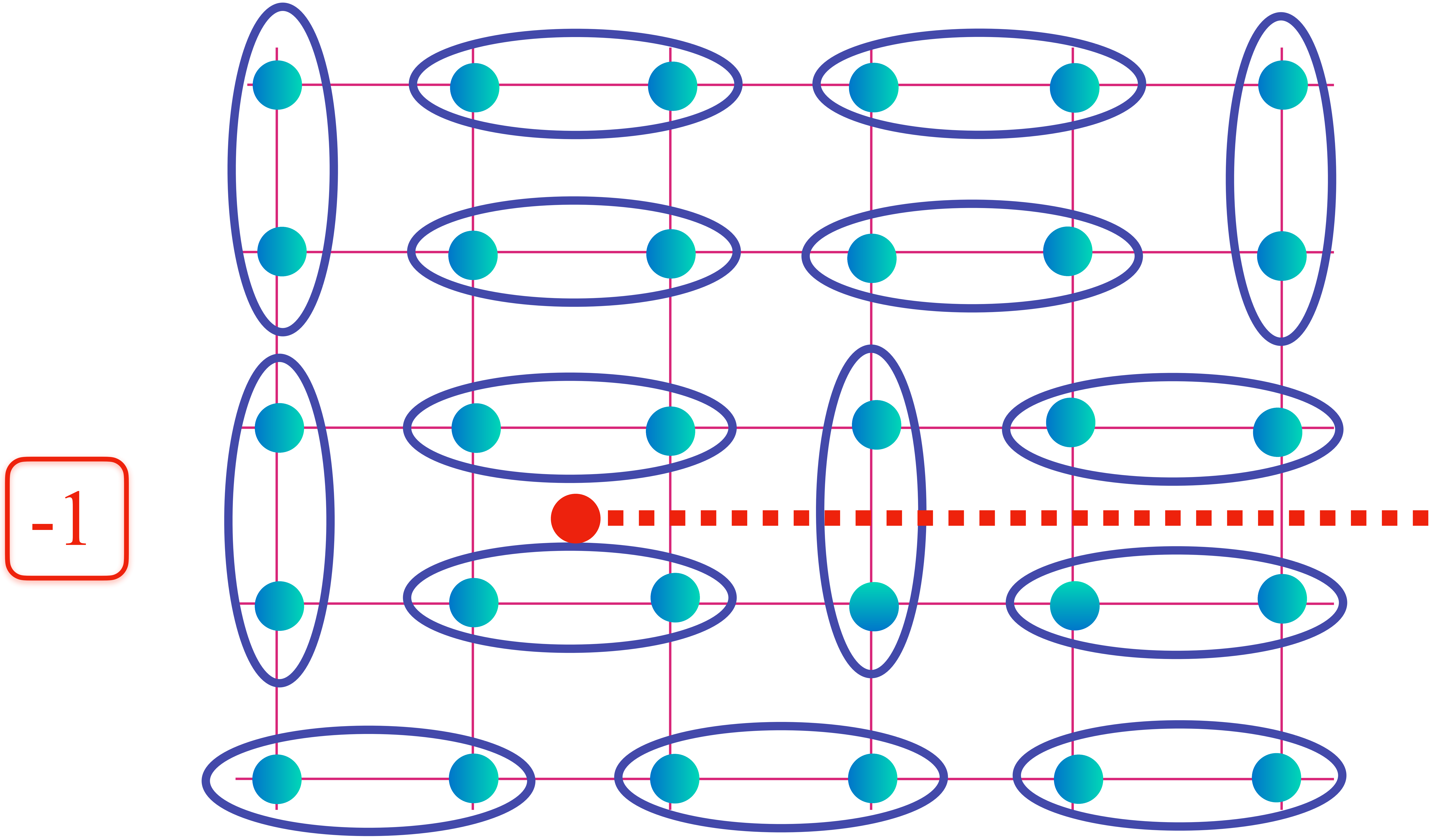

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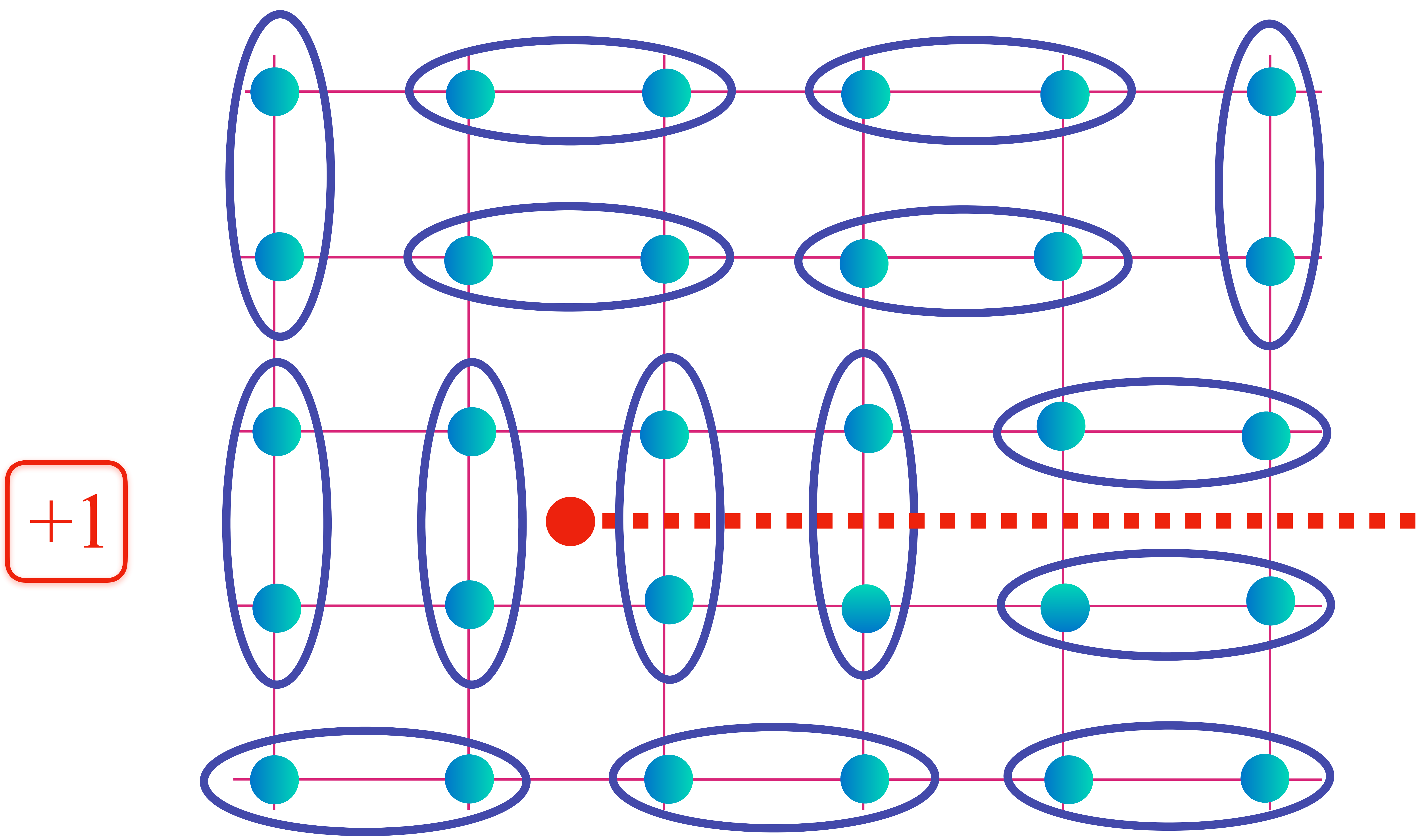

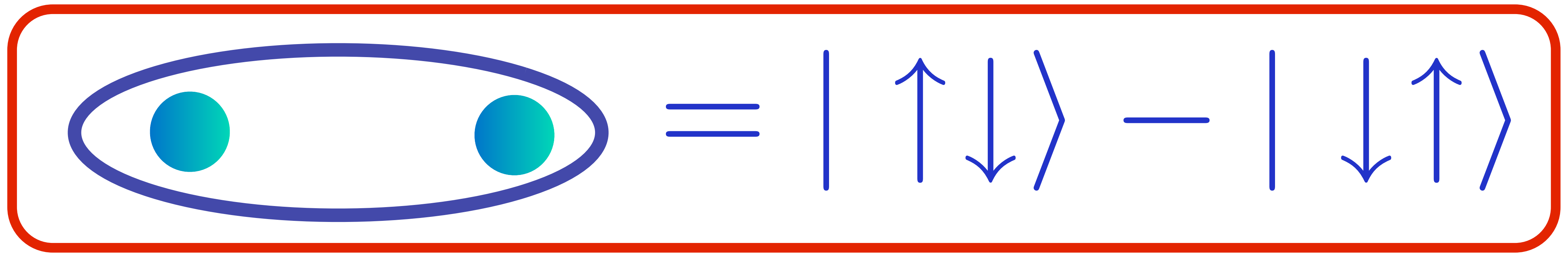

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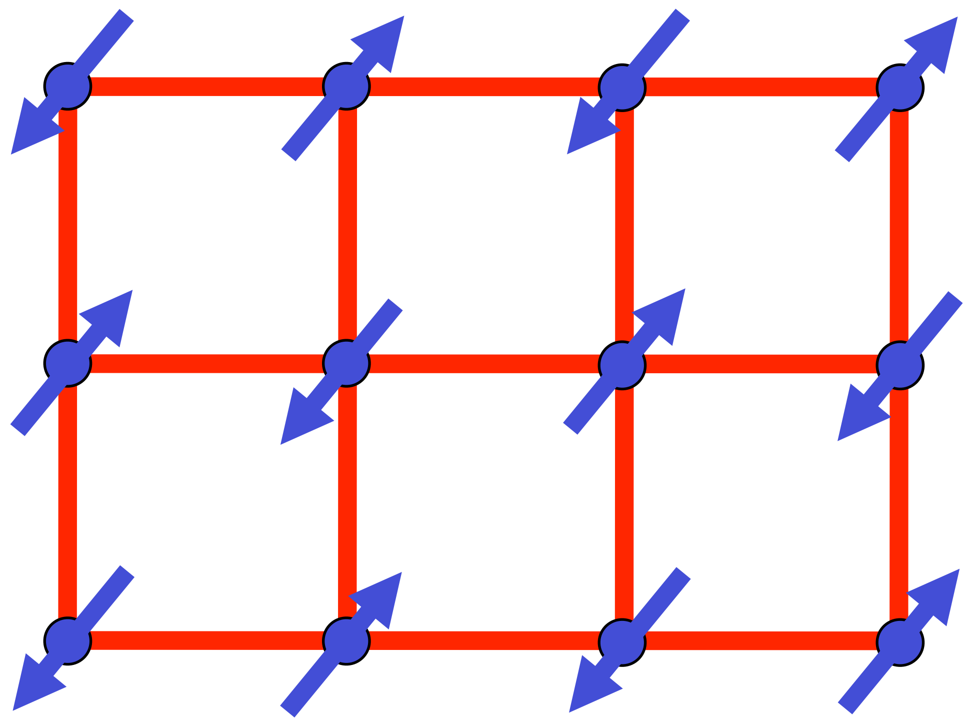
The topological structure of the $\mathbb{Z}_2$ spin liquid is identical to Kitaev's toric code, discovered later (spinons and visons correspond to the e and m particles).

Insulating S=1/2 antiferromagnet



$\langle b_\alpha \rangle \neq 0$:
Néel order

$\langle b_\alpha \rangle = 0$:
Spin liquid

$$H = \sum_{i<j} J_{ij} \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$

Schwinger bosons

$$\boldsymbol{S}_i = \frac{1}{2} b_{i\alpha}^\dagger \boldsymbol{\sigma}_{\alpha\beta} b_{i\beta} \,, \quad \sum_{\alpha=1}^{N=2} b_{i\alpha}^\dagger b_{i\alpha} = n_b = 2S$$

Mean-field spin liquid
with gapped bosonic spinons.

D.P. Arovas and A. Auerbach, *Phys. Rev. B* **38**, 316 (1988)

Insulating S=1/2 antiferromagnet

$$H = \sum_{i<j} J_{ij} \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$

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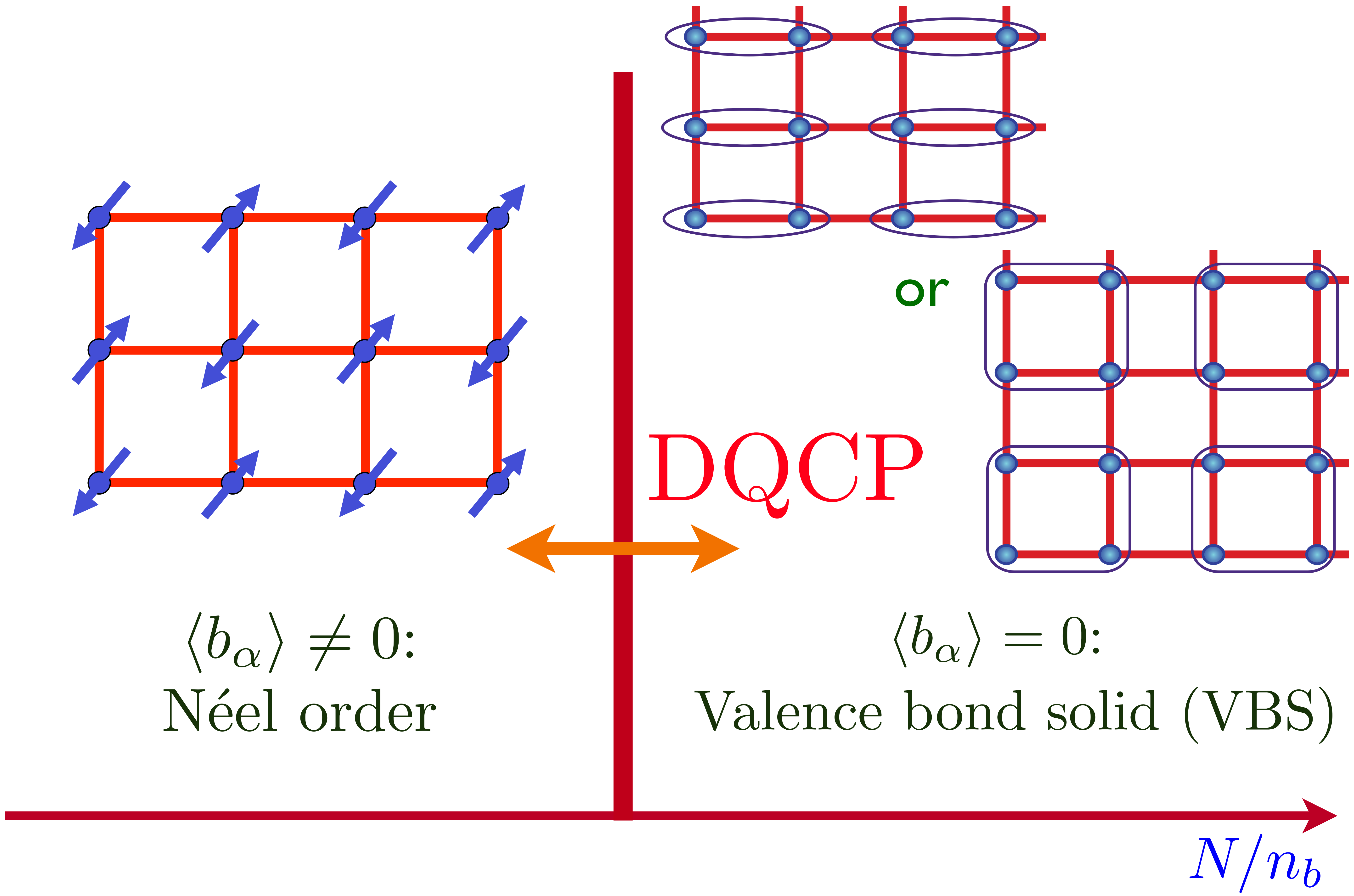


Mean-field spin liquid
with gapped bosonic spinons.

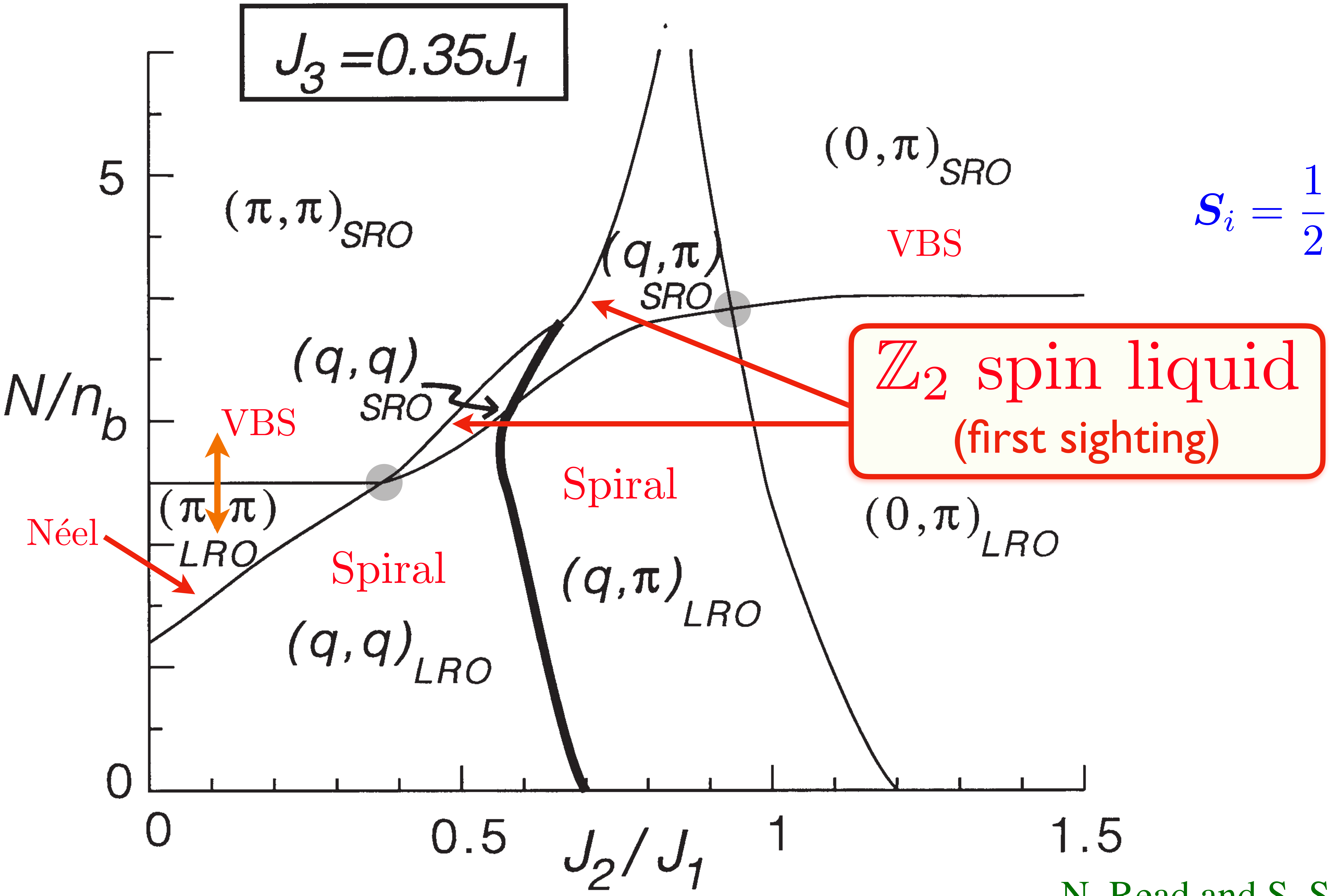
Low energy $\mathbb{CP}^1$ U(1) gauge theory

$$z_\alpha \sim b_{A\alpha} + \varepsilon_{\alpha\beta} b_{B\beta}^{\dagger}$$

$$\mathcal{L} = |(\partial_\mu - i a_\mu) z_\alpha|^2 + s|z_\alpha|^2 + u|z_\alpha|^4 + \mathcal{L}_{\mathrm{monopole}}$$

N. Read and S. Sachdev, Phys. Rev. Lett. **62**, 1694 (1989)
N. Read and S. Sachdev, Phys. Rev. B **42**, 4568 (1990)

Insulating S=1/2 antiferromagnet



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Schwinger bosons

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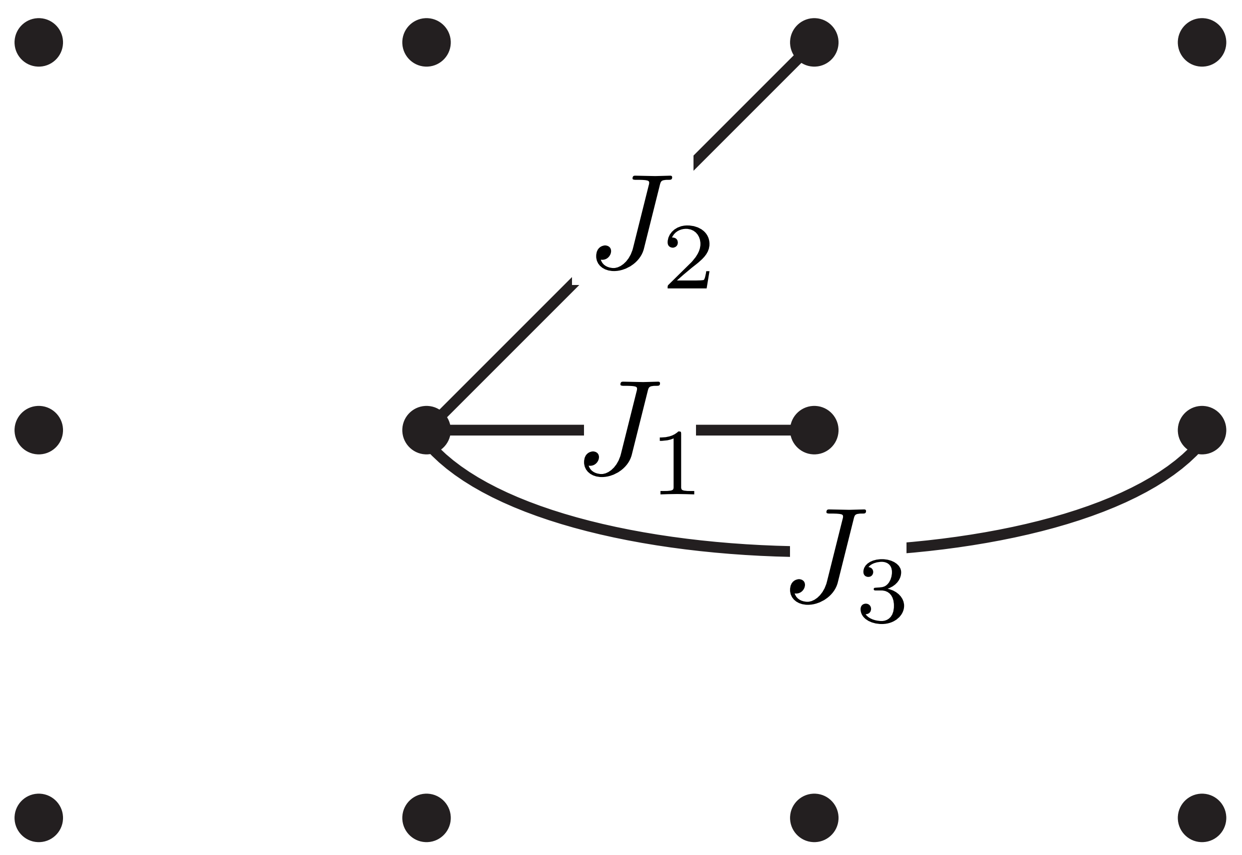


N. Read and S. Sachdev, Phys. Rev. Lett. **66,** 1773 (1991)
S. Sachdev and N. Read, Int. J. Mod. Phys. B 5, 219 (1991)

Insulating S=1/2 antiferromagnet

$$H = \sum_{i<j} J_{ij} \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$

Schwinger fermions

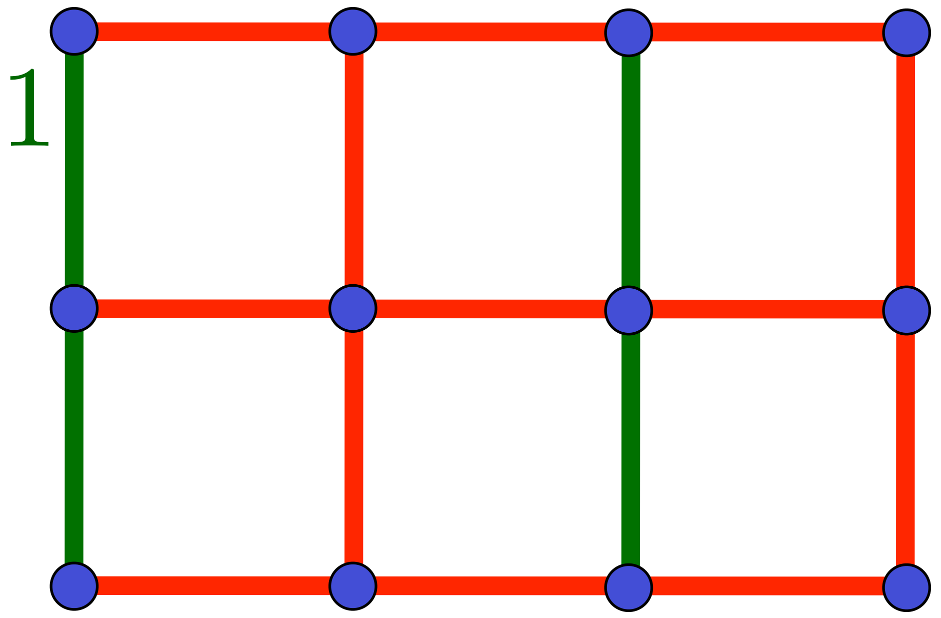
$$\boldsymbol{S}_i = \frac{1}{2} f_{i\alpha}^\dagger \boldsymbol{\sigma}_{\alpha\beta} f_{i\beta} \,, \quad \sum_{\alpha=\uparrow,\downarrow} f_{i\alpha}^\dagger f_{i\alpha} = 1$$

π-flux mean-field theory
with gapless spinons at 2 Dirac points.

I. Affleck and J.B. Marston, PRB **37**, 3774 (1988)

π-flux Spin liquid

$e_{ij} = -1$



$e_{ij} = 1$

$$H_f = iJ \sum_{\langle ij \rangle} e_{ij} \left(f_{i\alpha}^\dagger f_{j\alpha} - f_{j\alpha}^\dagger f_{i\alpha} \right) \quad , \quad \varepsilon_{\mathbf{k}} = 2J \sqrt{\sin^2(k_x) + \sin^2(k_y)}$$

Insulating S=1/2 antiferromagnet

$$H = \sum_{i<j} J_{ij} \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$

Schwinger fermions

$$\boldsymbol{S}_i = \frac{1}{2} f_{i\alpha}^\dagger \boldsymbol{\sigma}_{\alpha\beta} f_{i\beta} \,, \quad \sum_{\alpha=\uparrow,\downarrow} f_{i\alpha}^\dagger f_{i\alpha} = 1$$

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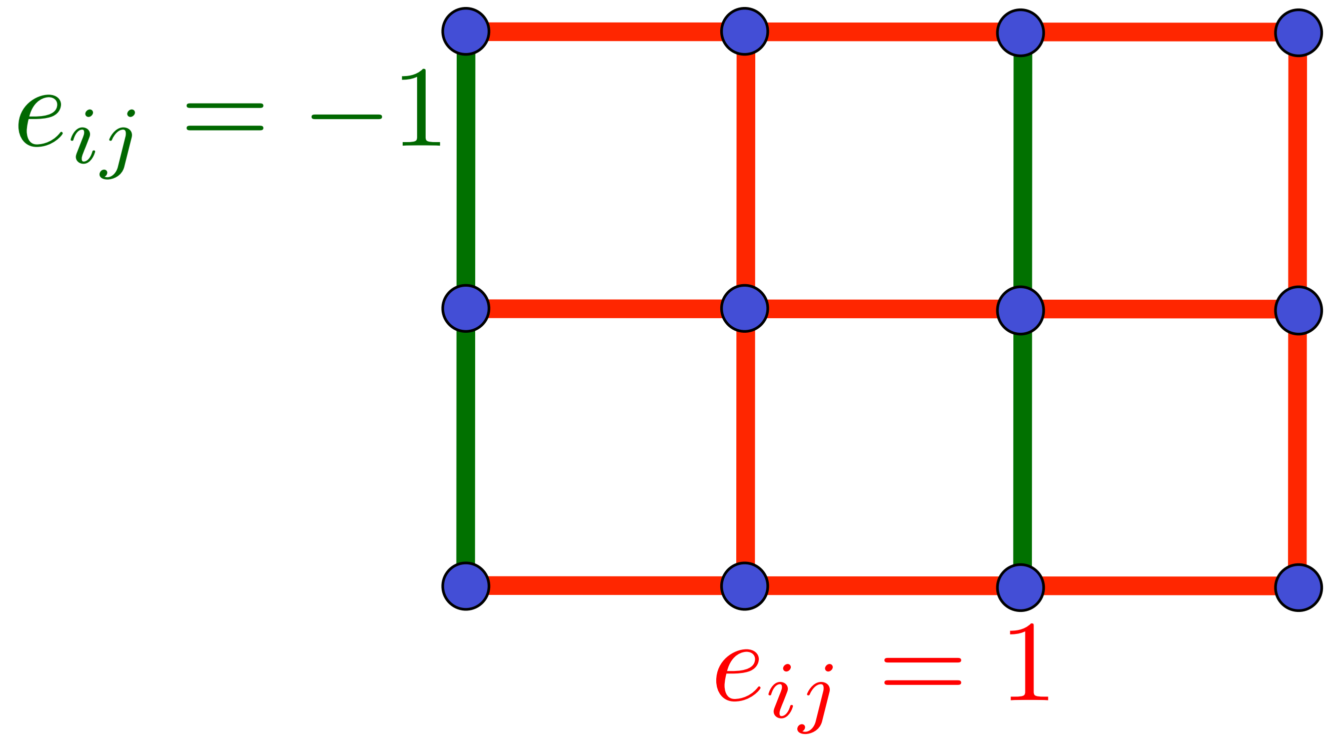
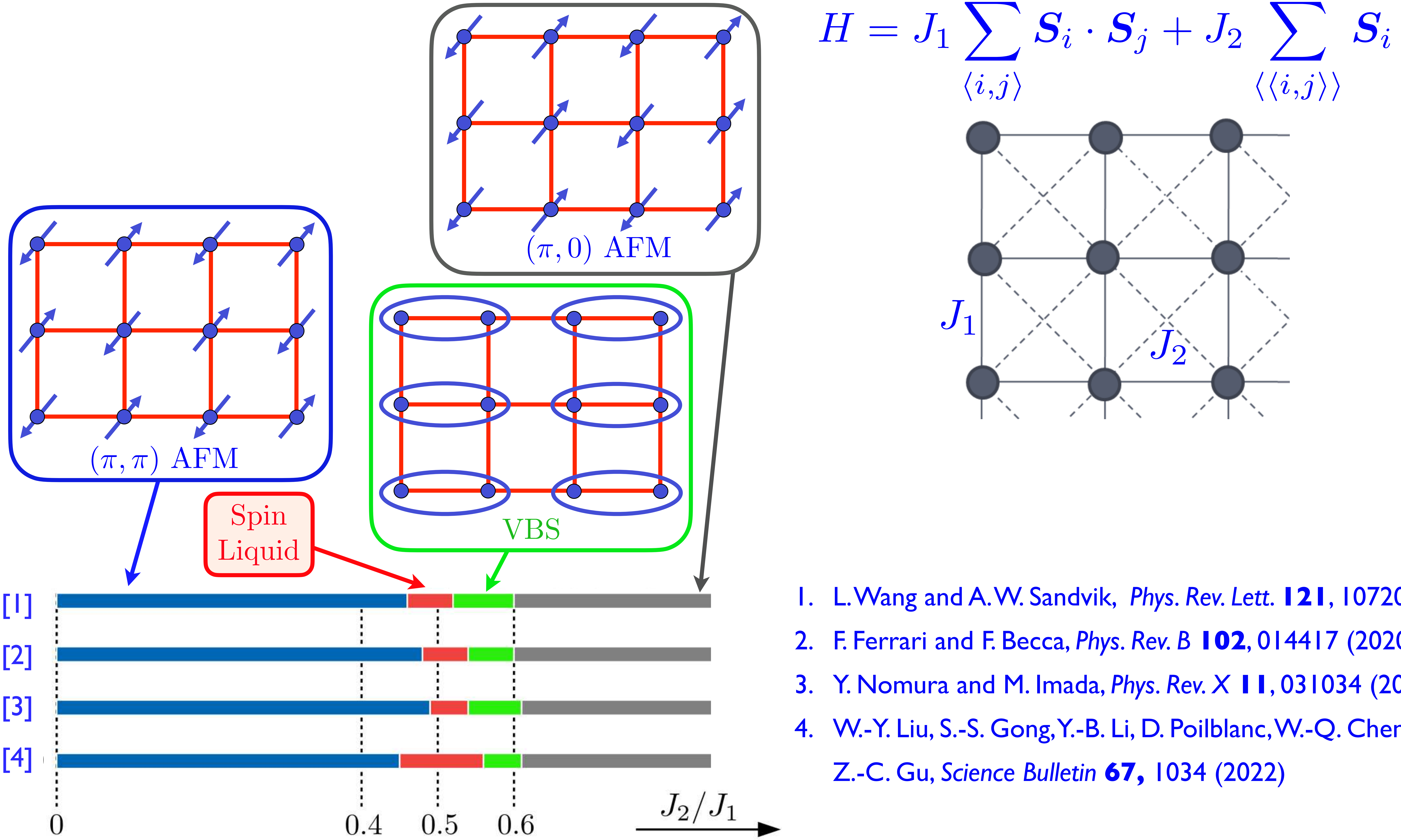


$$H_f = iJ \sum_{\langle ij \rangle} e_{ij} \left(f_{i\alpha}^\dagger f_{j\alpha} - f_{j\alpha}^\dagger f_{i\alpha} \right) \quad , \quad \varepsilon_{\mathbf{k}} = 2J\sqrt{\sin^2(k_x) + \sin^2(k_y)}$$

SU(2)$_N$ QCD with $N_f = 2$ massless fermions; $\mathcal{L} = i\overline{\Psi}_s \gamma_\mu D_\mu \Psi_s + \ldots$

$$H = J_1 \sum_{\langle i,j \rangle} \boldsymbol{S}_i \cdot \boldsymbol{S}_j + J_2 \sum_{\langle\langle i,j \rangle\rangle} \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$



1. L. Wang and A. W. Sandvik, *Phys. Rev. Lett.* **121**, 107202 (2018)
2. F. Ferrari and F. Becca, *Phys. Rev. B* **102**, 014417 (2020)
3. Y. Nomura and M. Imada, *Phys. Rev. X* **11**, 031034 (2021)
4. W.-Y. Liu, S.-S. Gong, Y.-B. Li, D. Poilblanc, W.-Q. Chen, and Z.-C. Gu, *Science Bulletin* **67,** 1034 (2022)

$\mathbb{CP}^1$ U(1) gauge theory

S=1/2
square
lattice anti-
ferromagnet

$\mathrm{SU}(2)_N$ gauge theory of $N_f = 2$
fundamental, massless, Dirac fermions.

Obtained from a saddle-point of
fermionic spinons moving in π-flux.

SO(5) non-linear σ-model
of Néel/VBS orders
with $k = 1$ WZW term

A. Tanaka and X. Hu, *Phys. Rev. Lett.* **95**, 036402 (2005); T. Senthil and M.P.A. Fisher *Phys. Rev. B* **74**, 064405 (2006);
Ying Ran and X.-G. Wen, cond-mat/0609620; C. Wang, A. Nahum, M. A. Metlitski, C. Xu, and T. Senthil, *Phys. Rev. X* **7**, 031051 (2017)

$\mathbb{CP}^1$ U(1) gauge theory

S=1/2 square lattice anti-ferromagnet

SU$(2)_N$ gauge theory of $N_f = 2$ fundamental, massless, Dirac fermions.

Obtained from a saddle-point of fermionic spinons moving in π-flux.

SO(5) non-linear σ-model of Néel/VBS orders with $k = 1$ WZW term

Many numerical works show that deconfined critical theory applies over a substantial length scale, but ultimately confines at the longest distances.

Zheng Zhou, Liangdong Hu, Wei Zhu, and Yin-Chen He, PRX **14**, 021044 (2024); S. M. Chester and N. Su, PRL **132**, 111601 (2024).
B.-B. Chen, X. Zhang, Y. Wang, K. Sun, and Z. Y. Meng, arXiv:2307.05307;
J. Takahashi, H. Shao, B. Zhao, W. Guo, and A. W. Sandvik, arXiv:2405.06607.

1. Identify spin liquid states of the insulator, describing the dynamics and symmetries of its anyons.

2. Work with metallic states in which the low energy anyons are essentially the same as those of the insulator, along with a 'trivial' fermion with the same quantum numbers as the electron.

1. Square lattice spin liquids

2. Spin liquids on the Kondo lattice: non-Luttinger volume Fermi surfaces (FL*)

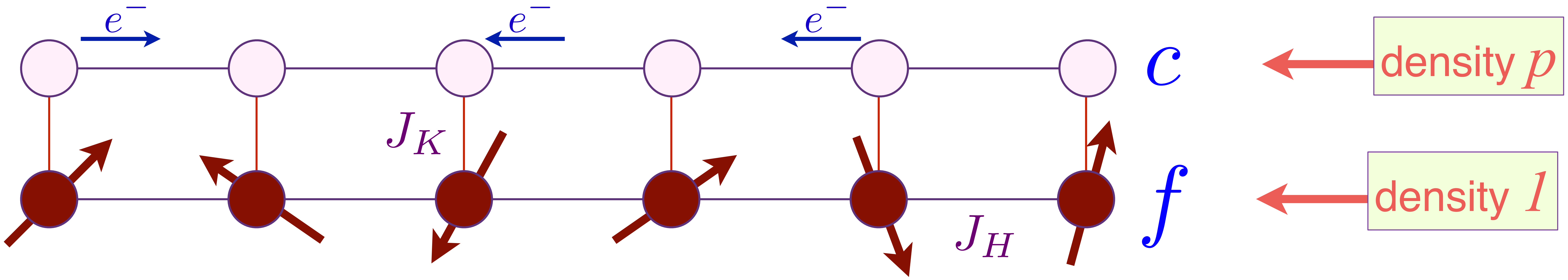
3. Doping square lattice spin liquids for t >> J: FL* in a single-band model

4. FL* theory of the pseudogap metal of the cuprates

5. Nodal fermionic quasiparticles in d-wave SC

6. Quantum oscillations in hole-doped cuprates

$$\mathcal{H}_{KL} = \sum_{\mathbf{p}} \varepsilon_{\mathbf{p}} c^\dagger_{\mathbf{p}\sigma} c_{\mathbf{p}\sigma} + \sum_i J_K \, c^\dagger_{i\sigma} \frac{\boldsymbol{\tau}_{\sigma\sigma'}}{2} c_{i\sigma'} \cdot \boldsymbol{S}_i + \sum_{\langle ij \rangle} J_H \, \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$

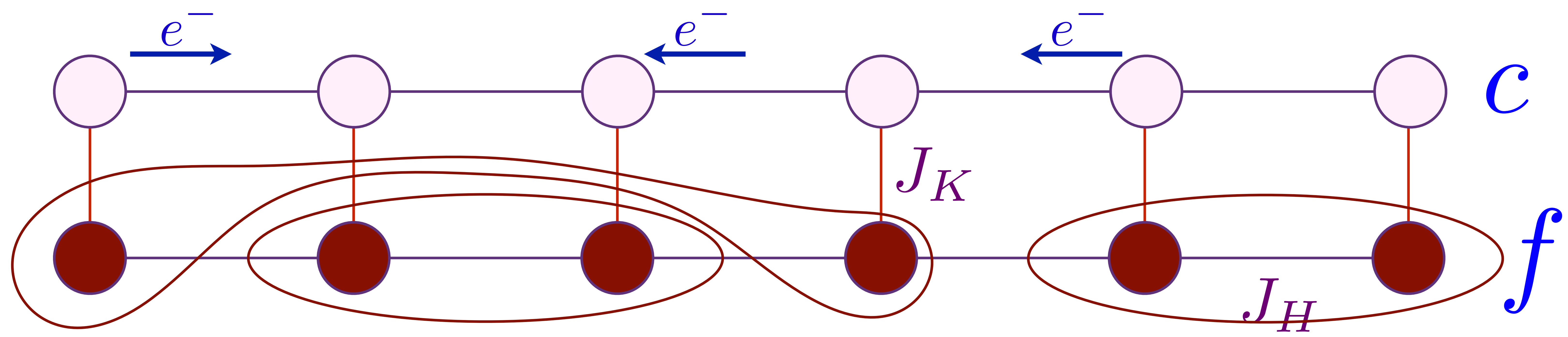


Assume J_H is chosen so that at $J_K = 0$
the $\boldsymbol{S}_i$ spins have a fractionalized spin liquid ground state.

Represent $\boldsymbol{S}_i$ by fermionic spinons: $\boldsymbol{S}_i = \frac{1}{2} f^\dagger_{i\sigma} \frac{\boldsymbol{\tau}_{\sigma\sigma'}}{2} f_{i\sigma'}$

$$\sum_\sigma f^\dagger_{i\sigma} f_{i\sigma} = 1 \text{ for all } i.$$

$$\mathcal{H}_{KL} = \sum_{\mathbf{p}} \varepsilon_{\mathbf{p}} c^\dagger_{\mathbf{p}\sigma} c_{\mathbf{p}\sigma} + \sum_i J_K \, c^\dagger_{i\sigma} \frac{\boldsymbol{\tau}_{\sigma\sigma'}}{2} c_{i\sigma'} \cdot \boldsymbol{S}_i + \sum_{\langle ij \rangle} J_H \, \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$



FL*

Small Fermi surface of size p

$|\mathrm{FL}*\rangle = [\text{Projection onto one } f \text{ per site}]$
$\bowtie |\text{Slater determinant of } f\rangle$
$\otimes |\text{Slater determinant of } c\rangle$

Spin liquid structure and Fermi surface size are stable for a finite range of J_K

Note: nevertheless, density of f electrons may change with J_K in an Anderson lattice model!

N. Andrei and P. Coleman, PRL **62**, 595 (1989); S. Burdin, D. R. Grempel, and A. Georges, PRB **66**, 045111 (2002); T. Senthil, M. Vojta, and S. Sachdev, PRB **69**, 035111 (2004); A. Paramekanti and A. Vishwanath, PRB **70**, 245118 (2004)

$$\mathcal{H}_{KL} = \sum_{\mathbf{p}} \varepsilon_{\mathbf{p}} c_{\mathbf{p}\sigma}^{\dagger} c_{\mathbf{p}\sigma} + \sum_{i} J_K \, c_{i\sigma}^{\dagger} \frac{\boldsymbol{\tau}_{\sigma\sigma'}}{2} c_{i\sigma'} \cdot \boldsymbol{S}_i + \sum_{\langle ij \rangle} J_H \, \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$

Small Fermi surface of size p

$|\mathrm{FL}*\rangle = $ [Projection onto one f per site] $\bowtie$ $|$Slater determinant of $f\rangle$ $\otimes$ $|$Slater determinant of $c\rangle$

Large Fermi surface of size $1 + p$

$|\mathrm{HFL}\rangle = $ [Projection onto one f per site] $\bowtie$ $|$Slater determinant of $(c, f)\rangle$

Coleman, Read, Newns, Millis, P.A. Lee, Auerbach, Levin

$$\mathcal{H}_{KL} = \sum_{\mathbf{p}} \varepsilon_{\mathbf{p}} c^\dagger_{\mathbf{p}\sigma} c_{\mathbf{p}\sigma} + \sum_i J_K \, c^\dagger_{i\sigma} \frac{\boldsymbol{\tau}_{\sigma\sigma'}}{2} c_{i\sigma'} \cdot \boldsymbol{S}_i + \sum_{\langle ij \rangle} J_H \, \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$

$$H_{\mathrm{mf}} = -\sum_{i,j} t_{ij} c^\dagger_{i\sigma} c_{j\sigma} - \sum_{i,j} t_{1,ij} f^\dagger_{i\sigma} f_{j\sigma} - \sum_i \Phi \left(c^\dagger_{i\sigma} f_{i\sigma} + f^\dagger_{i\sigma} c_{i\sigma} \right)$$

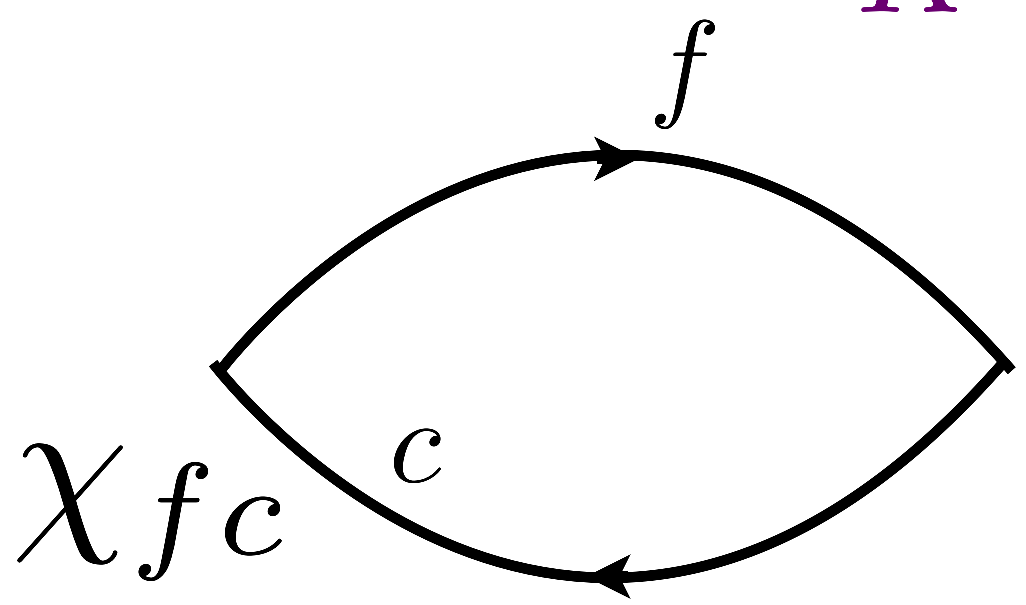
T. Senthil, M. Vojta, and S. Sachdev, PRB **69**, 035111 (2004)

$$\mathcal{H}_{KL} = \sum_{\mathbf{p}} \varepsilon_{\mathbf{p}} c^{\dagger}_{\mathbf{p}\sigma} c_{\mathbf{p}\sigma} + \sum_{i} J_K \, c^{\dagger}_{i\sigma} \frac{\boldsymbol{\tau}_{\sigma\sigma'}}{2} c_{i\sigma'} \cdot \boldsymbol{S}_i + \sum_{\langle ij \rangle} J_H \, \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$

Effective action for Φ: $\mathcal{L}[\Phi] = s|\Phi|^2 + \dots$ with $s \sim \dfrac{1}{J_K} - \chi_{fc}$.

