



Happy Birthday Louis!



Happy Birthday

Louis!

Decoding the phases of the cuprates using fractionalization and spinons

Key Advances in Quantum Materials
Jouvence, Canada
October 15, 2024
Subir Sachdev

Maine Christos, Zhu-Xi Luo, Henry Shackleton, Ya-Hui Zhang,
Mathias Scheurer, S. S., *PNAS* **120**, e2302701120 (2023)

A. Nikolaenko, J. v. Milczewski, D. G. Joshi, S.S., *PRB* **108**, 045123 (2023)

Maine Christos and S.S., *npj Quantum Materials* **9**, 4 (2024)

M. Christos, H. Shackleton, S. S., and Zhu-Xi Luo, *PRR* **6**, 033018 (2024)

Pietro M. Bonetti, Maine Christos, S.S. (BCS), arXiv: 2405.08817

Jia-Xin Zhang, S. Sachdev, arXiv:2406.12964

PHYSICS



HARVARD





Maine Christos



Zhu-Xi Luo



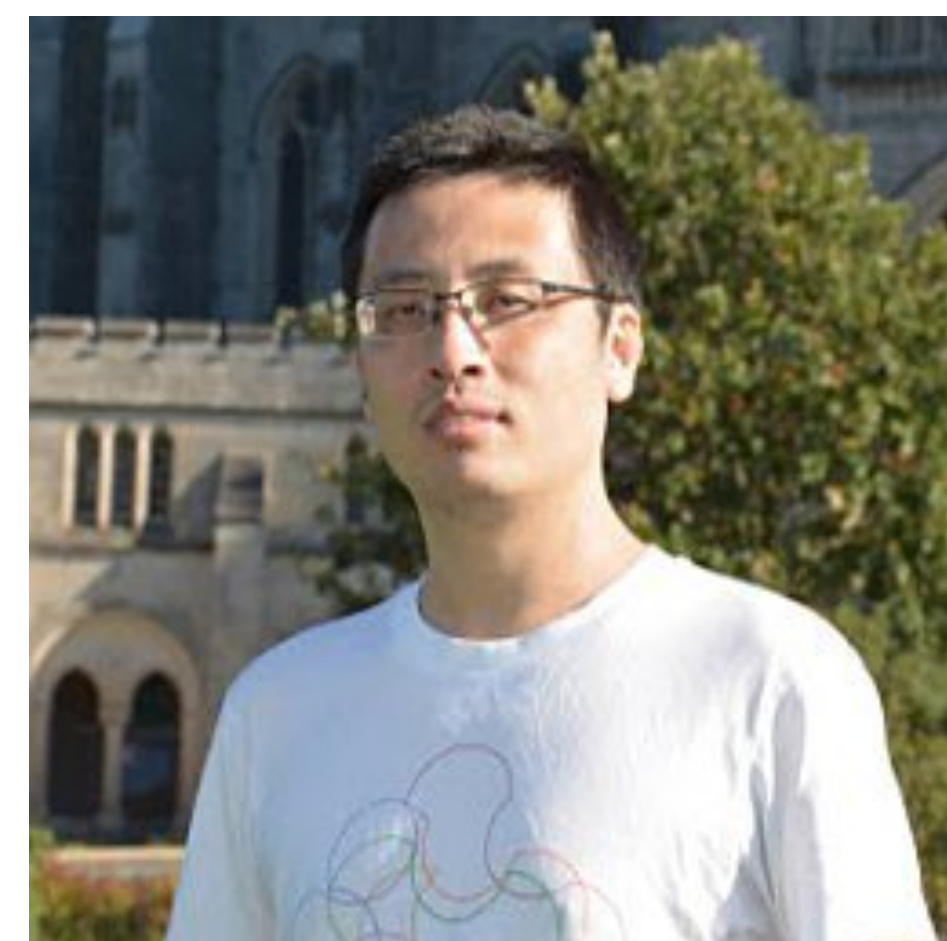
Maria
Tikhanovskaya



Mathias
Scheurer



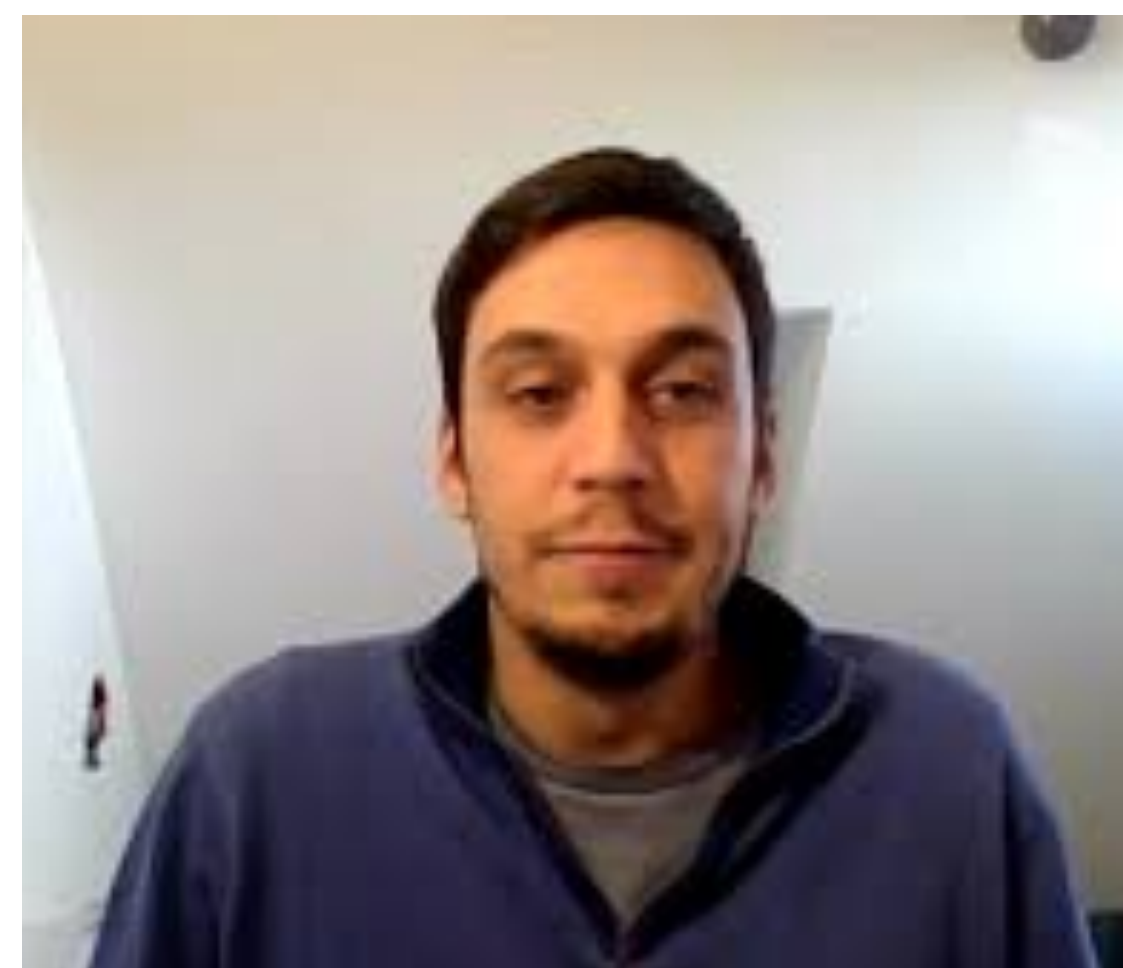
Jia-Xin Zhang



Ya-Hui Zhang



Alexander
Nikolaenko



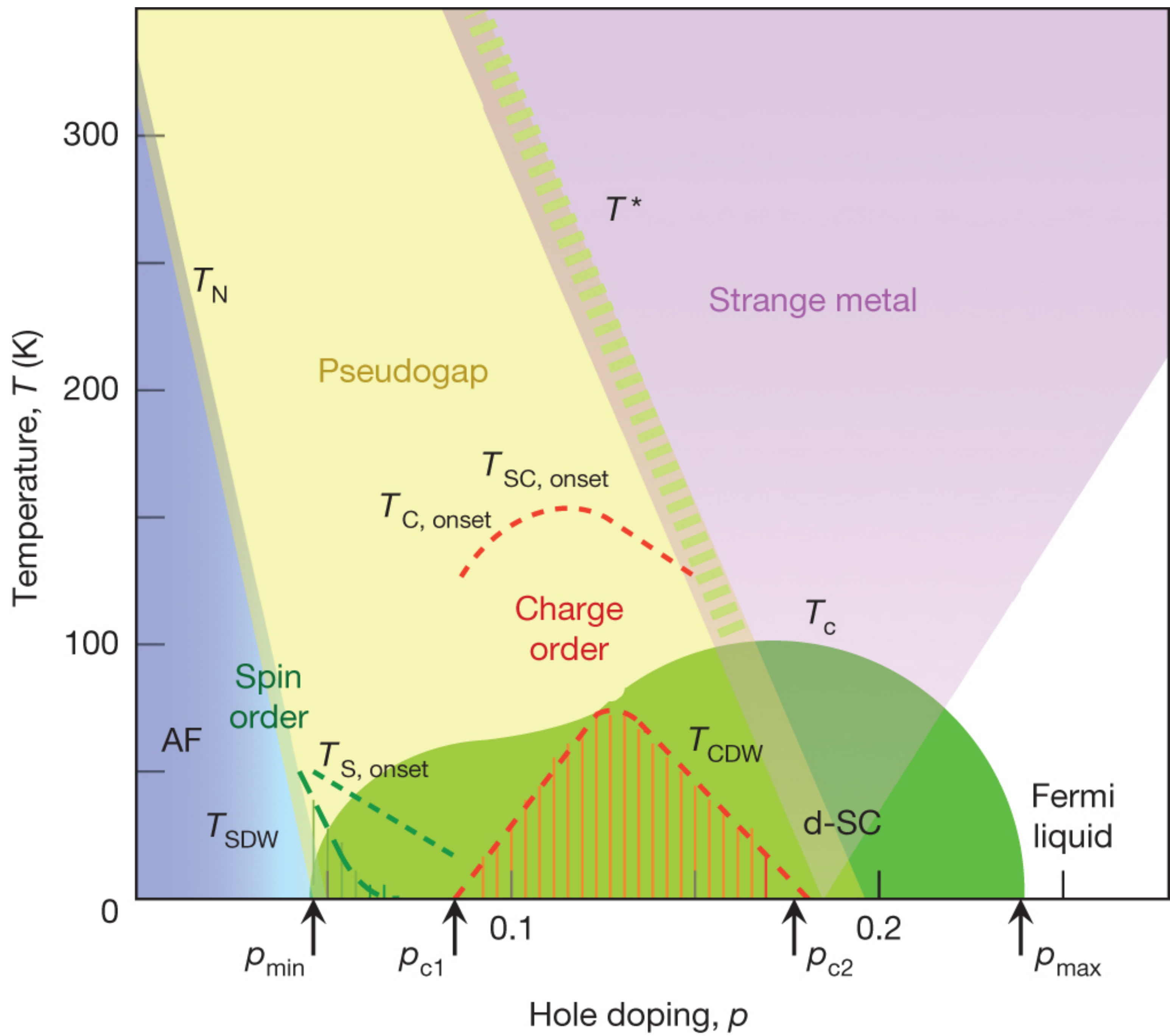
Pietro Bonetti



Henry
Shackleton



Darshan
Joshi

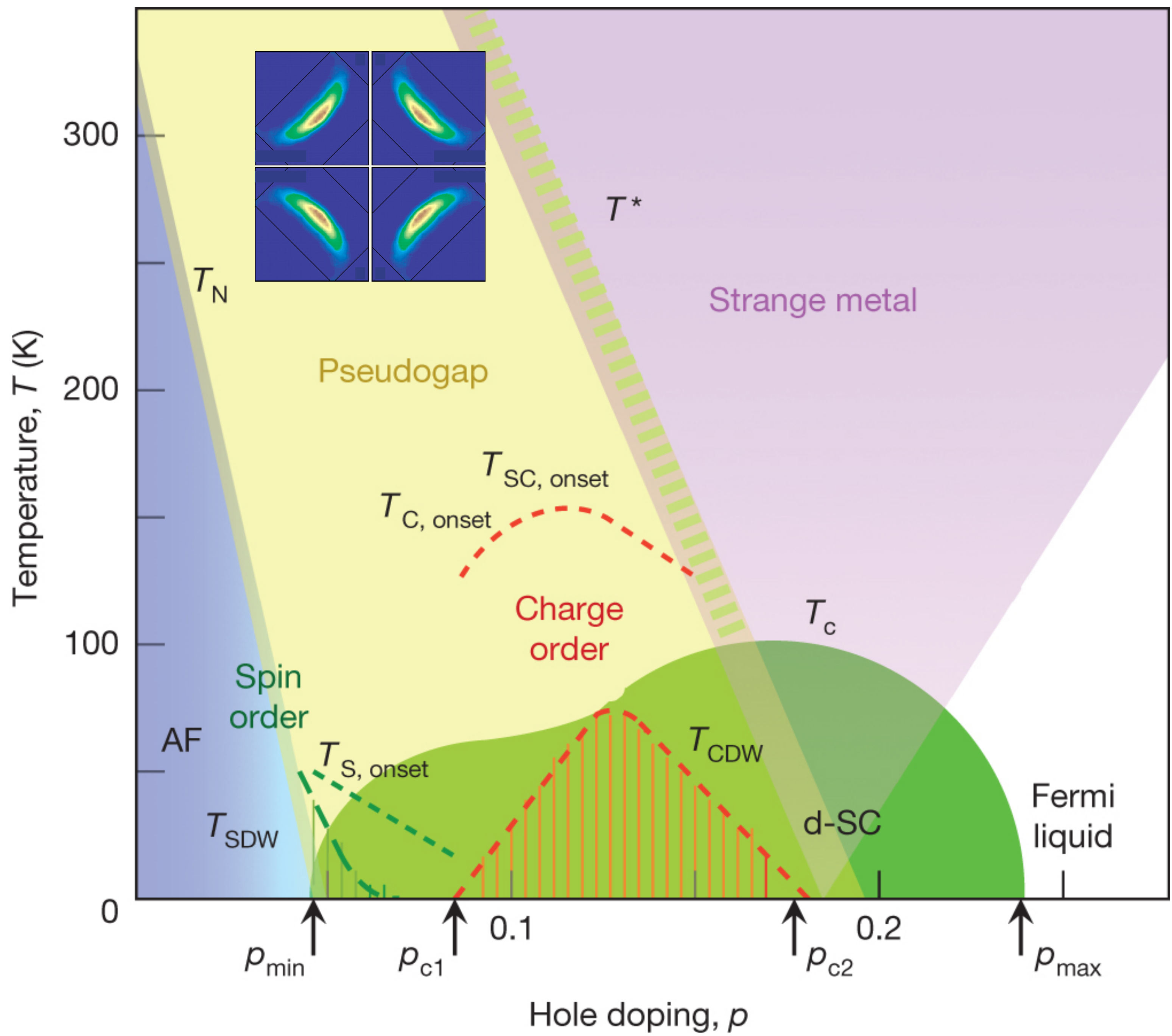


Pseudogap puzzles:

- A. Photoemission: Fermi arcs
- B. RIXS: Intense paramagnons
- C. Quantum oscillations
- D. Velocities of Bogoliubov quasiparticles
in the d-wave superconductor