No singularity in χ_{fc} because Fermi surfaces do not match.

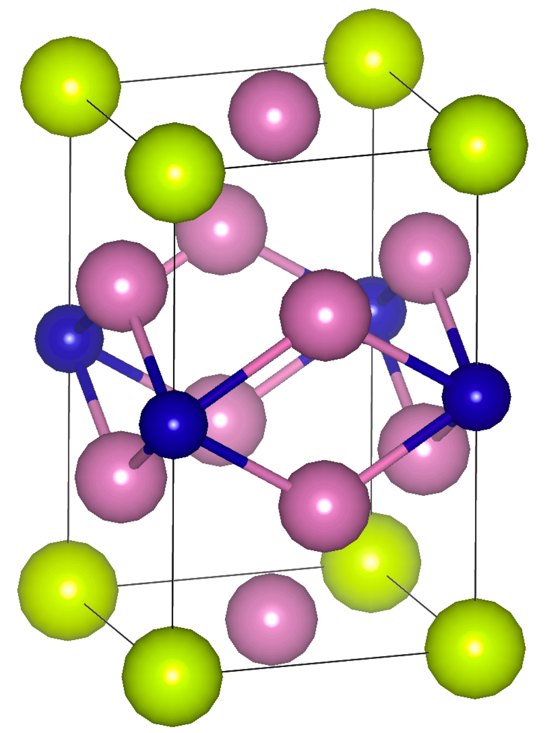
$\Rightarrow$ FL* is stable for small J_K

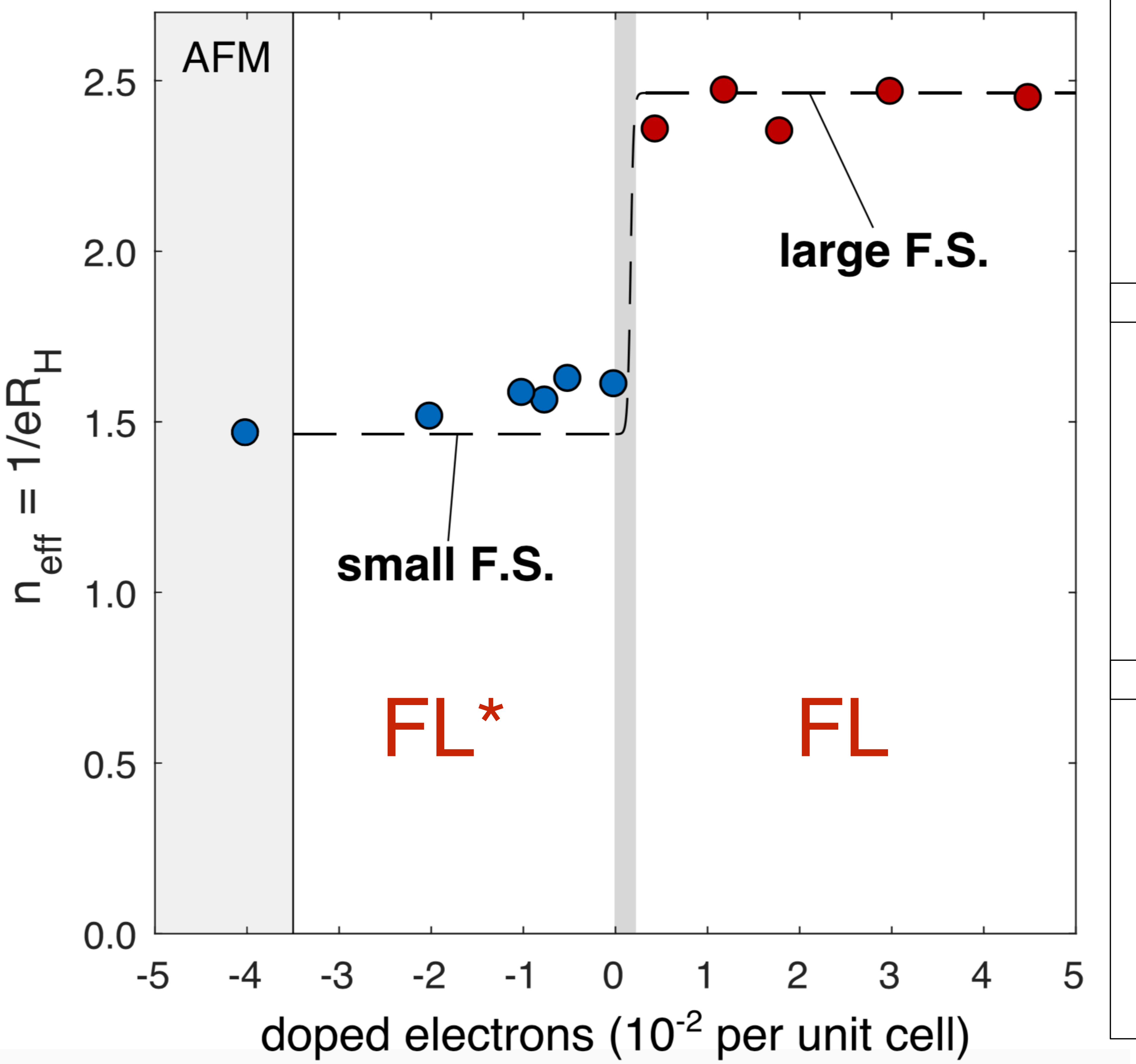


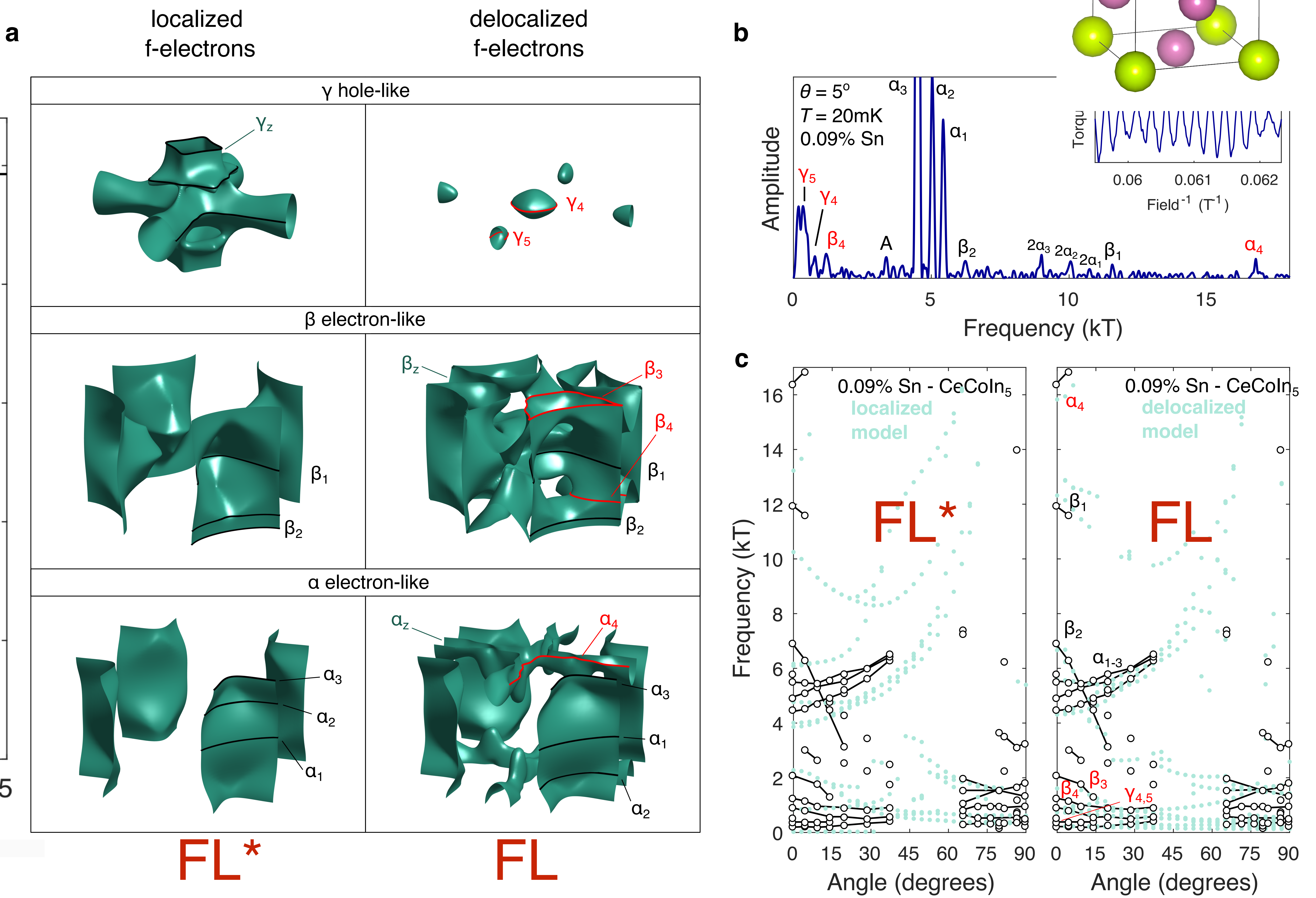
T. Senthil, M. Vojta, and S. Sachdev, PRB **69**, 035111 (2004)

Evidence for a delocalization quantum phase transition without symmetry breaking in $CeCoIn_5$

Nikola Maksimovic, Daniel H. Eilbott, Tessa Cookmeyer…….Ehud Altman, Alessandra Lanzara,James G. Analytis,
Science **375**, 76 (2021)







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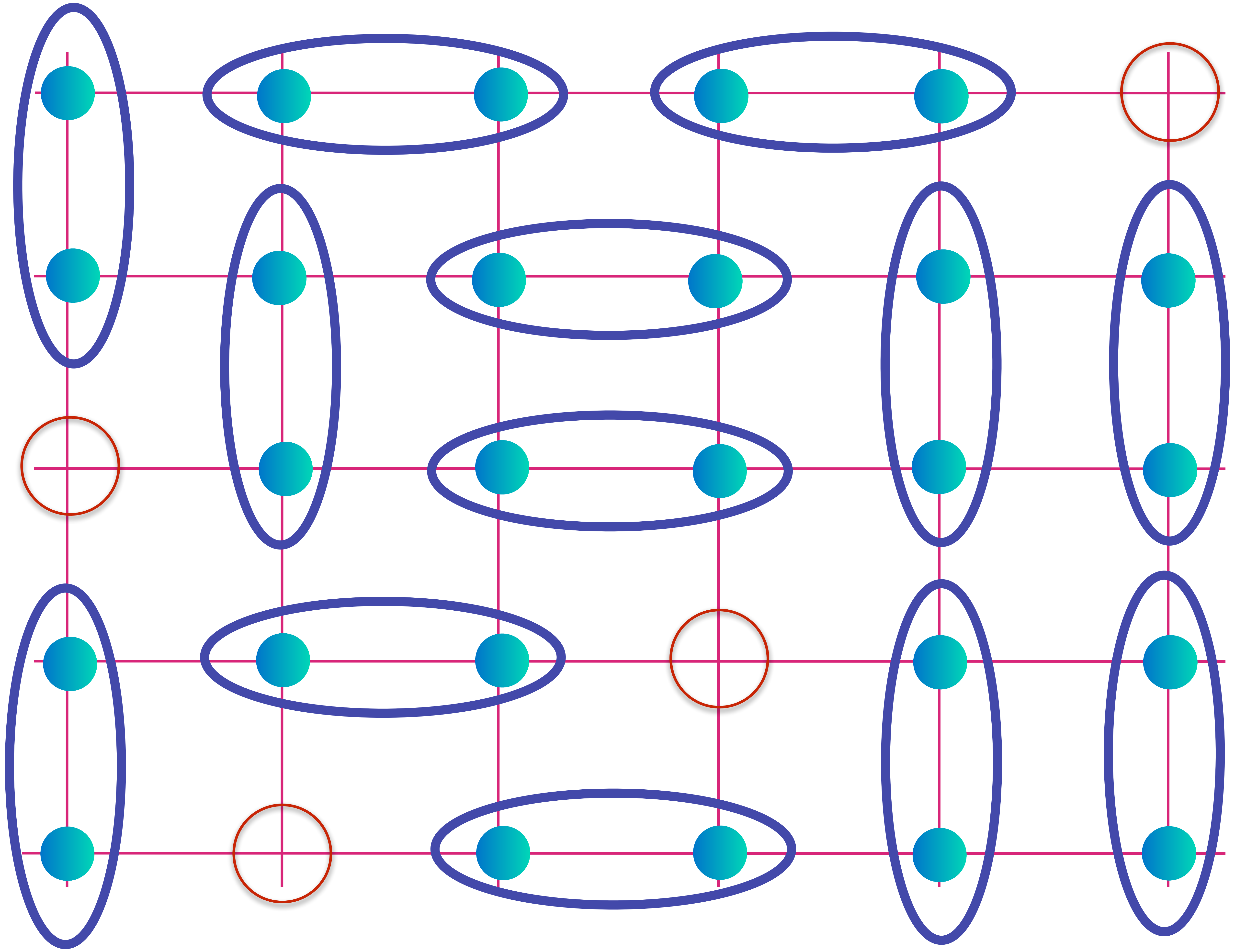
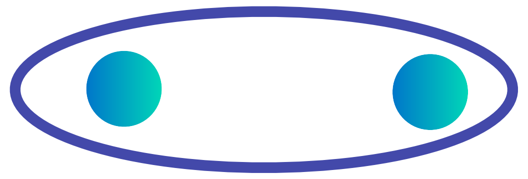
5. Nodal fermionic quasiparticles in d-wave SC

6. Quantum oscillations in hole-doped cuprates

Holons

G. Baskaran, Z. Zou, P.W. Anderson, *Solid State Comm.* **63**, 973 (1987)
S.A. Kivelson, D.S. Rokhsar and J.P. Sethna, *Phys. Rev. B* **35**, 8865 (1987)
D. Rokhsar and S.A. Kivelson, *Phys. Rev. Lett.* **61**, 2376 (1988)



 $= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$

Spin liquid

Fractionalized holon excitations with spin S=0 and charge +e.

Holons

G. Baskaran, Z. Zou, P.W. Anderson, *Solid State Comm.* **63**, 973 (1987)
S.A. Kivelson, D.S. Rokhsar and J.P. Sethna, *Phys. Rev. B* **35**, 8865 (1987)
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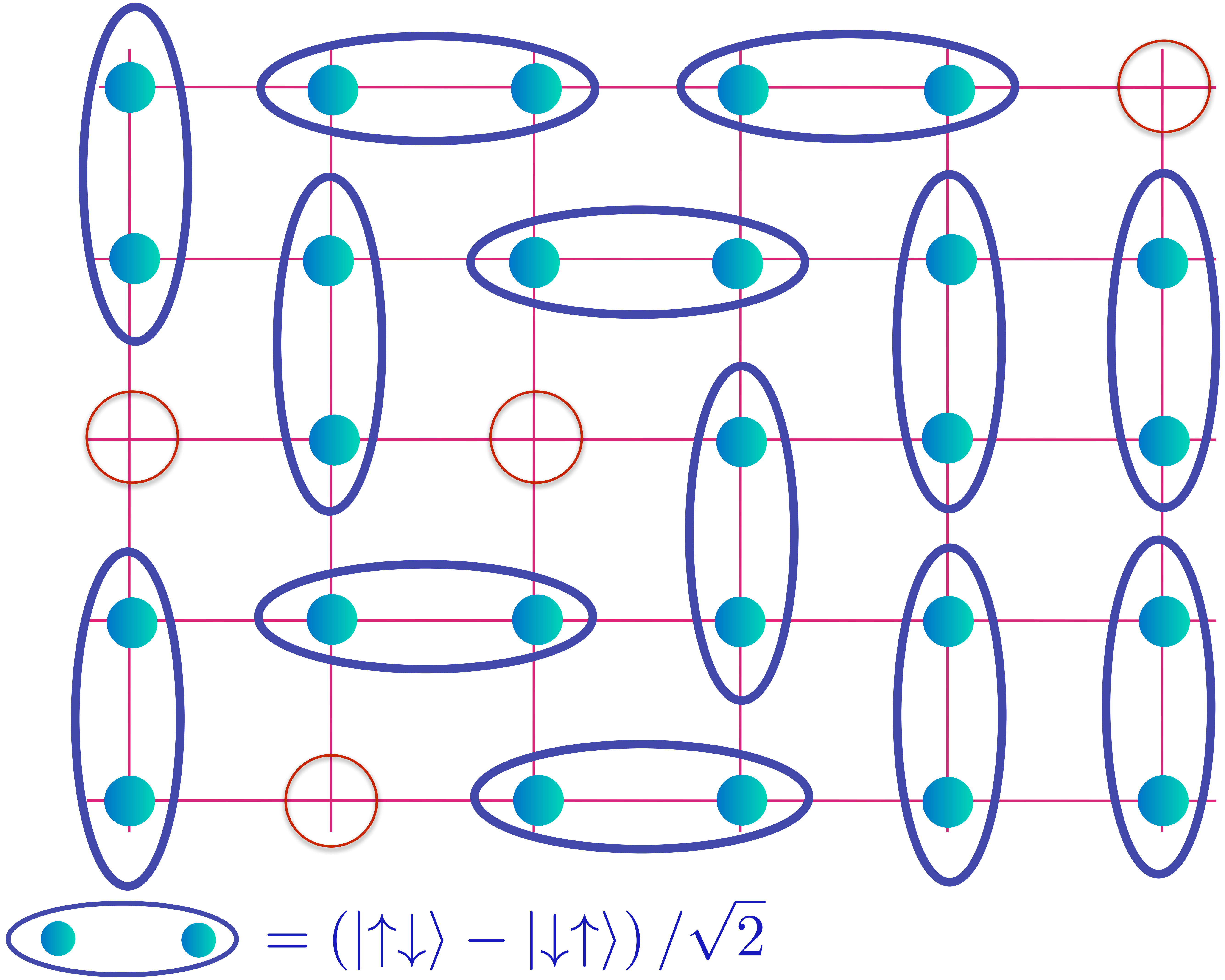

$$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$$

Spin liquid

Fractionalized holon excitations with spin S=0 and charge +e.

Holon metal

G. Baskaran, Z. Zou, P.W. Anderson, *Solid State Comm.* **63**, 973 (1987)
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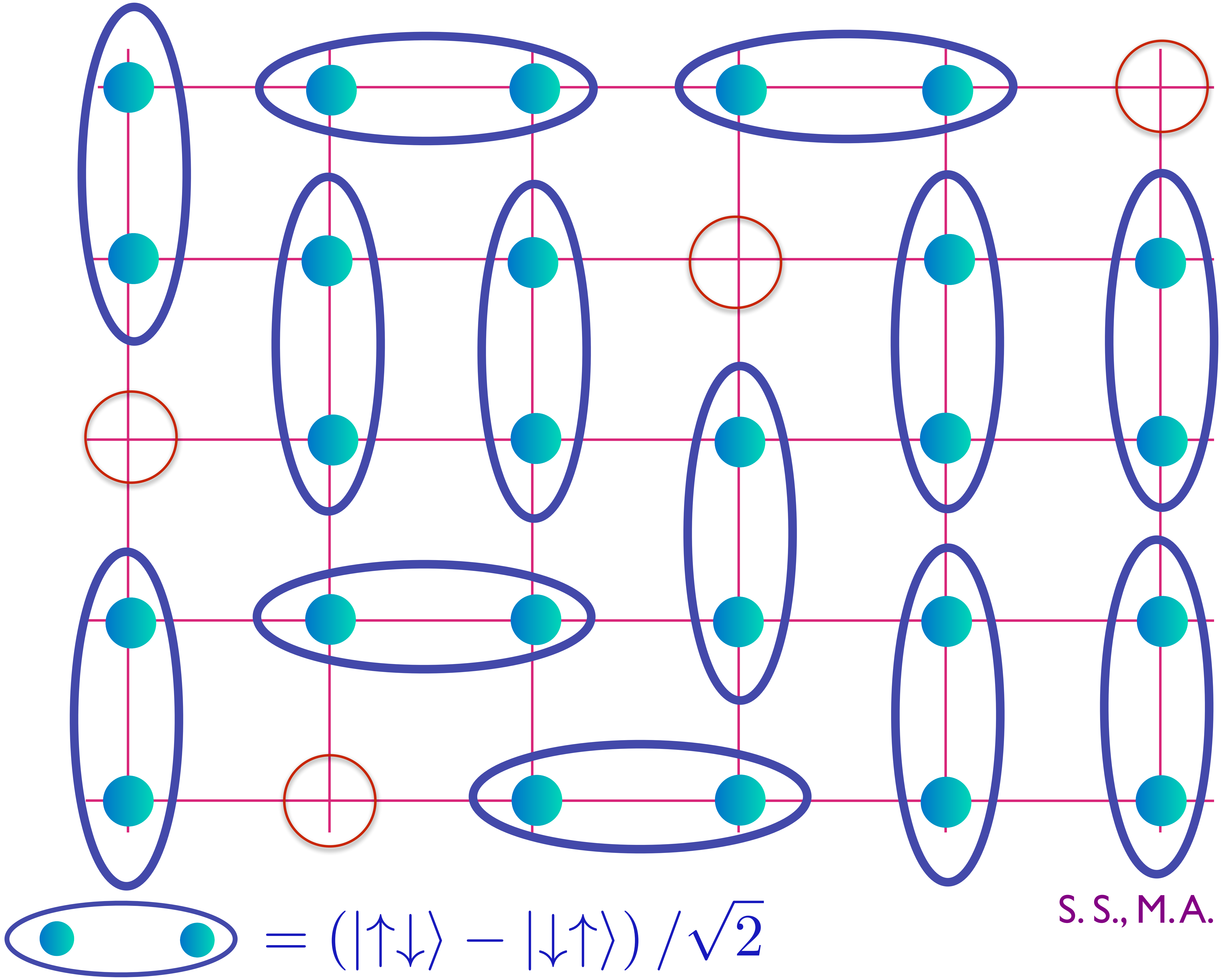

$$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$$

If each holon is a fermion, we obtain a Fermi surface of holons of size p

S. S., M.A. Metlitski, Y. Qi, and C. Xu, PRB **80**, 155129 (2009)
M. S. Scheurer, S. Chatterjee, Wei Wu, M. Ferrero, A. Georges, and S. S., PNAS **115**, E3665 (2018)
Pietro M. Bonetti and Walter Metzner, PRB **106**, 205152 (2022)

Holon metal

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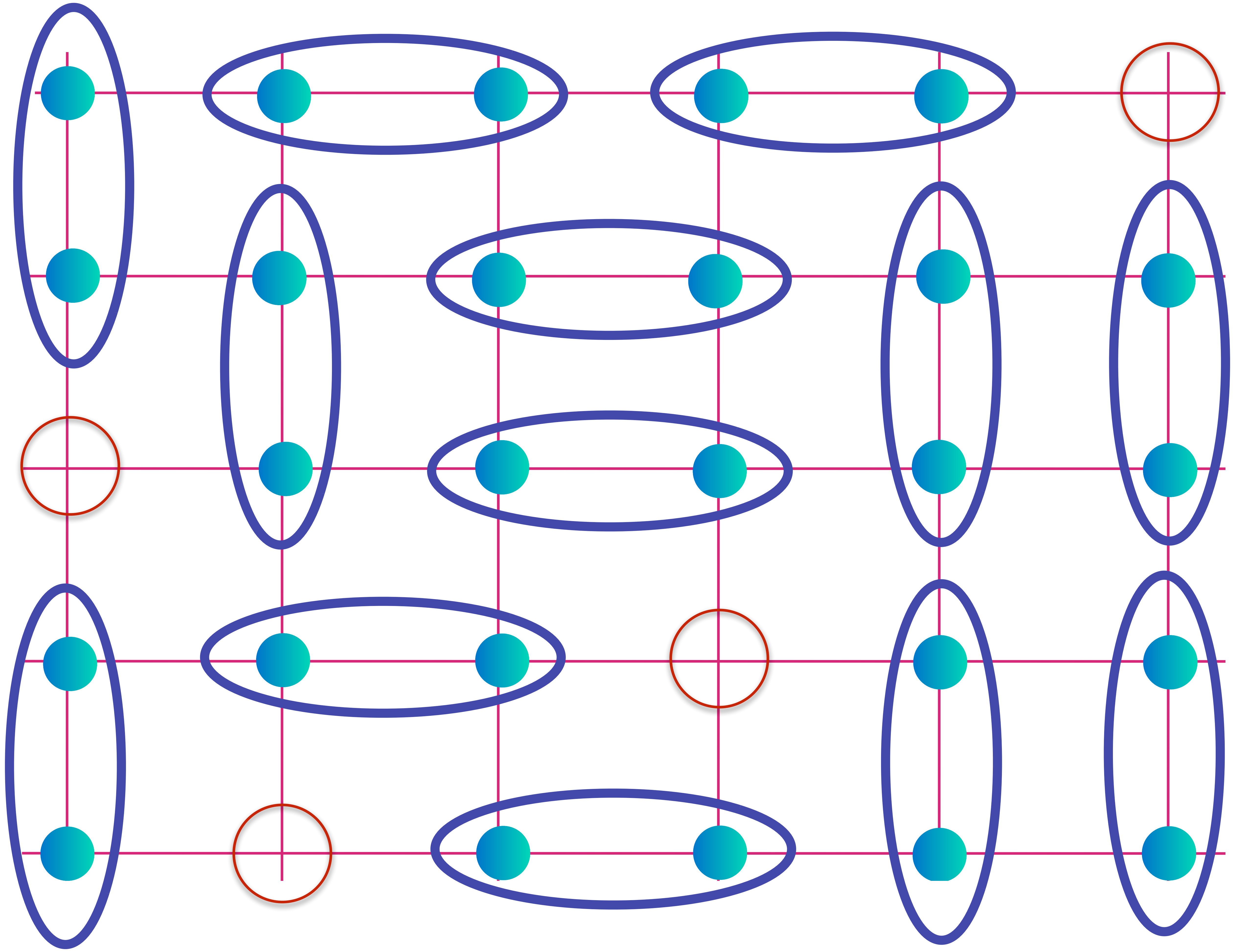

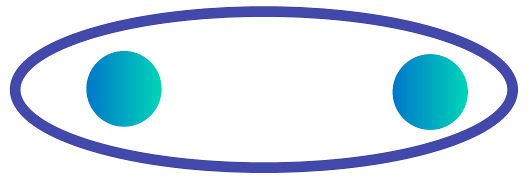 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right) / \sqrt{2}$

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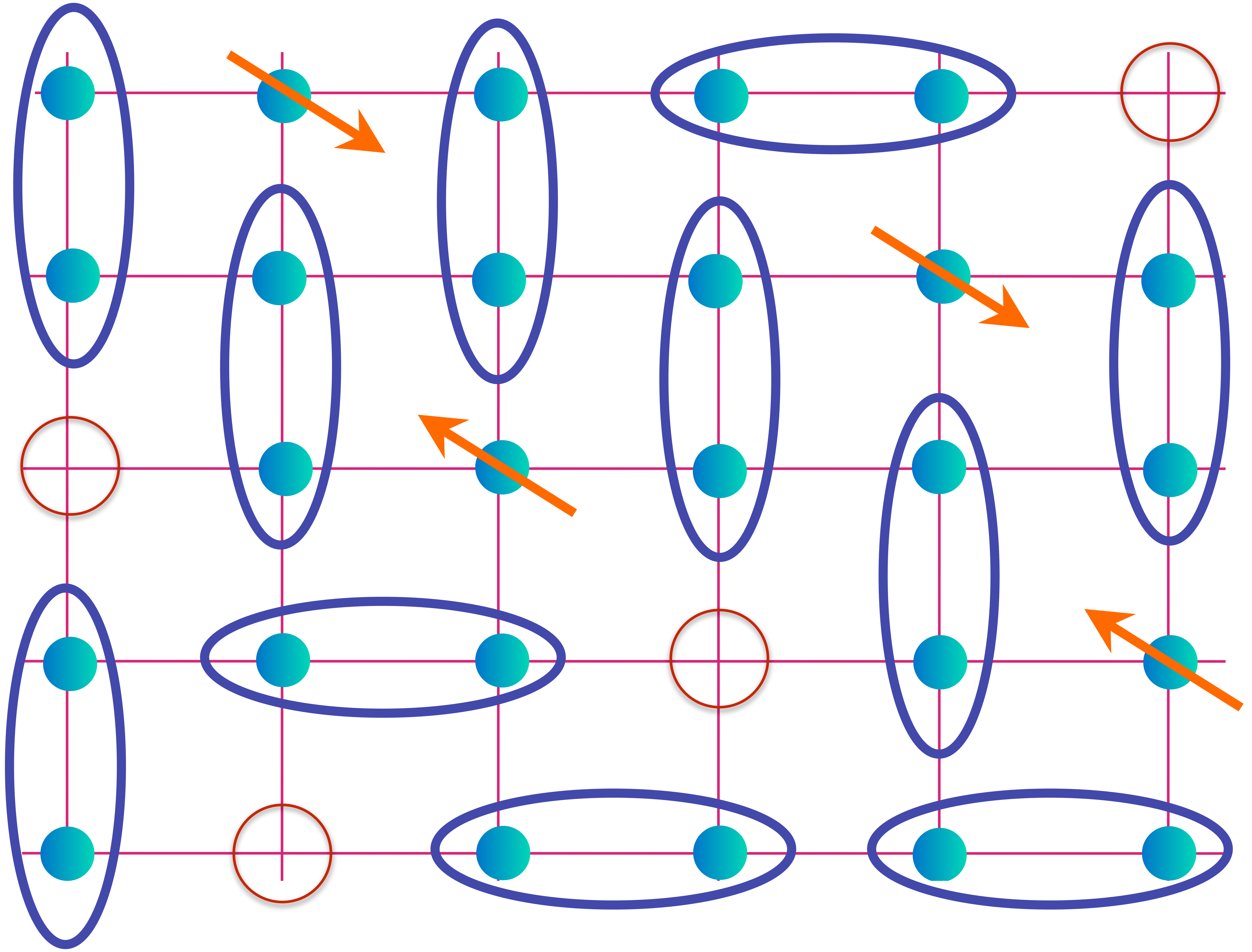
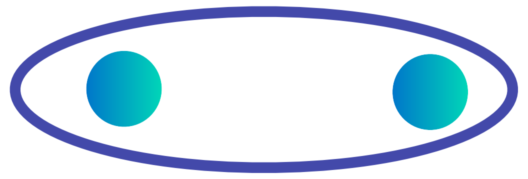


There is an energy cost of order J to create spinons in a holon metal

 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

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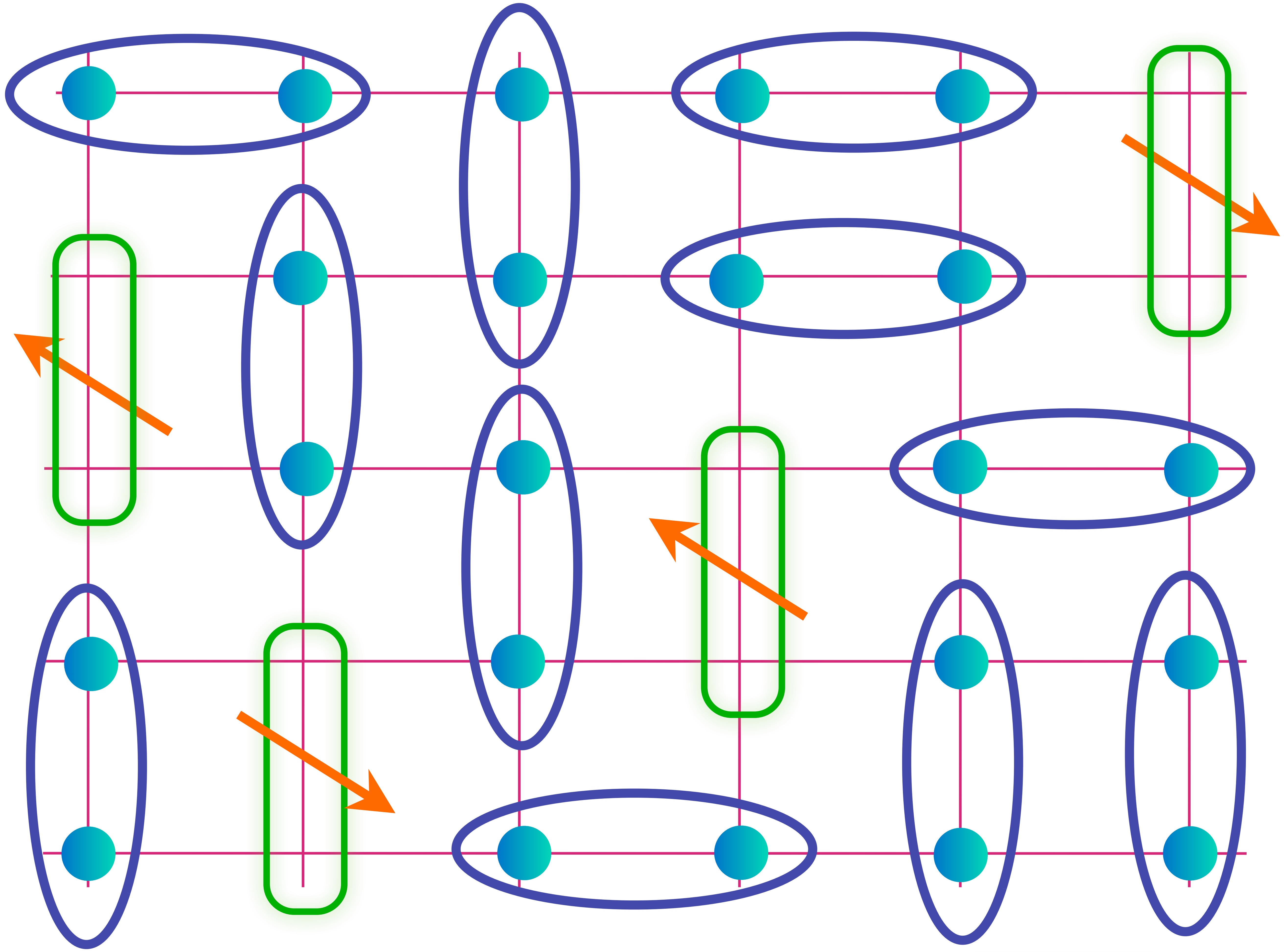
FL* in a ***one-band*** model

S. Sachdev *Phys. Rev. B* **49**, 6770 (1994); X.-G. Wen and P.A. Lee *Phys. Rev. Lett.* **76**, 503 (1996)
R. K. Kaul, A. Kolezhuk, M. Levin, S. Sachdev, and T. Senthil, *Phys. Rev. B* **75**, 235122 (2007)



There is an energy gain of order t in the formation of spinon-holon bound states

Spinon-holon bound state

$$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$$

$$= \left(|\uparrow\circ\rangle + |\circ\uparrow\rangle\right)/\sqrt{2}$$

Yang Qi and S. Sachdev, *Phys. Rev. B* **81**, 115129 (2010); M. Punk, A. Allais, and S. Sachdev, *Proc. Nat. Acad. Sci.* **112**, 9552 (2015)

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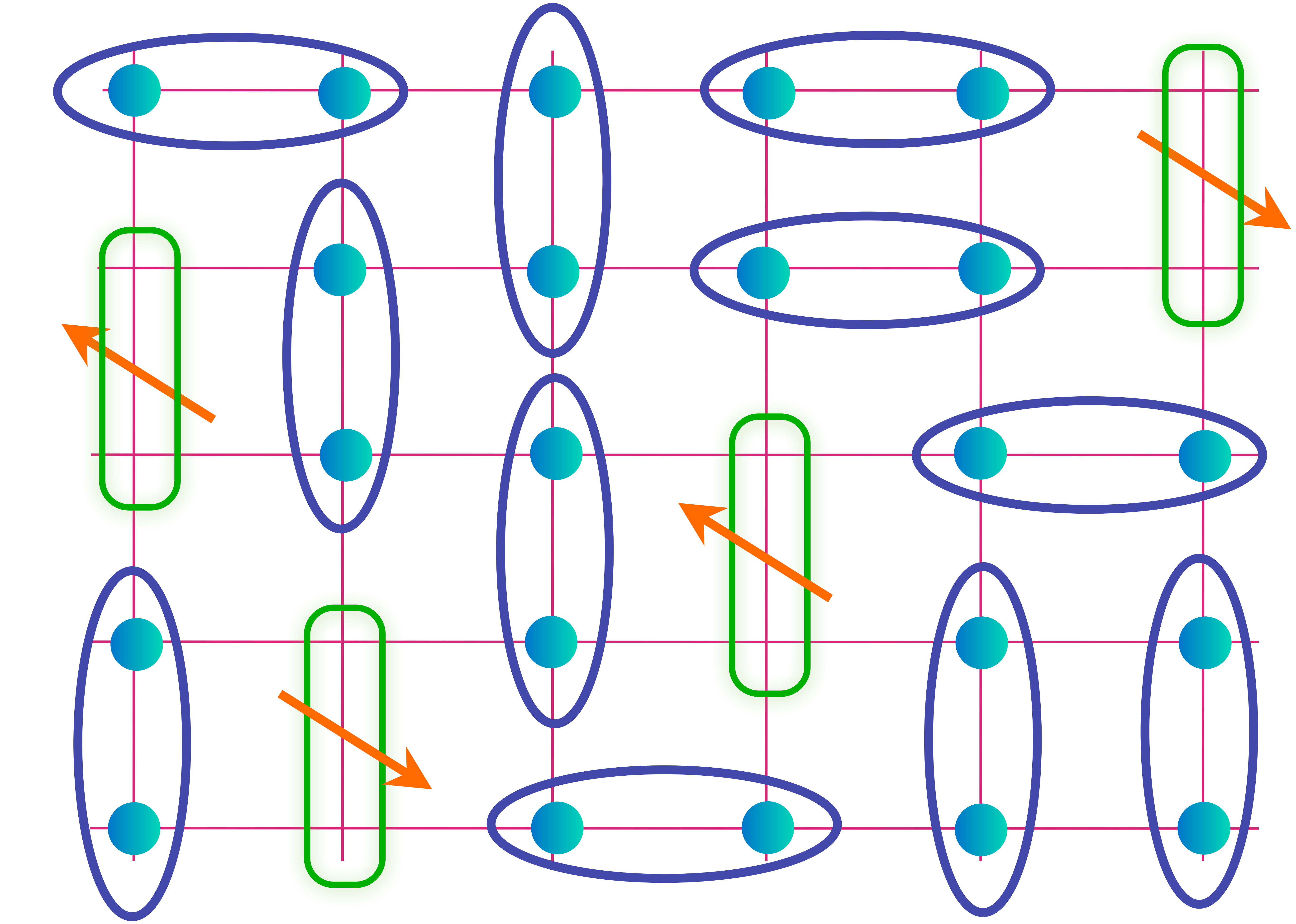

Metal with electron-like quasiparticles on a Fermi surface of size p, and emergent gauge fields

Spinon-holon bound state

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$= \left(|\uparrow \circ\rangle + |\circ \uparrow\rangle\right)/\sqrt{2}$

Yang Qi and S. Sachdev, *Phys. Rev. B* **81**, 115129 (2010); M. Punk, A. Allais, and S. Sachdev, *Proc. Nat. Acad. Sci.* **112**, 9552 (2015)

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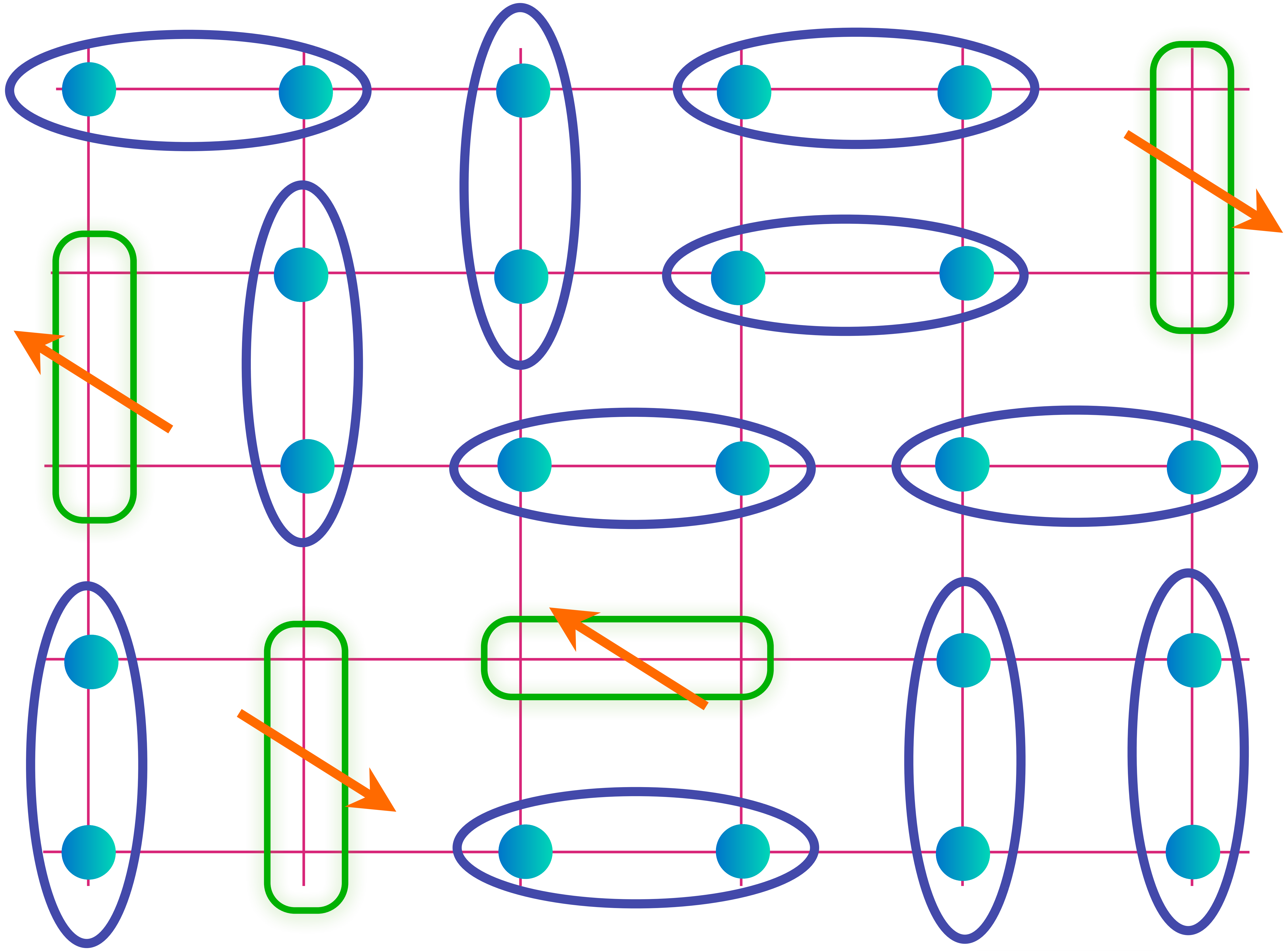

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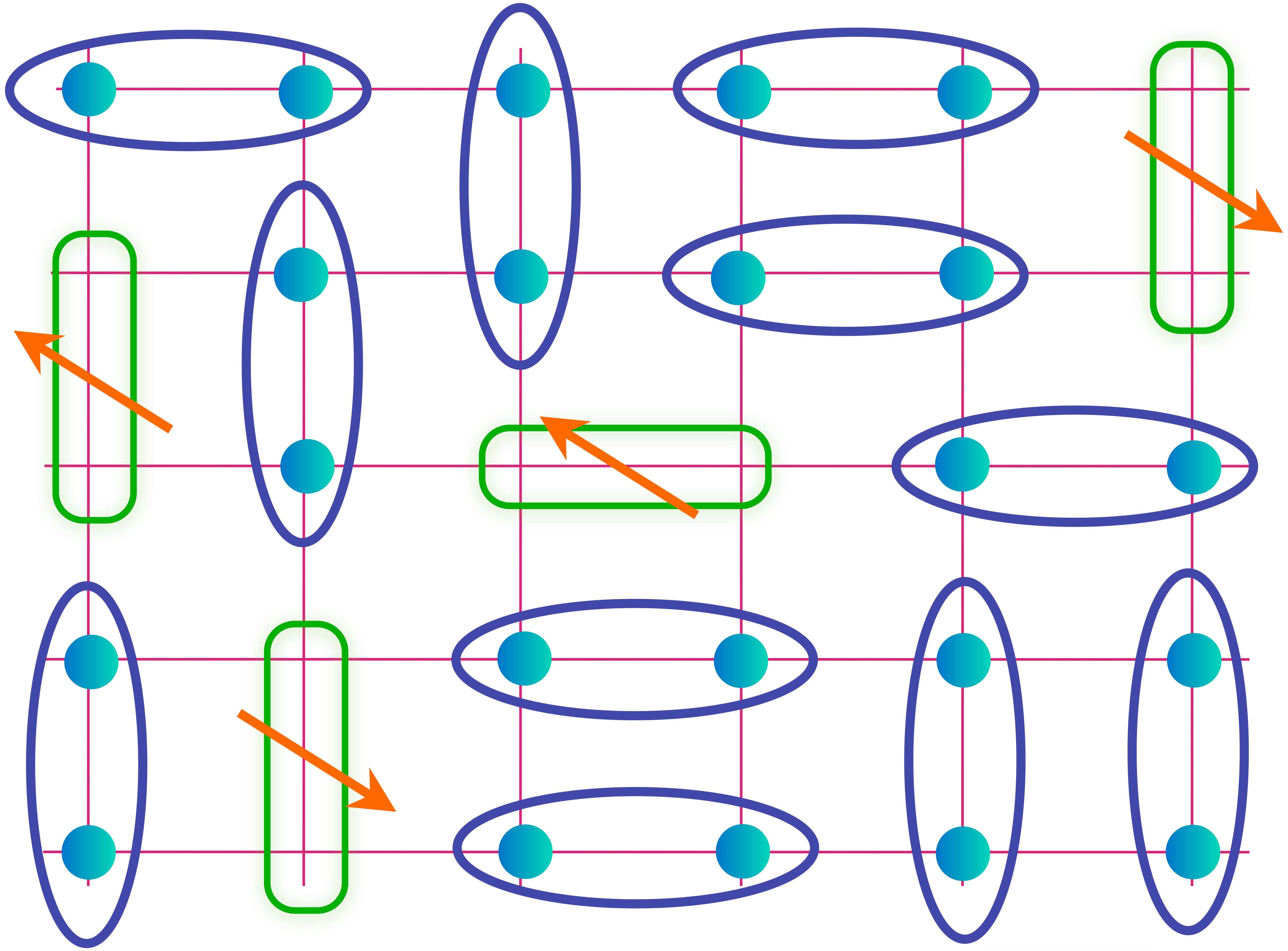
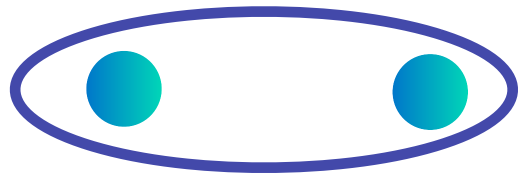
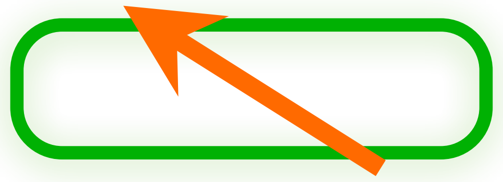


Metal with electron-like quasiparticles on a Fermi surface of size p, and emergent gauge fields

Spinon-holon bound state

 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

 $= \left(|\uparrow \circ\rangle + |\circ \uparrow\rangle\right)/\sqrt{2}$

Yang Qi and S. Sachdev, *Phys. Rev. B* **81**, 115129 (2010); M. Punk, A. Allais, and S. Sachdev, *Proc. Nat. Acad. Sci.* **112**, 9552 (2015)

FL* in a ***one-band*** model

S. Sachdev *Phys. Rev. B* **49**, 6770 (1994); X.-G. Wen and P.A. Lee *Phys. Rev. Lett.* **76**, 503 (1996)
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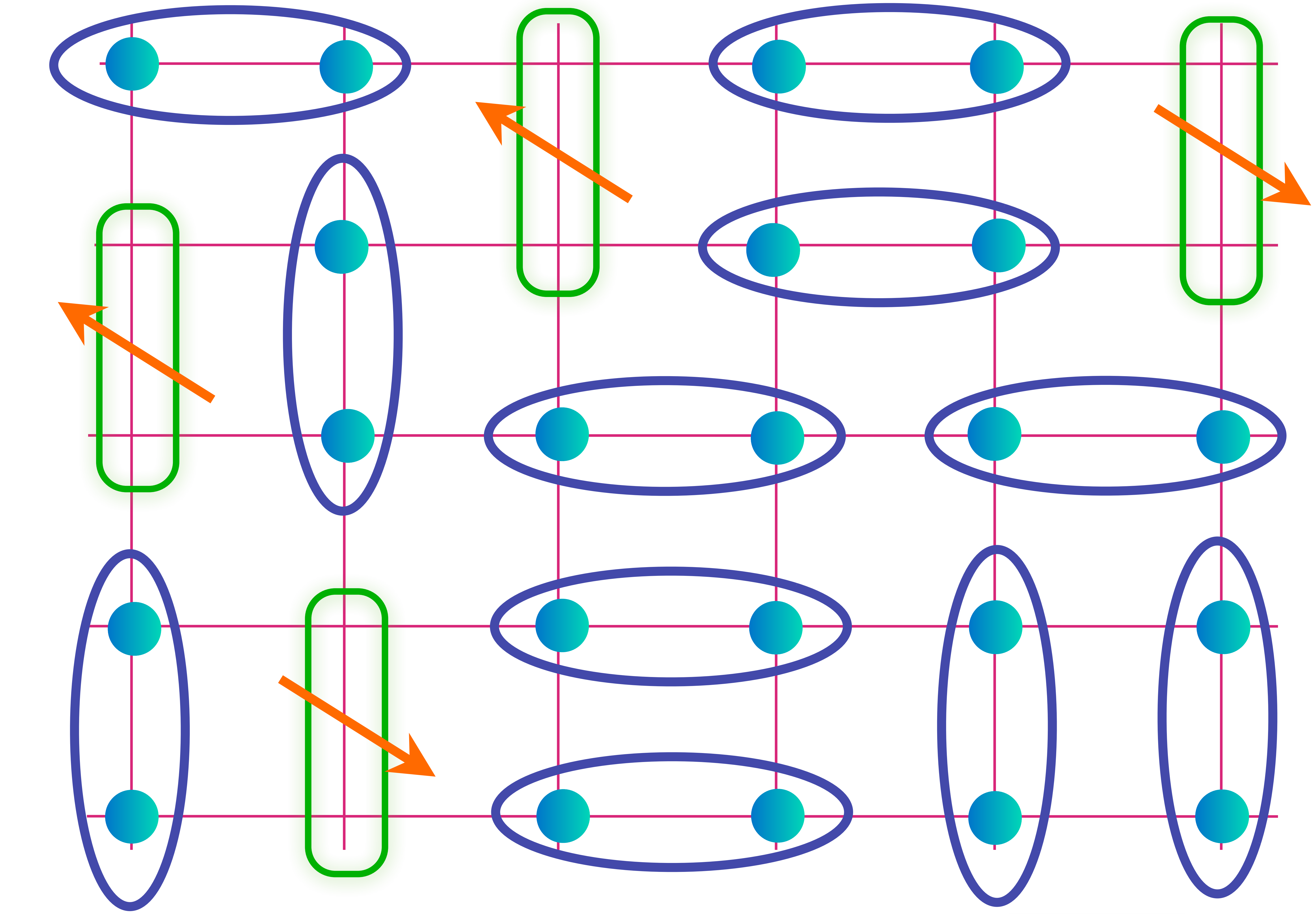

Metal with electron-like quasiparticles on a Fermi surface of size p, and emergent gauge fields

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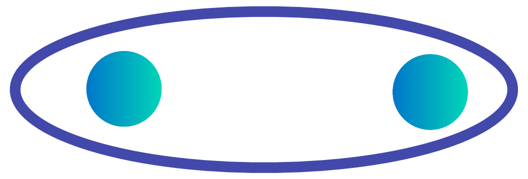
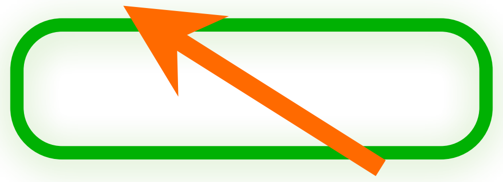
 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

 $= \left(|\uparrow \circ\rangle + |\circ \uparrow\rangle\right)/\sqrt{2}$

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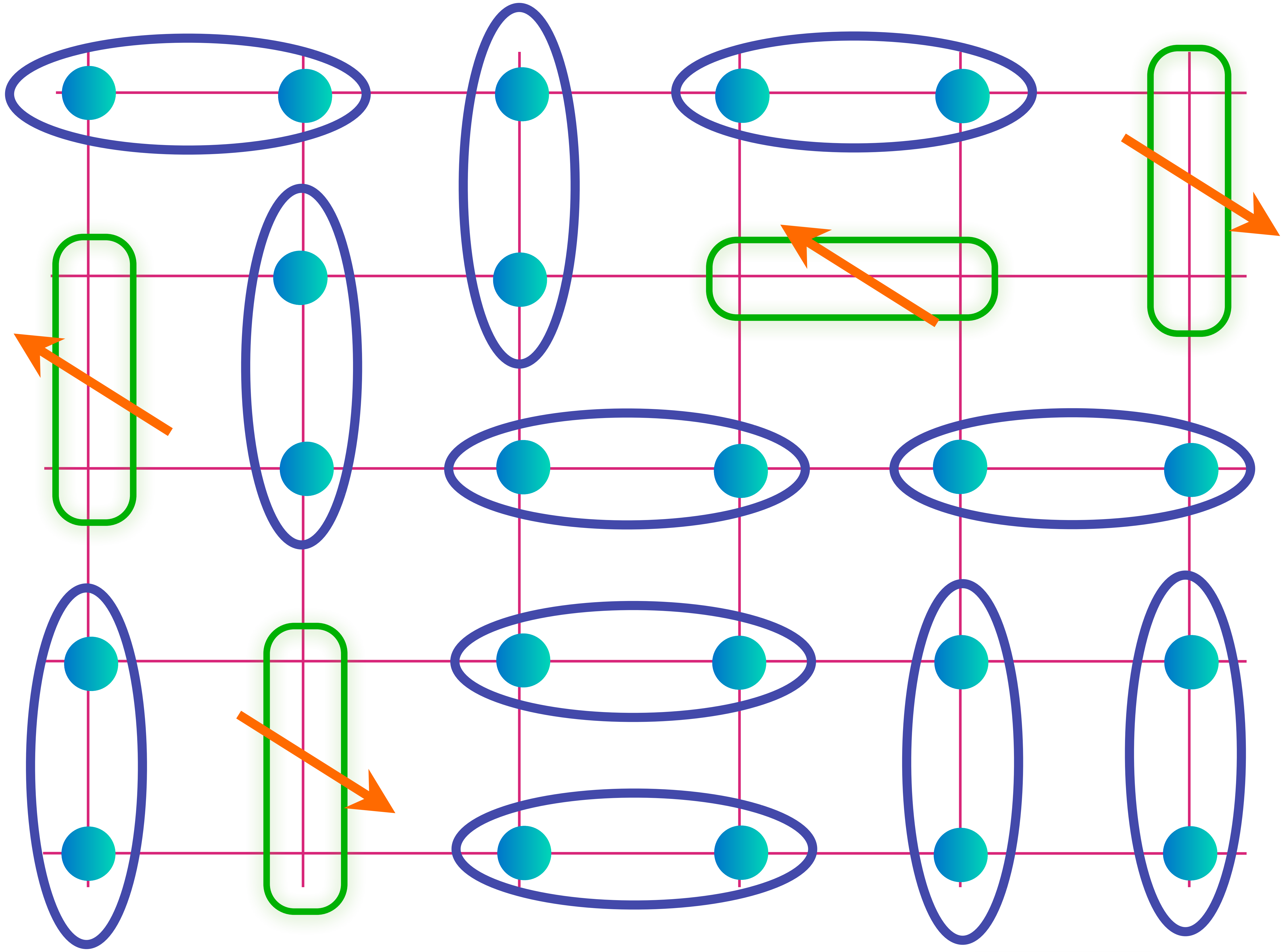
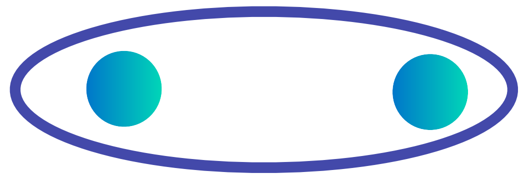
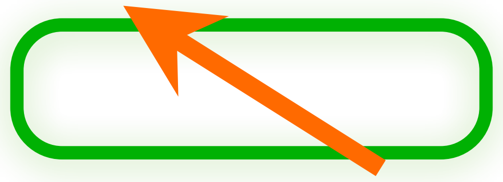


Metal with electron-like quasiparticles on a Fermi surface of size p, and emergent gauge fields

Spinon-holon bound state

 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

 $= \left(|\uparrow \circ\rangle + |\circ \uparrow\rangle\right)/\sqrt{2}$

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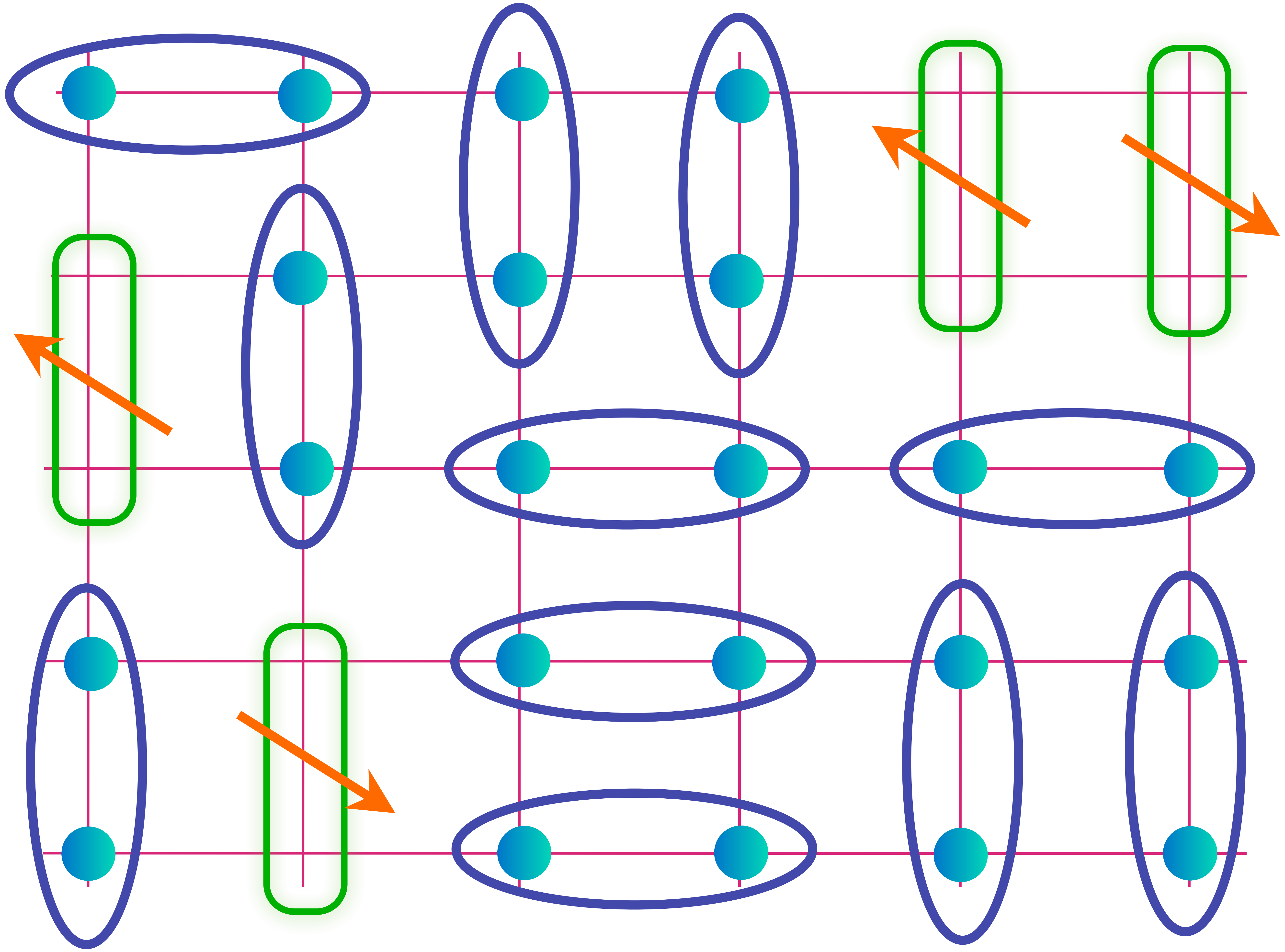

Metal with electron-like quasiparticles on a Fermi surface of size p, and emergent gauge fields

Spinon-holon bound state

$= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$ $\quad$ $= (|\uparrow \circ\rangle + |\circ \uparrow\rangle)/\sqrt{2}$

Yang Qi and S. Sachdev, *Phys. Rev. B* **81**, 115129 (2010); M. Punk, A. Allais, and S. Sachdev, *Proc. Nat. Acad. Sci.* **112**, 9552 (2015)

FL* in a *one-band* model

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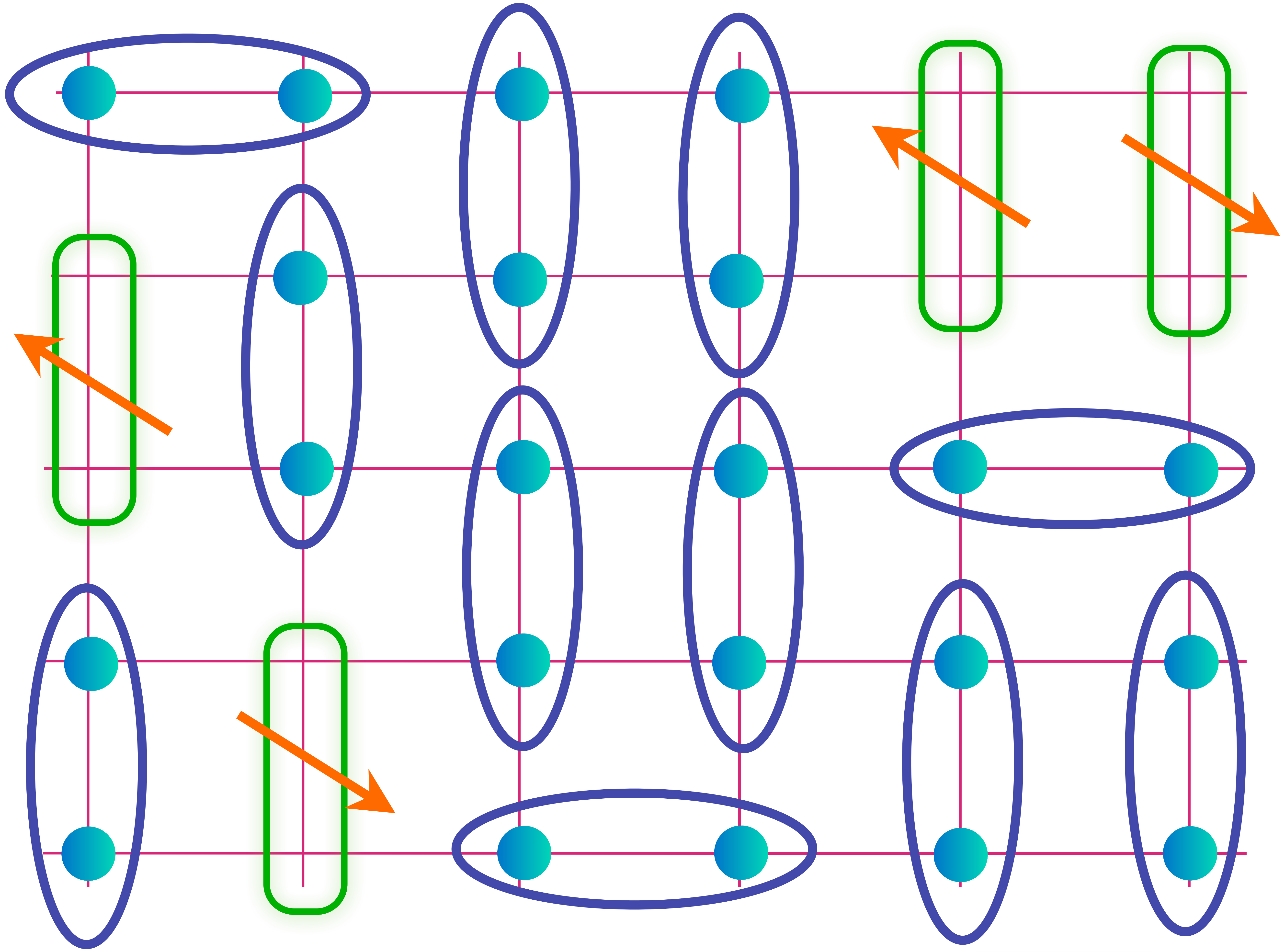
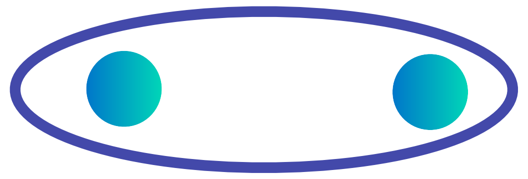
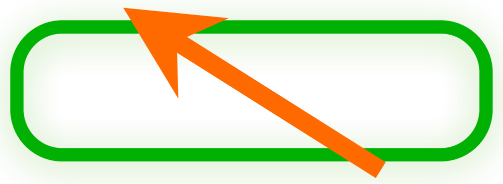


Metal with electron-like quasiparticles on a Fermi surface of size *p*, and emergent gauge fields

Spinon-holon bound state

 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

 $= \left(|\uparrow \circ\rangle + |\circ \uparrow\rangle\right)/\sqrt{2}$

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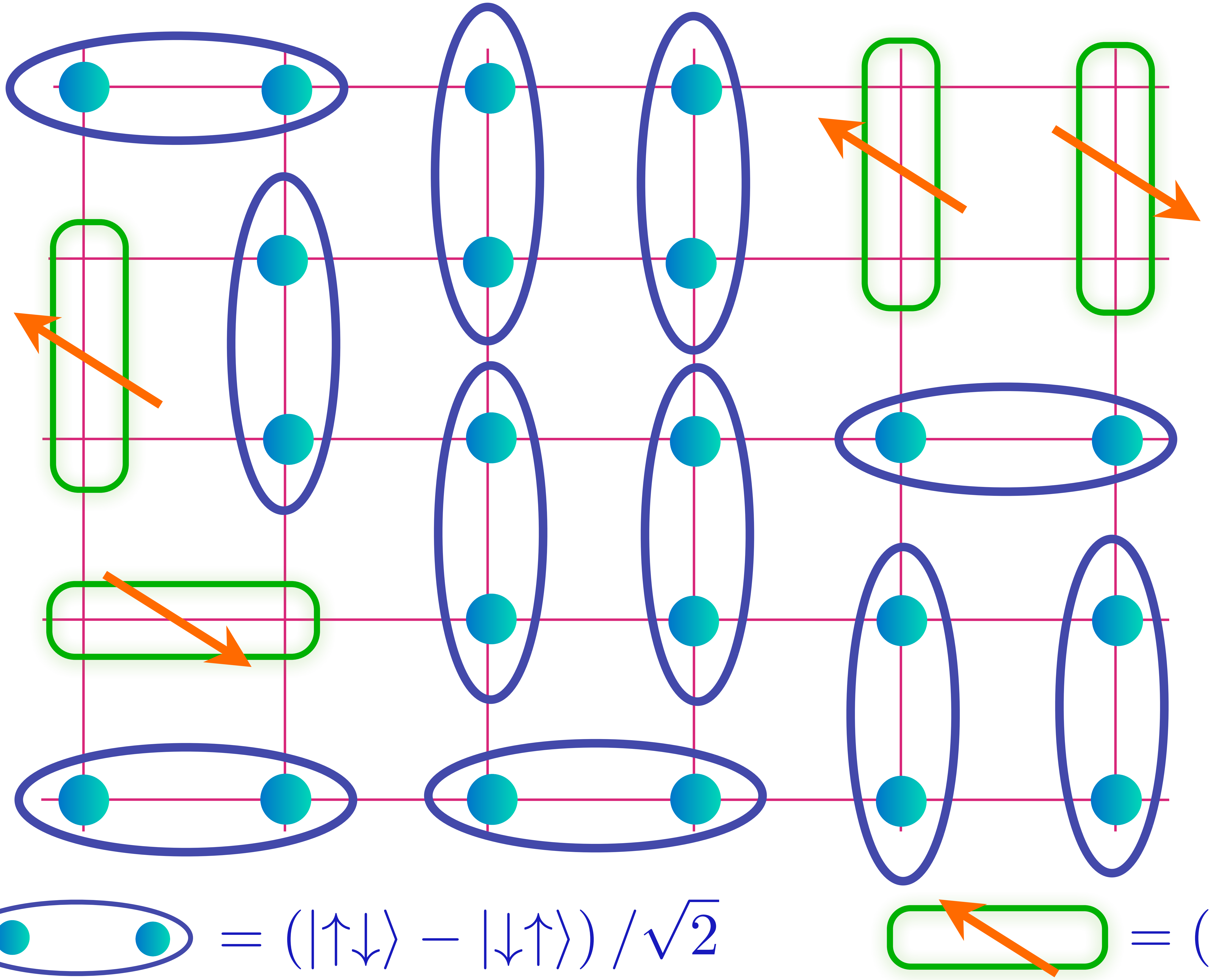

Metal with electron-like quasiparticles on a Fermi surface of size p, and emergent gauge fields

Spinon-holon bound state

$$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$$

$$= \left(|\uparrow \circ\rangle + |\circ \uparrow\rangle\right)/\sqrt{2}$$

Yang Qi and S. Sachdev, *Phys. Rev. B* **81**, 115129 (2010); M. Punk, A. Allais, and S. Sachdev, *Proc. Nat. Acad. Sci.* **112**, 9552 (2015)

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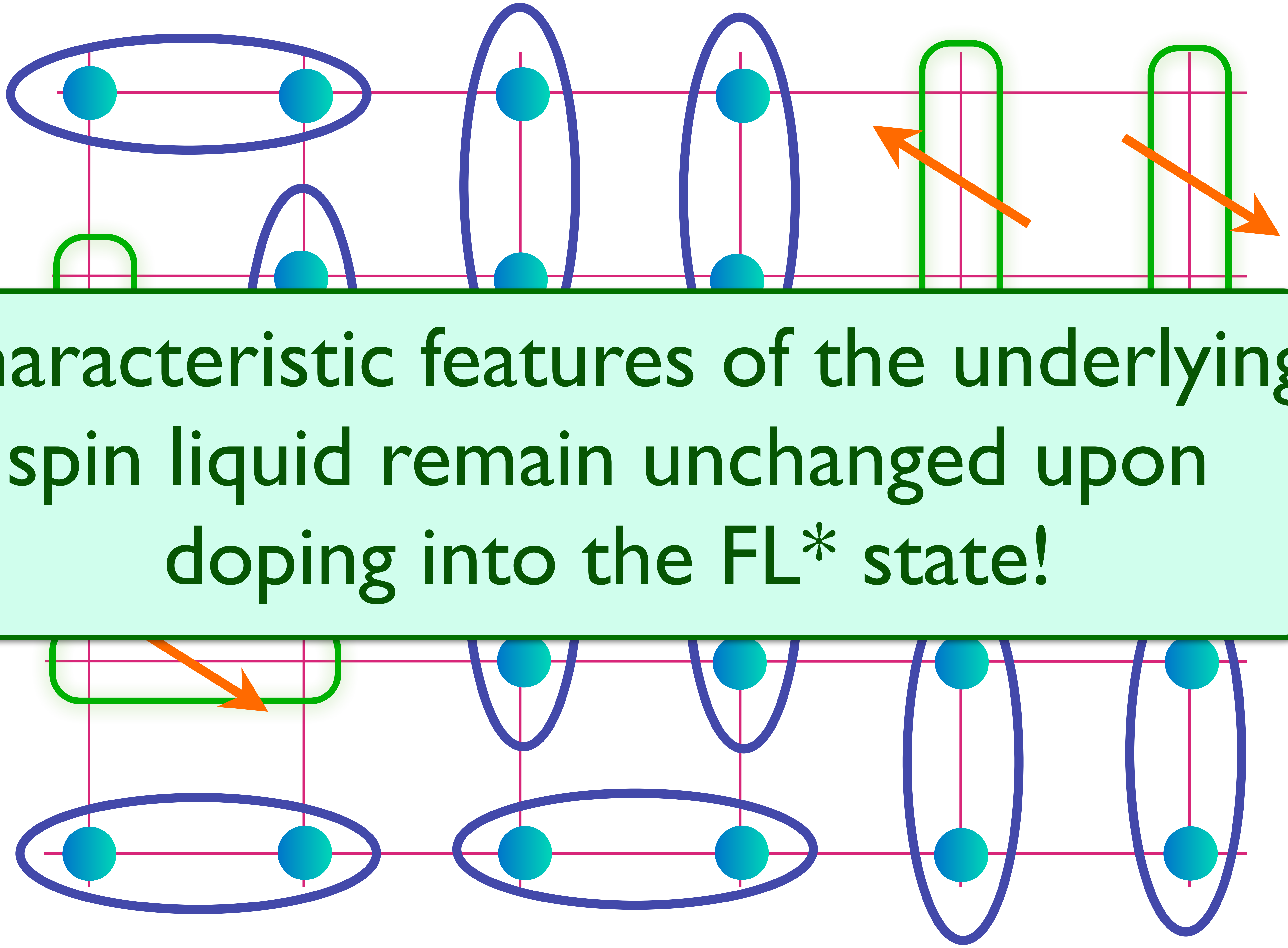
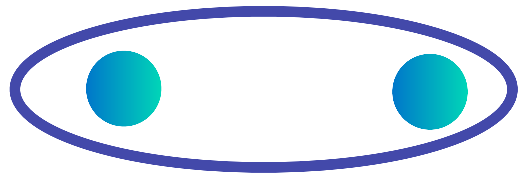
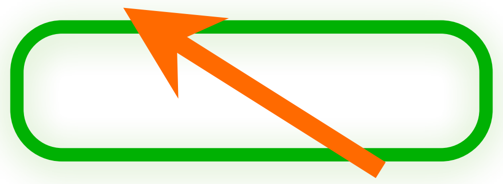


Characteristic features of the underlying spin liquid remain unchanged upon doping into the FL* state!

Metal with electron-like quasiparticles on a Fermi surface of size *p*, and emergent gauge fields

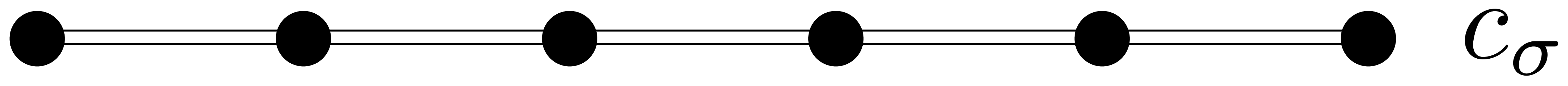
Spinon-holon bound state

 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

 $= \left(|\uparrow\circ\rangle + |\circ\uparrow\rangle\right)/\sqrt{2}$

E. G. Moon and S. Sachdev, *Phys. Rev. B* **83**, 224508 (2011); M. Punk, A. Allais, and S. Sachdev, *Proc. Nat. Acad. Sci.* **112**, 9552 (2015)

Ancilla theory of the Hubbard model

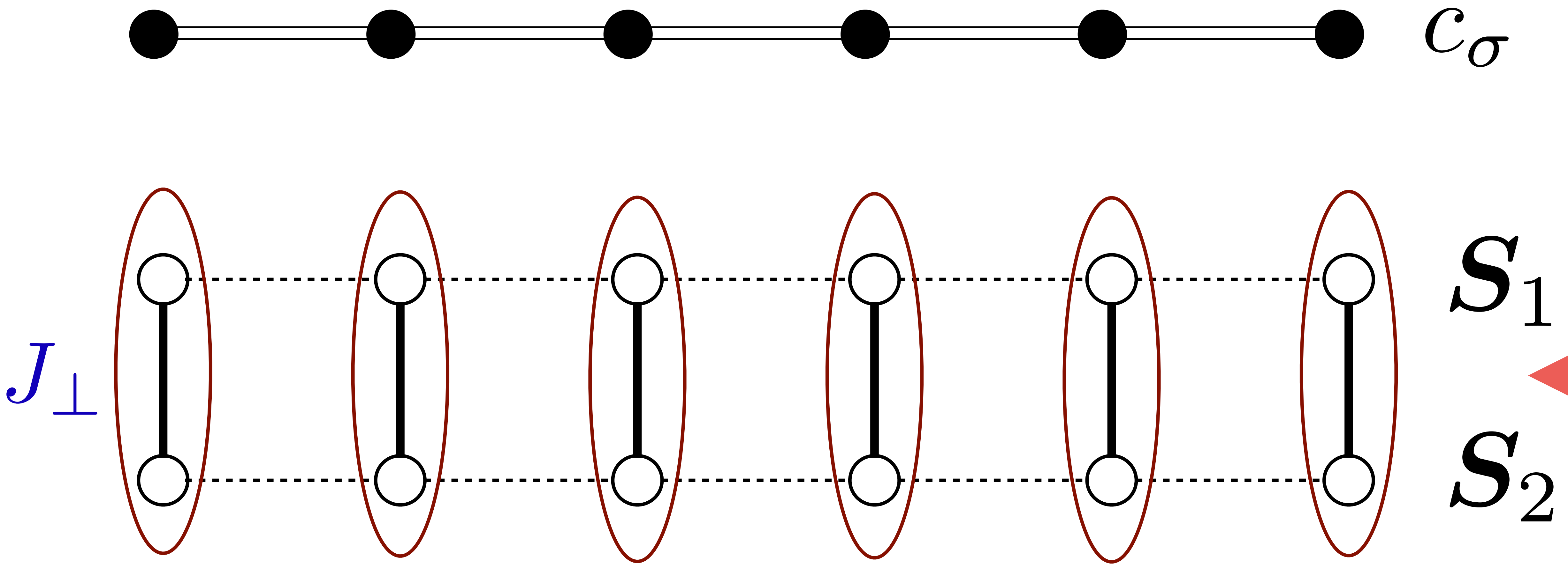


Hubbard model of hole density 1+p

Ancilla theory of the Hubbard model

(Luttinger-Oshikawa anomalies for dummies)

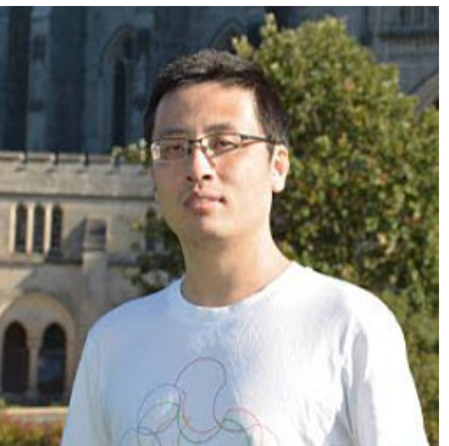
Hubbard model of hole density 1+p



Ancilla qubits in a "trivial" gapped insulator

$$\mathcal{H}_{\rm Hubbard} + \mathcal{H}_{\rm trivial\ \ insulator}$$

Ya-Hui Zhang



Ya-Hui Zhang and S. Sachdev, PRR **2**, 023172 (2020)
A. Nikolaenko, M. Tikhanovskaya, S. Sachdev, and Ya-Hui Zhang, PRB **103**, 235138 (2021)

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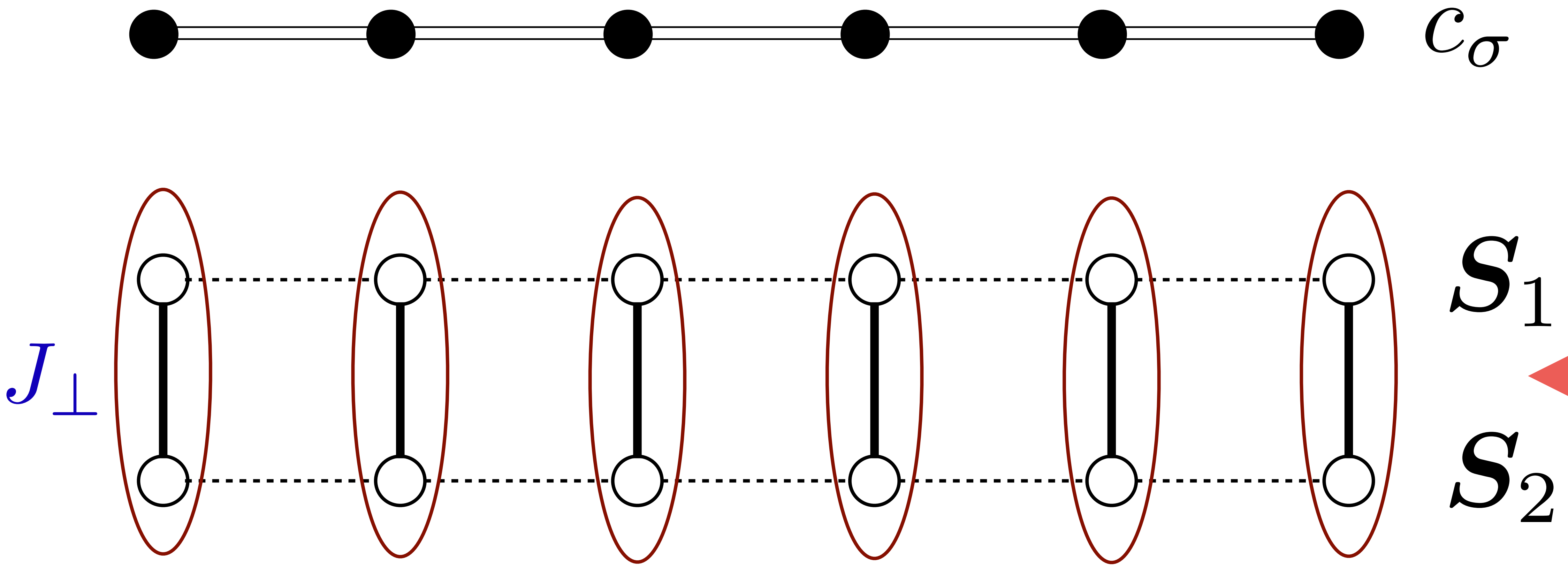
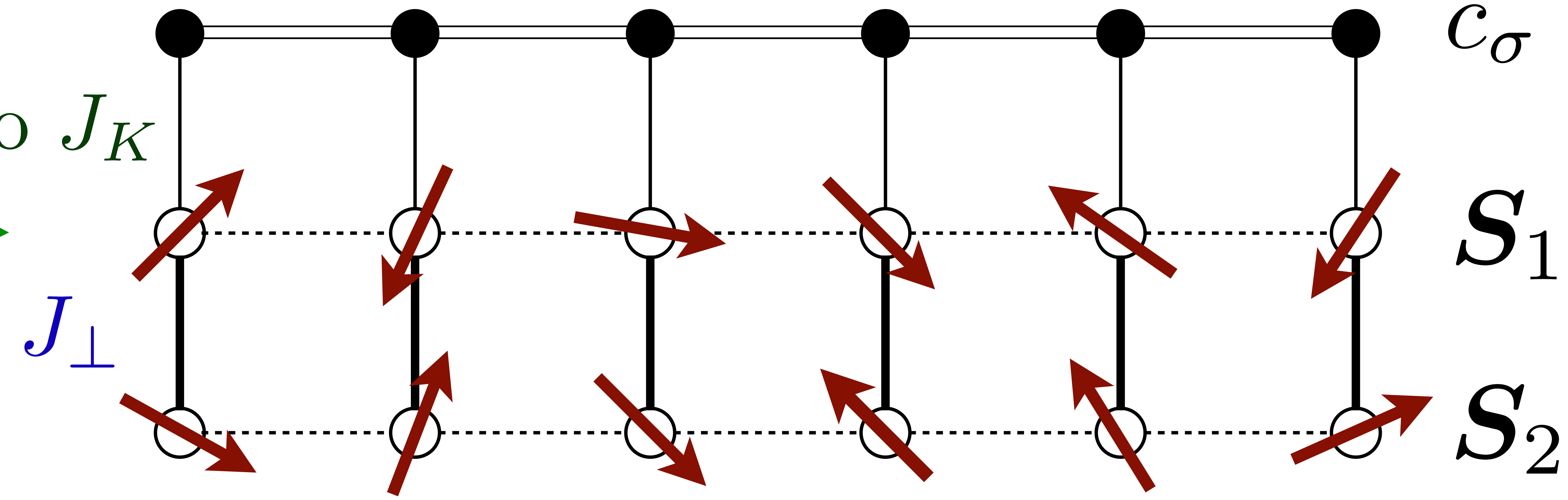


Ancilla qubits in a "trivial" gapped insulator

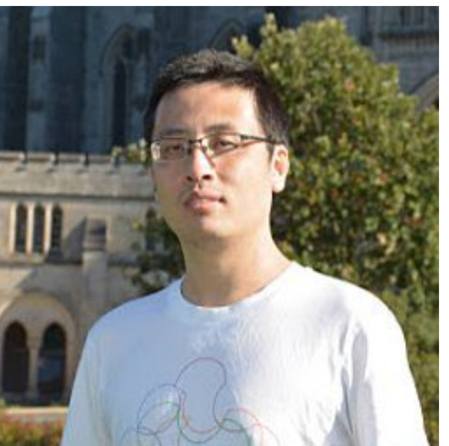
$$\boxed{\mathcal{U}\left(\mathcal{H}_{\rm Hubbard} + \mathcal{H}_{\rm trivial\ \ insulator}\right)\mathcal{U}^{-1} = \mathcal{H}_{\rm ancilla}}$$

Antiferromagnetic Kondo J_K



Free holes of density 1+p

Ya-Hui Zhang



Ya-Hui Zhang and S. Sachdev, PRR **2**, 023172 (2020)

A. Nikolaenko, M. Tikhanovskaya, S. Sachdev, and Ya-Hui Zhang, PRB **103**, 235138 (2021)

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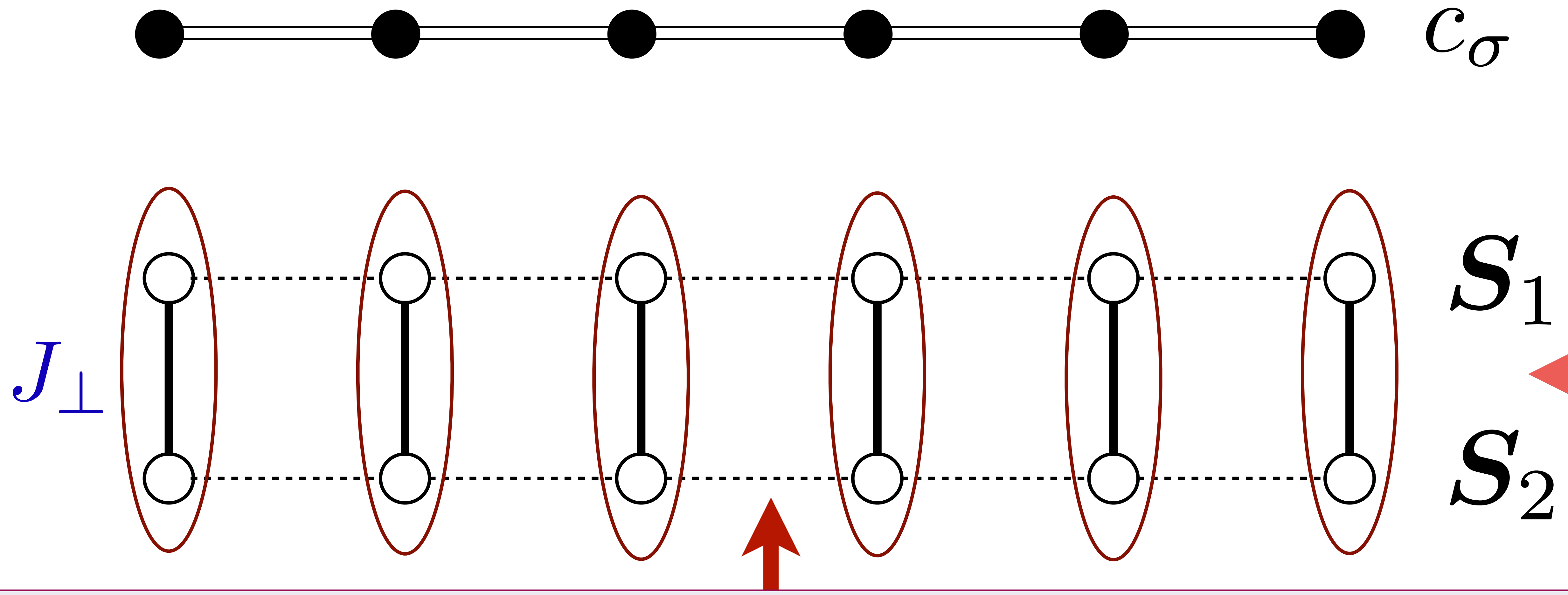
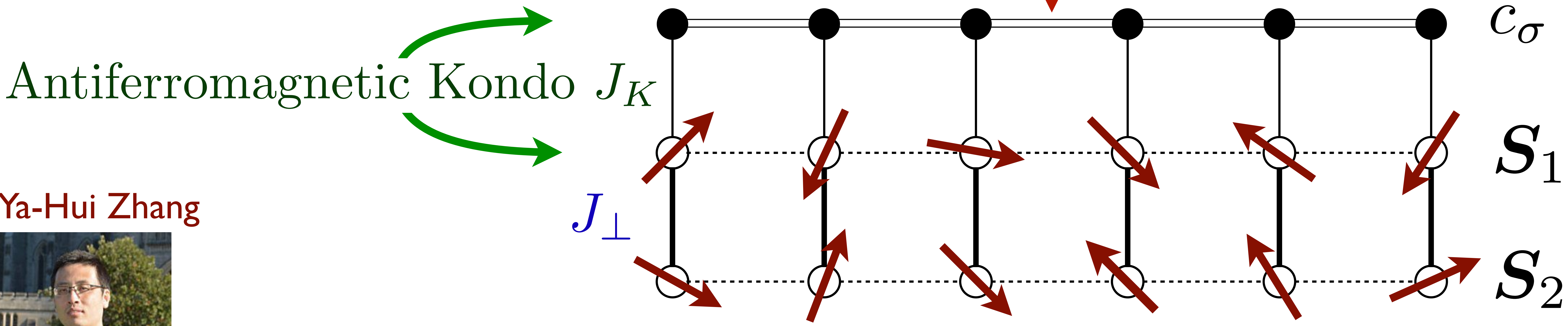