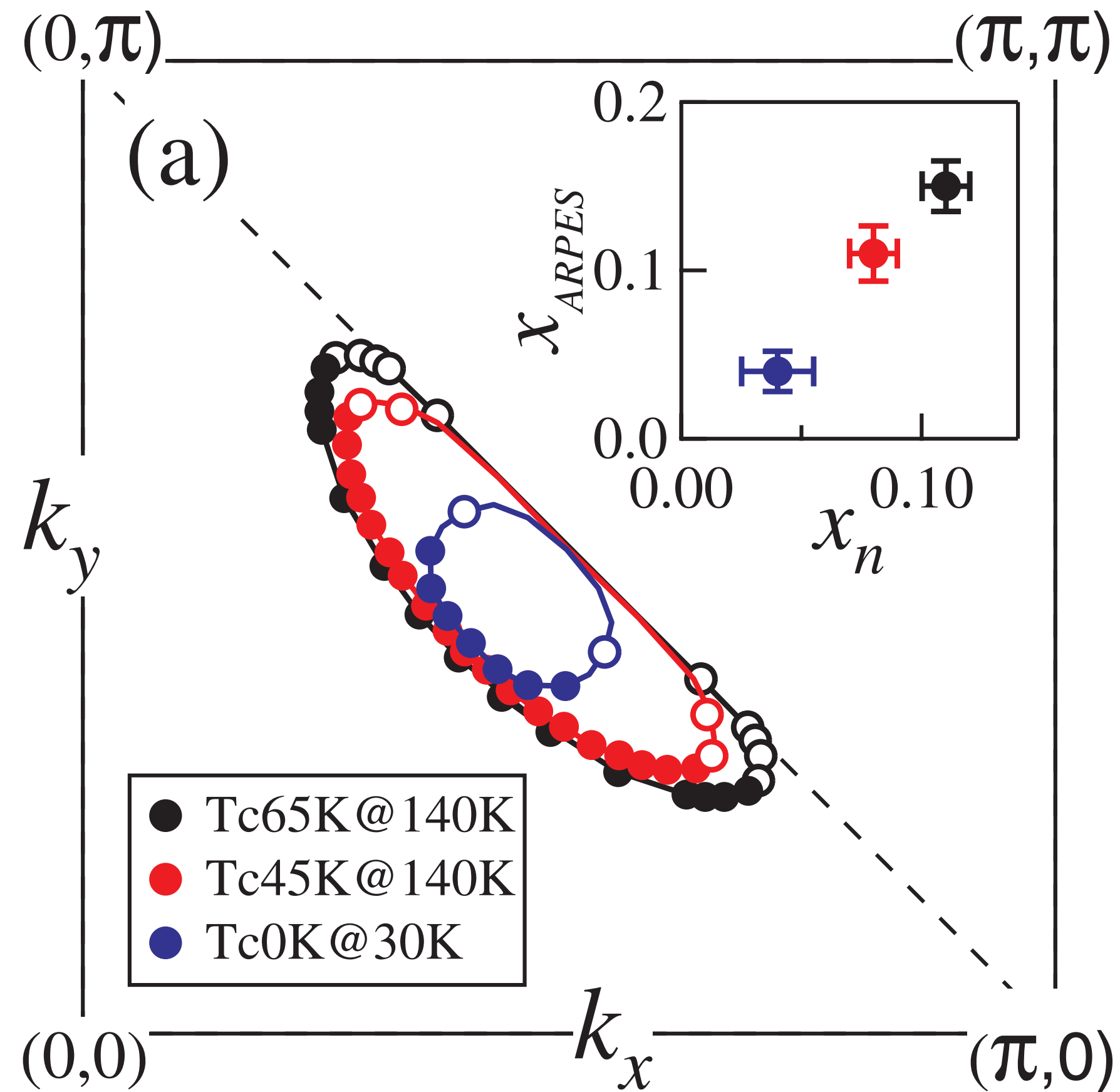
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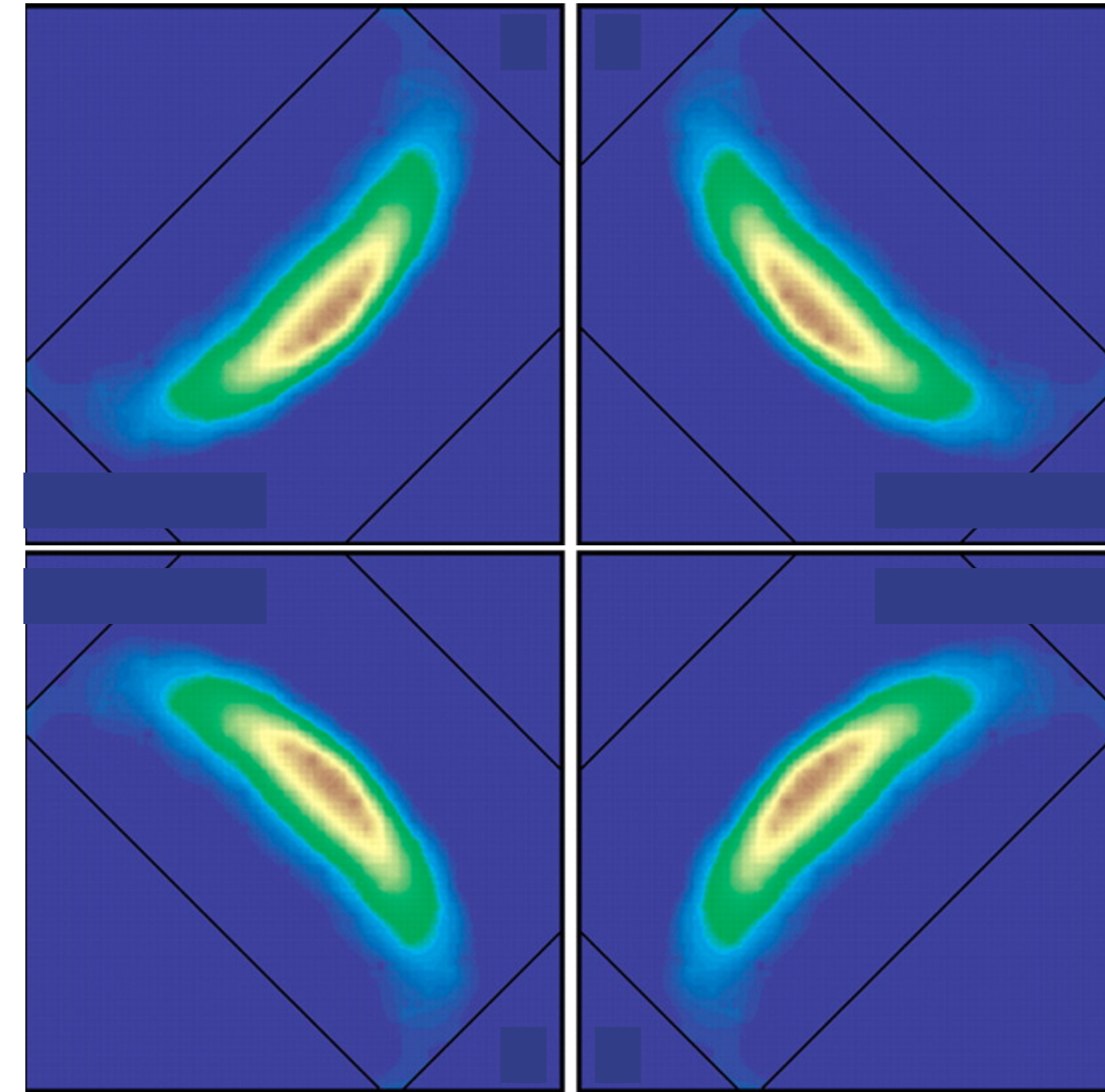


Pseudogap metal
with “Fermi arcs”