Hubbard model of hole density 1+p

Ancilla qubits in a "trivial" gapped insulator

Schrieffer-Wolff transformation at large $J_\perp$ yields $U = \dfrac{3J_K^2}{8J_\perp} + \dfrac{3J_K^3}{16J_\perp^2} + \ldots$



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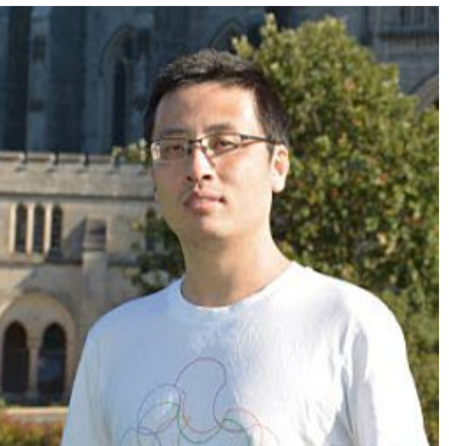
Ya-Hui Zhang



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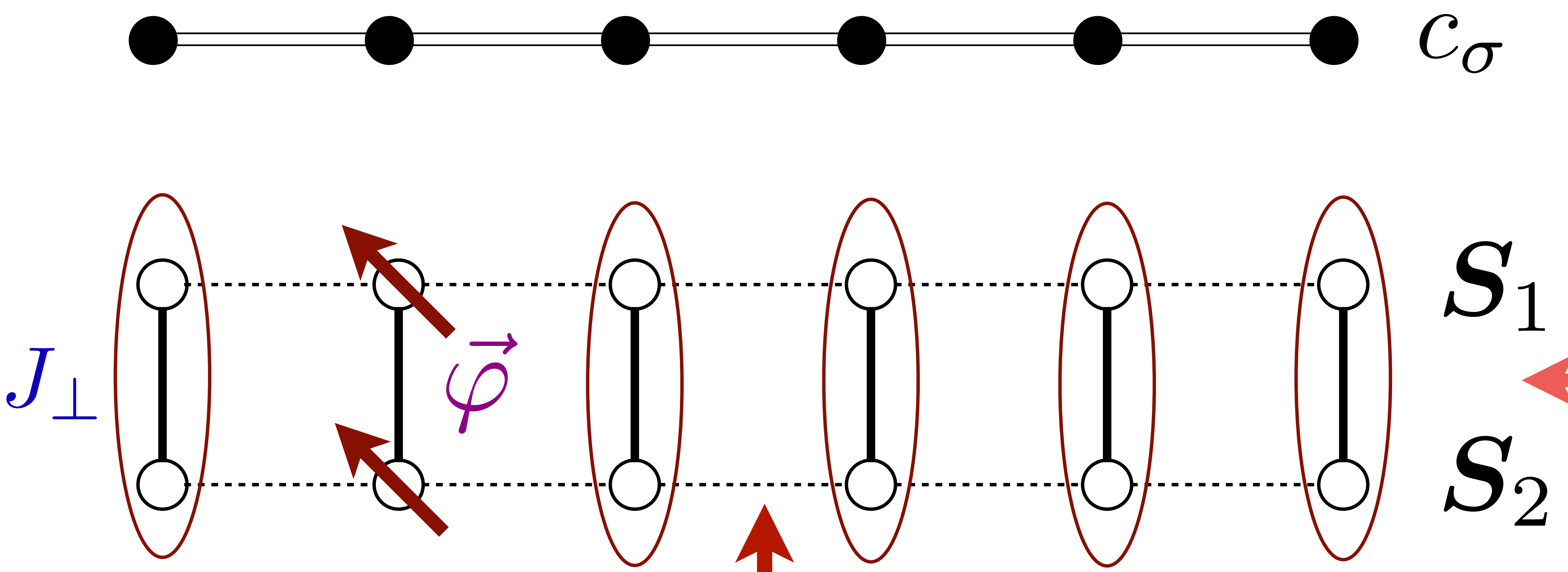
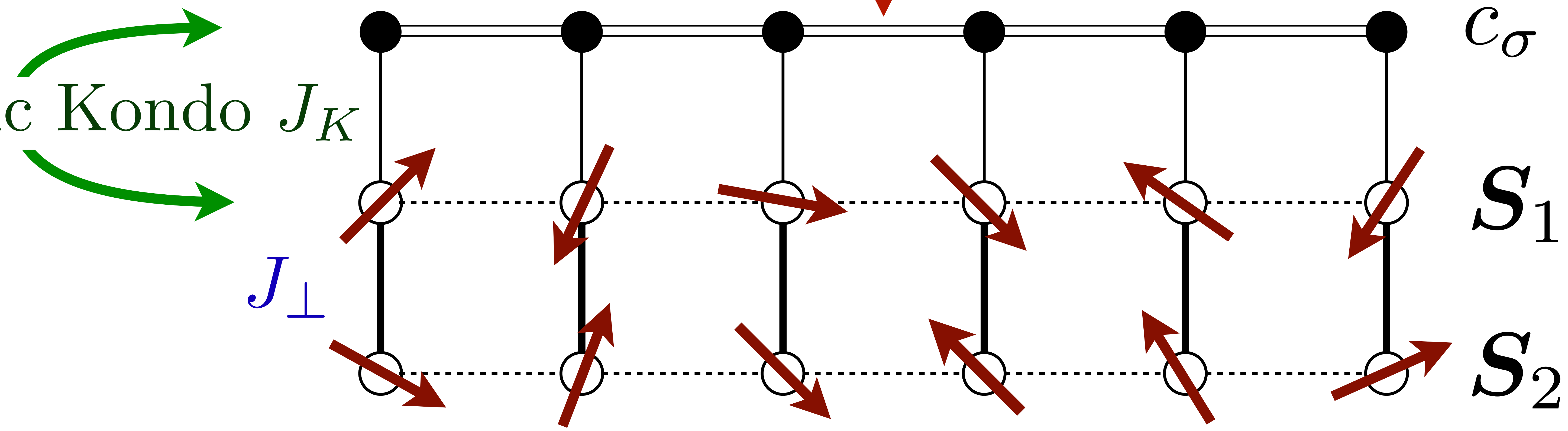


Hubbard model of hole density 1+p

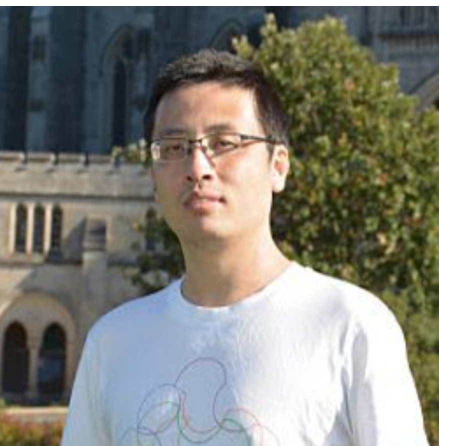
Ancilla qubits in a "trivial" gapped insulator

Hubbard-Stratonovich transformation:
Paramagnon $\vec{\varphi}$ fractionalized into a pair of $S = 1/2$ spins, $\boldsymbol{S}_1$ and $\boldsymbol{S}_2$.



Free holes of density 1+p

Ya-Hui Zhang



Ya-Hui Zhang and S. Sachdev, PRR **2**, 023172 (2020)
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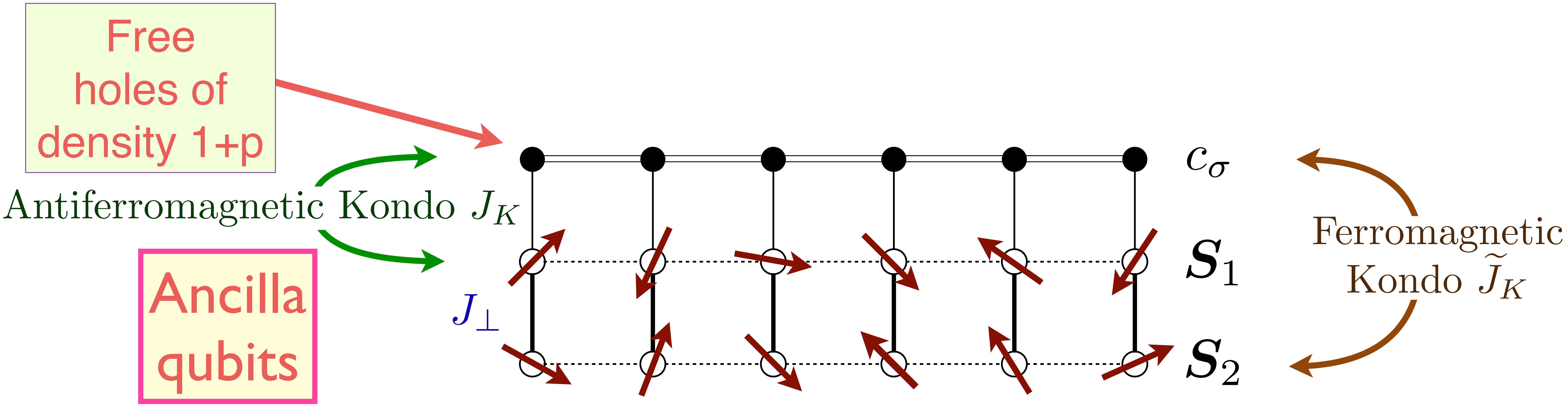
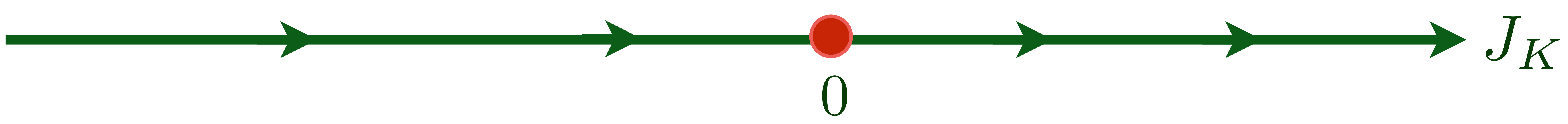
Ancilla theory of the Hubbard model

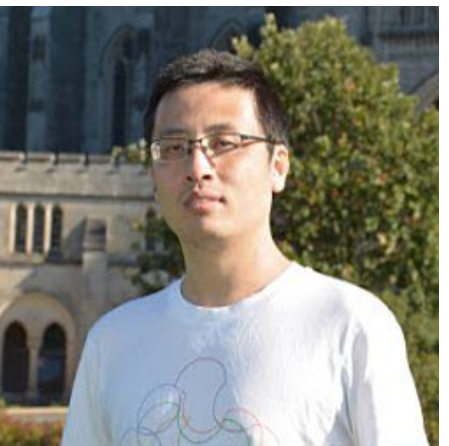
Ya-Hui Zhang and S. Sachdev,
Phys. Rev. Res. **2**, 023172 (2020)





Kondo RG

Ya-Hui Zhang

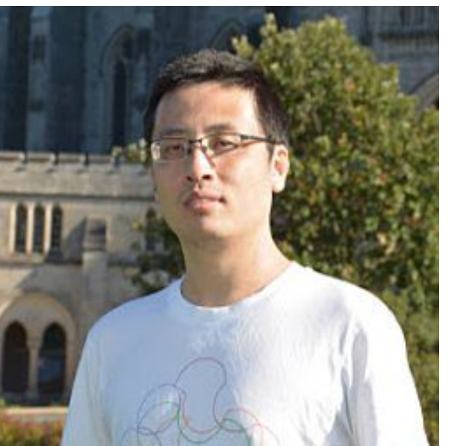


$$\mathcal{H}_{\text{ancilla}} = \sum_{\mathbf{p}} \varepsilon_{\mathbf{p}} c^\dagger_{\mathbf{p}\sigma} c_{\mathbf{p}\sigma} + J_K \sum_i c^\dagger_{i\sigma} \frac{\boldsymbol{\tau}_{\sigma\sigma'}}{2} c_{i\sigma'} \cdot \boldsymbol{S}_{1i} + J_\perp \sum_i \boldsymbol{S}_{1i} \cdot \boldsymbol{S}_{2i}$$

Ancilla theory of the Hubbard model

Ya-Hui Zhang and S. Sachdev,
Phys. Rev. Res. **2**, 023172 (2020)

Ya-Hui Zhang



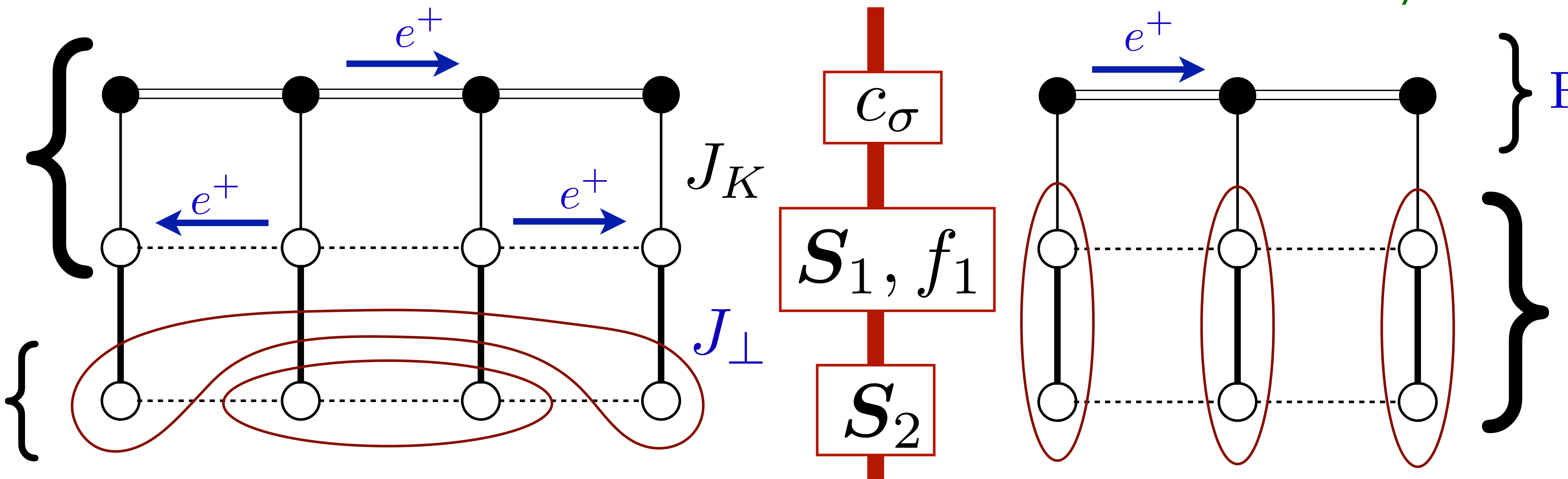
Ancilla theory of the Hubbard model

Ya-Hui Zhang and S. Sachdev,
Phys. Rev. Res. **2**, 023172 (2020)

Kondo lattice heavy Fermi liquid. Size $1+p+1$ $= p \pmod 2$. *Small* Fermi surface!

$\langle \Phi \rangle \neq 0$

Spin liquid



Large Fermi surface. Size: $1+p$

Trivial insulator

FL* $\quad \langle \Phi \rangle \neq 0 \quad$ | $\quad \langle \Phi \rangle = 0 \quad$ FL

J_K

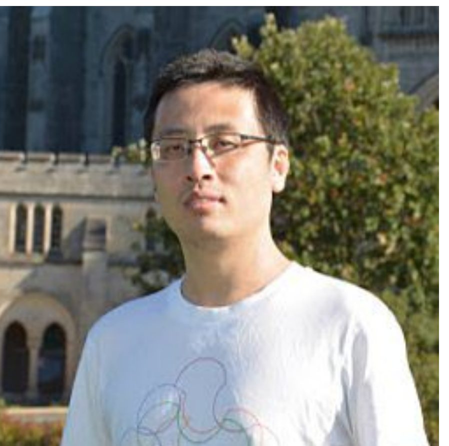
doping p

Pseudogap metal = Kondo Lattice Heavy Fermi Liquid $\oplus$ Spin Liquid

Fractionalized excitations of layer $\boldsymbol{S}_1$ confined by condensation of Higgs boson $\Phi \sim f_{1\sigma}^\dagger c_\sigma$.

Frationalized excitations of layer $\boldsymbol{S}_2$ remain deconfined

Ya-Hui Zhang



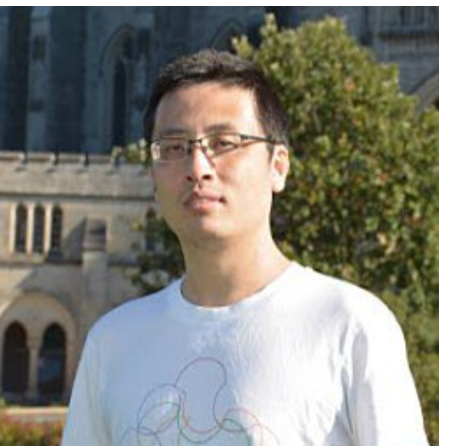
Ancilla theory of the Hubbard model

Ya-Hui Zhang and S. Sachdev, *Phys. Rev. Res.* **2**, 023172 (2020)

Pseudogap metal =
Kondo Lattice Heavy
Fermi Liquid
$\oplus$
Spin Liquid

$$H_{\rm mf} = -\sum_{i,j} t_{ij} c_{i\sigma}^\dagger c_{j\sigma} - \sum_{i,j} t_{1,ij} f_{1i\sigma}^\dagger f_{1j\sigma}$$
$$- \sum_i \Phi \left(c_{i\sigma}^\dagger f_{1i\sigma} + f_{1i\sigma}^\dagger c_{i\sigma} \right)$$

Ya-Hui Zhang

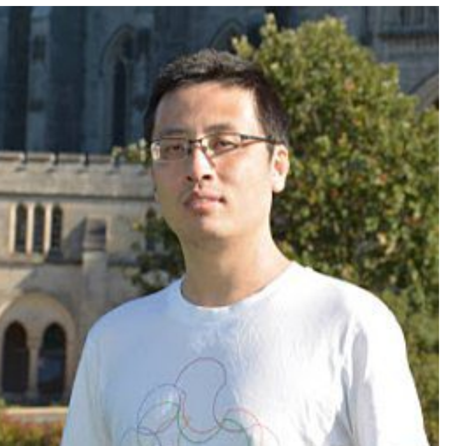


Ancilla theory of the Hubbard model

Ya-Hui Zhang and S. Sachdev,
Phys. Rev. Res. **2**, 023172 (2020)

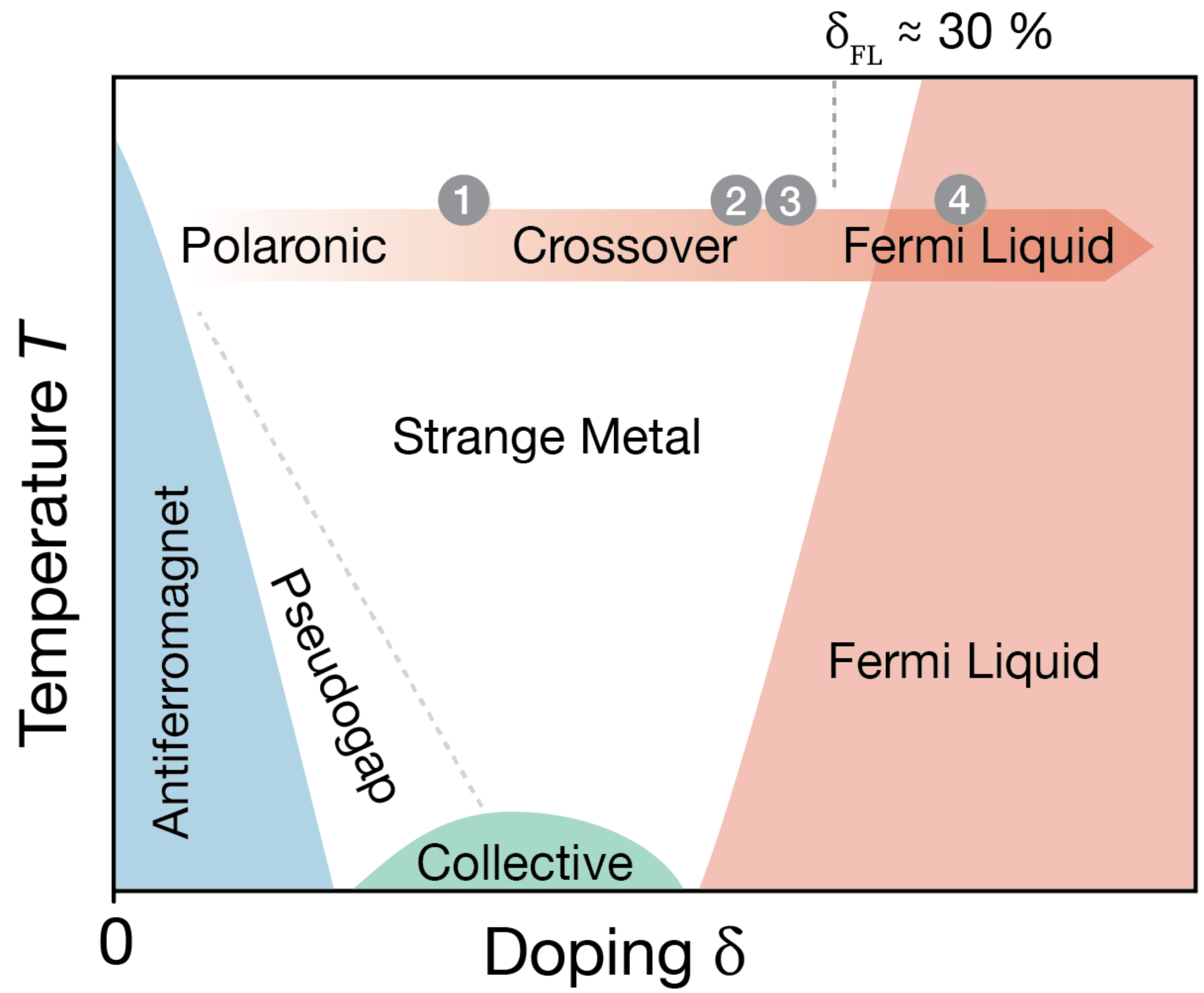
Pseudogap metal = Kondo Lattice Heavy Fermi Liquid $\oplus$ Spin Liquid	$\lvert \text{FL}* \rangle = [\text{Projection onto rung singlets of } \boldsymbol{S}_1, \boldsymbol{S}_2]$ $\bowtie \lvert \text{Slater determinant of } (c, f_1) \rangle$ $\otimes \lvert \text{Spin liquid of } \boldsymbol{S}_2 \rangle$

Ya-Hui Zhang



Replacement for "vanilla" Gutzwiller-projected Fermi liquid in the underdoped regime

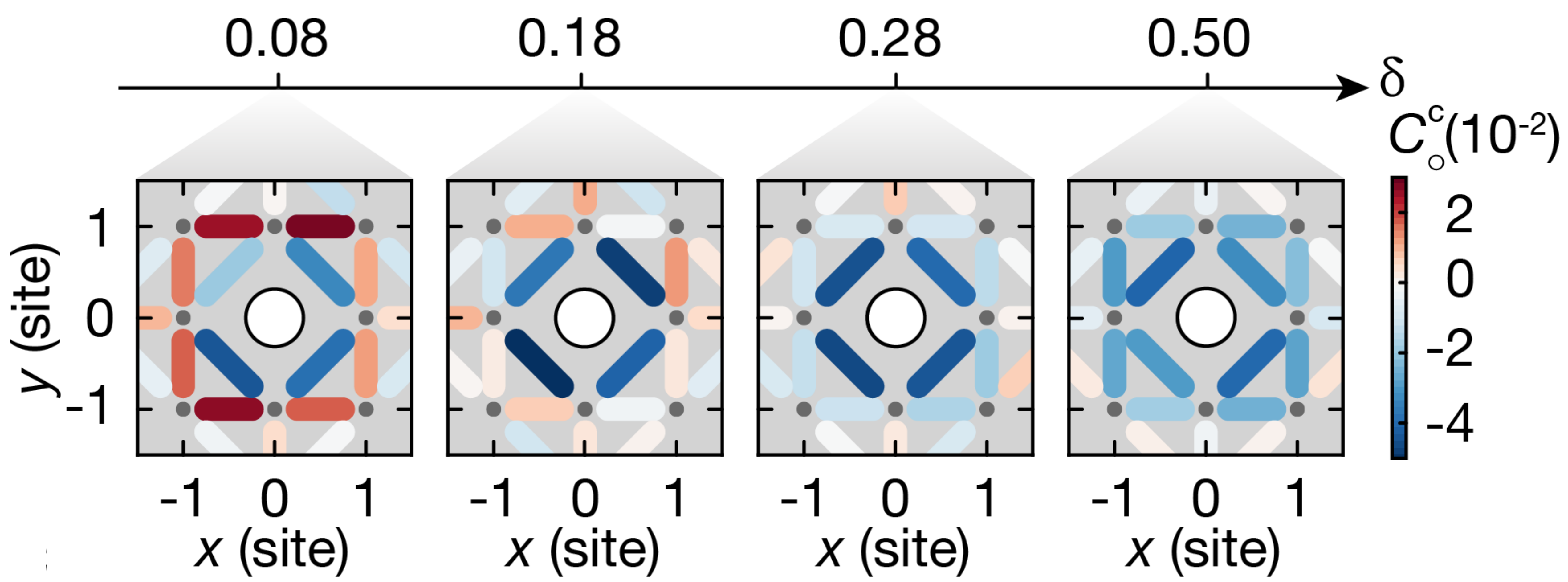
Ultracold fermionic atoms in optical lattices



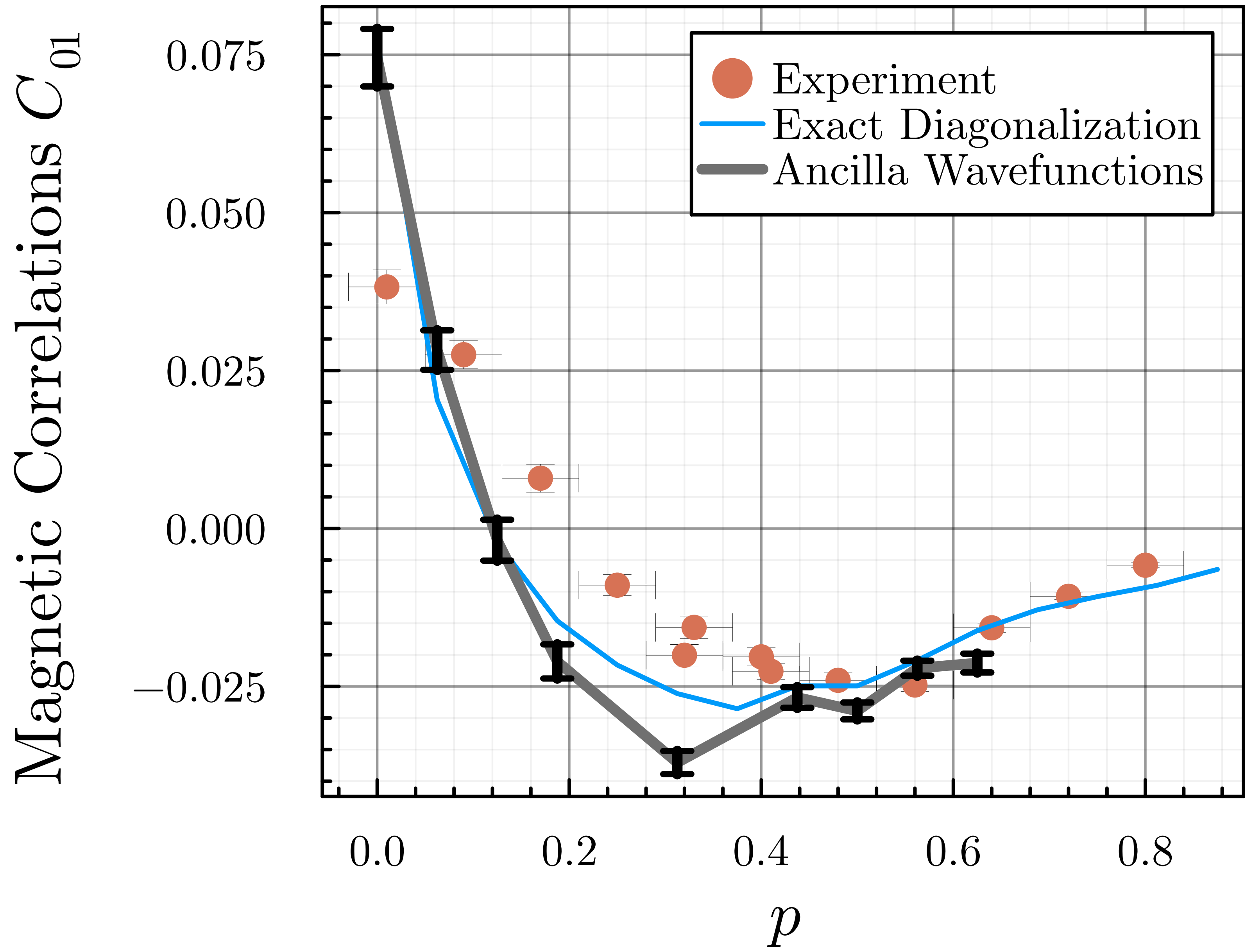
Microscopic evolution of doped Mott insulators from polaronic metal to Fermi liquid

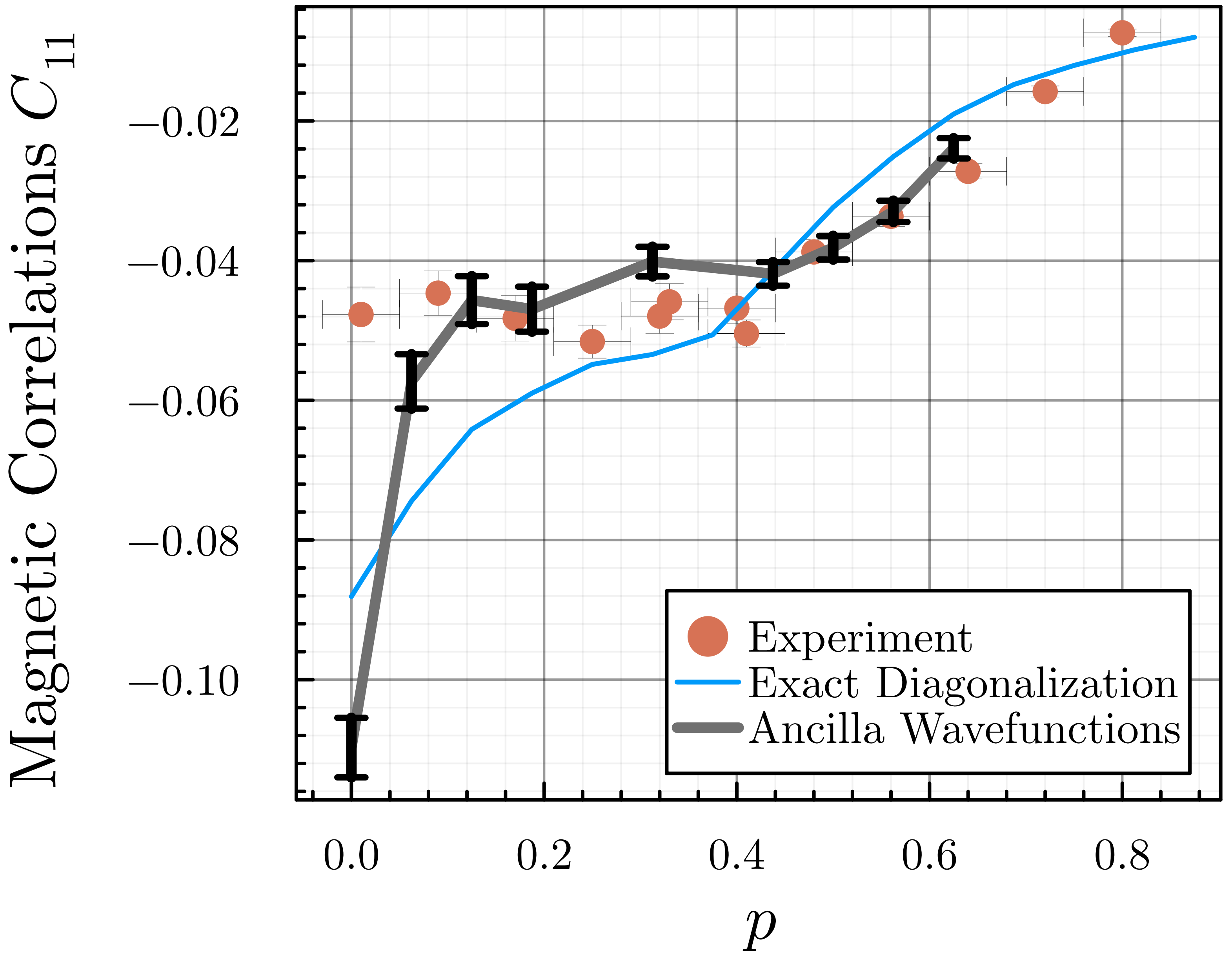
Joannis Koepsell, Dominik Bourgund, Pimonpan Sompet, Sarah Hirthe, Annabelle Bohrdt, Yao Wang, Fabian Grusdt, Eugene Demler, Guillaume Salomon, Christian Gross, Immanuel Bloch

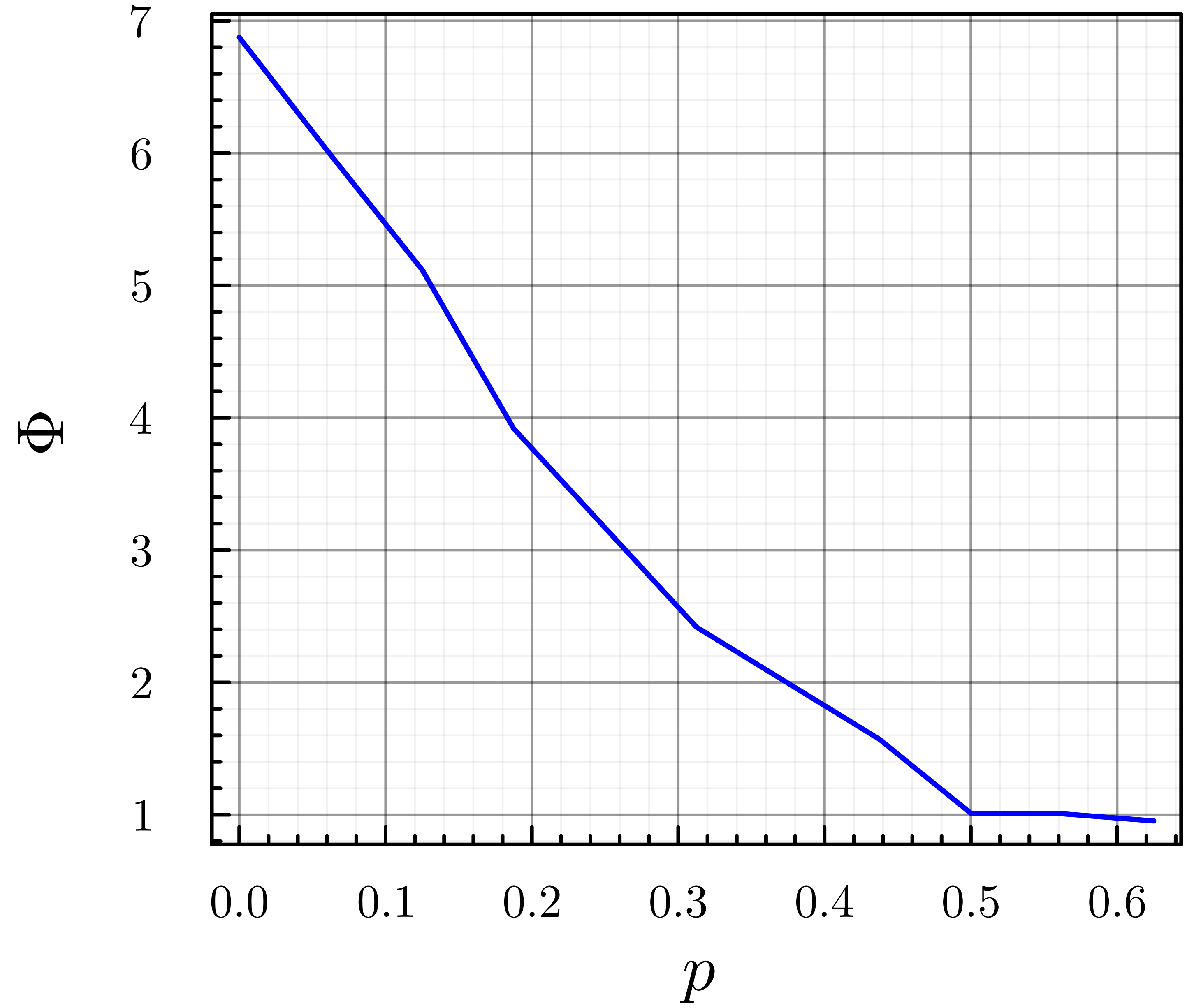
Science **374** (2021) 82



Max Planck Institute of Quantum Optics, Garching

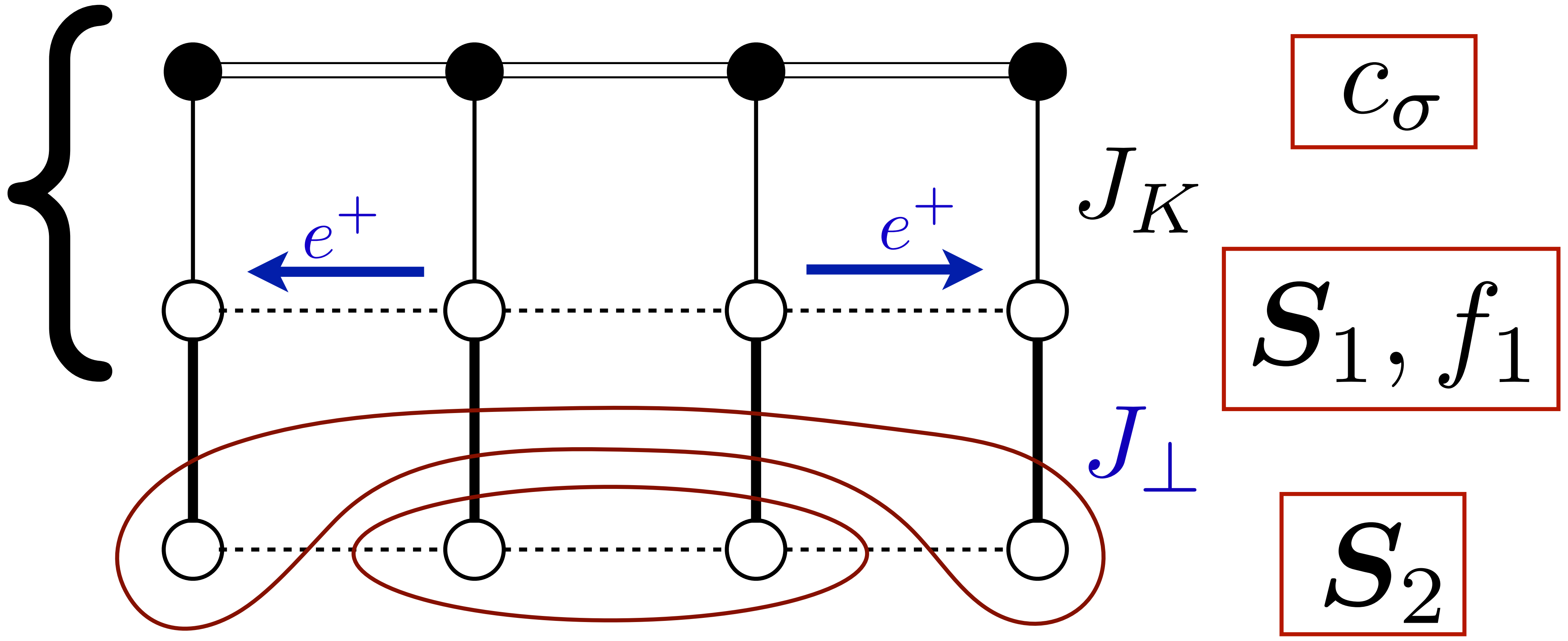






Higgs boson $\Phi \sim f^\dagger_{1\sigma} c_\sigma$.

$\langle \Phi \rangle \neq 0$






H. Shackleton and Shiwei Zhang, arXiv:2408.02190
(Tobias Müller, Yasir Iqbal, S.S., Ronny Thomale, arXiv:2408.01492)

1. Square lattice spin liquids

2. Spin liquids on the Kondo lattice: non-Luttinger volume Fermi surfaces (FL*)

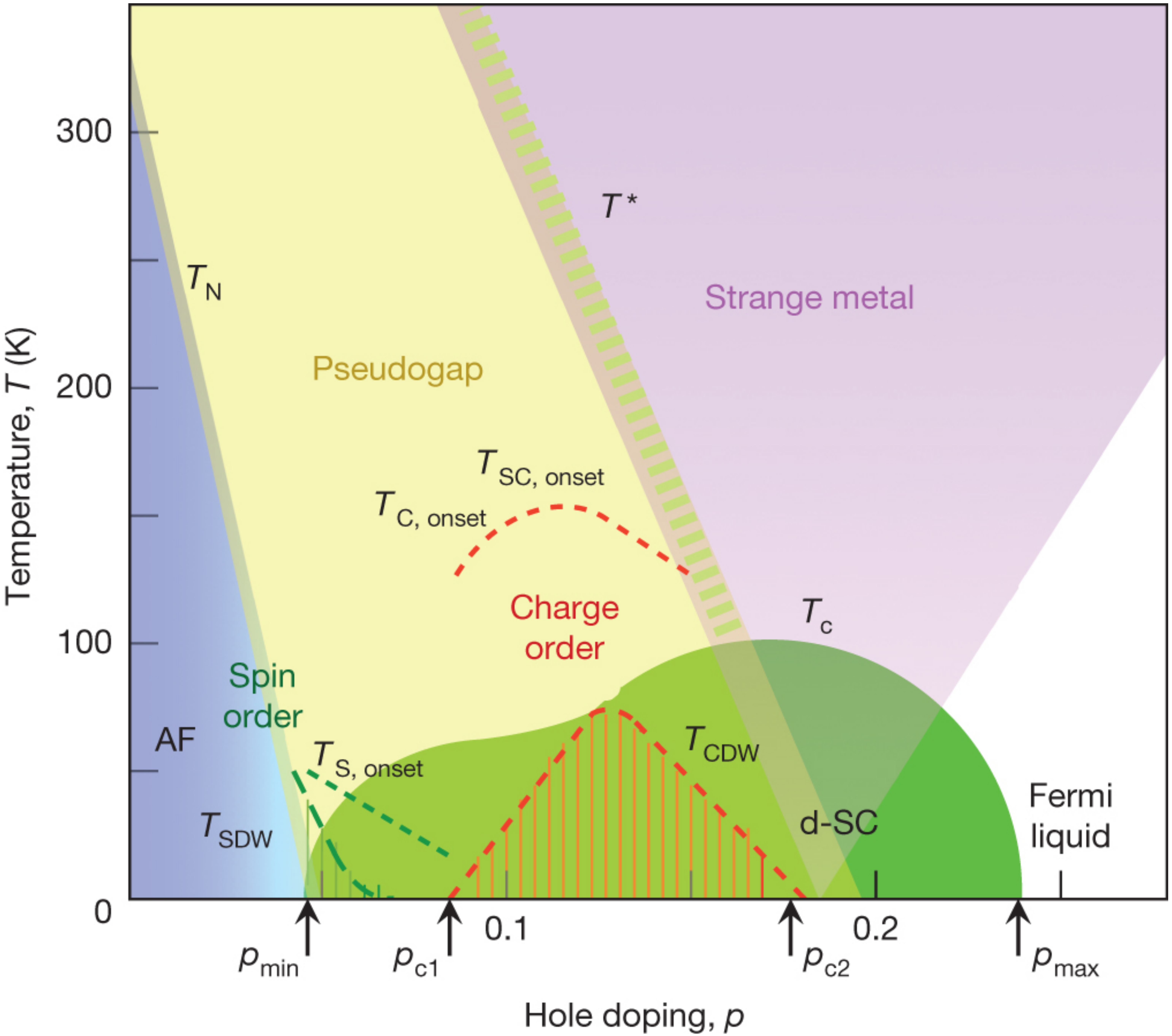
3. Doping square lattice spin liquids for $t >> J$: FL* in a single-band model

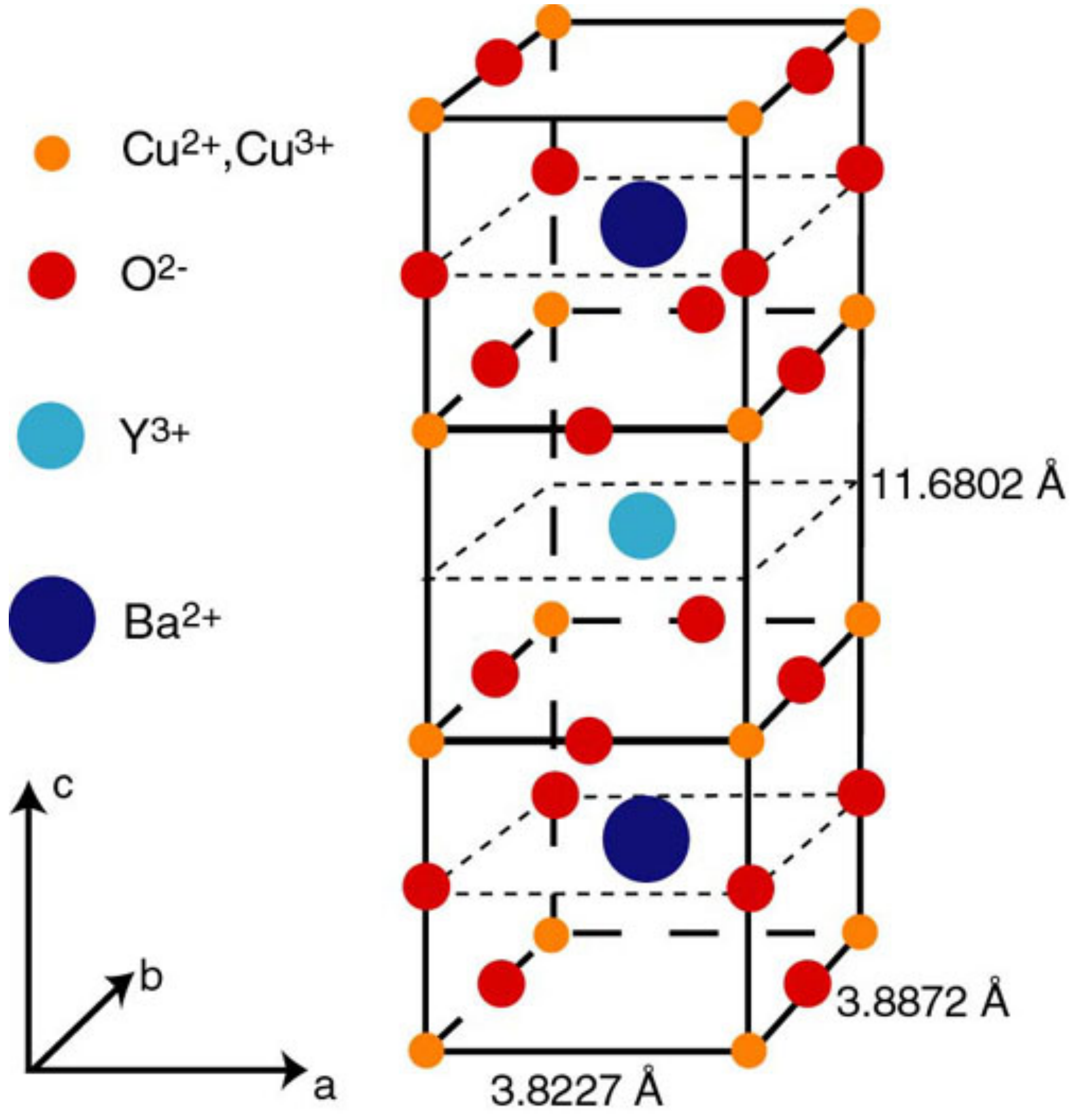
4. FL* theory of the pseudogap metal of the cuprates

5. Nodal fermionic quasiparticles in d-wave SC

6. Quantum oscillations in hole-doped cuprates

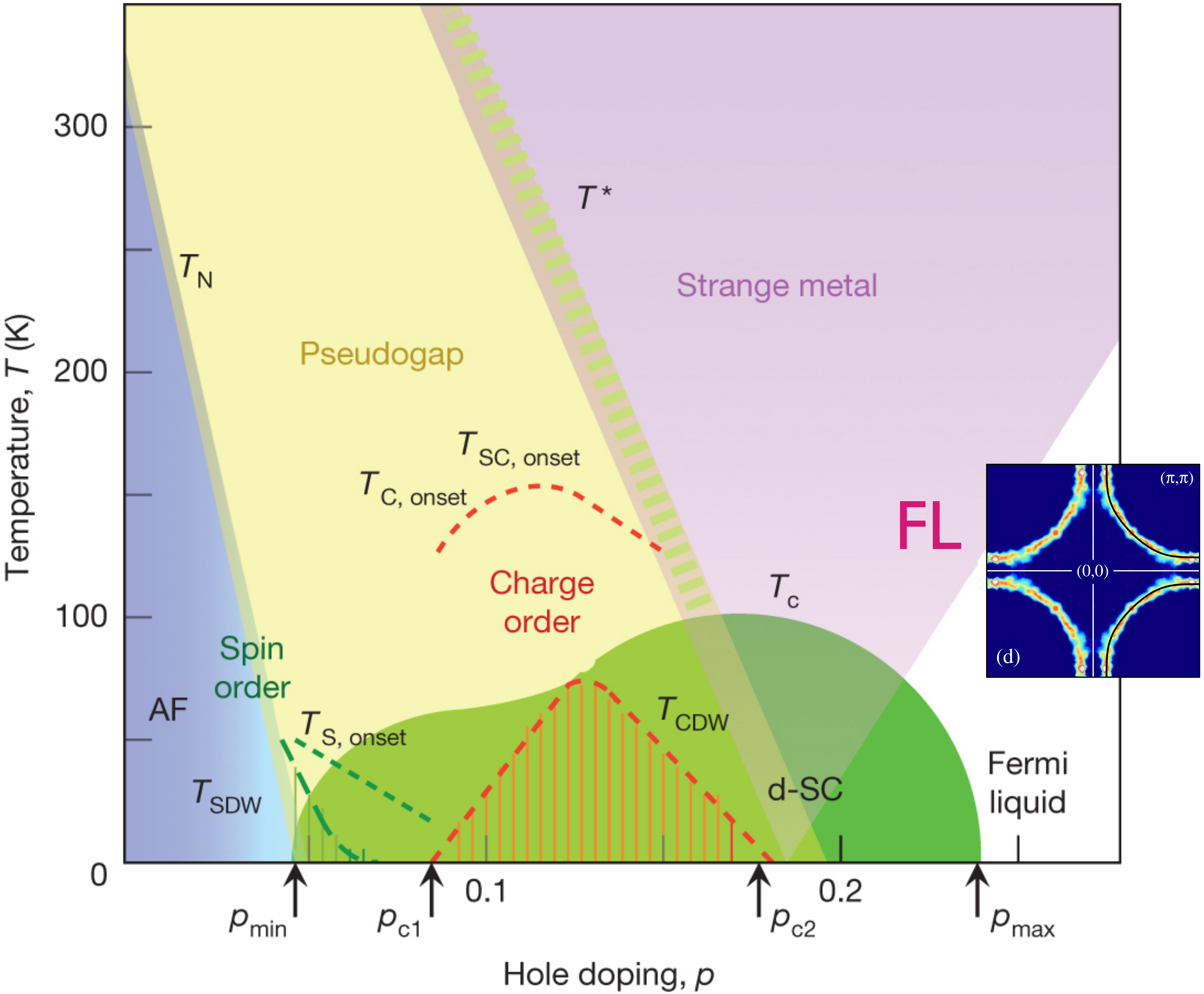
Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)





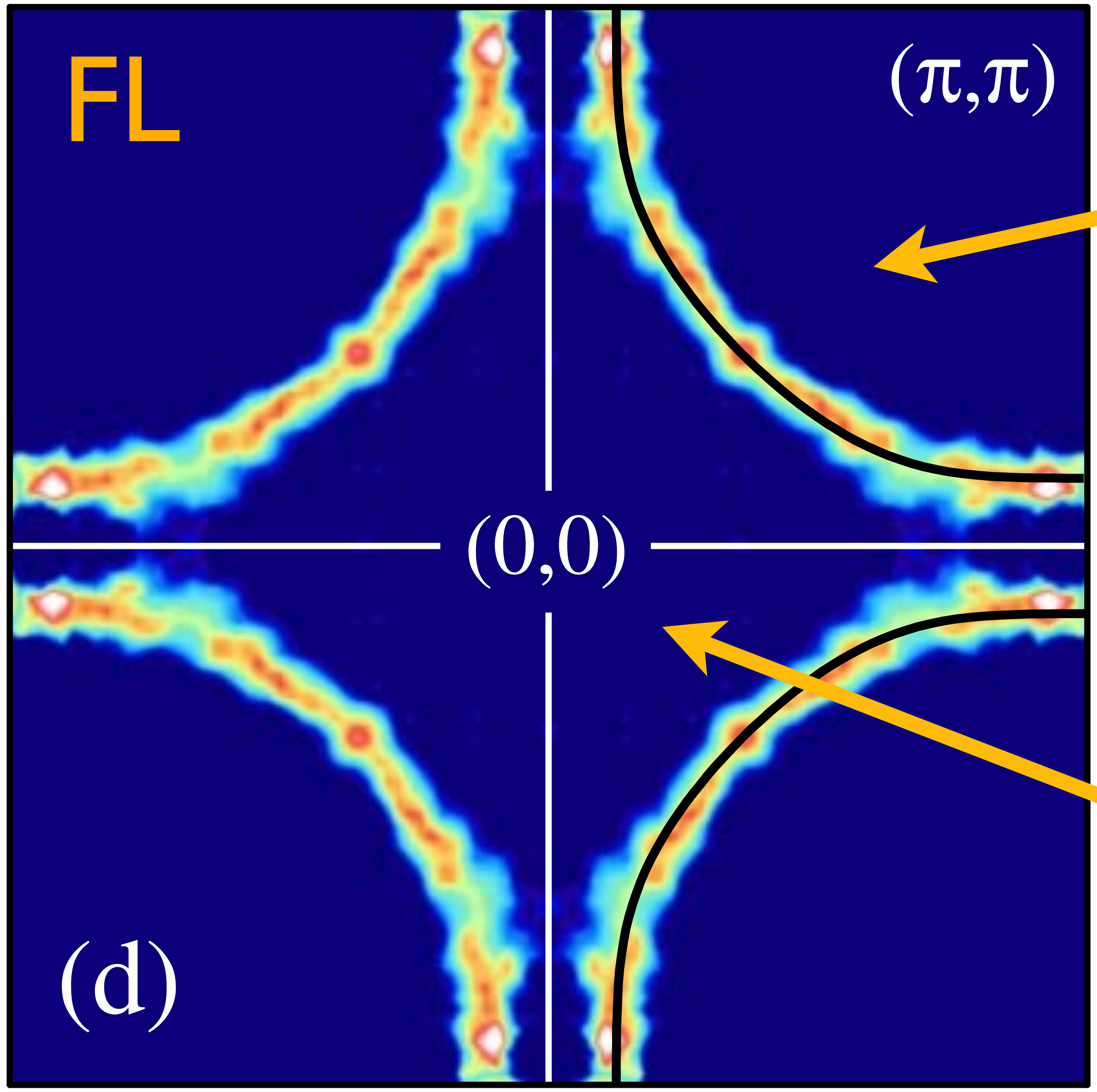
$YBa_2Cu_3O_{6+x}$

Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)



Fermi liquid
in the
overdoped metal

Photoemission at large p



$1+p$ holes

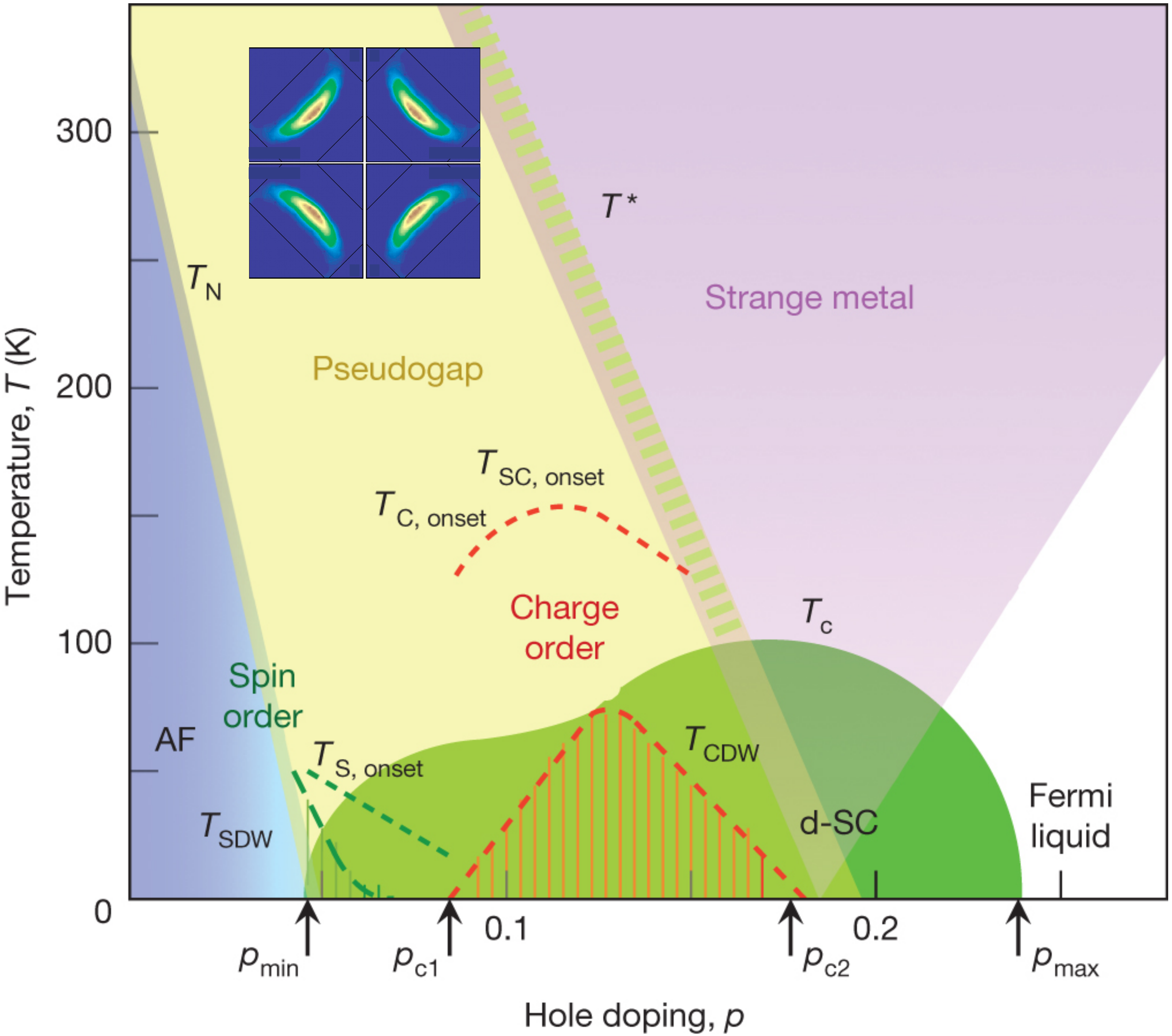
1-p electrons

Overdoped $Tl_2Ba_2CuO_{6+\delta}$
$T_c = 30K$

$1+p$ mobile holes in a filled band of 2 electrons per site

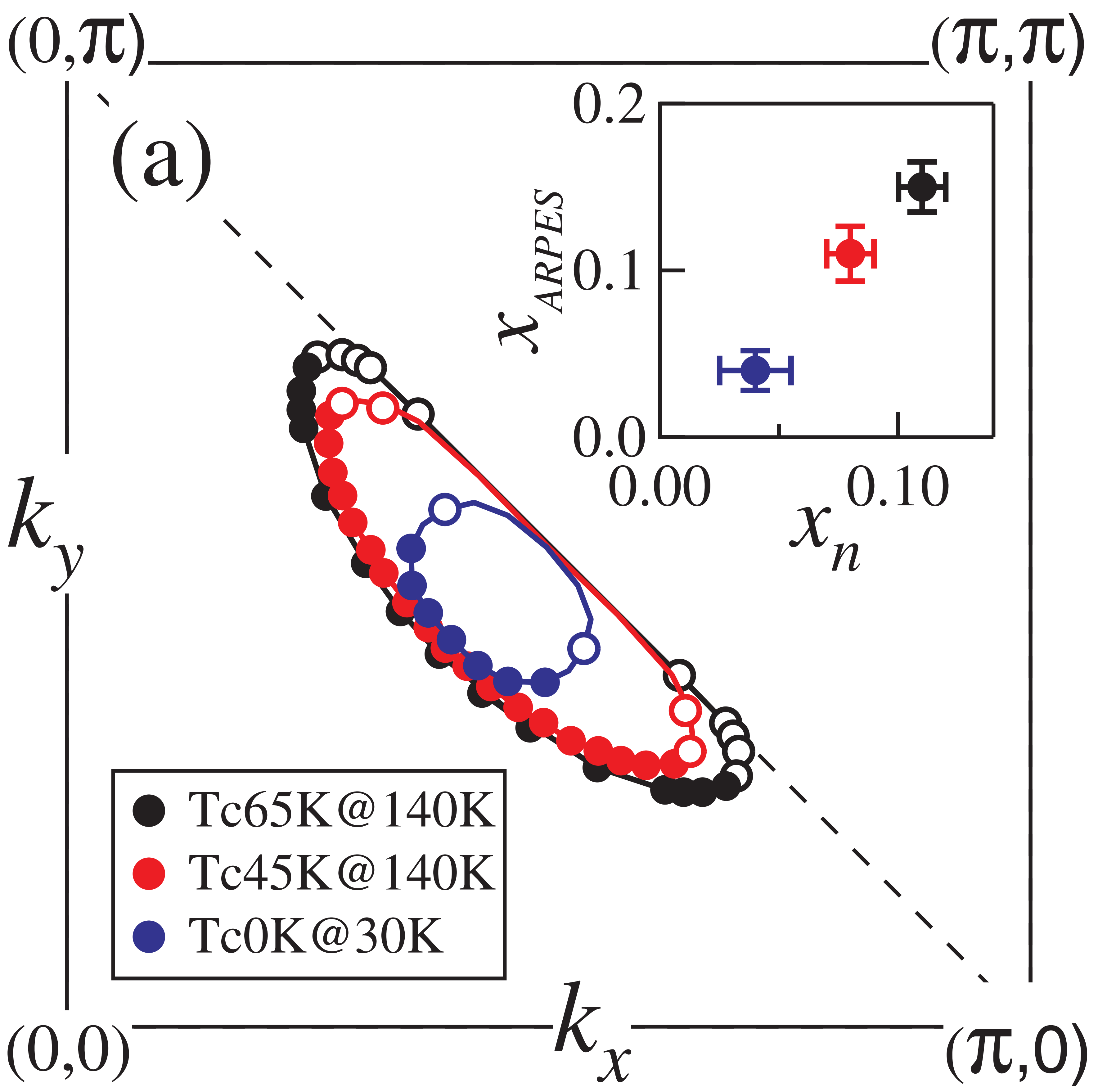
M. Platé, J. D. F. Mottershead, I. S. Elfimov, D. C. Peets, Ruixing Liang, D. A. Bonn, W. N. Hardy, S. Chiuzbaian, M. Falub, M. Shi, L. Patthey, and A. Damascelli, Phys. Rev. Lett. **95**, 077001 (2005)

Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)

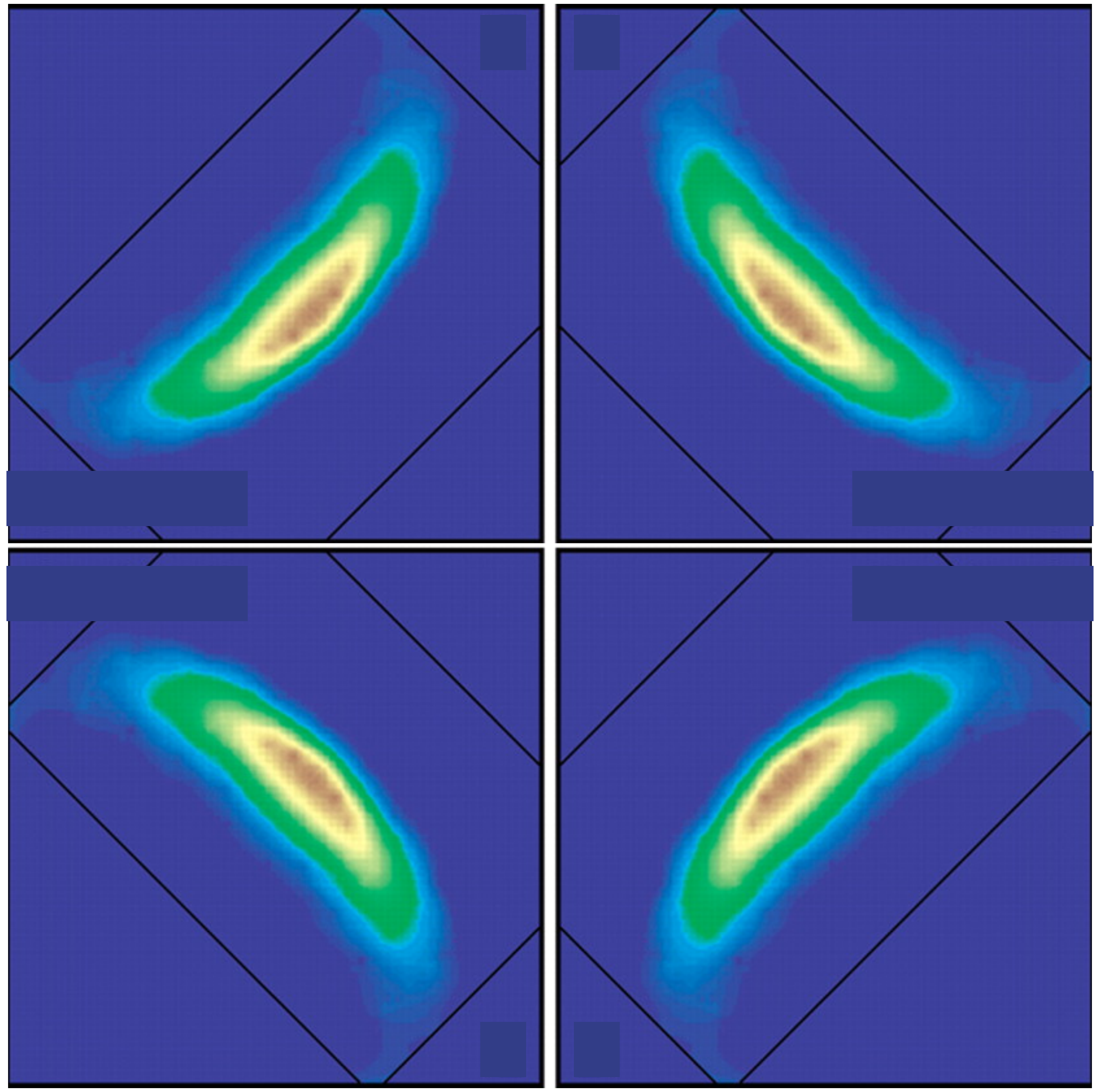


Pseudogap metal with "Fermi arcs"

Photoemission expts at small p



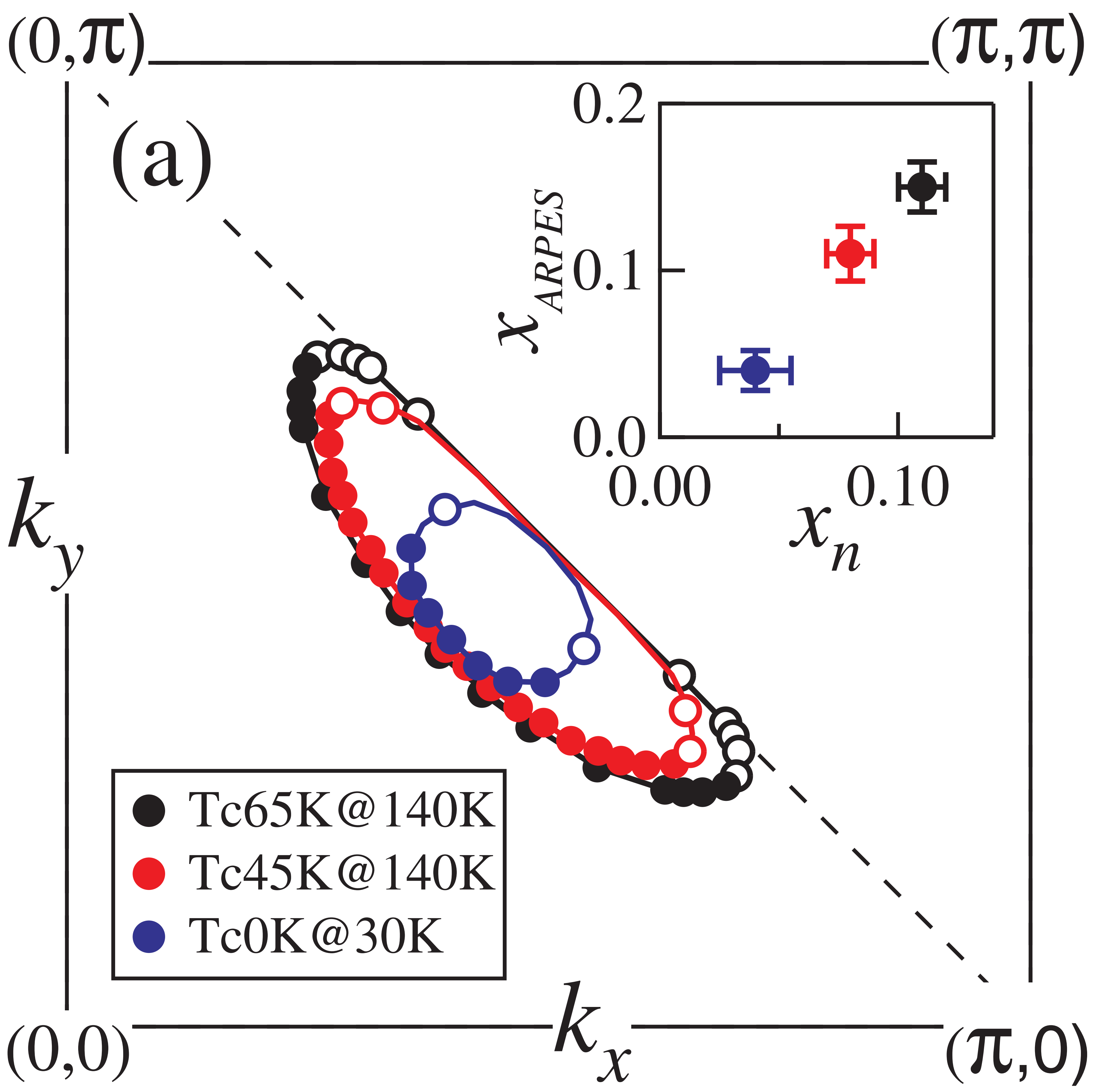
Reconstructed Fermi Surface of Underdoped $Bi_2Sr_2CaCu_2O_{8+\delta}$ Cuprate Superconductors, H.-B. Yang, J. D. Rameau, Z.-H. Pan, G. D. Gu, P. D. Johnson, H. Claus, D. G. Hinks, and T. E. Kidd, PRL **107**, 047003 (2011).



$Ca_{2-x}Na_xCuO_2Cl_2$ at $x = 0.10$

Kyle M. Shen, F. Ronning, D. H. Lu, F. Baumberger, N. J. C. Ingle, W. S. Lee, W. Meevasana, Y. Kohsaka, M. Azuma, M. Takano, H. Takagi, Z.-X. Shen, Science **307**, 901 (2005)

Photoemission expts at small p

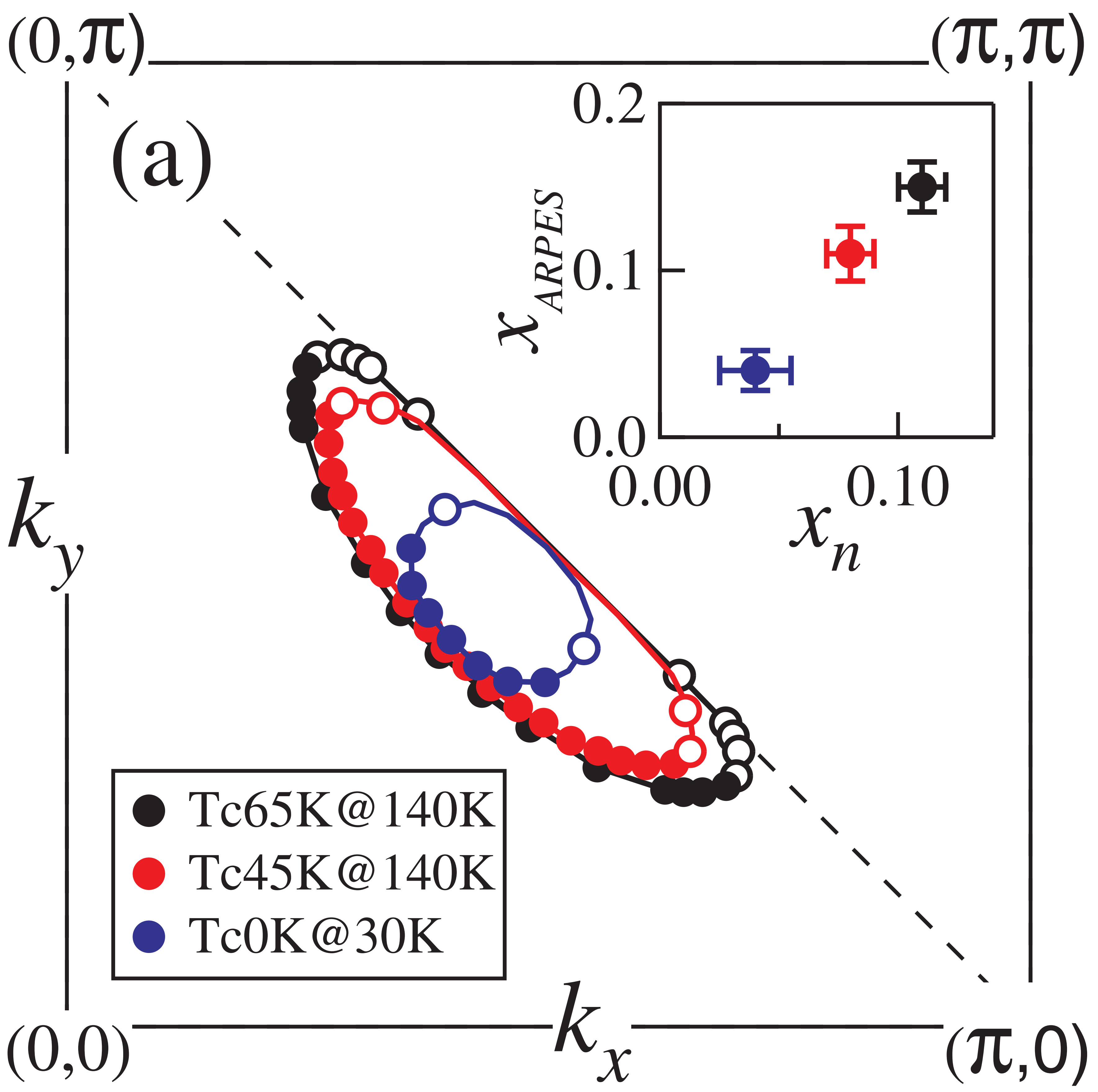


Reconstructed Fermi Surface of Underdoped $Bi_2Sr_2CaCu_2O_{8+\delta}$ Cuprate Superconductors, H.-B. Yang, J. D. Rameau, Z.-H. Pan, G. D. Gu, P. D. Johnson, H. Claus, D. G. Hinks, and T. E. Kidd, PRL **107**, 047003 (2011).

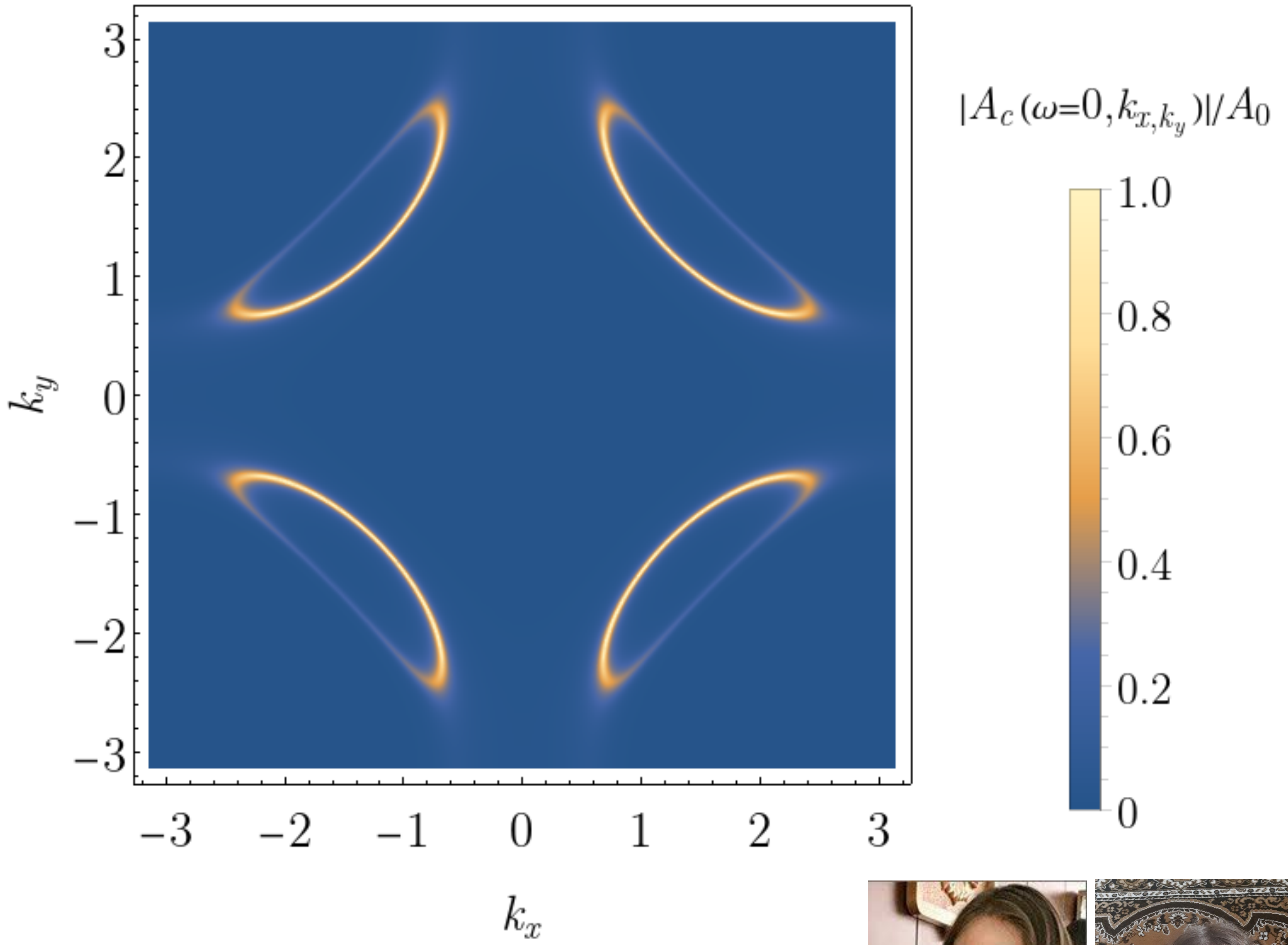
Kai-Yu Yang, T. M. Rice, Fu-Chun Zhang, *Phys. Rev. B* **73**, 174501 (2006).
T. D. Stanescu and G. Kotliar, *Phys. Rev. B* **74**, 125110 (2006).
C. Berthod, T. Giamarchi, S. Biermann, and A. Georges, *Phys. Rev. Lett.* **97**, 136401 (2006).
S. Sakai, Y. Motome, M. Imada, *Phys. Rev. Lett.* **102**, 056404 (2009).
J. Skolimowski and M. Fabrizio, *Phys. Rev. B* **106**, 045109 (2022).
N. Wagner….A. Georges, G. Sangiovanni, *Nature Communication* **14**, 7531 (2023)
Jinchao Zhao, Gabriele La Nave, Philip Phillips, *Phys. Rev. B* **108**, 165135
Jing-Yu Zhao, Zheng-Yu Weng, arXiv:2309.11556

Ancilla theory of photoemission

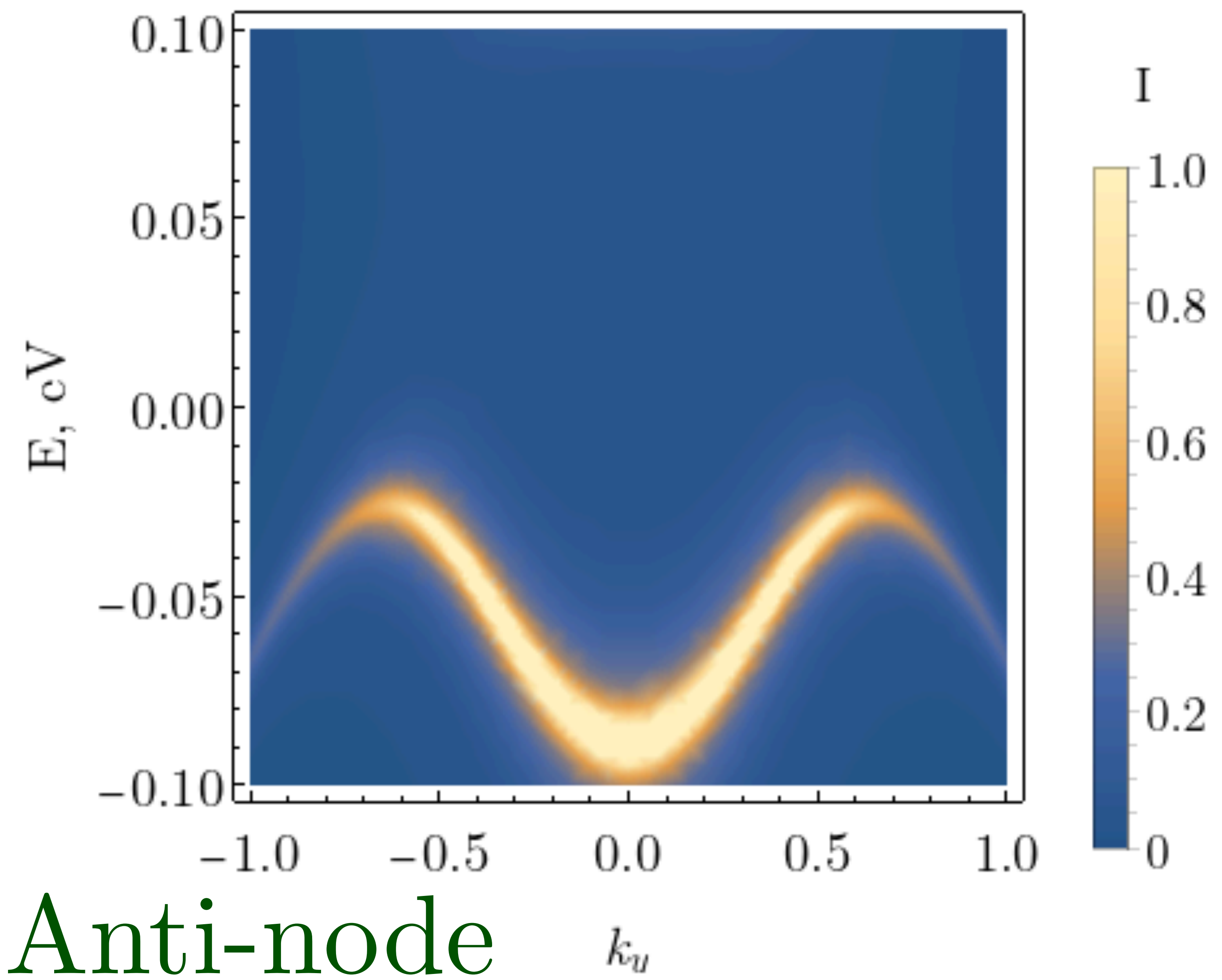
E. Mascot, A. Nikolaenko, M. Tikhanovskaya, Ya-Hui Zhang, D. K. Morr, and S. S., PRB **105**, 075146 (2022)



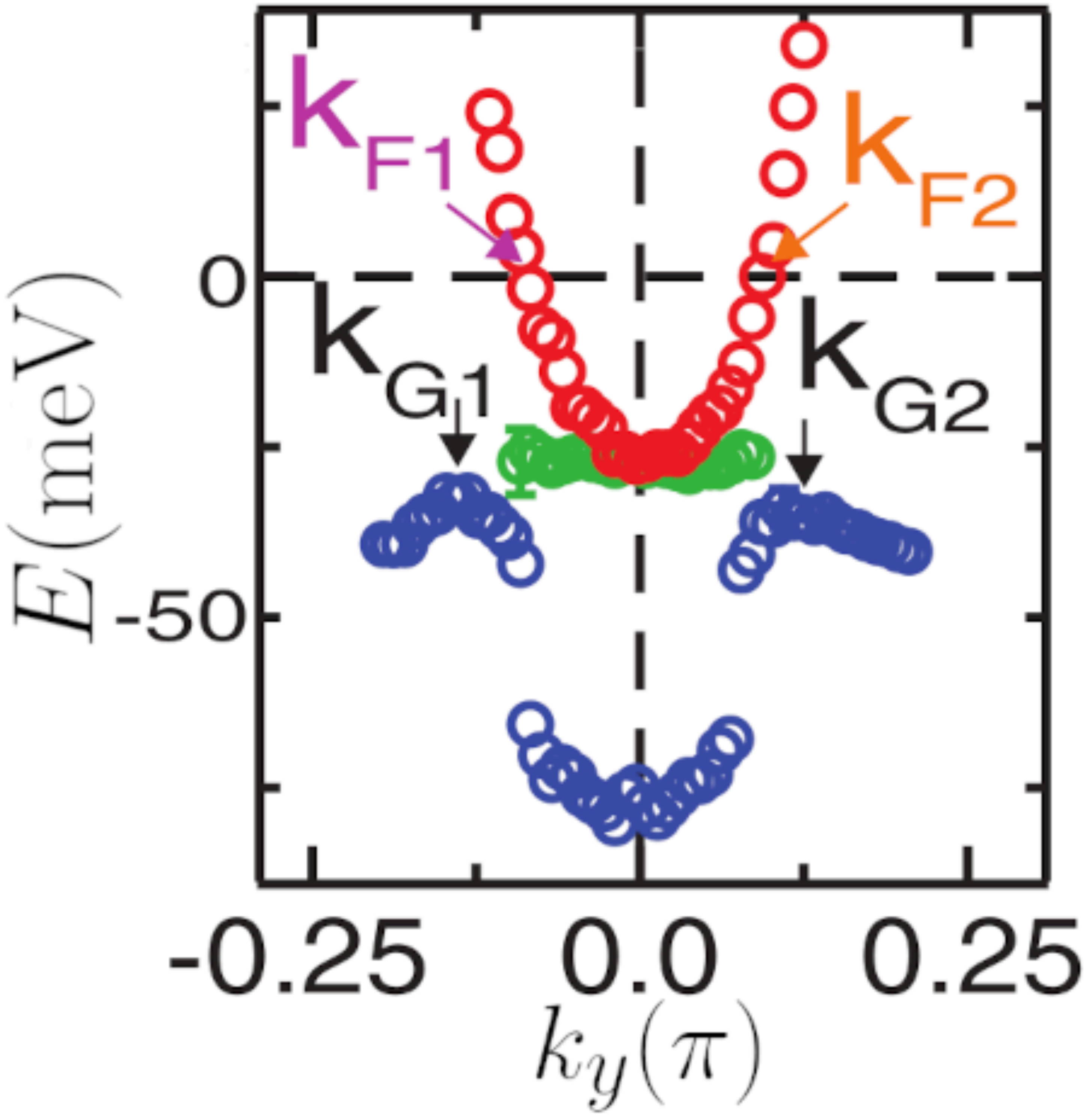
Reconstructed Fermi Surface of Underdoped $Bi_2Sr_2CaCu_2O_{8+\delta}$ Cuprate Superconductors,
H.-B. Yang, J. D. Rameau, Z.-H. Pan, G. D. Gu, P. D. Johnson, H. Claus, D. G. Hinks, and T. E. Kidd, PRL **107**, 047003 (2011).



$$H_{\rm mf} = -\sum_{i,j} t_{ij} c_{i\sigma}^\dagger c_{j\sigma} - \sum_{i,j} t_{1,ij} f_{1i\sigma}^\dagger f_{1j\sigma} - \sum_i \Phi \left(c_{i\sigma}^\dagger f_{1i\sigma} + f_{1i\sigma}^\dagger c_{i\sigma}\right)$$