Photoemission expts at small p



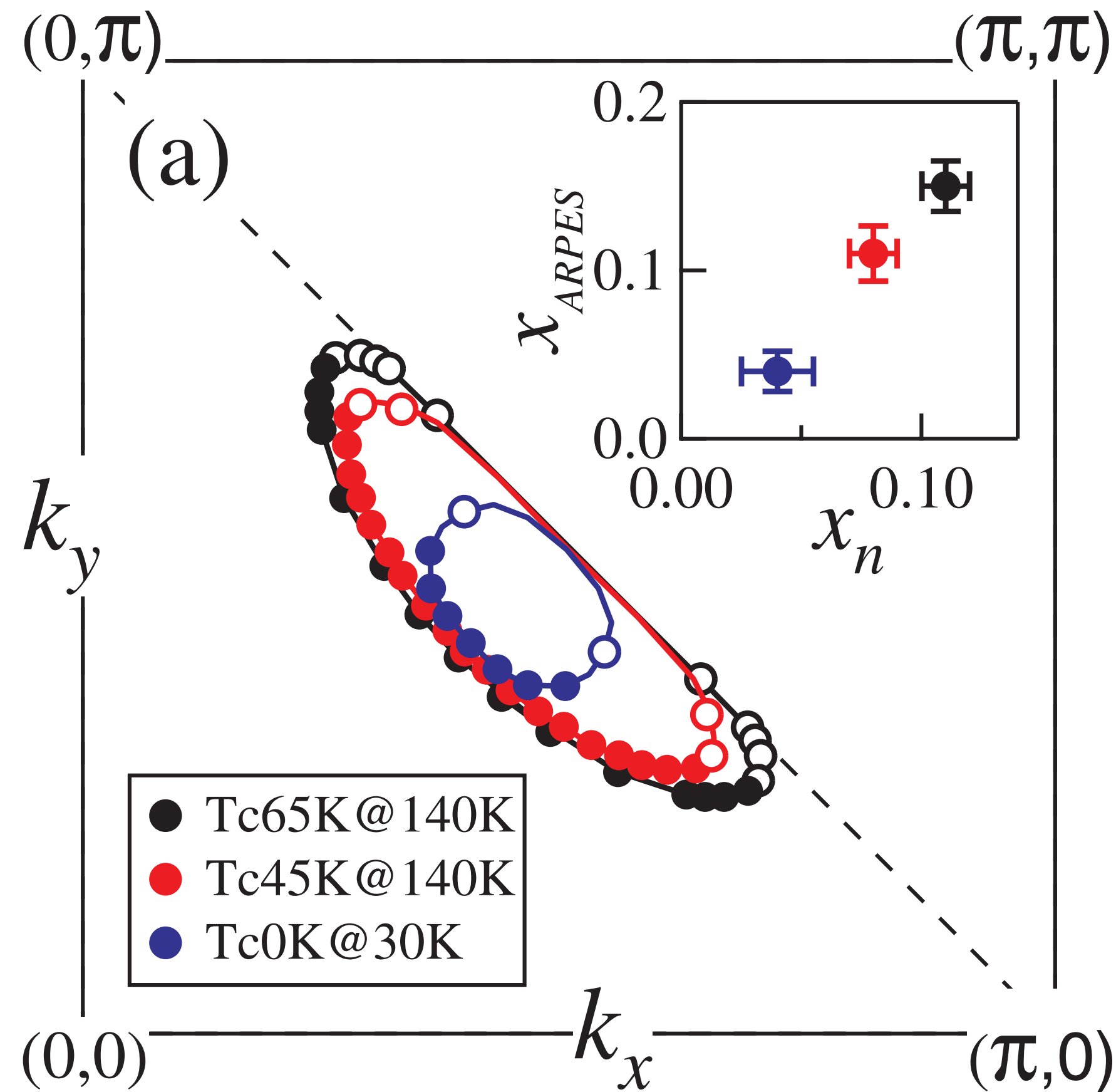
Reconstructed Fermi Surface of Underdoped $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$ Cuprate Superconductors,
H.-B. Yang, J. D. Rameau, Z.-H. Pan, G. D. Gu,
P. D. Johnson, H. Claus, D. G. Hinks,
and T. E. Kidd, PRL **107**, 047003 (2011).



$\text{Ca}_{2-x}\text{Na}_x\text{CuO}_2\text{Cl}_2$ at $x = 0.10$

Kyle M. Shen, F. Ronning, D. H. Lu, F. Baumberger,
N. J. C. Ingle, W. S. Lee, W. Meevasana, Y. Kohsaka, M. Azuma,
M. Takano, H. Takagi, Z.-X. Shen, Science **307**, 901 (2005)

Photoemission expts at small p

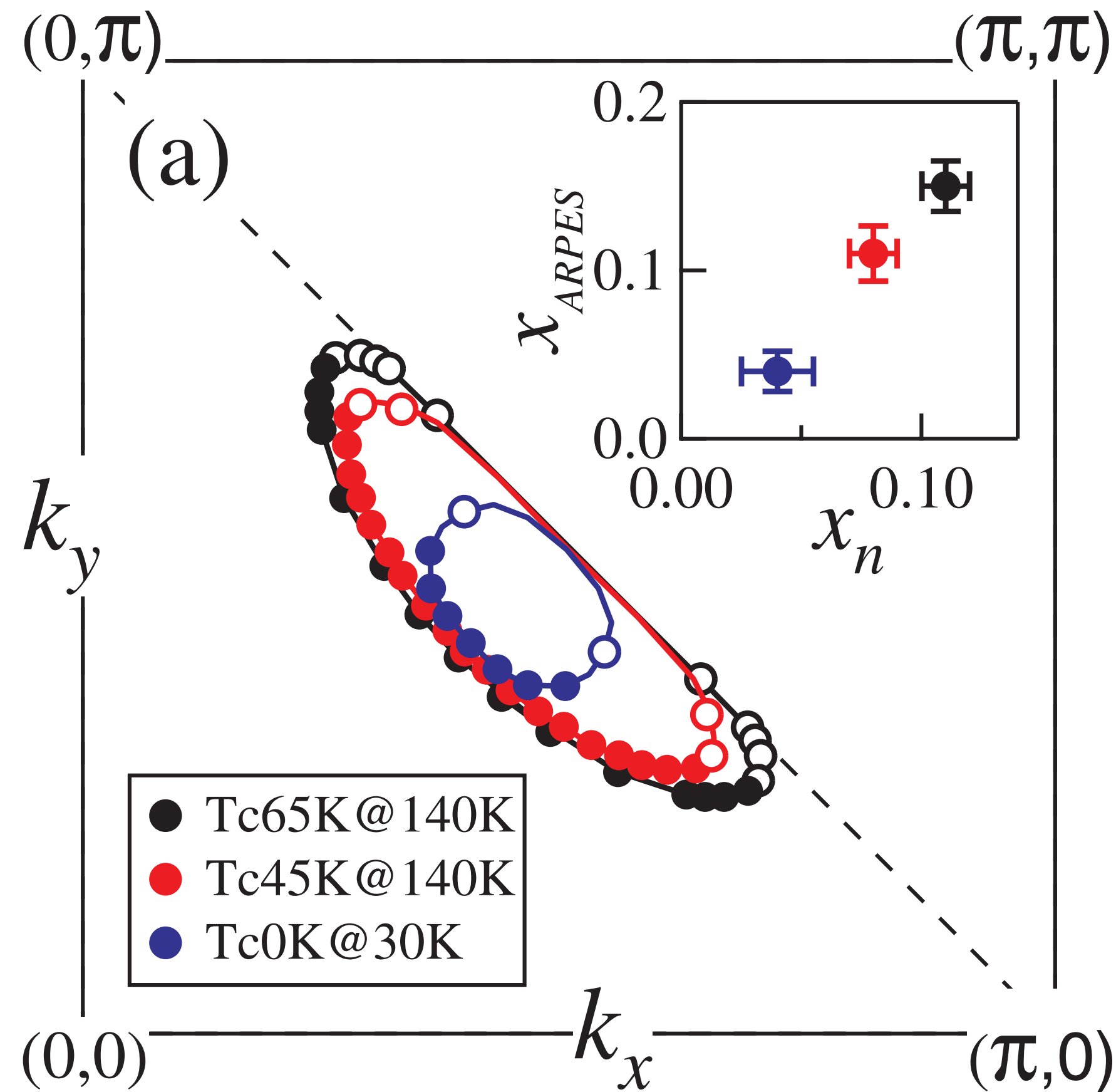


Reconstructed Fermi Surface of Underdoped $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$ Cuprate Superconductors,
H.-B. Yang, J. D. Rameau, Z.-H. Pan, G. D. Gu,
P. D. Johnson, H. Claus, D. G. Hinks,
and T. E. Kidd, PRL **107**, 047003 (2011).

Non-Luttinger volume Fermi surfaces
from various self-consistent Green's
function approaches.

Kai-Yu Yang, T. M. Rice, Fu-Chun Zhang,
Phys. Rev. B **73**, 174501 (2006).
T. D. Stanescu and G. Kotliar,
Phys. Rev. B **74**, 125110 (2006).
C. Berthod, T. Giamarchi, S. Biermann, and A. Georges,
Phys. Rev. Lett. **97**, 136401 (2006).
S. Sakai, Y. Motome, M. Imada,
Phys. Rev. Lett. **102**, 056404 (2009).
J. Skolimowski and M. Fabrizio,
Phys. Rev. B **106**, 045109 (2022).
N. Wagner...A. Georges, G. Sangiovanni,
Nature Communication **14**, 7531 (2023)
Jinchao Zhao, Gabriele La Nave, Philip Phillips,
Phys. Rev. B **108**, 165135
Jing-Yu Zhao, Zheng-Yu Weng, arXiv:2309.11556

Photoemission expts at small p



Reconstructed Fermi Surface of Underdoped $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$ Cuprate Superconductors,
H.-B. Yang, J. D. Rameau, Z.-H. Pan, G. D. Gu,
P. D. Johnson, H. Claus, D. G. Hinks,
and T. E. Kidd, PRL **107**, 047003 (2011).

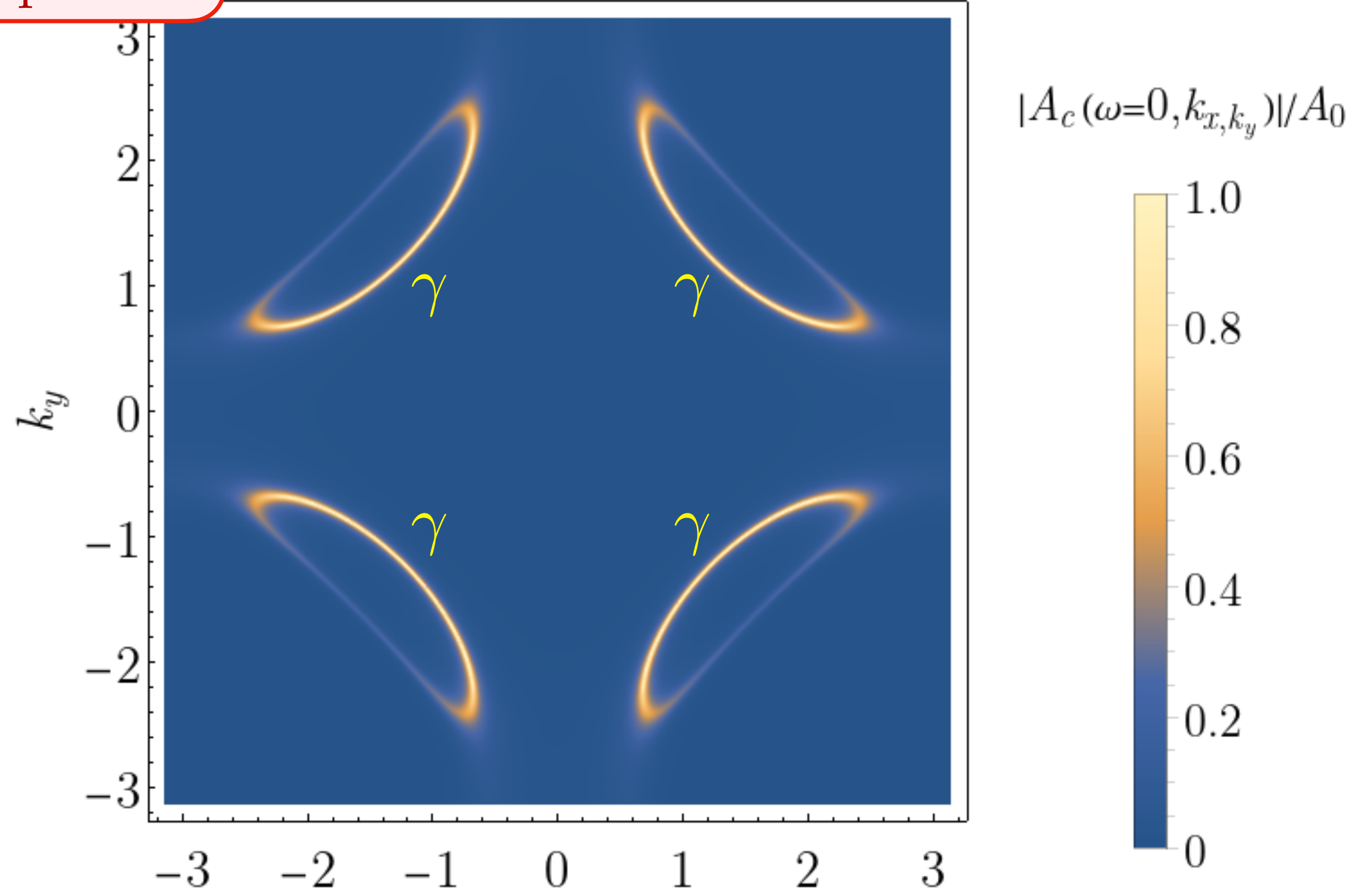
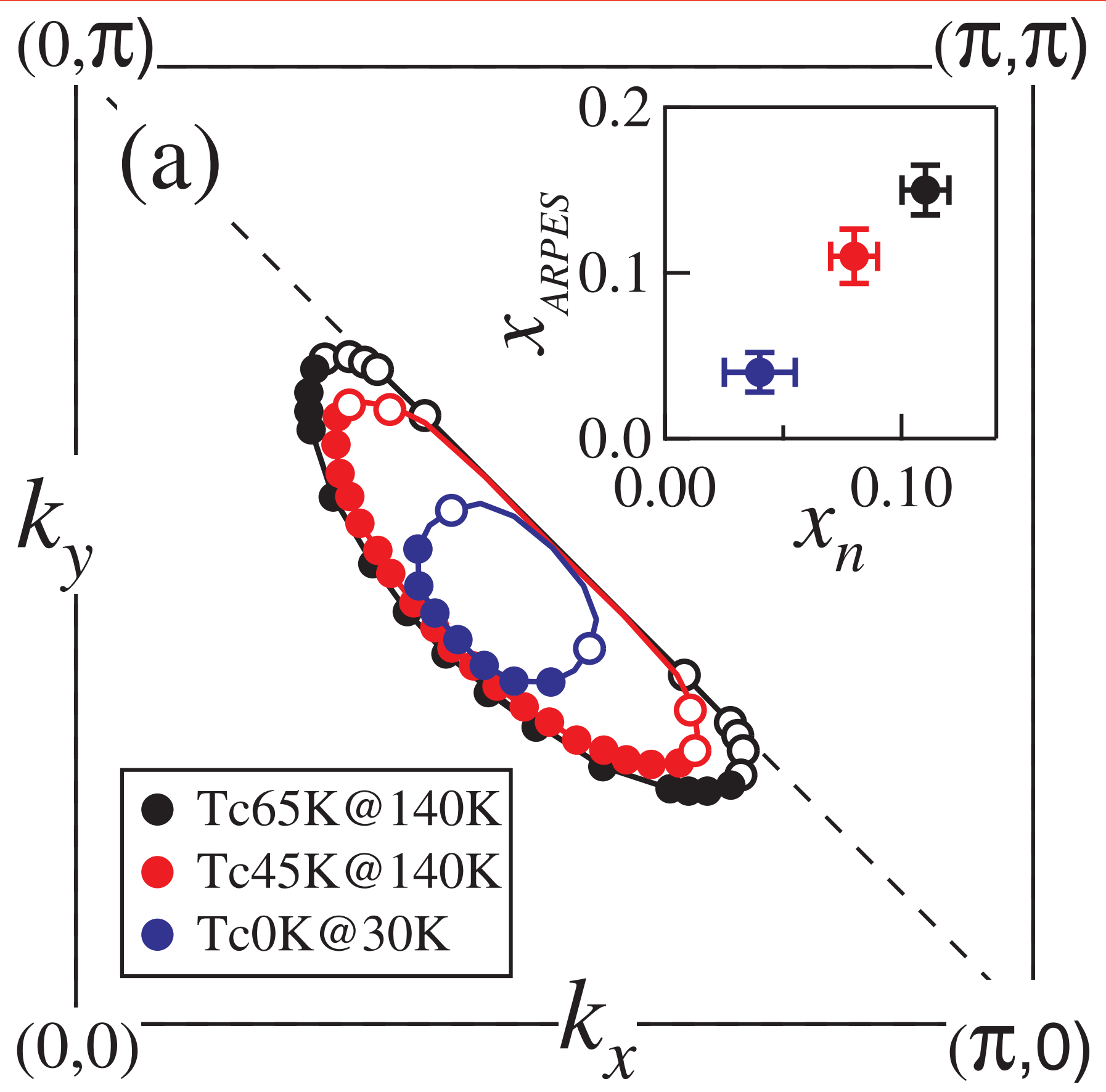
Non-Luttinger volume Fermi surfaces
from various self-consistent Green's
function approaches.