Anti-node



E. Mascot, A. Nikolaenko, M. Tikhanovskaya, Ya-Hui Zhang, D. K. Morr, and S. S., PRB **105**, 075146 (2022)

Ancilla theory of photoemission

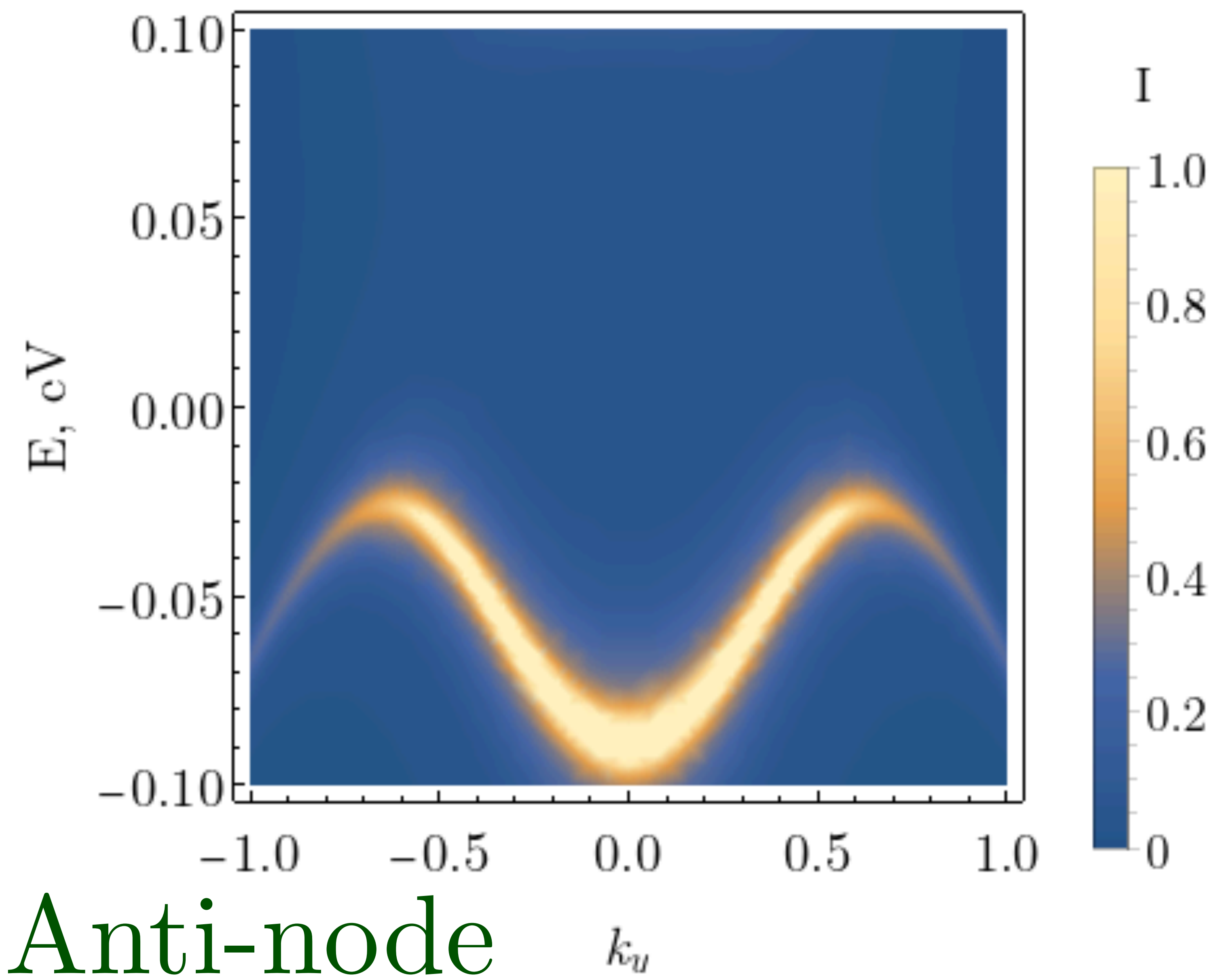
$$H_{\mathrm{mf}} = -\sum_{i,j} t_{ij} c_{i\sigma}^{\dagger} c_{j\sigma} - \sum_{i,j} t_{1,ij} f_{1i\sigma}^{\dagger} f_{1j\sigma} - \sum_{i} \Phi \left(c_{i\sigma}^{\dagger} f_{1i\sigma} + f_{1i\sigma}^{\dagger} c_{i\sigma} \right)$$

R.-H. He, M. Hashimoto, H. Karapetyan, J. D. Koralek, J. P. Hinton, J. P. Testaud, V. Nathan, Y. Yoshida, H. Yao, K. Tanaka, W. Meevasana, R. G. Moore, D. H. Lu, S. K. Mo, M. Ishikado, H. Eisaki, Z. Hussain, T. P. Devereaux, S. A. Kivelson, J. Orenstein, A. Kapitulnik, and Z.-X. Shen, *Science* **331**, 1579 (2011)

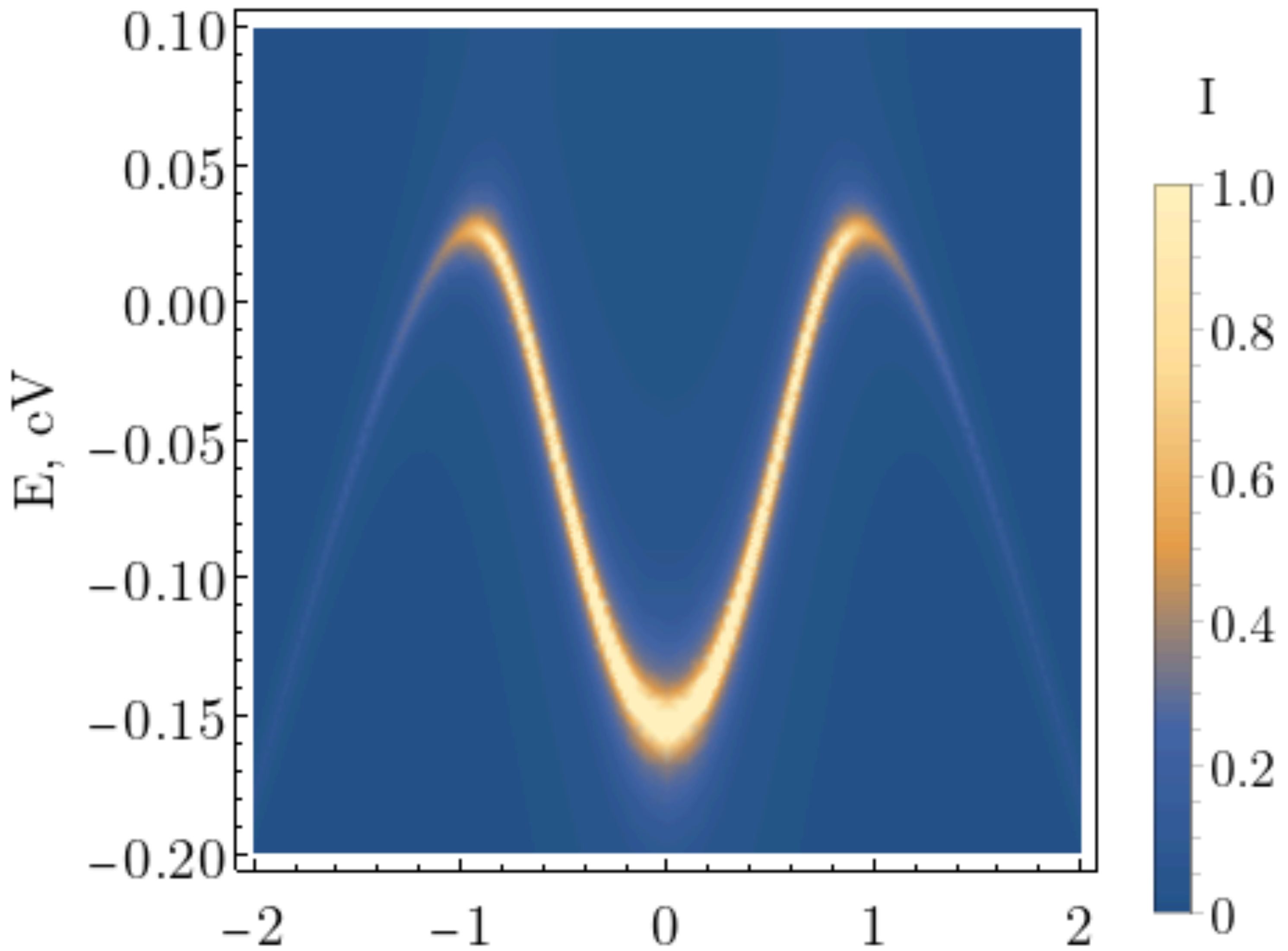
ARPES on Bi2201

E. Mascot, A. Nikolaenko, M. Tikhanovskaya, Ya-Hui Zhang, D. K. Morr, and S. S., PRB **105**, 075146 (2022)

Ancilla theory of photoemission

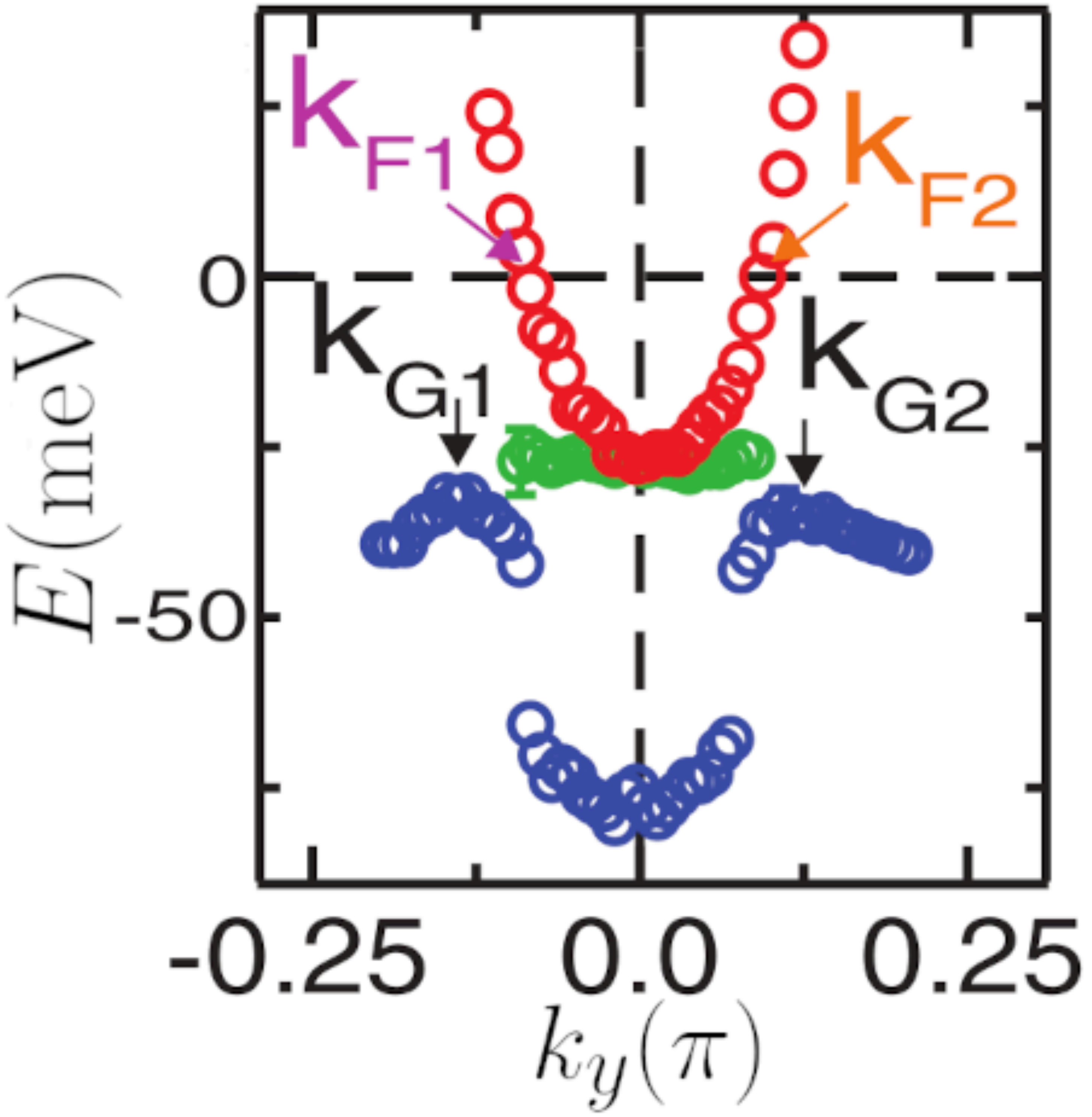


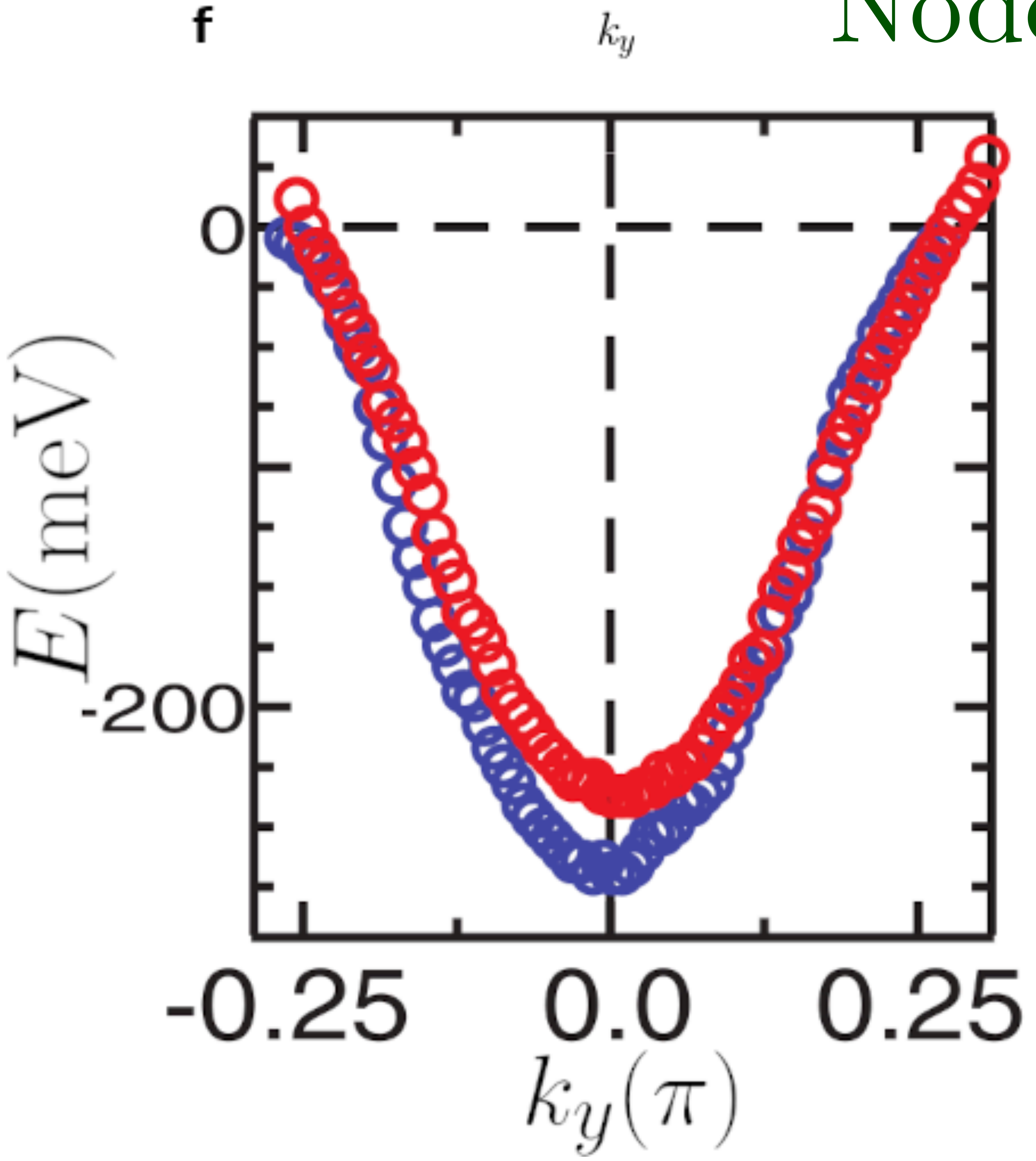
Anti-node



Node

$$H_{\mathrm{mf}} = -\sum_{i,j} t_{ij} c_{i\sigma}^\dagger c_{j\sigma} - \sum_{i,j} t_{1,ij} f_{1i\sigma}^\dagger f_{1j\sigma} - \sum_i \Phi \left(c_{i\sigma}^\dagger f_{1i\sigma} + f_{1i\sigma}^\dagger c_{i\sigma} \right)$$

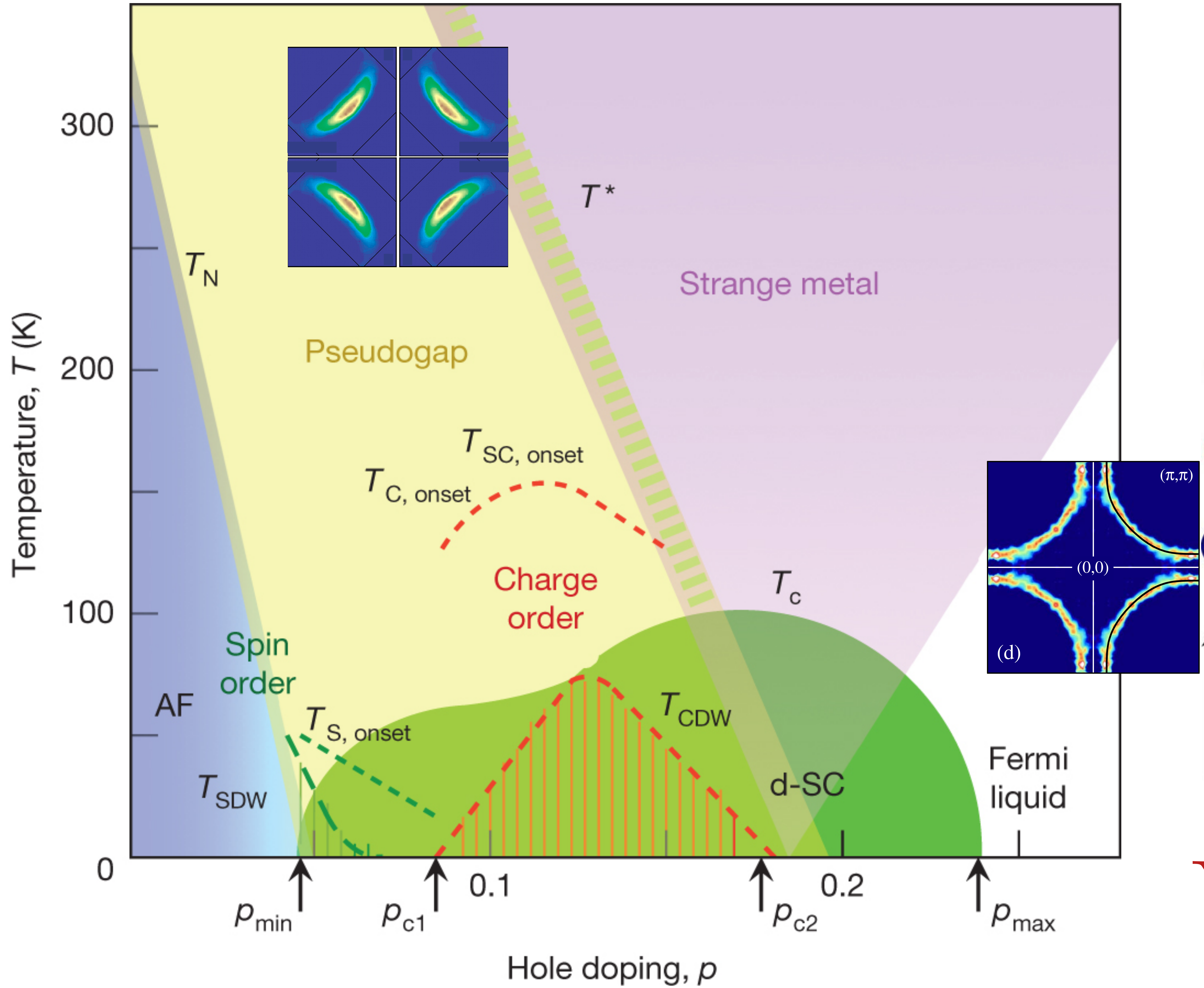


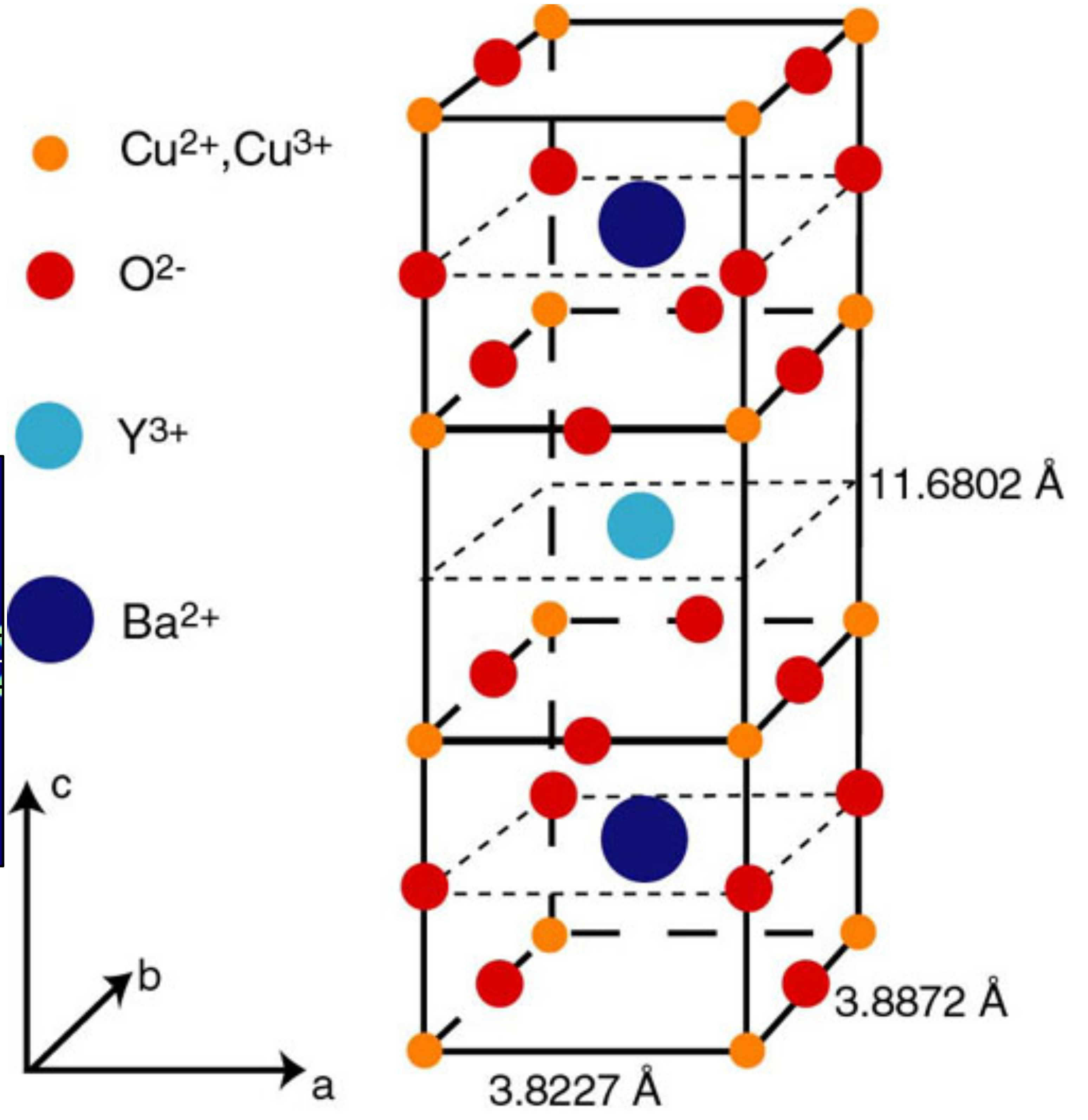


R.-H. He, M. Hashimoto, H. Karapetyan, J. D. Koralek, J. P. Hinton, J. P. Testaud, V. Nathan, Y. Yoshida, H. Yao, K. Tanaka, W. Meevasana, R. G. Moore, D. H. Lu, S. K. Mo, M. Ishikado, H. Eisaki, Z. Hussain, T. P. Devereaux, S. A. Kivelson, J. Orenstein, A. Kapitulnik, and Z.-X. Shen, *Science* **331**, 1579 (2011)

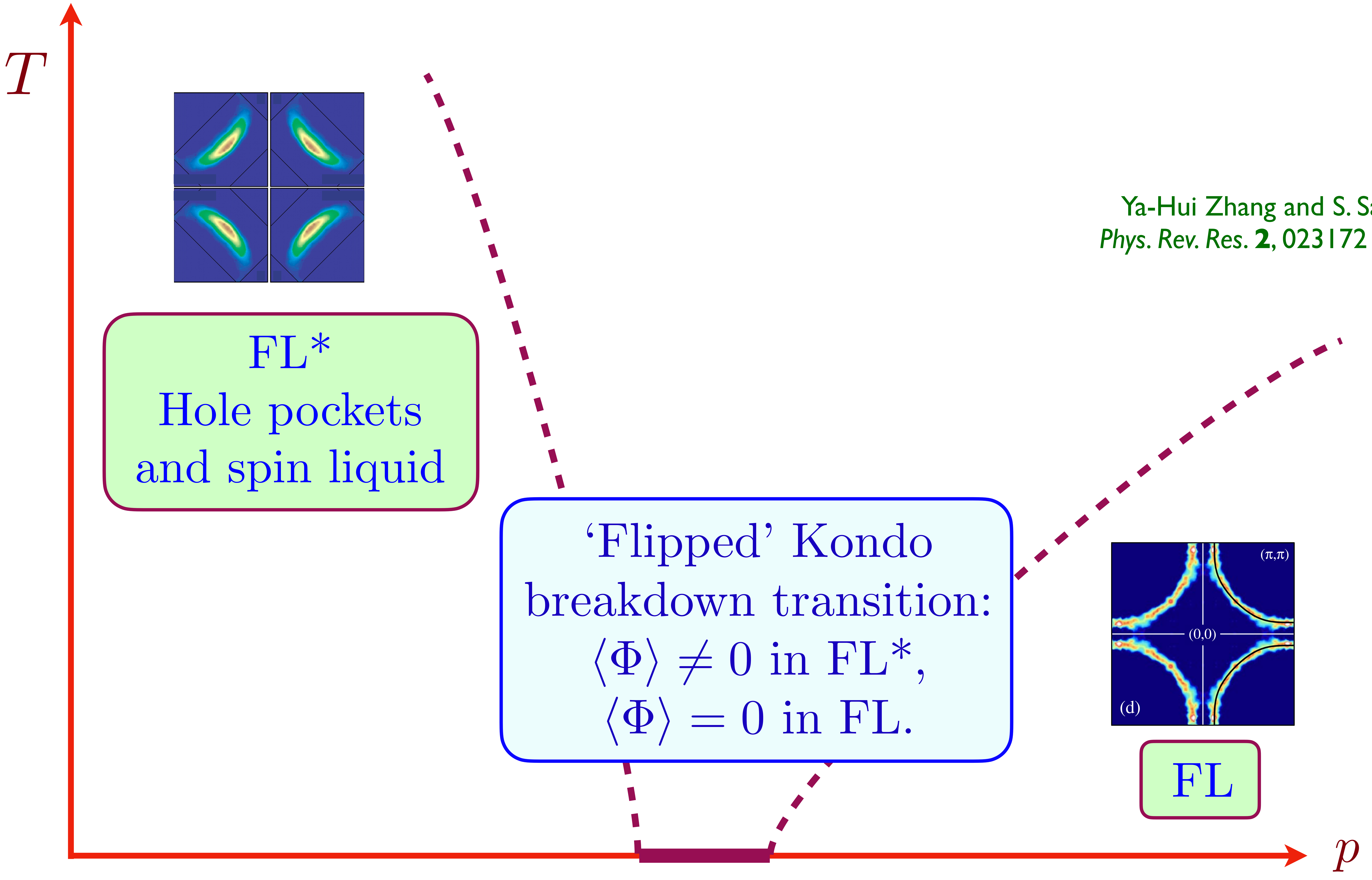
ARPES on Bi2201

Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)





$YBa_2Cu_3O_{6+x}$



Ya-Hui Zhang and S. Sachdev,
Phys. Rev. Res. **2**, 023172 (2020)

Universal 2d-Yukawa-Sachdev-Ye-Kitaev theory
for strange metal in the presence of disorder.
Confinement transition from FL* to FL.
Aavishkar A. Patel, Haoyu Guo,
Ilya Esterlis, S. Sachdev,
Science **381**, 790 (2023)
T
Higgs boson Φ
FL*
Hole pockets
and spin liquid
'Flipped' Kondo
breakdown transition:
$\langle \Phi \rangle \neq 0$ in FL*,
$\langle \Phi \rangle = 0$ in FL.
(π,π)
$(0,0)$
(d)
FL
p

1. Square lattice spin liquids

2. Spin liquids on the Kondo lattice:
non-Luttinger volume Fermi surfaces (FL*)

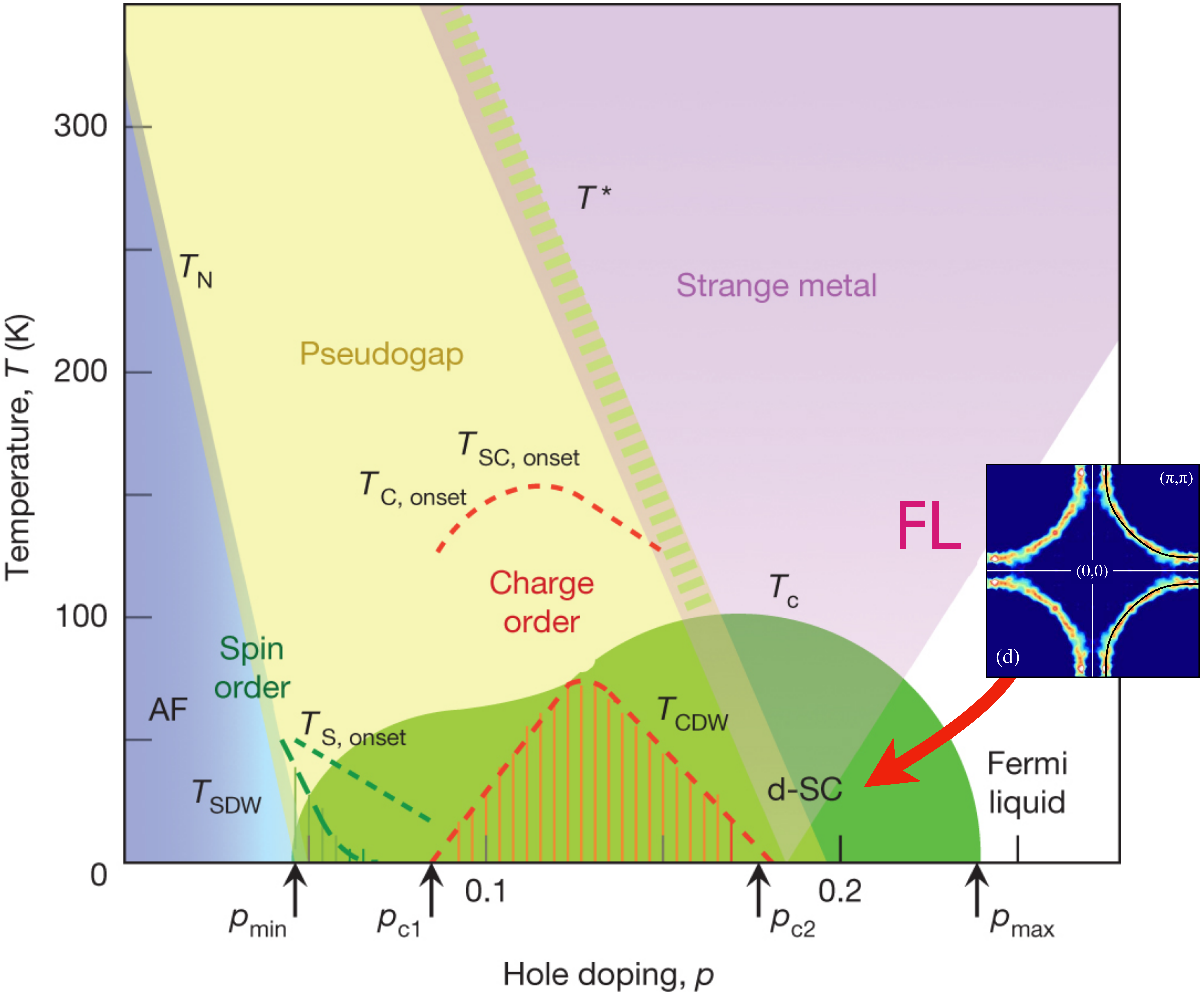
3. Doping square lattice spin liquids for t >> J:
FL* in a single-band model

4. FL* theory of the pseudogap metal of the cuprates

5. Nodal fermionic quasiparticles in d-wave SC

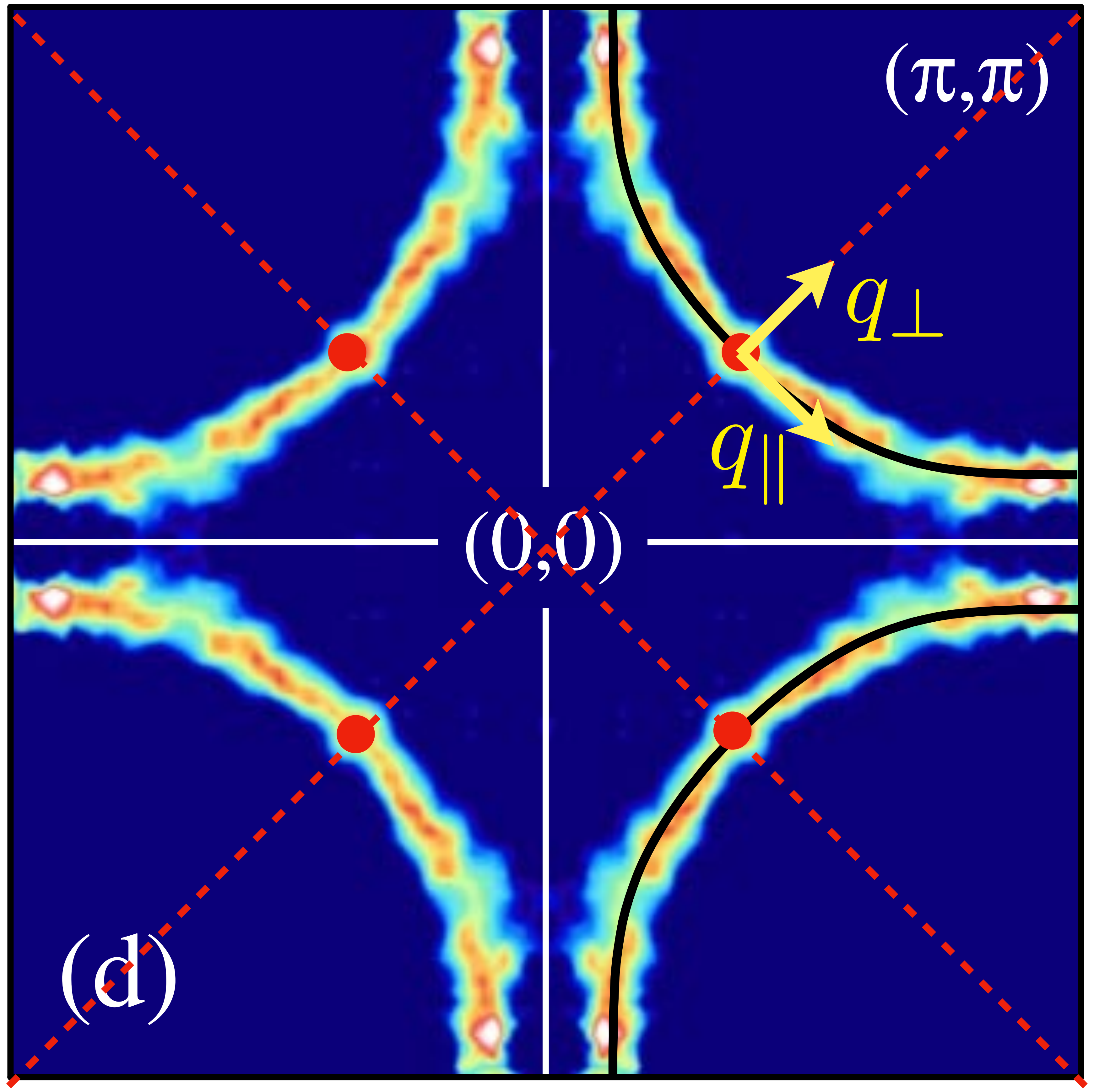
6. Quantum oscillations in hole-doped cuprates

Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)



BCS-type theory of *d*-wave superconductivity (and charge order) induced by antiferromagnetic spin fluctuations.

FL→dSC



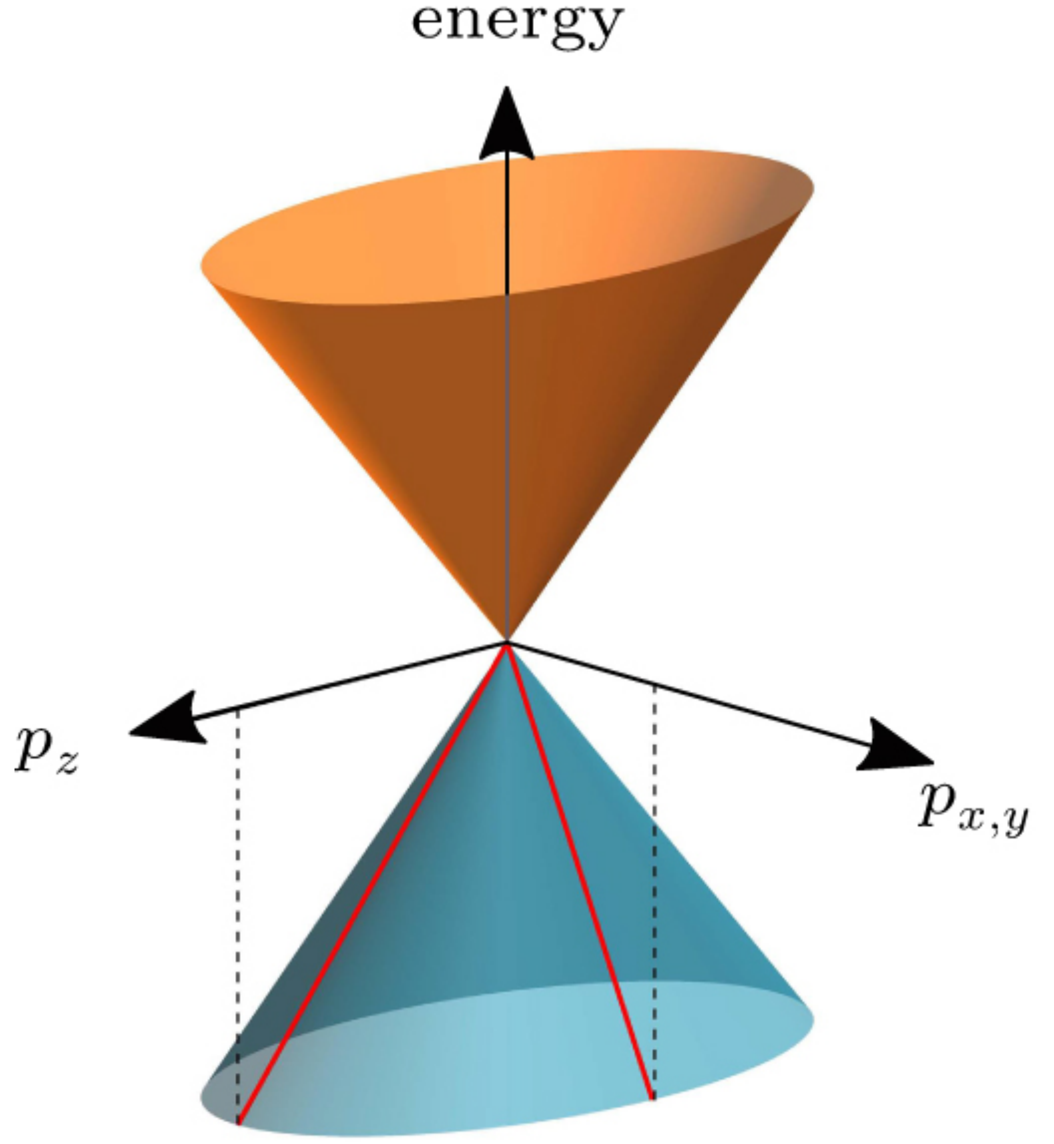
BCS/Bogoliubov quasiparticles
in a d-wave superconductor

$$E_{\boldsymbol{k}} = \left(\varepsilon_{\boldsymbol{k}}^2 + \Delta_{\boldsymbol{k}}^2\right)^{1/2}$$
$$\Delta_{\boldsymbol{k}} = \Delta_0 \left(\cos k_x - \cos k_y\right)$$

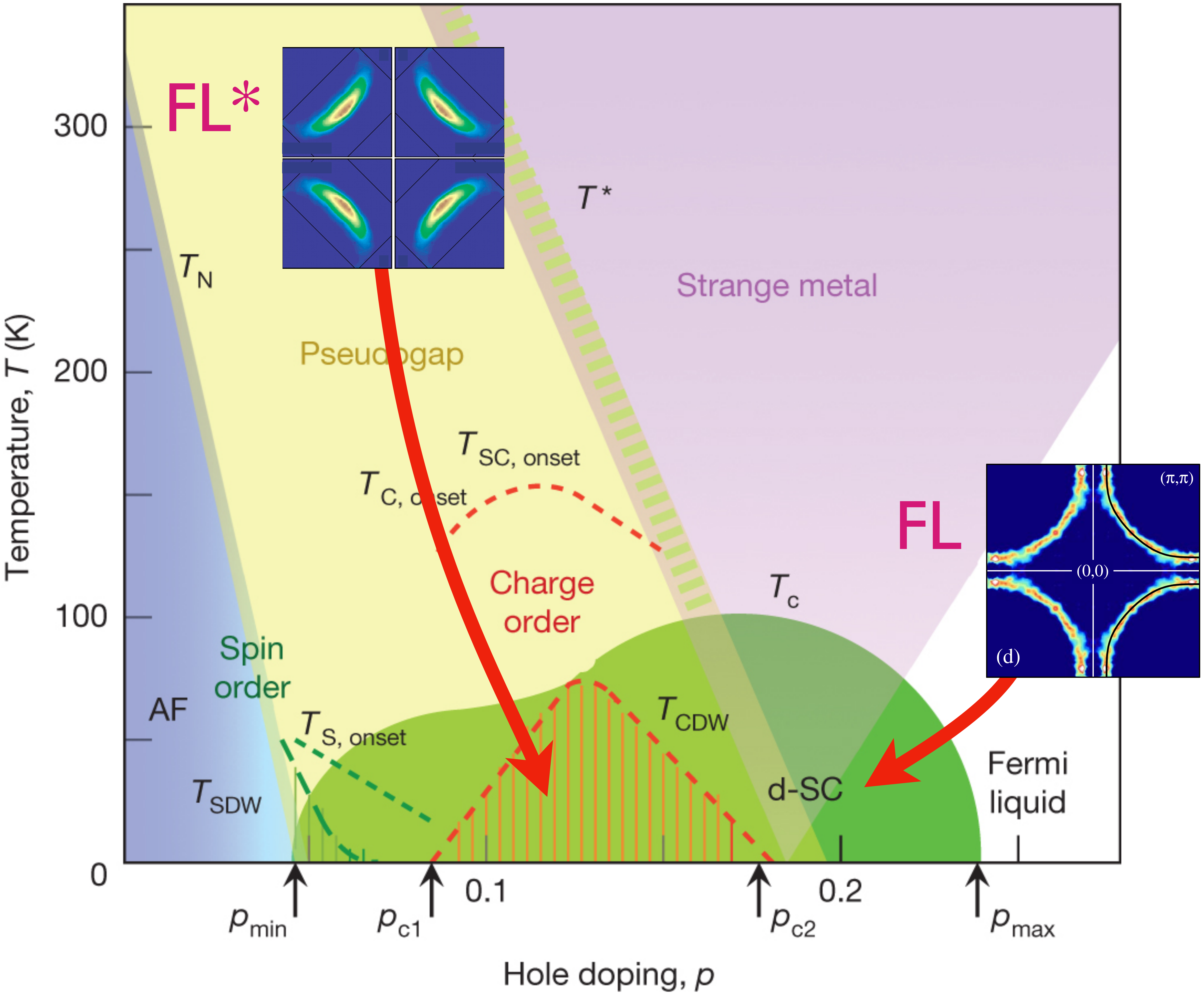
4 nodal points where

$$E_{\boldsymbol{k}_0+\boldsymbol{q}} = \left(v_F^2 q_\perp^2 + v_\Delta^2 q_\parallel^2\right)^{1/2}$$

with $v_F \gg v_\Delta$.



Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)



Obtain d-wave superconductor and charge order from a theory of *confinement* instabilities of FL*.