Oshikawa's topological Luttinger
argument implies that
non-Luttinger Fermi surfaces
must be accompanied by
fractionalized spinon excitations

T. Senthil, M. Vojta, S.S., PRB **69**, 035111 (2004)
R. K. Kaul, A. Kolezhuk, M. Levin, S. S., T. Senthil, PRB **75**, 235122 (2007)
Y. Qi, S. S., PRB **81**, 115129 (2010)

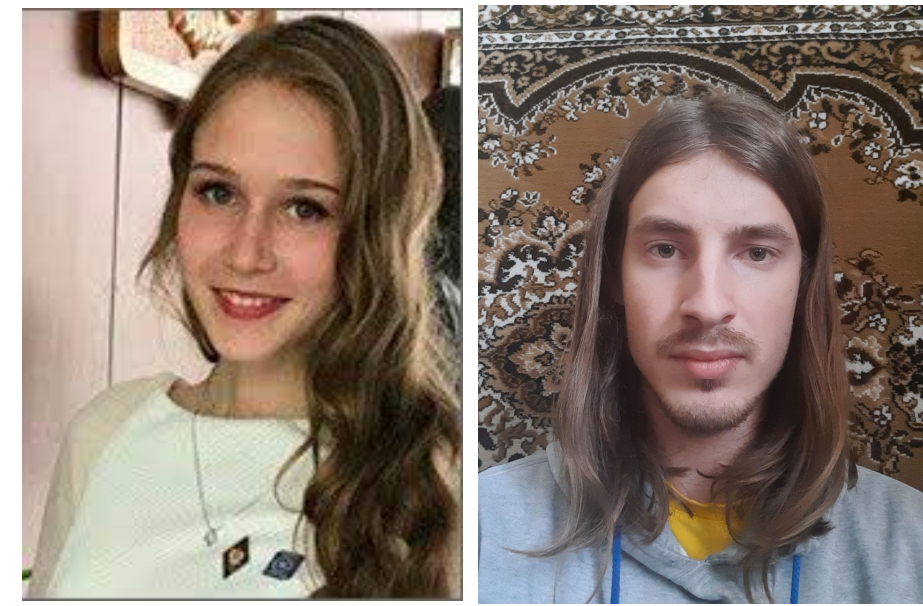
Ancilla theory of pseudogap metal: FL*—
 electrons hybridized by a half-filled band of fermions (f_1),
 and exchange coupled to a spin liquid layer with spinons.

E. Mascot, A. Nikolaenko, M. Tikhanovskaya, Ya-Hui Zhang,
 D. K. Morr, and S. S., PRB **105**, 075146 (2022)

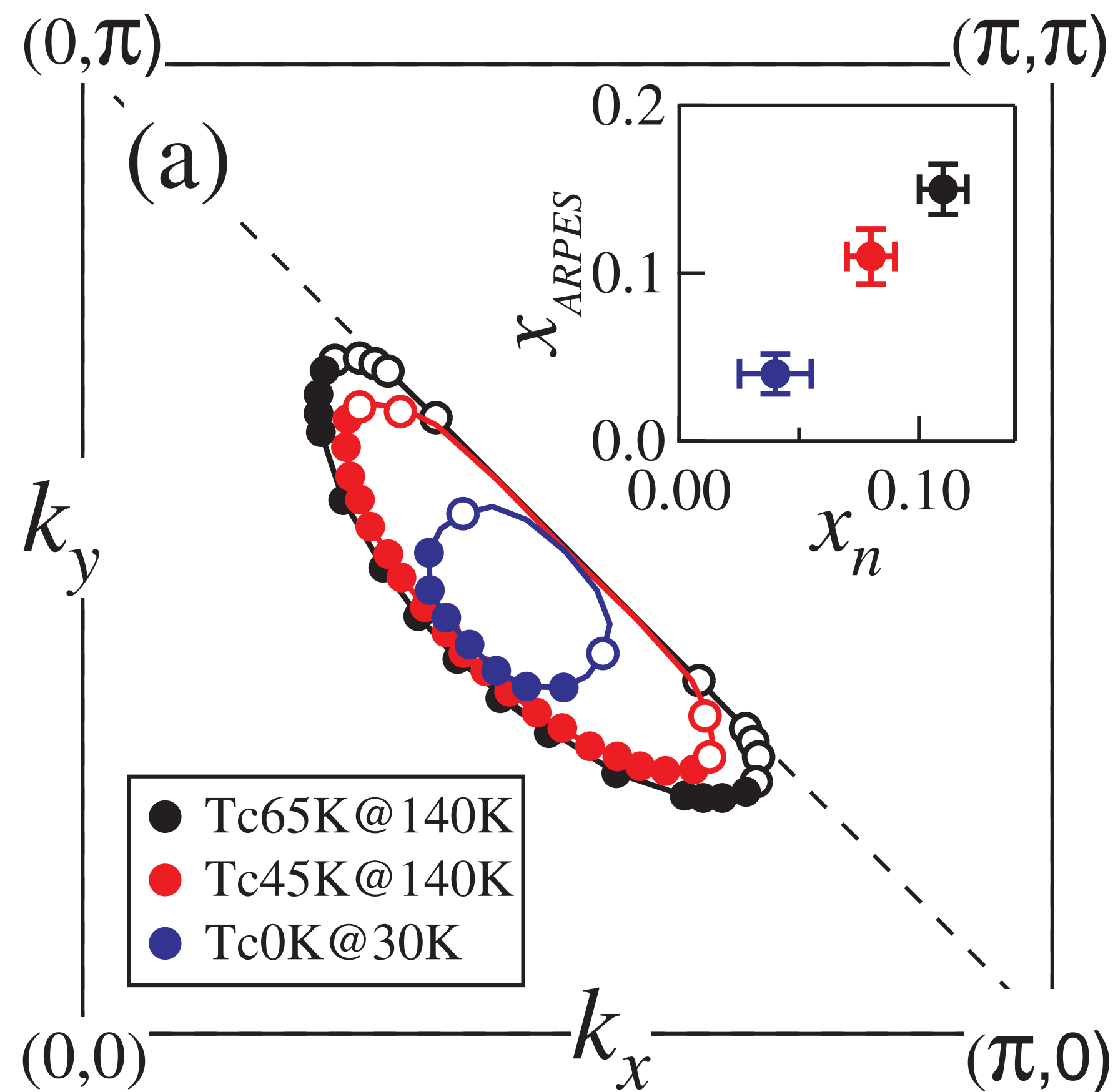


*Reconstructed Fermi Surface of Underdoped
 $Bi_2Sr_2CaCu_2O_{8+\delta}$ Cuprate Superconductors,*
 H.-B. Yang, J. D. Rameau, Z.-H. Pan, G. D. Gu,
 P. D. Johnson, H. Claus, D. G. Hinks,
 and T. E. Kidd, PRL **107**, 047003 (2011).

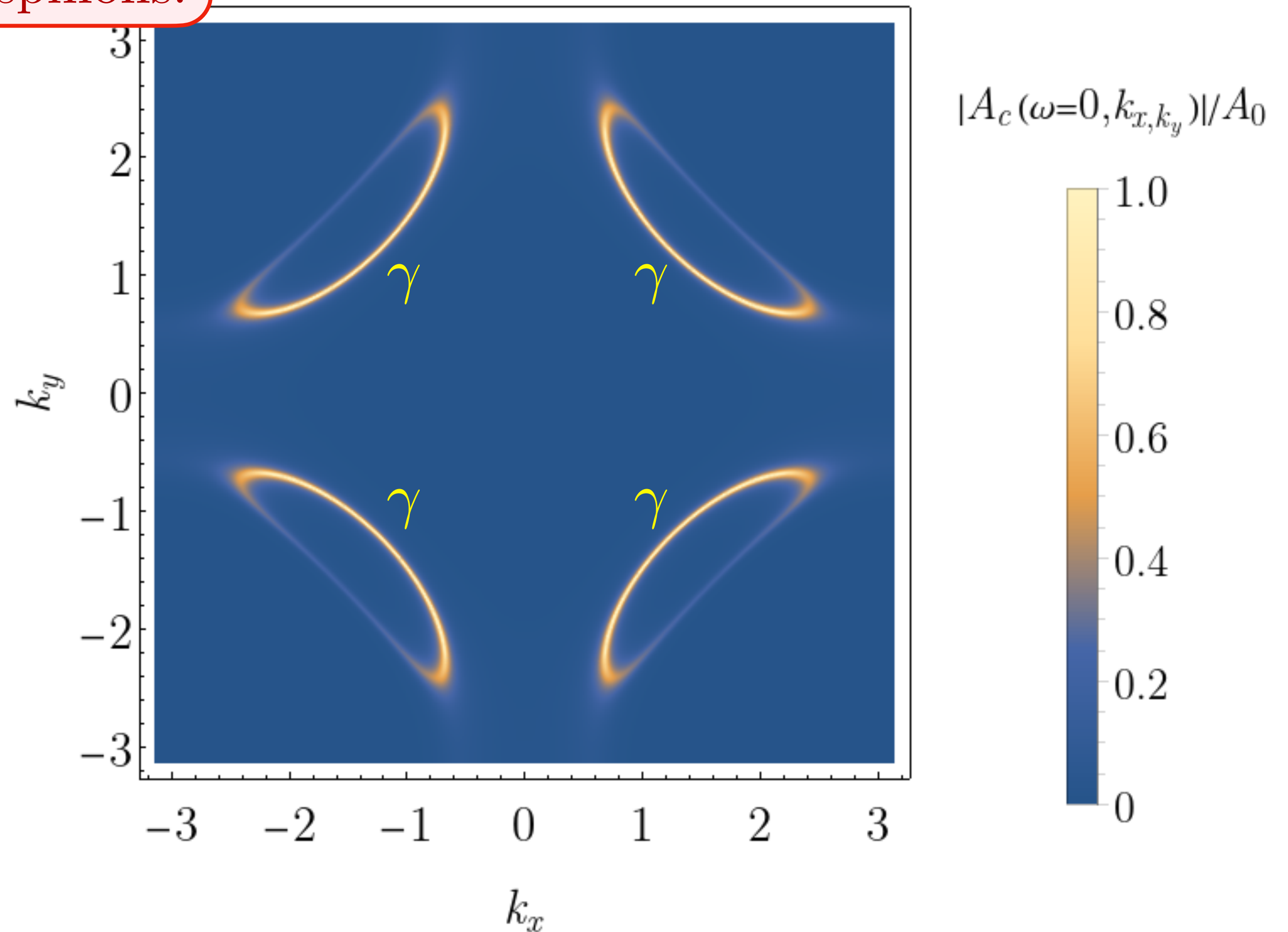
$$H_{mf} = - \sum_{i,j} t_{ij} c_{i\sigma}^\dagger c_{j\sigma} - \sum_{i,j}^{k_x} t_{1,ij} f_{1i\sigma}^\dagger f_{1j\sigma} - \sum_i \Phi (c_{i\sigma}^\dagger f_{1i\sigma} + f_{1i\sigma}^\dagger c_{i\sigma})$$



Ancilla theory of pseudogap metal: FL*—
electrons hybridized by a half-filled band of fermions (f_1),
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Reconstructed Fermi Surface of Underdoped $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$ Cuprate Superconductors,
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P. D. Johnson, H. Claus, D. G. Hinks,
and T. E. Kidd, PRL **107**, 047003 (2011).



Puzzle:
why is the photoemission intensity
on the “backsides” of the hole pockets so low?

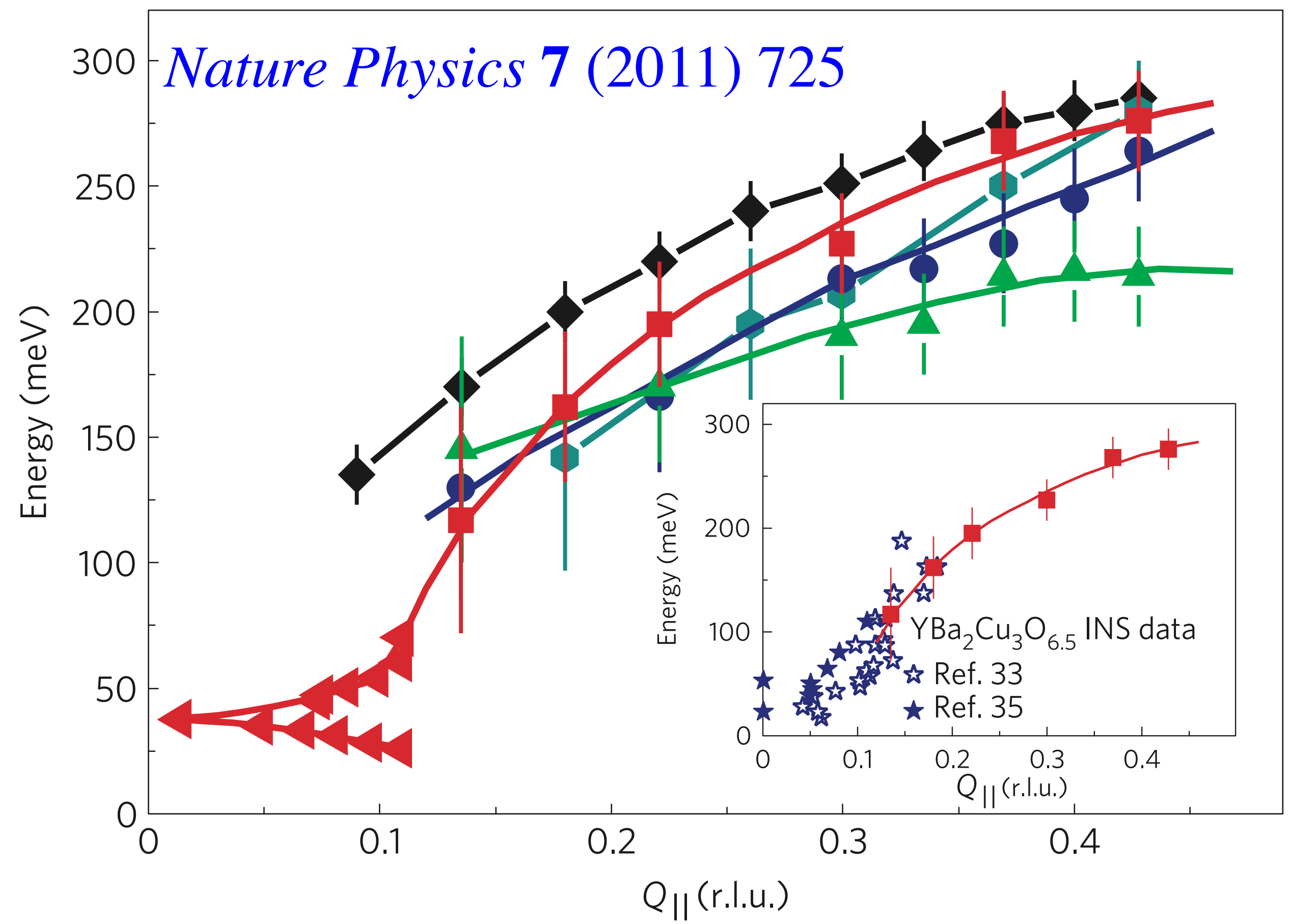
Pseudogap puzzles:

- A. Photoemission: Fermi arcs
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- C. Quantum oscillations
- D. Velocities of Bogoliubov quasiparticles
in the d-wave superconductor

Intense paramagnon excitations in a large family of high-temperature superconductors

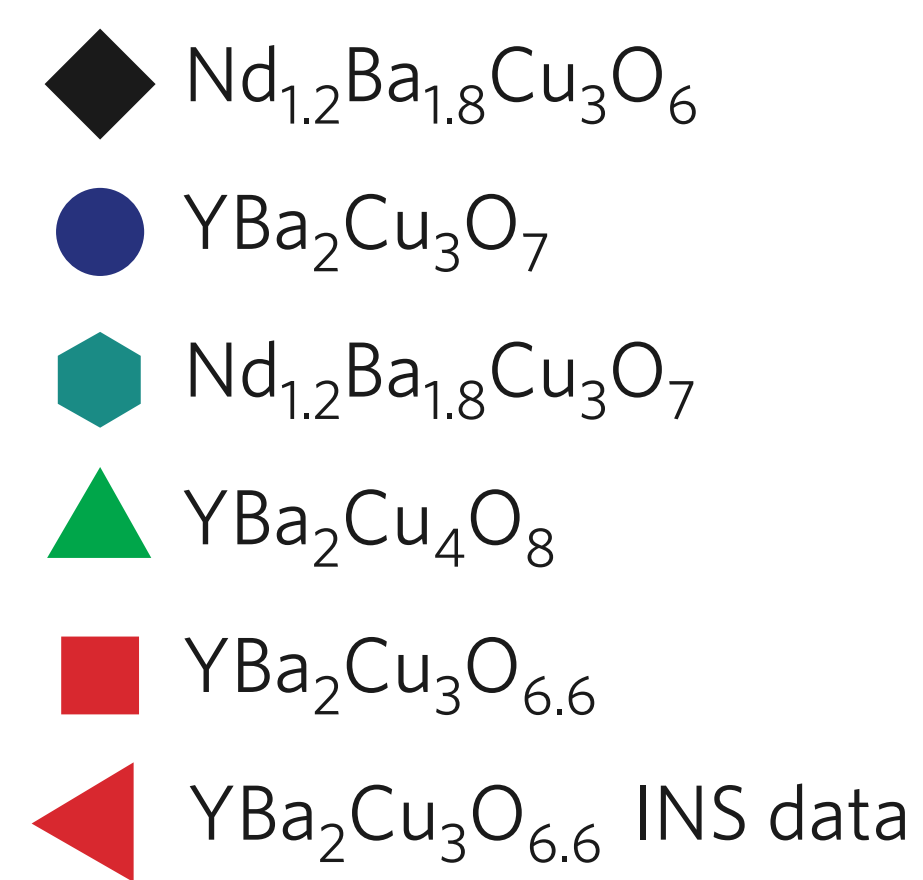
M. Le Tacon^{1*}, G. Ghiringhelli², J. Chaloupka¹, M. Moretti Sala², V. Hinkov^{1,3}, M. W. Haverkort¹, M. Minola², M. Bakr¹, K. J. Zhou⁴, S. Blanco-Canosa¹, C. Monney⁴, Y. T. Song¹, G. L. Sun¹, C. T. Lin¹, G. M. De Luca⁵, M. Salluzzo⁵, G. Khaliullin¹, T. Schmitt⁴, L. Braicovich² and B. Keimer^{1*}