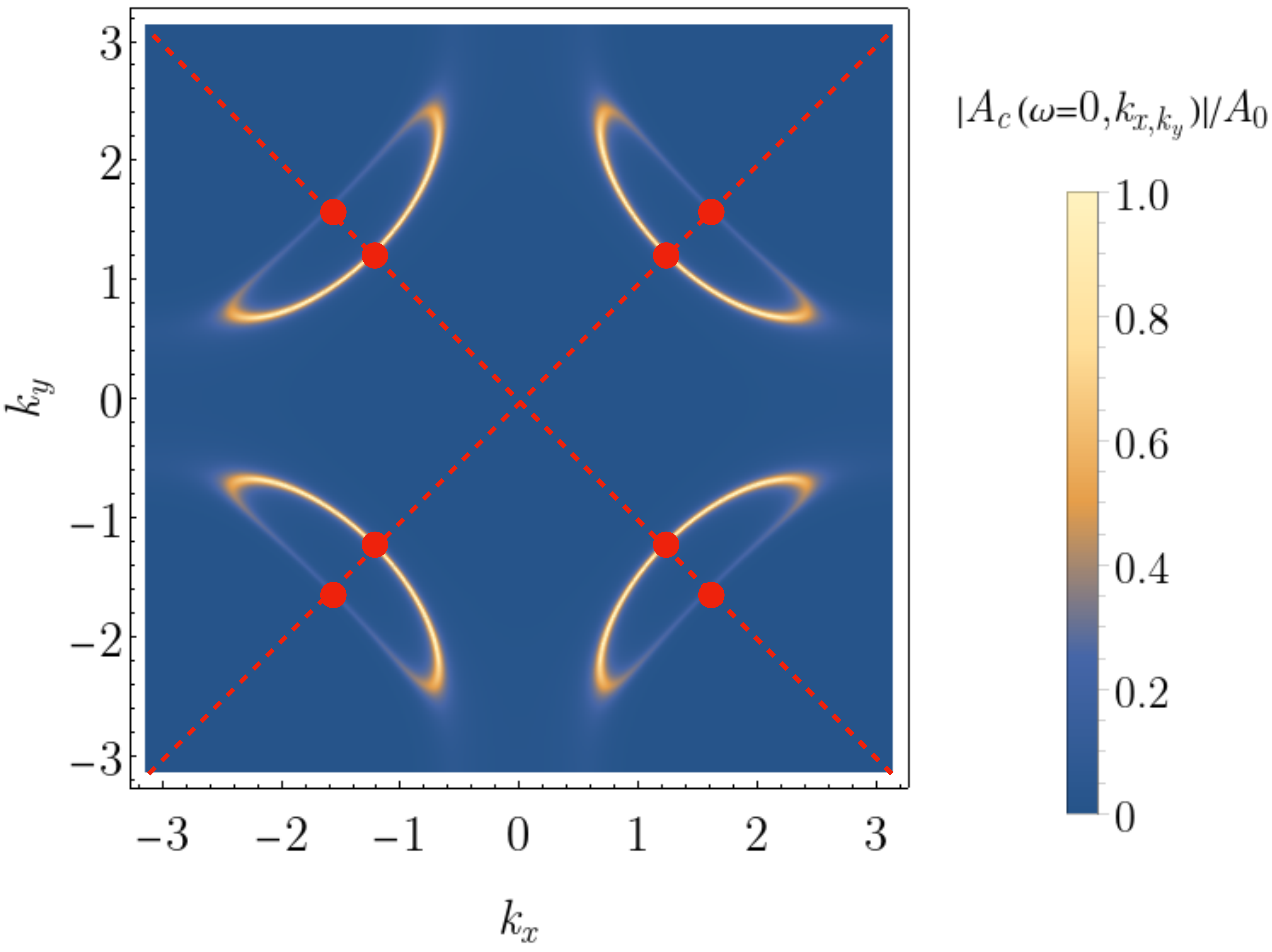
The resulting low T ordered states should be adiabatically connected to the corresponding states obtained from instabilities of FL.

Early work described the emergence of nodal d-wave superconductivity from a spin liquid with spinons with a Dirac dispersion: in this approach, the spinons of the spin liquid are directly transformed into the nodal Bogoliubov quasiparticles of the d-wave superconductor.

F.C. Zhang, C. Gros, T.M. Rice and H. Shiba, Superconductor Sci. Tech. **1** (1988) 36 [cond-mat/0311604].
D. A. Ivanov, T. Senthil, PRB **66**, 115111 (2002).

This leads to a d-wave superconductor with $v_F \approx v_\Delta$, in contrast to observations.

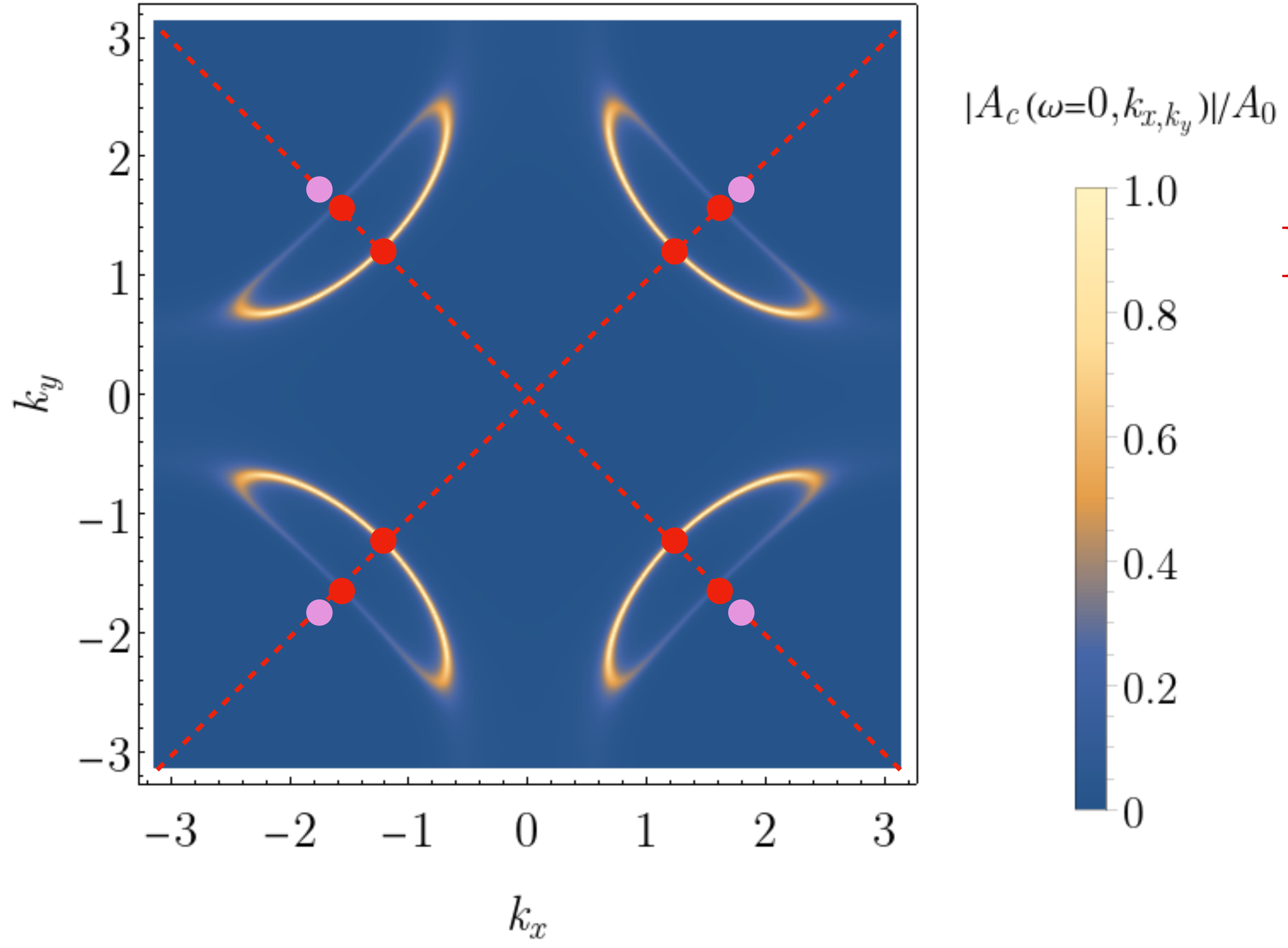
FL*→dSC*



$$E_{\boldsymbol{k}} = \left(\varepsilon_{\boldsymbol{k}}^2 + \Delta_{\boldsymbol{k}}^2\right)^{1/2}$$
$$\Delta_{\boldsymbol{k}} = \Delta_0 \left(\cos k_x - \cos k_y\right)$$

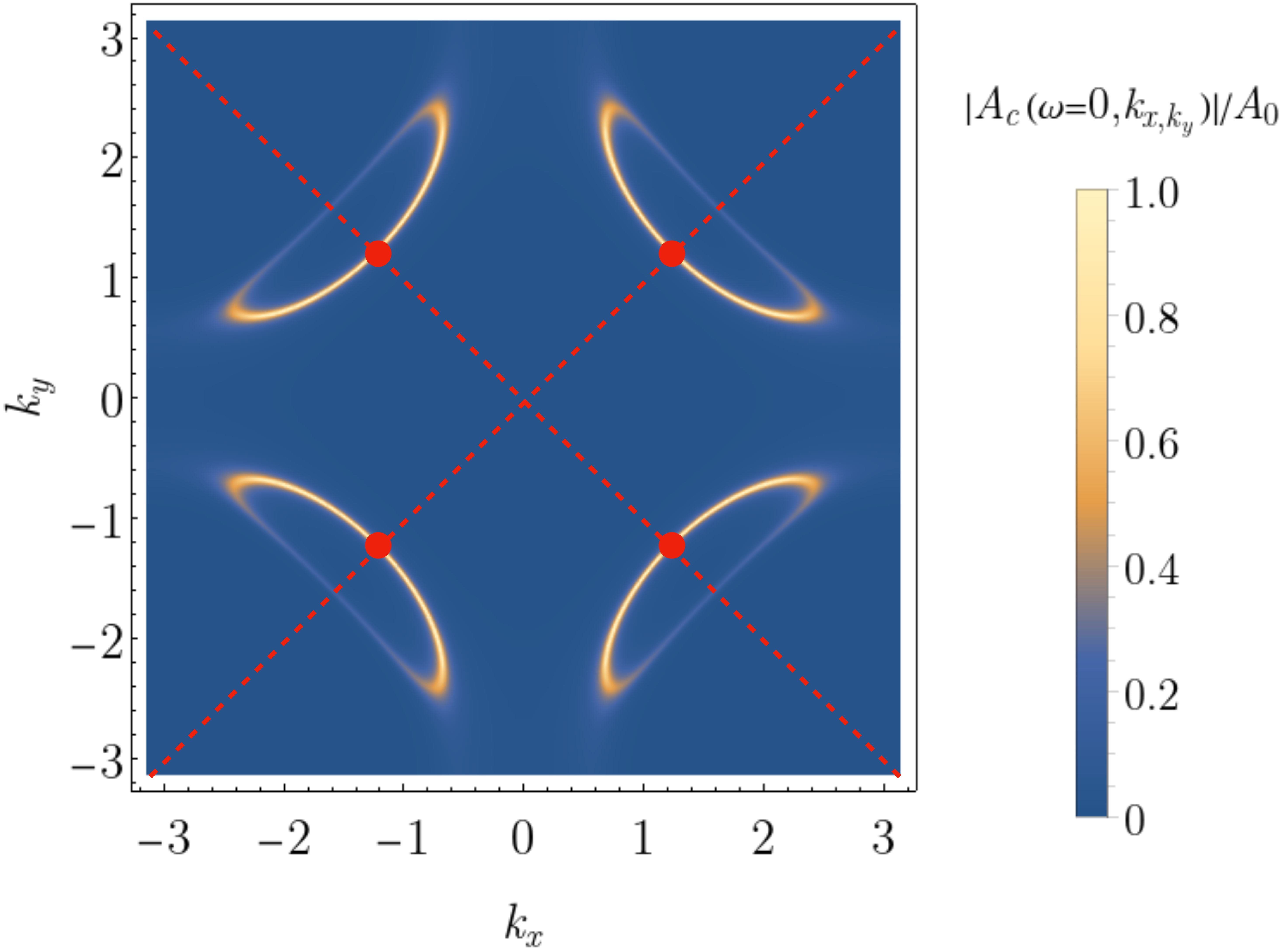
Adding d-wave pairing
to the hole pockets
leads to 8 nodal points???

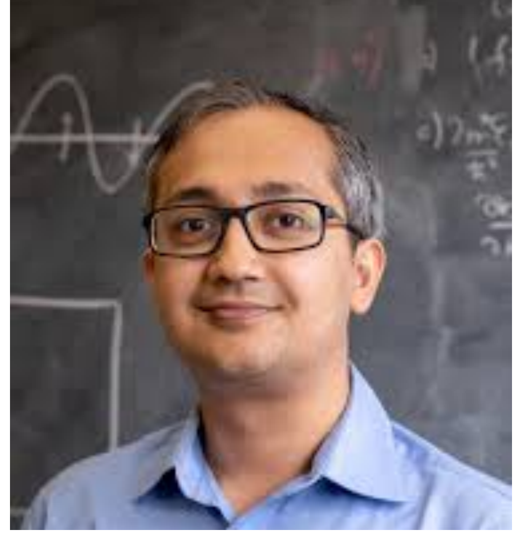
FL*→dSC*



8 nodal points of
Bogoliubov quasiparticles
from the Fermi pockets
and
4 nodal points of
spinons from
the π-flux spin liquid

FL*→dSC



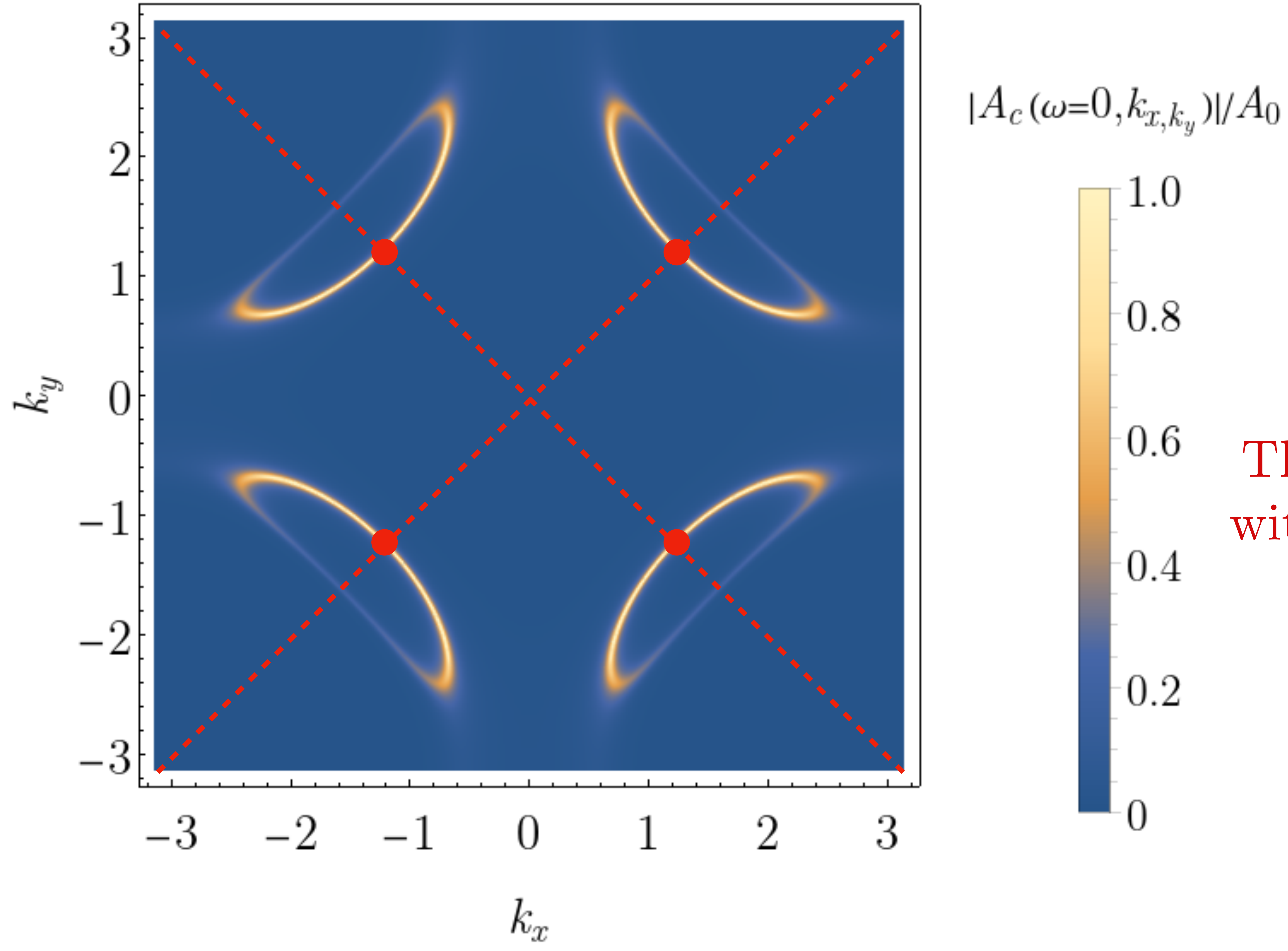


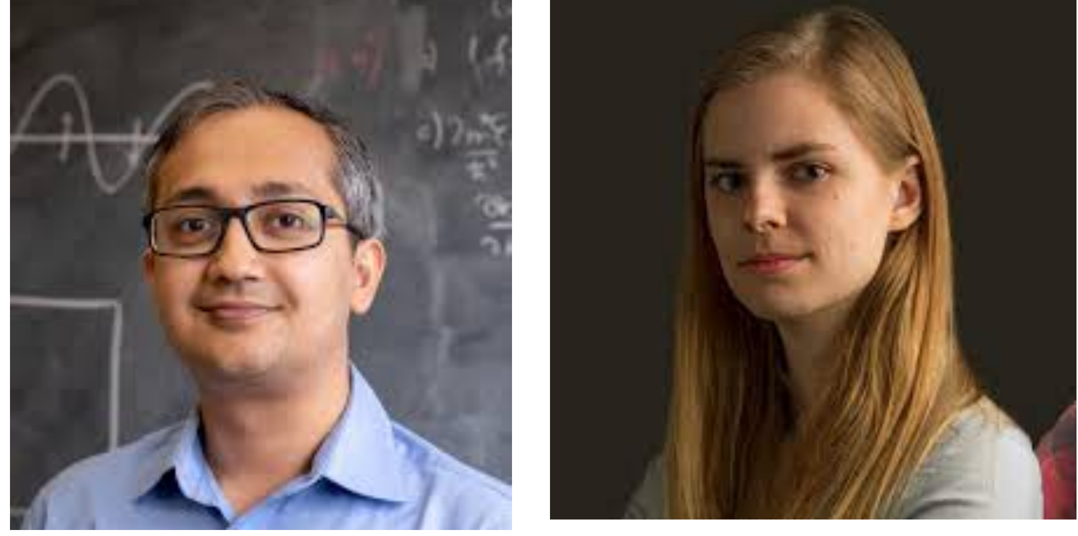
A B Higgs condensate allows spinons and Bogoliubov quasiparticles to hybridize:

8 nodal points annihilate each other, leaving 4 nodal points with anisotropic velocities, just as in a BCS d-wave state.

Shubhayu Chatterjee and S. Sachdev, PRB **94**, 205117 (2016)
Maine Christos and S.Sachdev, npj Quantum Materials **9**, 4 (2024)

FL*→dSC





The spinons do *not* become the Bogoliubov quasiparticles, they *annihilate* the unwanted Bogoliubov quasiparticles. This leads to a d-wave superconductor with 4 nodal Bogoliubov quasiparticles, with $v_F \gg v_\Delta$, consistent with observations.

Shubhayu Chatterjee and S. Sachdev, PRB **94**, 205117 (2016)
Maine Christos and S.Sachdev, npj Quantum Materials **9**, 4 (2024)

Ancilla theory of the Hubbard model

Ya-Hui Zhang and S. Sachdev,
Phys. Rev. Res. **2**, 023172 (2020)

1. Choose Néel-VBS DQCP spin liquid: f_σ spinons moving in π-flux coupled to SU(2)$_N$ gauge field. Note $\Phi \sim f_{1\sigma}^\dagger c_\sigma$ is condensed.

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Phys. Rev. Res. **2**, 023172 (2020)

1. Choose Néel-VBS DQCP spin liquid: f_σ spinons moving in π-flux coupled to $\mathrm{SU}(2)_N$ gauge field. Note $\Phi \sim f^\dagger_{1\sigma} c_\sigma$ is condensed.

2. Confine spin liquid by condensing charge e, $\mathrm{SU}(2)_N$ fundamental, Higgs boson

$$B \sim \begin{pmatrix} f^\dagger_{1\sigma} f_\sigma \\ \varepsilon_{\sigma\sigma'} f^\dagger_{1\sigma} f^\dagger_{\sigma'} \end{pmatrix}$$

Boson with same quantum numbers in X.-G. Wen and P.A. Lee, PRL **76**, 503 (1996)

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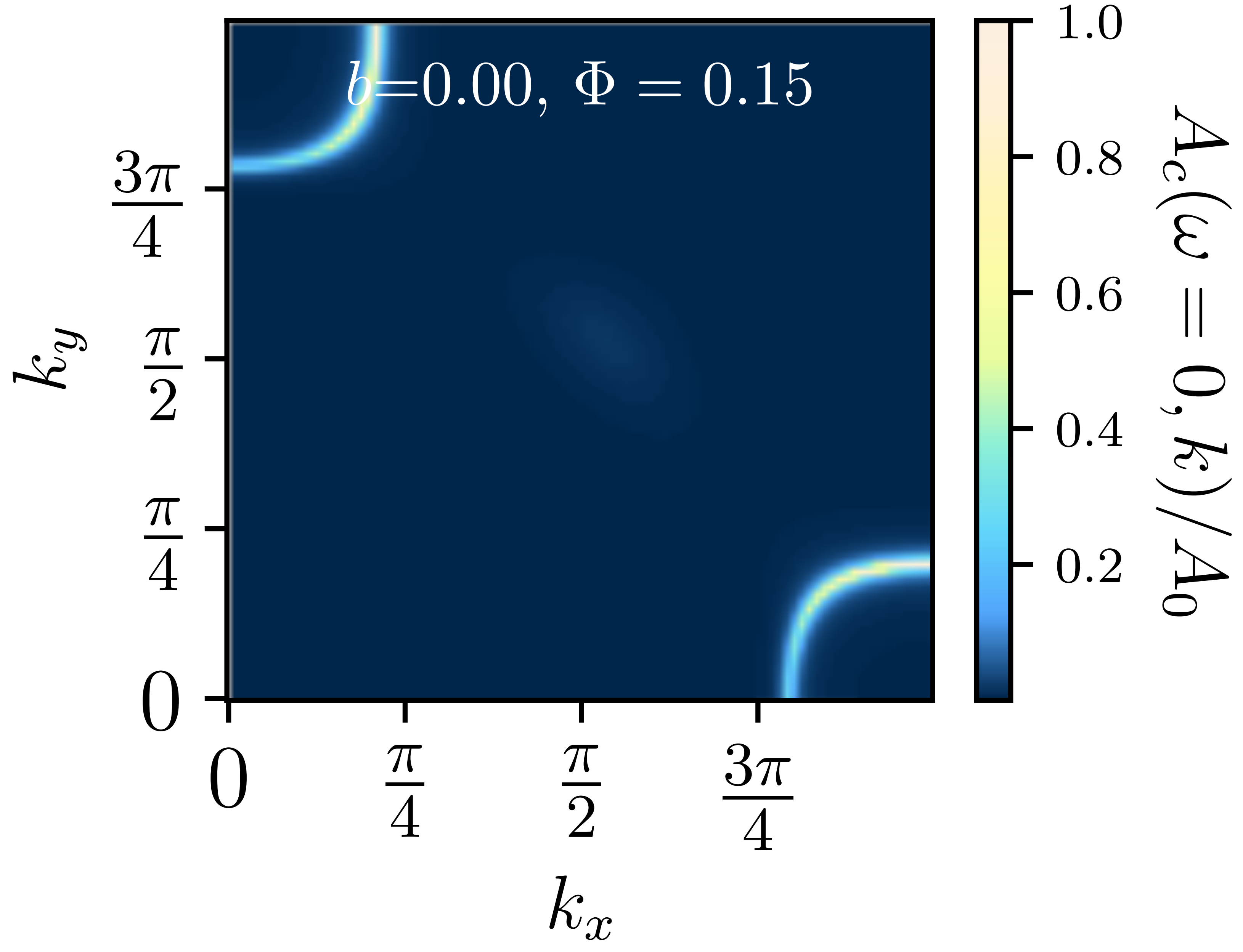
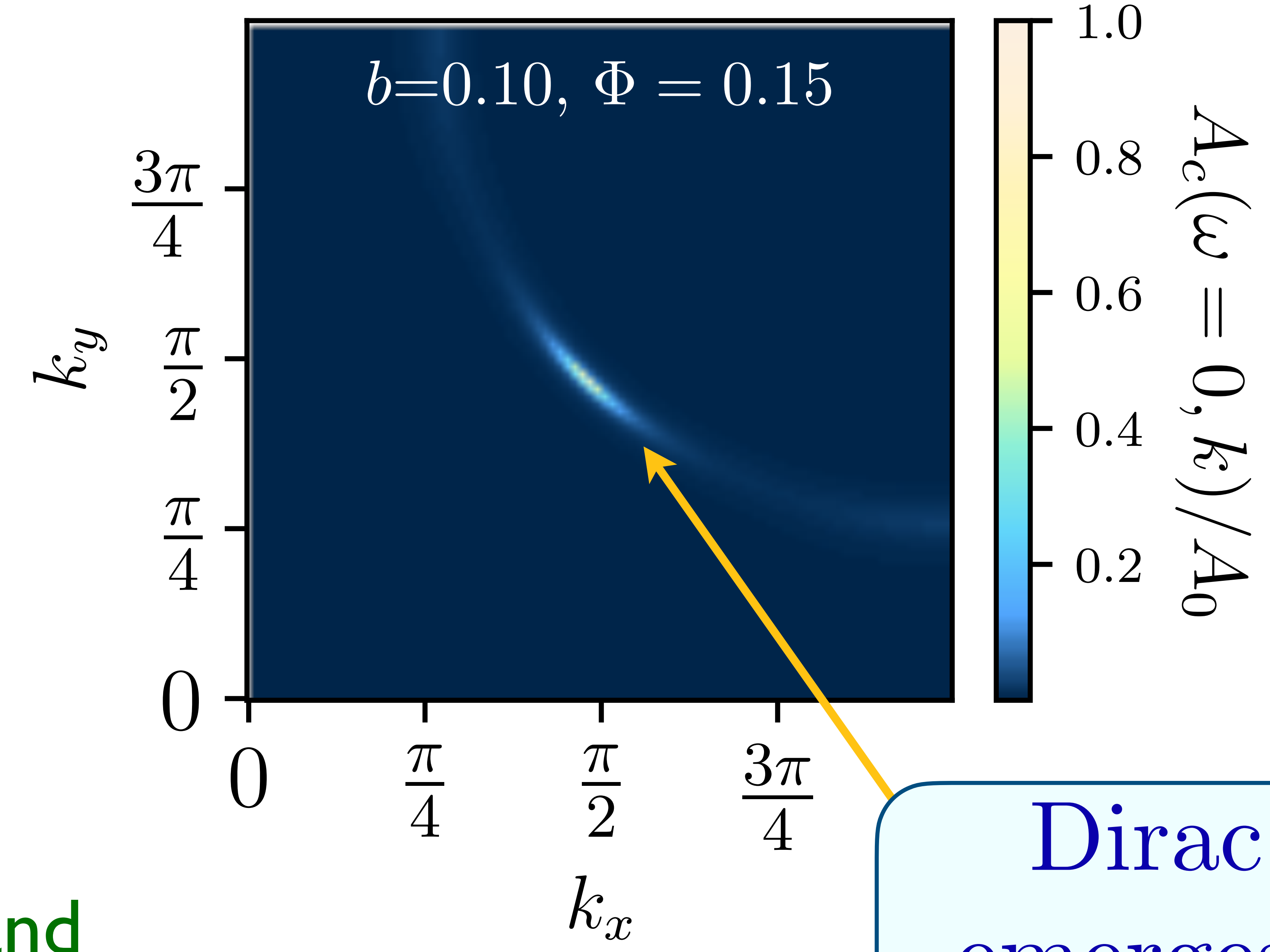
Ya-Hui Zhang and S. Sachdev,
Phys. Rev. Res. **2**, 023172 (2020)

$$H_{\rm mf} = -\sum_{i,j} t_{ij} c_{i\sigma}^\dagger c_{j\sigma} - \sum_{i,j} t_{1,ij} f_{1i\sigma}^\dagger f_{1j\sigma} - \sum_{i,j} \widetilde{t}_{ij} f_{i\sigma}^\dagger f_{j\sigma}$$

$$- \sum_i \Phi \left(c_{i\sigma}^\dagger f_{1i\sigma} + f_{1i\sigma}^\dagger c_{i\sigma} \right) - \sum_i \left(B_{1i}^* f_{1i\sigma}^\dagger f_{i\sigma} + B_{2i}^* \varepsilon_{\sigma\sigma'} f_{1i\sigma}^\dagger f_{i\sigma'}^\dagger + \text{H.c.} \right)$$

Electron spectral density in electron-doped cuprates





Dirac node emerges inside normal state gap

Maine Christos and S.Sachdev, npj Quantum Materials **9**, 4 (2024)

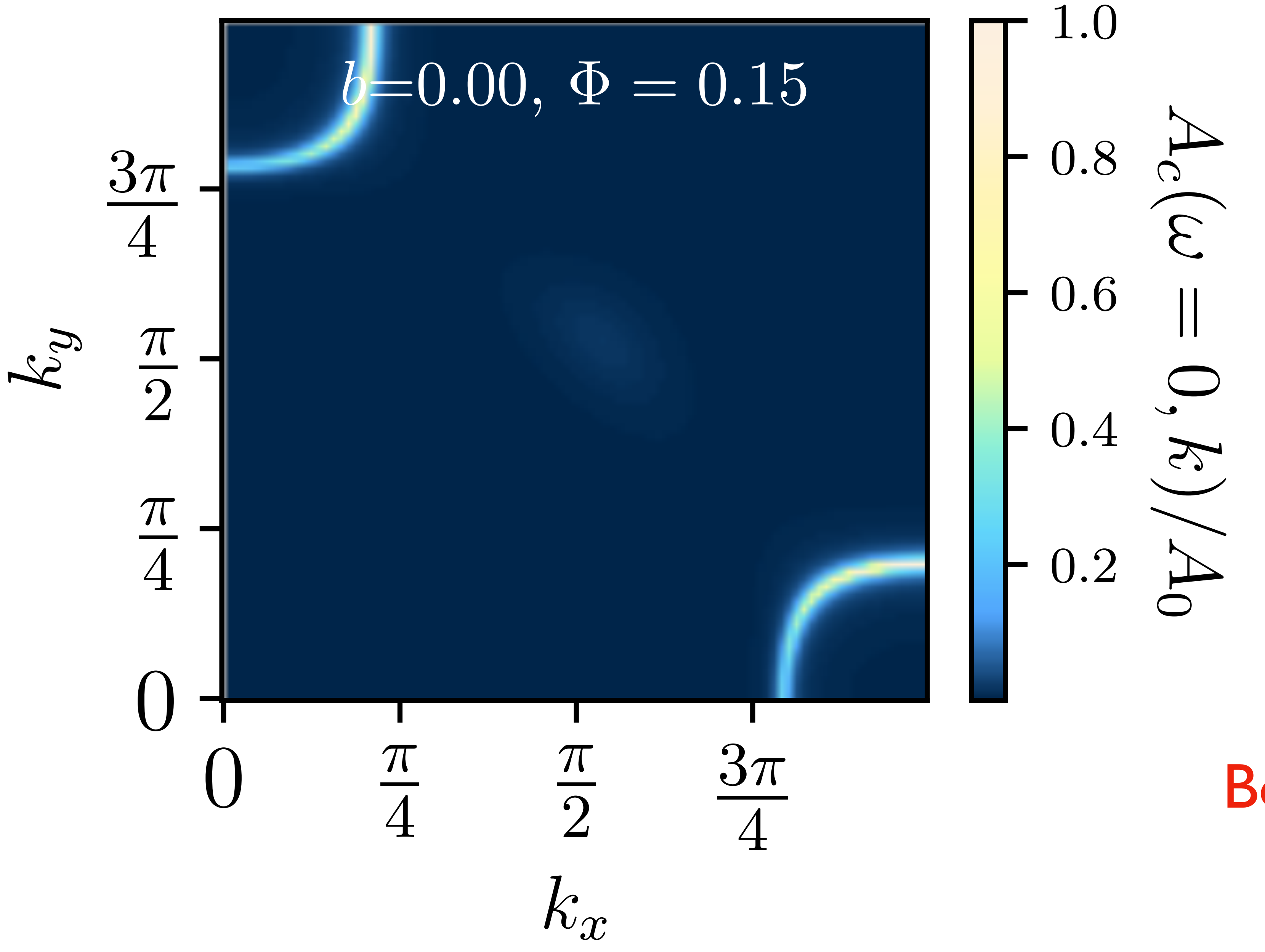
FL*

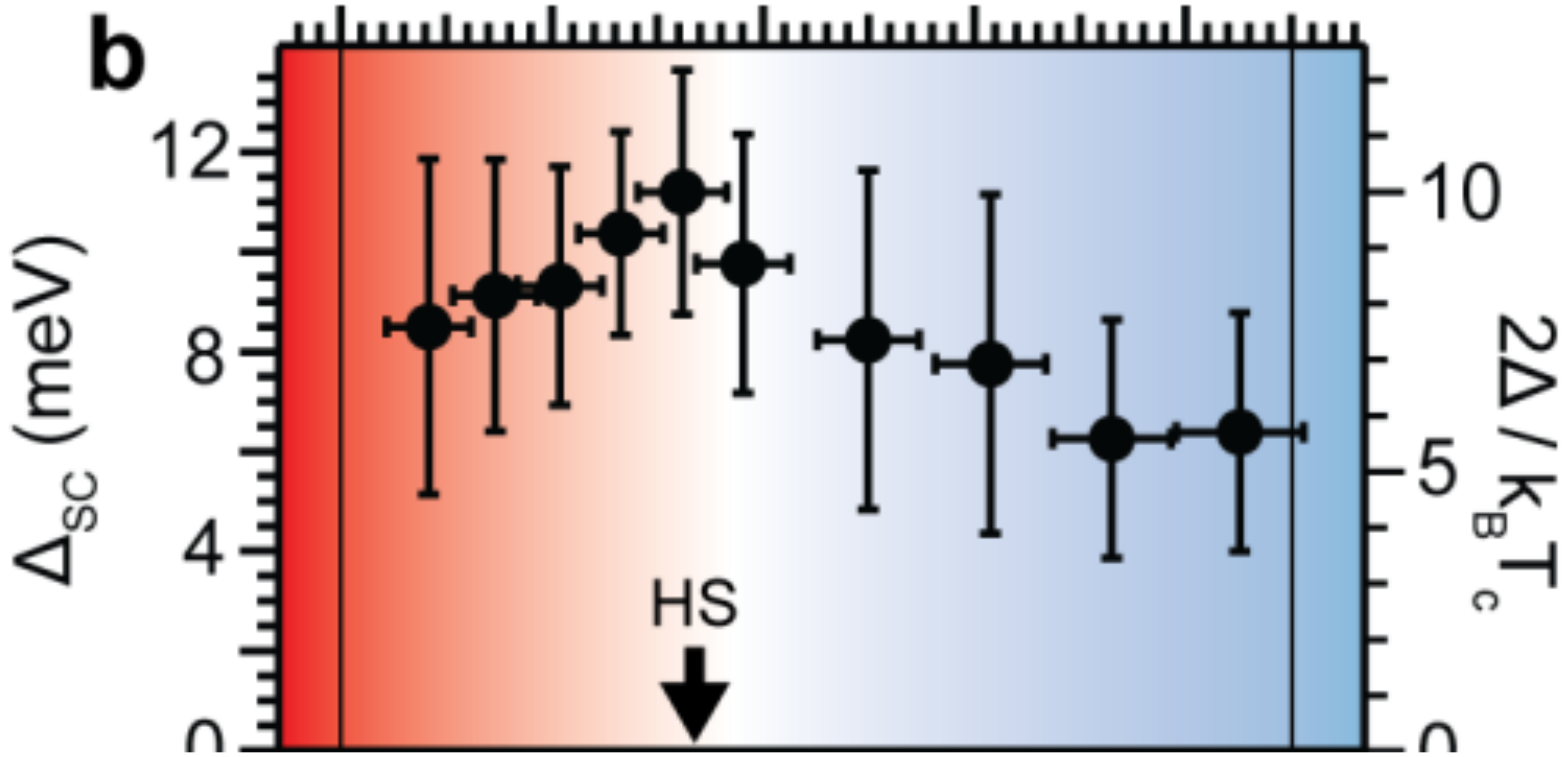
d-wave superconductor obtained by condensing charge-e, SU(2) fundamental boson.

dSC

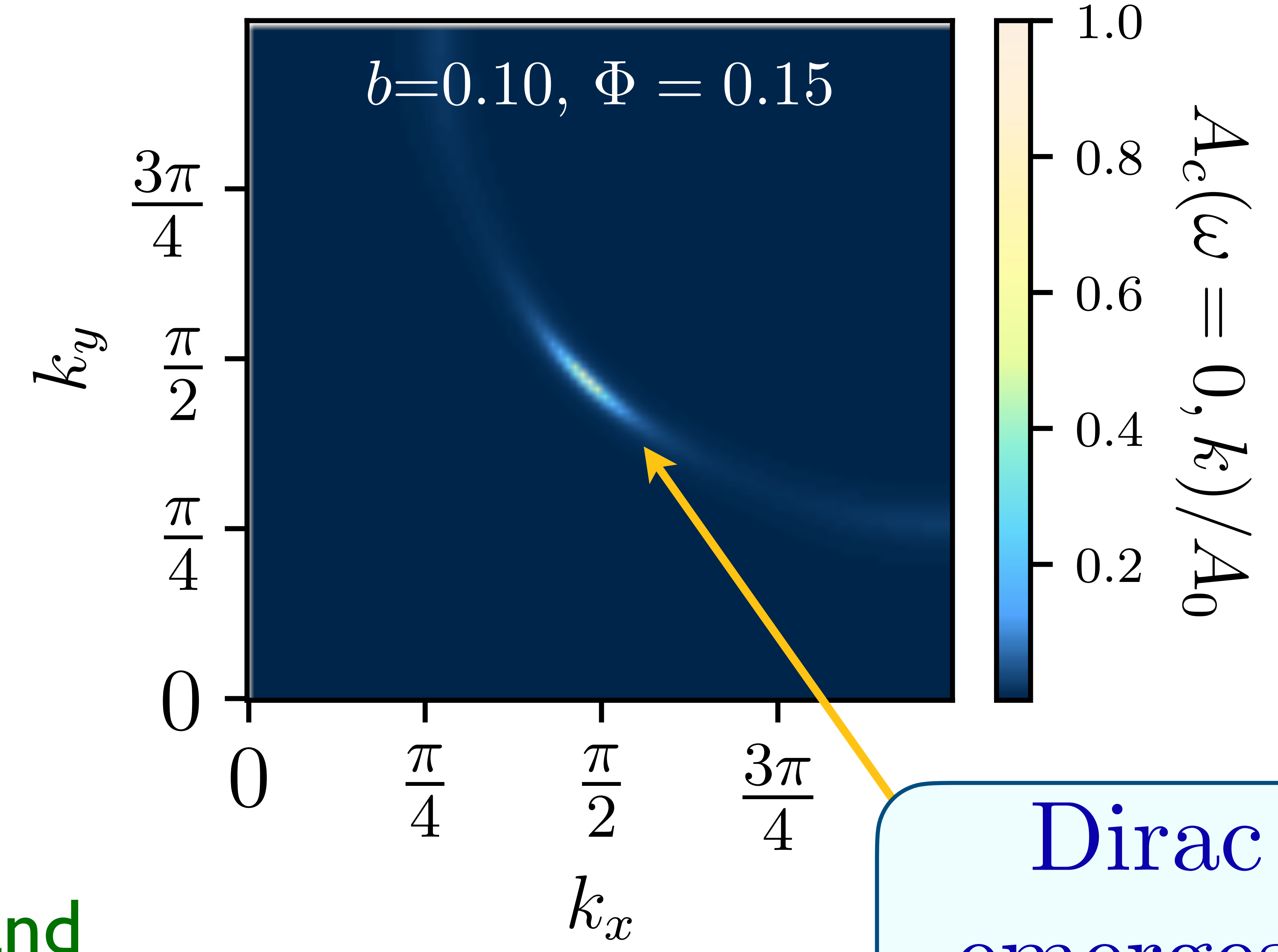
In the electron-doped case, nodes of the d-wave superconductor *are* remnants of the spinons of the π-flux state.

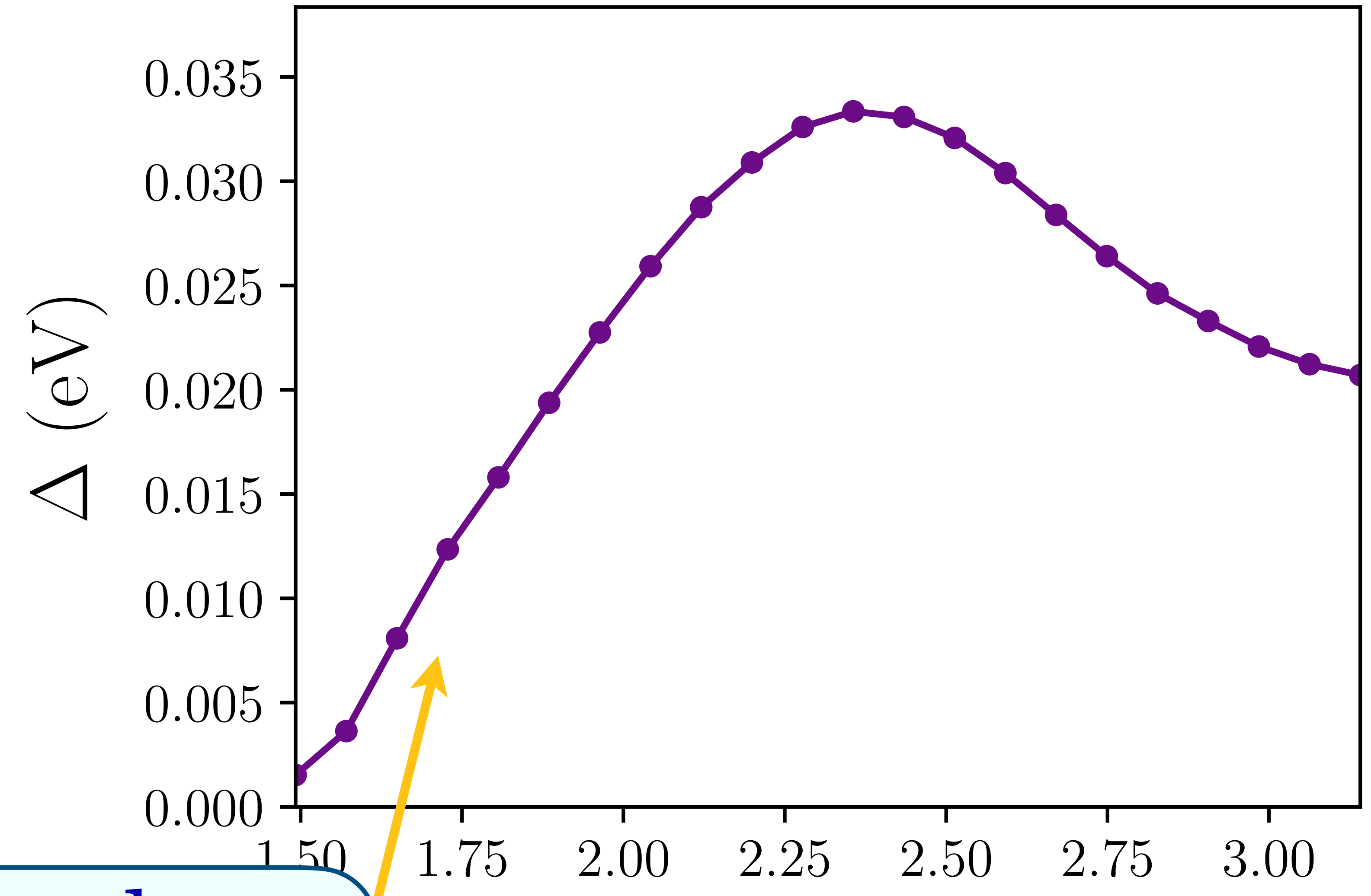
Electron spectral density in electron-doped cuprates





Bogoliubov Quasiparticle on the Gossamer Fermi Surface in Electron-Doped Cuprates, Ke-Jun Xu……Z.-X. Shen, arXiv:2308.05313; Nature Physics





Maine Christos and S.Sachdev, npj Quantum Materials **9**, 4 (2024)

Dirac node emerges inside normal state gap

1. Square lattice spin liquids

2. Spin liquids on the Kondo lattice: non-Luttinger volume Fermi surfaces (FL*)

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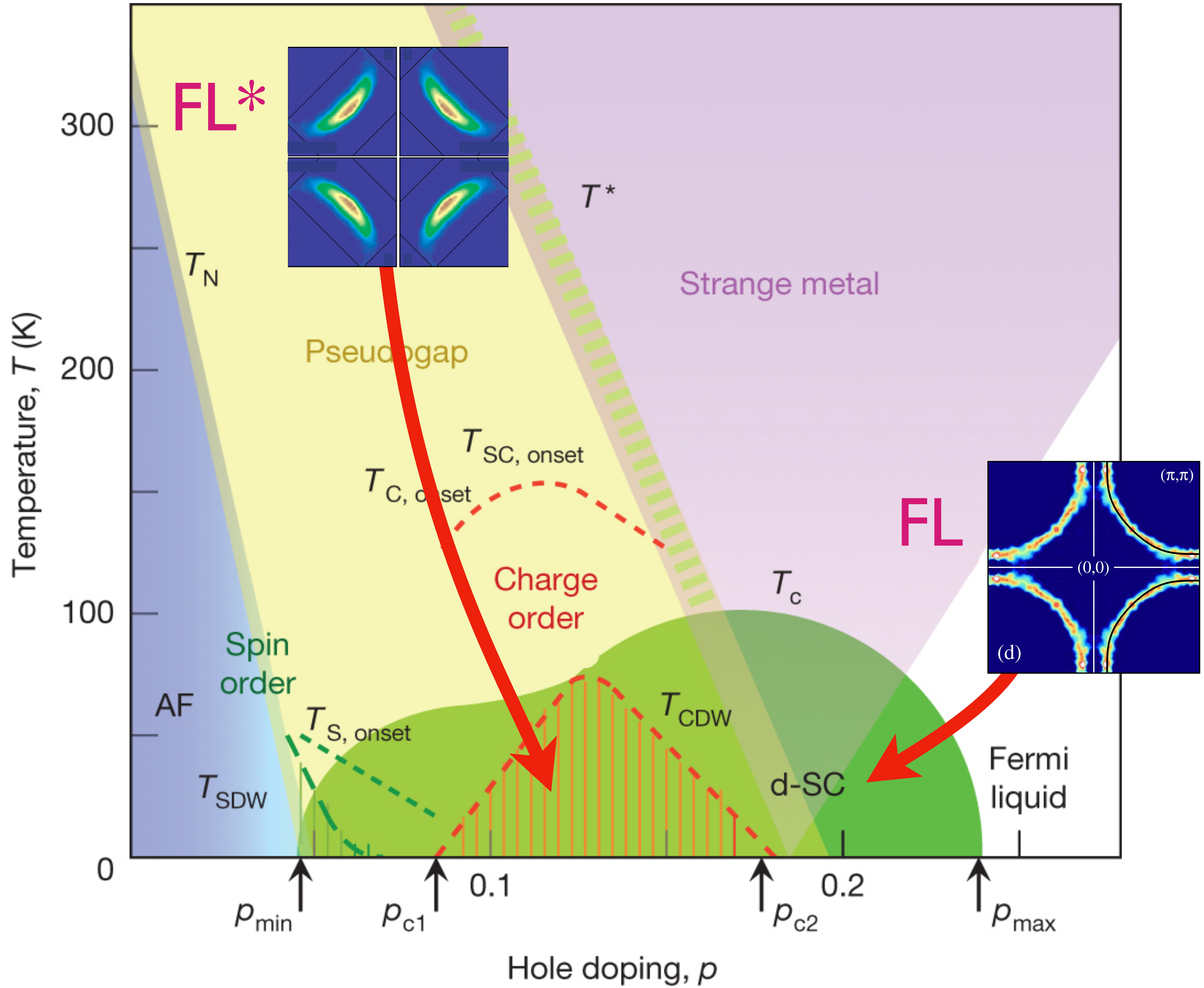
Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)



Obtain *d*-wave superconductor and charge order from a theory of *confinement* instabilities of FL*.

The resulting low T ordered states should be adiabatically connected to the corresponding states obtained from instabilities of FL.

C. Proust and L. Taillefer, *The Remarkable Underlying Ground States of Cuprate Superconductors*, Annual Review of Condensed Matter Physics **10**, 409 (2019),

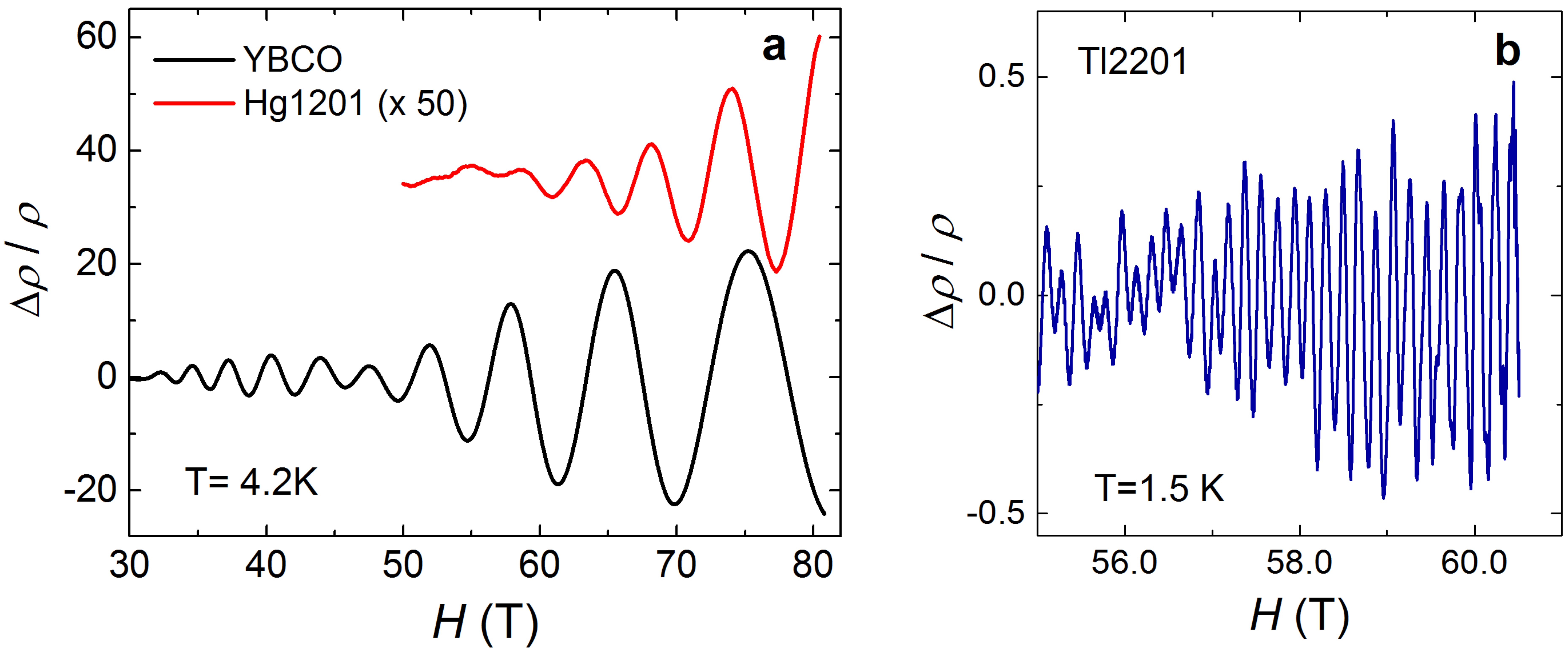
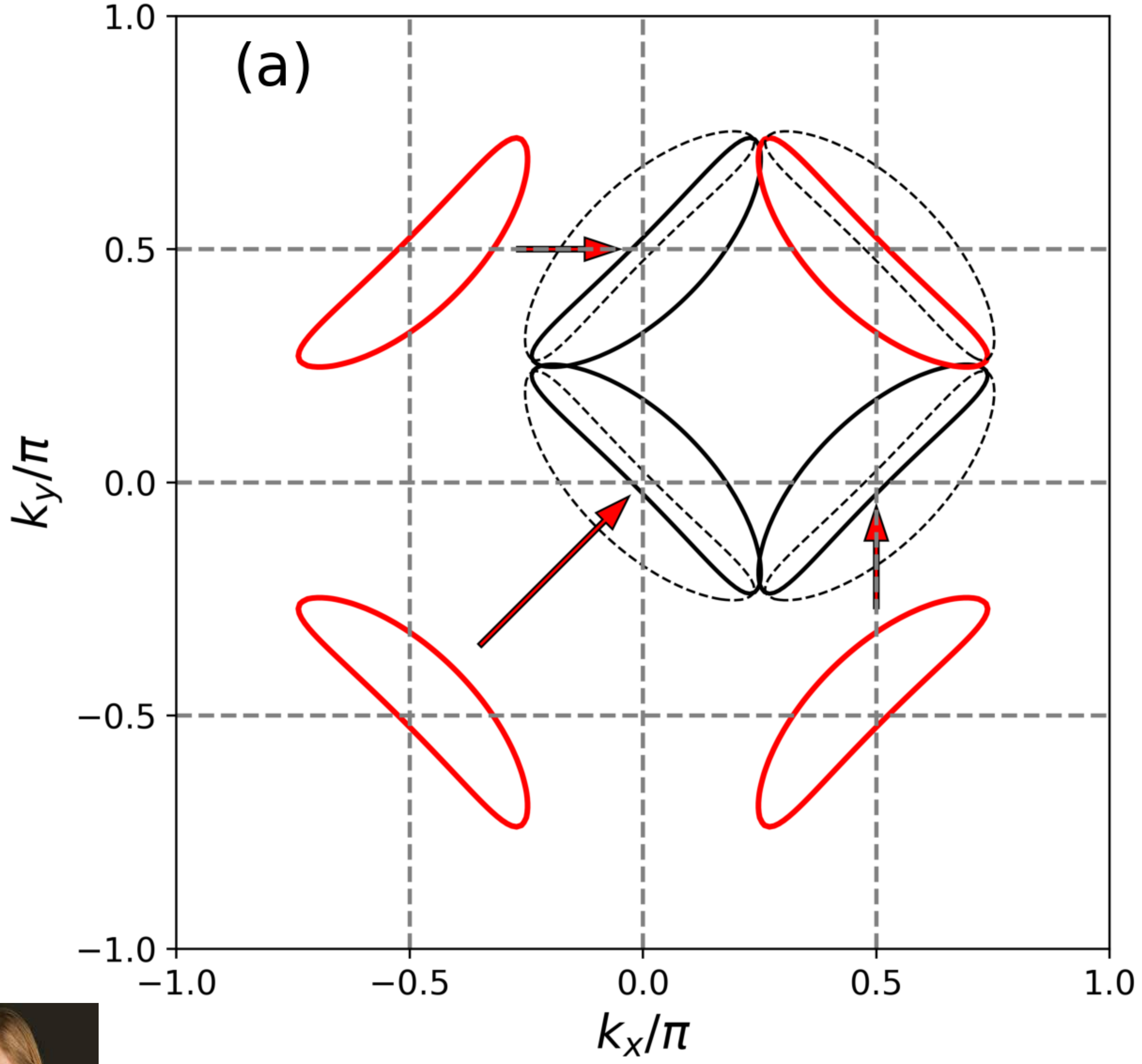


Figure 4

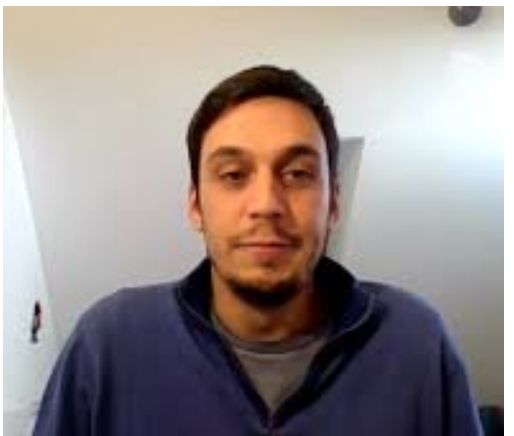
Quantum oscillations in cuprates. a) Underdoped YBCO ($p = 0.11$, $T_c = 62$ K) (black (22)) and Hg1201 ($p \simeq 0.1$, $T_c = 72$ K) (red (33), $\times 50$). **b)** Overdoped Tl2201 ($p \simeq 0.3$, $T_c \approx 10$ K) (28).

22. Vignolle B, et al. 2013. *C. R. Physique* 14:39–52
28. Vignolle B, et al. 2008. *Nature* 455:952–955
33. Barisic N, et al. 2013. *Nat. Phys.* 9:761–764

FL*



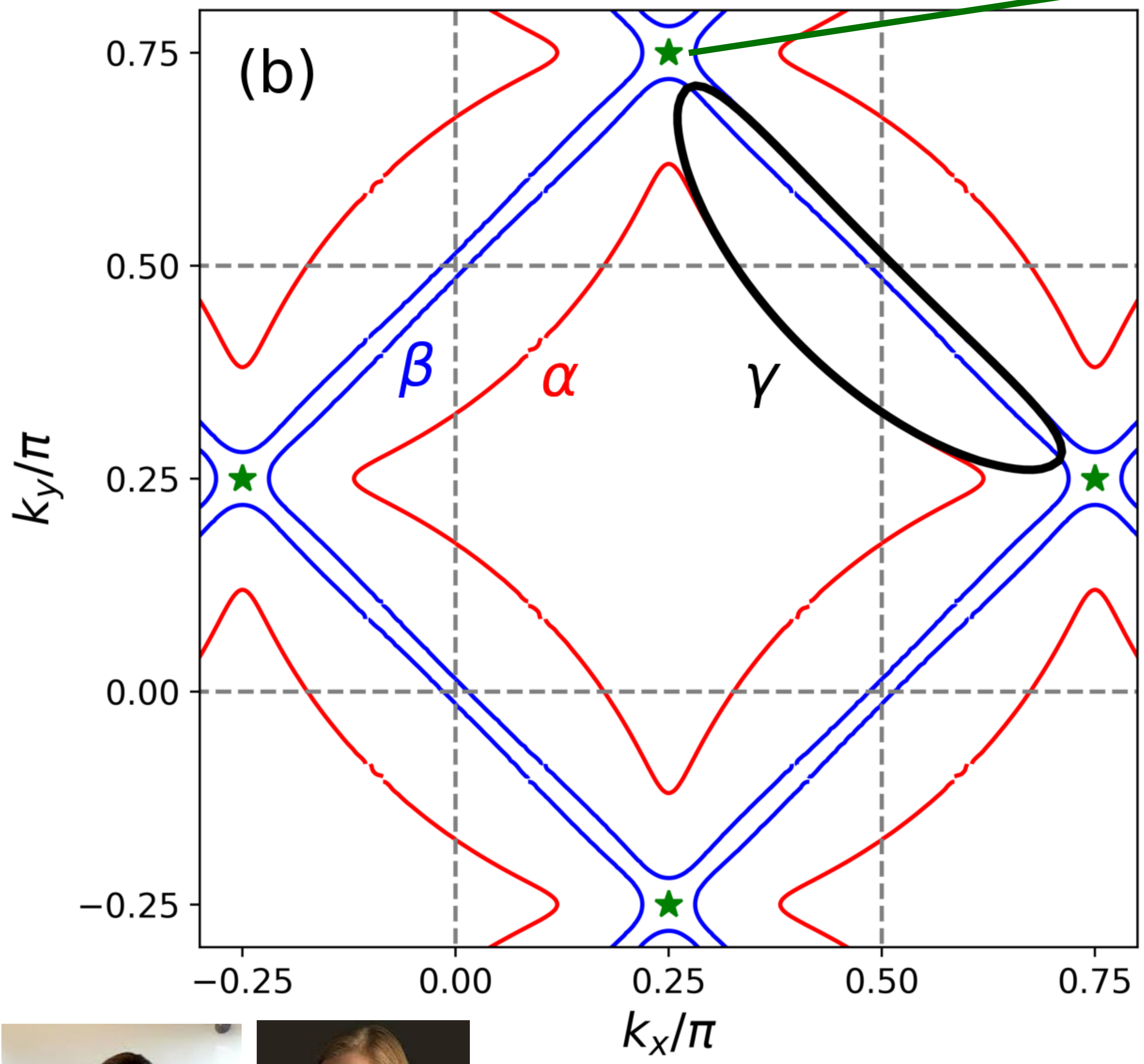
N. Harrison and S. Sebastian
electron pocket
(PRL **106**, 226402 (2011))

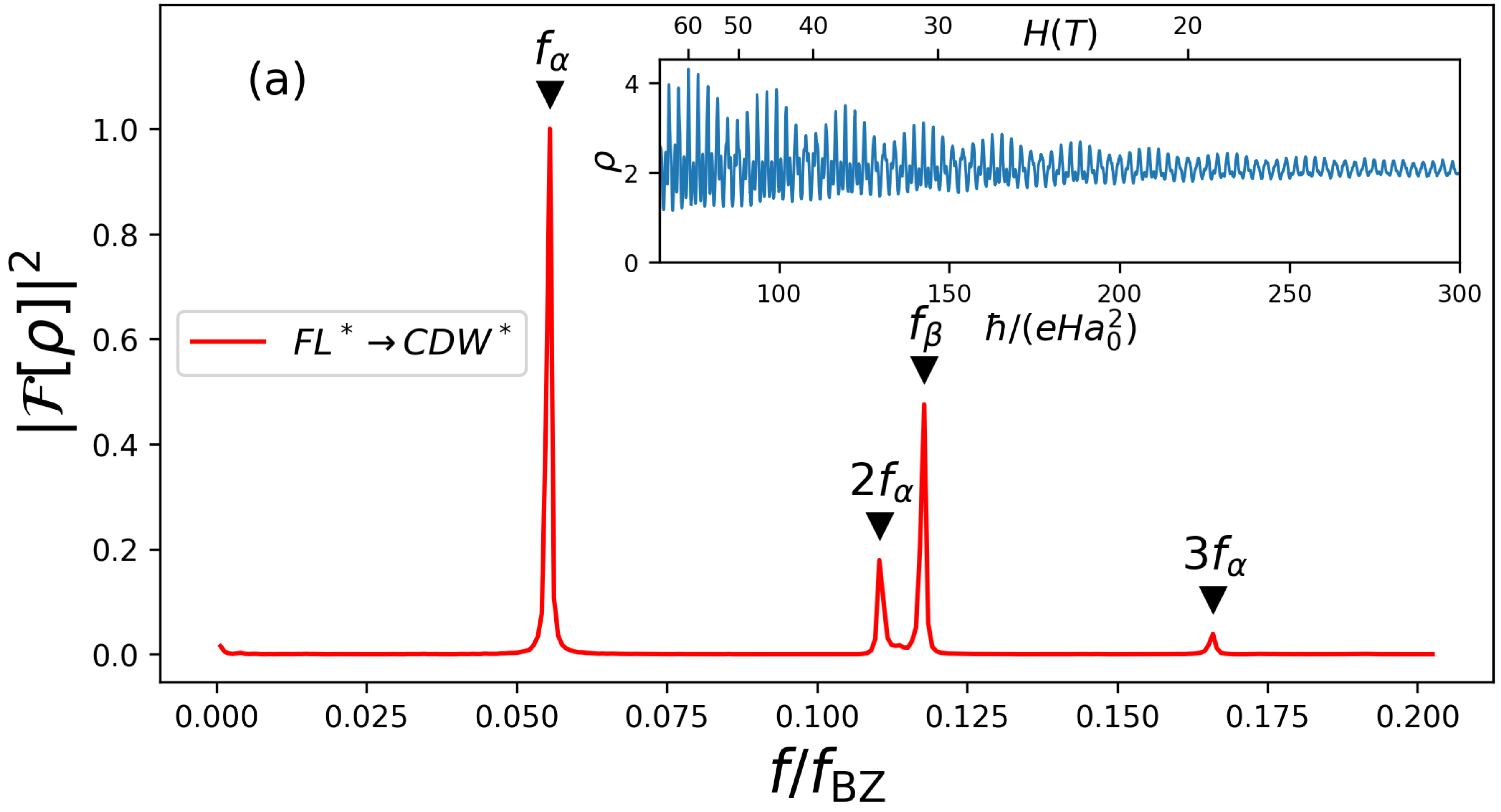


Pietro Bonetti, Maine Christos and S.S. (**BCS**), arXiv:2405.08817

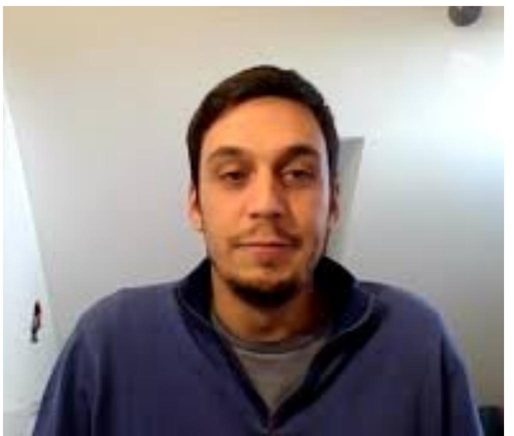
FL*→CDW*

Spinon





Computation does not account for spinons.
α and β pockets
show clear quantum oscillations.
Long Zhang and Jia-Wei Mei,
EPL **114**, 47008 (2016)



Pietro Bonetti, Maine Christos and S.S. (**BCS**), arXiv:2405.08817

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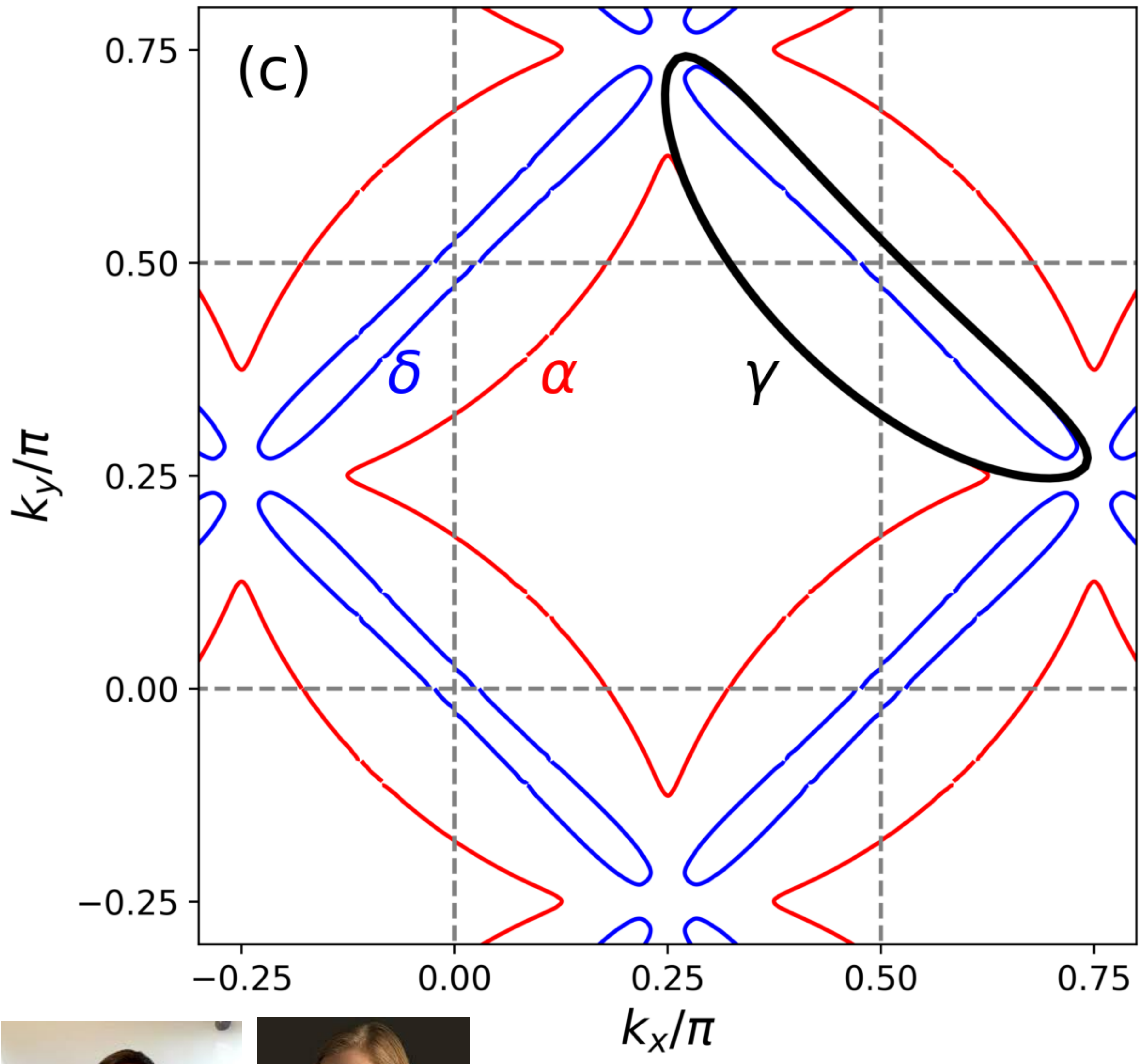
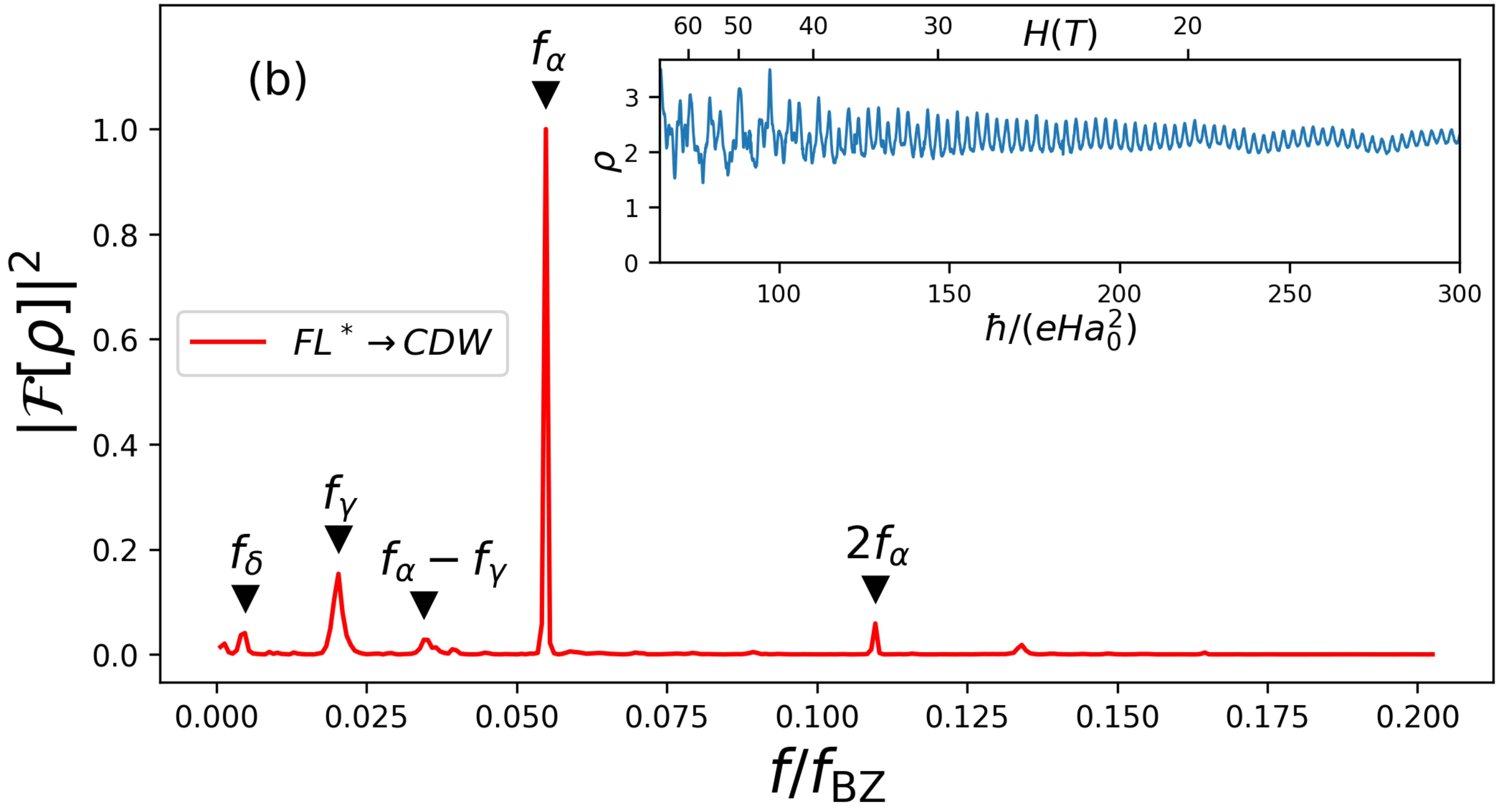
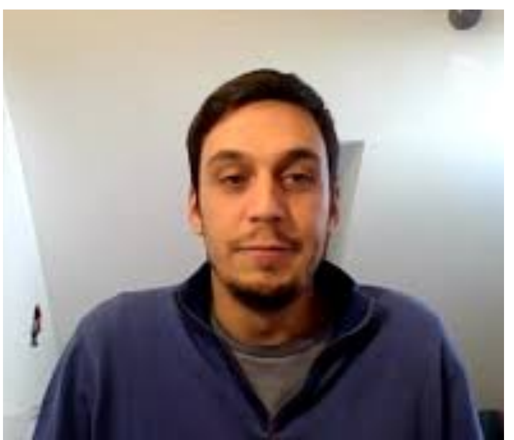




Period 4 CDW obtained by condensing B. SU(2) gauge theory allows CDW with vanishing Δ_{ij} and J_{ij}.

Mixing between electrons and spinons removes β pocket.



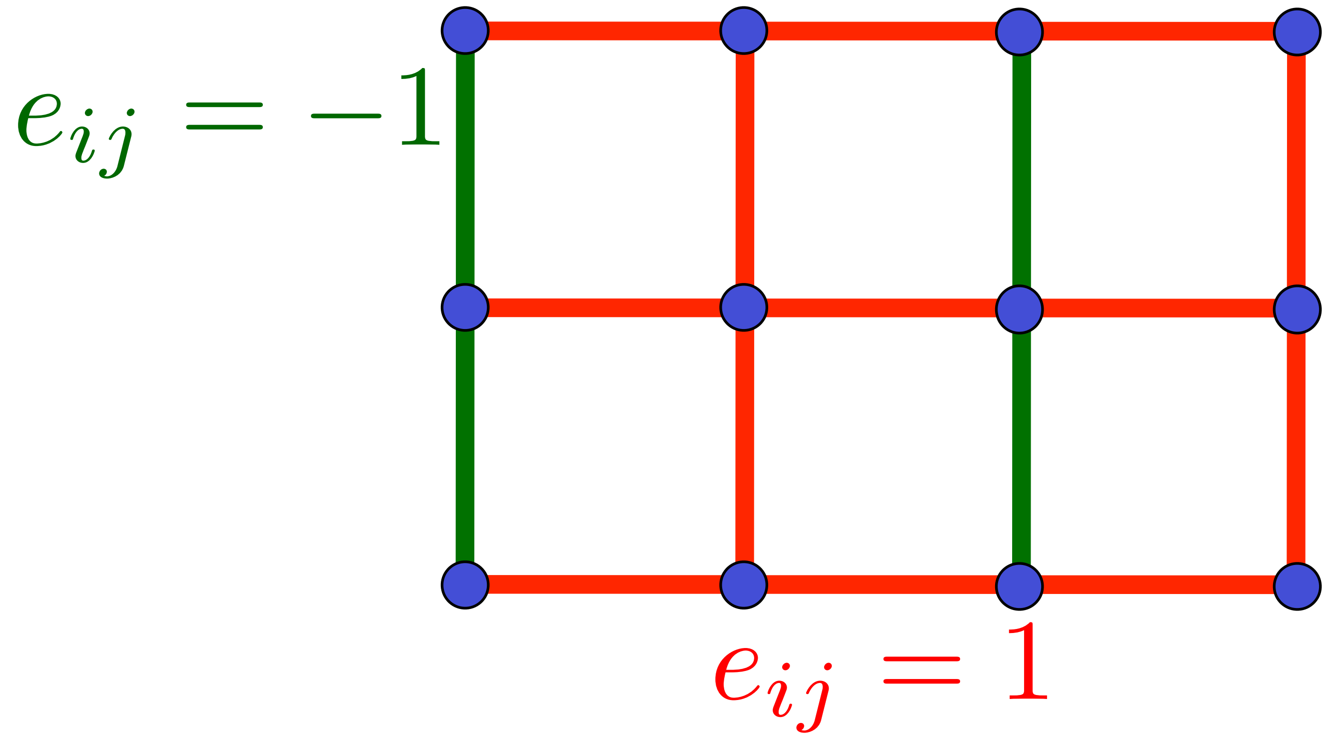
Pietro Bonetti, Maine Christos and S.S. (**BCS**), arXiv:2405.08817

Confinement of SU(2)$_N$ gauge theory by charge fluctuations

- Begin with the π-flux spin liquid in the fermionic spinon description.

$$H_f = iJ \sum_{\langle ij \rangle} e_{ij} \left(f_{i\sigma}^\dagger f_{j\sigma} - f_{j\sigma}^\dagger f_{i\sigma} \right) = \boxed{iJ \sum_{\langle ij \rangle} e_{ij} \left(\Psi_i^\dagger U_{ij} \Psi_j - \Psi_j^\dagger U_{ji} \Psi_i \right)} \; ; \quad \Psi_i = \begin{pmatrix} f_{i\uparrow} \\ f_{i\downarrow}^\dagger \end{pmatrix}$$

H_f is invariant under SU(2) rotations in spin and SU(2)$_N$ rotations in Nambu space; U_{ij} is the SU(2)$_N$ gauge field.



M. Christos, Zhu-Xi Luo,
H. Shackleton, Ya-Hui Zhang,
M. Scheurer, and S. S., *PNAS*
120, e2302701120 (2023)

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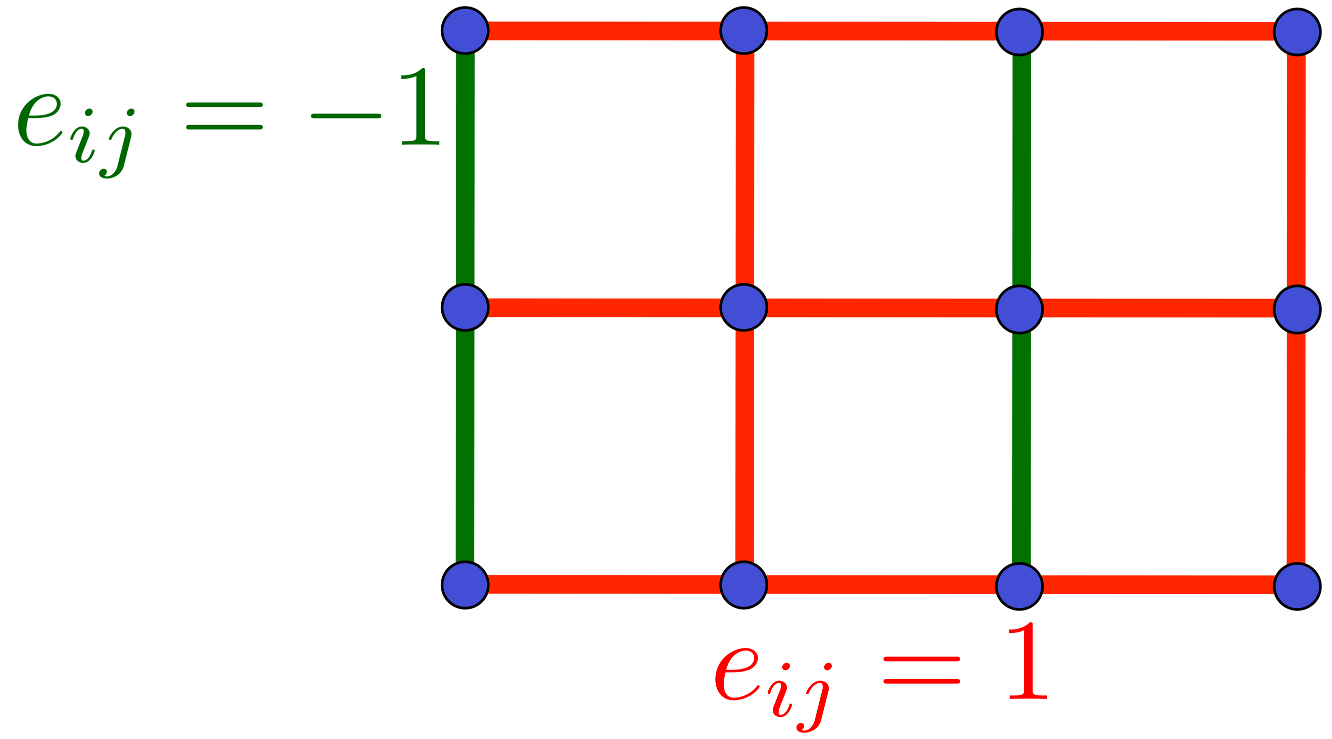


- Introduce a charge e, SU(2)$_N$ fundamental boson B_i such that the composite of B_i and Ψ_i is an electron. The projective symmetries require

$$\boxed{H_B = r \sum_i B_i^\dagger B_i + iw \sum_{\langle ij \rangle} e_{ij} \left(B_i^\dagger U_{ij} B_j - B_j^\dagger U_{ji} B_i \right) + \ldots}$$

M. Christos, Zhu-Xi Luo, H. Shackleton, Ya-Hui Zhang, M. Scheurer, and S. S., *PNAS* **120**, e2302701120 (2023)

Confinement of SU(2)$_N$ gauge theory by charge fluctuations

$$\mathcal{L}(B) = H_B + \frac{u}{2}\sum_{\boldsymbol{i}} \rho_{\boldsymbol{i}}^2 + V_1 \sum_{\boldsymbol{i}} \rho_{\boldsymbol{i}} \left(\rho_{\boldsymbol{i}+\hat{\boldsymbol{x}}} + \rho_{\boldsymbol{i}+\hat{\boldsymbol{y}}}\right) + g \sum_{\langle \boldsymbol{ij}\rangle} |\Delta_{\boldsymbol{ij}}|^2 + J_1 \sum_{\langle \boldsymbol{ij}\rangle} Q_{\boldsymbol{ij}}^2 + K_1 \sum_{\langle \boldsymbol{ij}\rangle} J_{\boldsymbol{ij}}^2 .$$

site charge density: $\left\langle c_{\boldsymbol{i}\alpha}^\dagger c_{\boldsymbol{i}\alpha} \right\rangle \sim \rho_{\boldsymbol{i}} = B_{\boldsymbol{i}}^\dagger B_{\boldsymbol{i}}$

bond density: $\left\langle c_{\boldsymbol{i}\alpha}^\dagger c_{\boldsymbol{j}\alpha} + c_{\boldsymbol{j}\alpha}^\dagger c_{\boldsymbol{i}\alpha} \right\rangle \sim Q_{\boldsymbol{ij}} = Q_{\boldsymbol{ji}} = \mathrm{Im}\left(B_{\boldsymbol{i}}^\dagger e_{\boldsymbol{ij}} U_{\boldsymbol{ij}} B_{\boldsymbol{j}}\right)$

bond current: $i\left\langle c_{\boldsymbol{i}\alpha}^\dagger c_{\boldsymbol{j}\alpha} - c_{\boldsymbol{j}\alpha}^\dagger c_{\boldsymbol{i}\alpha} \right\rangle \sim J_{\boldsymbol{ij}} = -J_{\boldsymbol{ji}} = \mathrm{Re}\left(B_{\boldsymbol{i}}^\dagger e_{\boldsymbol{ij}} U_{\boldsymbol{ij}} B_{\boldsymbol{j}}\right)$

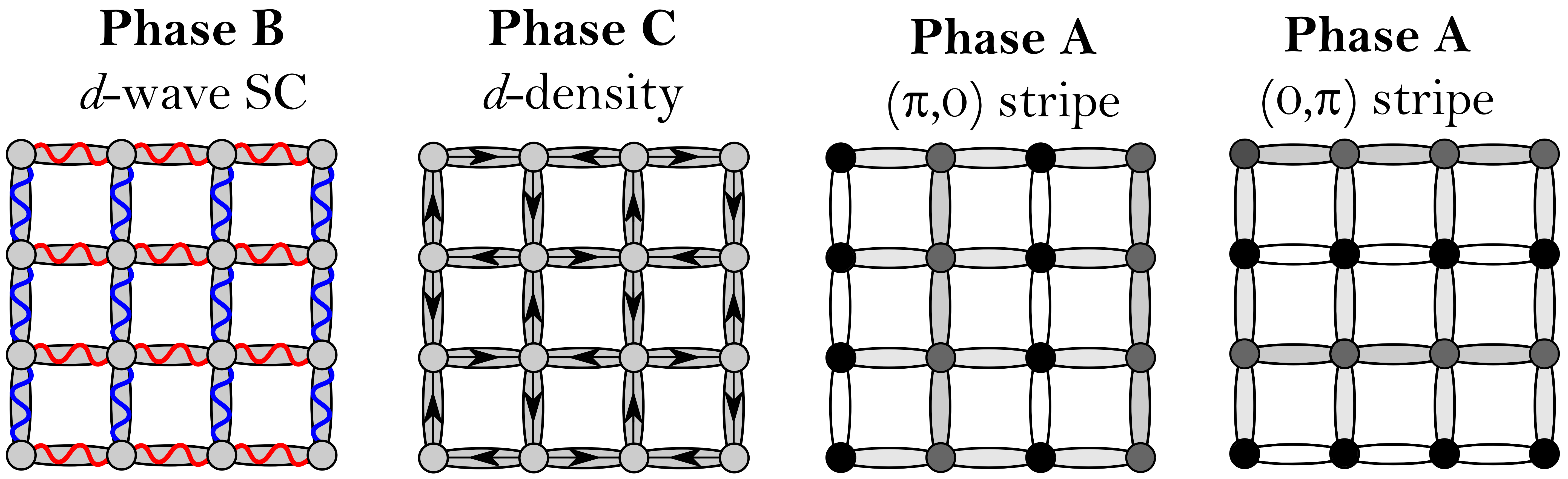
Pairing: $\langle \varepsilon_{\alpha\beta} c_{i\alpha} c_{j\beta}\rangle \sim \Delta_{\boldsymbol{ij}} = \Delta_{\boldsymbol{ji}} = \varepsilon_{ab} B_{a\boldsymbol{i}} e_{\boldsymbol{ij}} U_{\boldsymbol{ij}} B_{b\boldsymbol{j}}$.

M. Christos, Zhu-Xi Luo, H. Shackleton, Ya-Hui Zhang, M. Scheurer, and S. S., *PNAS* **120**, e2302701120 (2023)

Global phase diagram of SU(2)$_N$ gauge theory

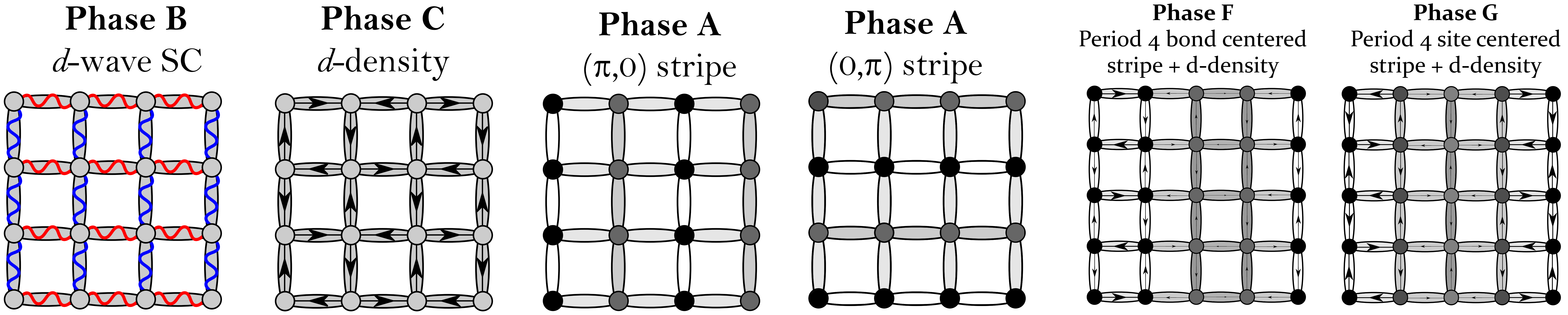
M. Christos, Zhu-Xi Luo, H. Shackleton, Ya-Hui Zhang, M. Scheurer, and S. S., *PNAS* **120**, e2302701120 (2023)

Global phase diagram of $SU(2)_N$ gauge theory



<u>At half filling:</u>
possible CFT/DQCP with
$N_f = 2$ Dirac fermions and
$N_b = 2$ complex scalars coupled
to $SU(2)_N$ gauge field.
M. Christos, H. Shackleton, S. S.,
and Zhu-Xi Luo, PRR **6**, 033018 (2024).

Global phase diagram of SU(2)$_N$ gauge theory



M. Christos, Zhu-Xi Luo, H. Shackleton, Ya-Hui Zhang, M. Scheurer, and S. S., *PNAS* **120**, e2302701120 (2023)

Summary

Ancilla theory of pseudogap metal with hole pockets and underlying π-flux spin liquid yields:

- Theory for Fermi arcs in hole-doped pseudogap metal.
- ADMR in pseudogap.
- Anti-nodal and nodal electronic dispersion.
- d-wave superconductor with 4 nodal points in both electron- and hole-doped cuprates.
- Near-equality of dSC and charge order onset temperatures
- Multipoint correlators measured by cold atom experiments
- Theory for strange metal in the crossover from FL* to FL