- ◆ $\text{Nd}_{1.2}\text{Ba}_{1.8}\text{Cu}_3\text{O}_6$
- $\text{YBa}_2\text{Cu}_3\text{O}_7$
- ◆ $\text{Nd}_{1.2}\text{Ba}_{1.8}\text{Cu}_3\text{O}_7$
- ▲ $\text{YBa}_2\text{Cu}_4\text{O}_8$
- $\text{YBa}_2\text{Cu}_3\text{O}_{6.6}$
- ◀ $\text{YBa}_2\text{Cu}_3\text{O}_{6.6}$ INS data

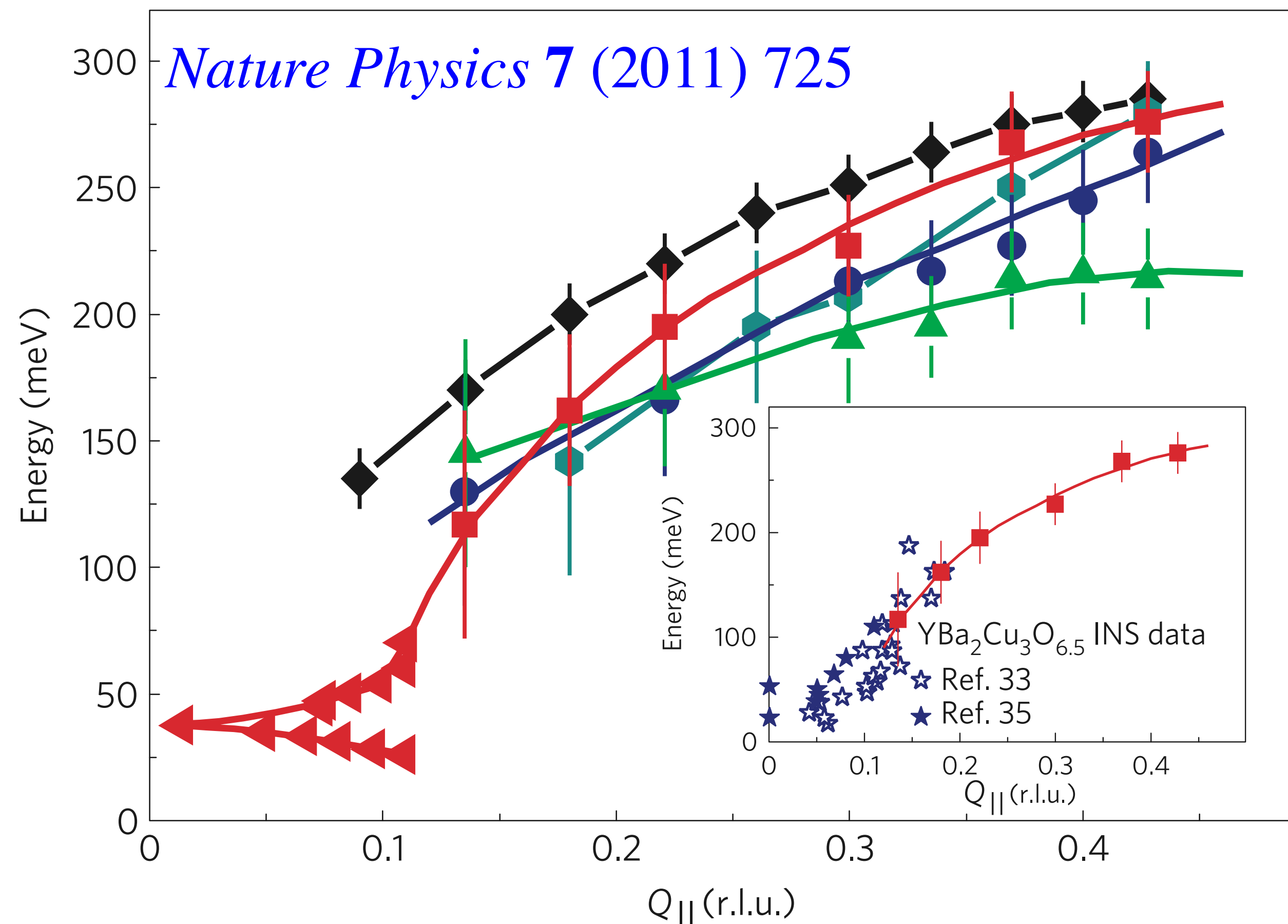


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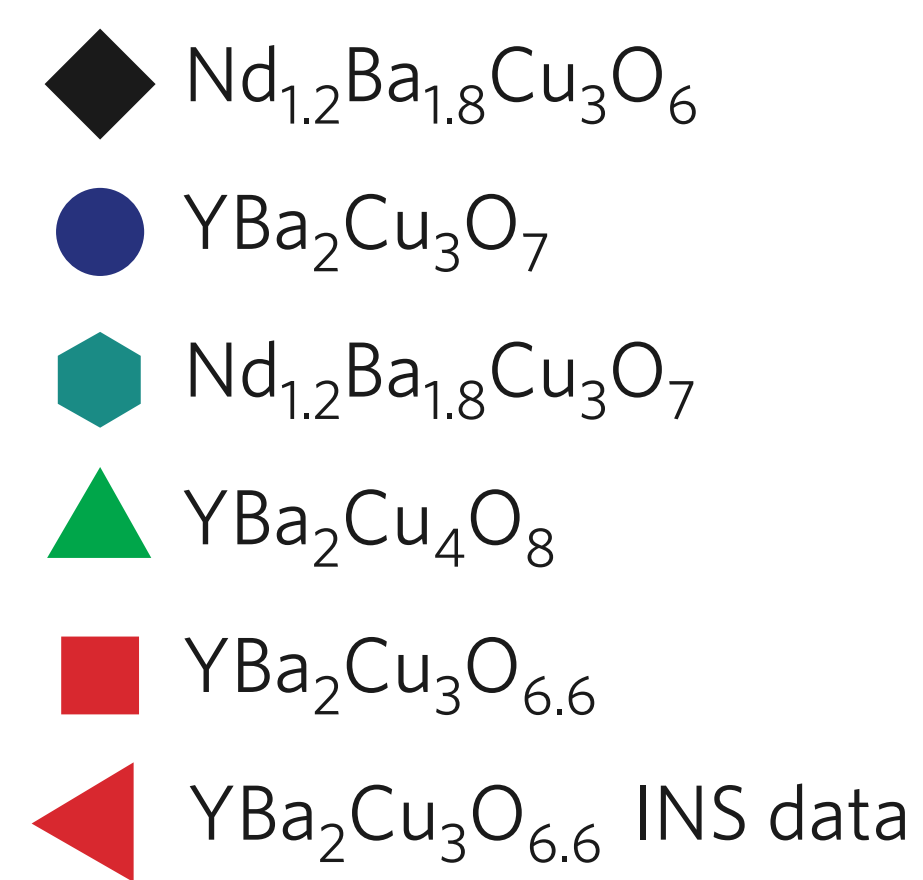


- Difficult to have intense paramagnons from a small Fermi surface.

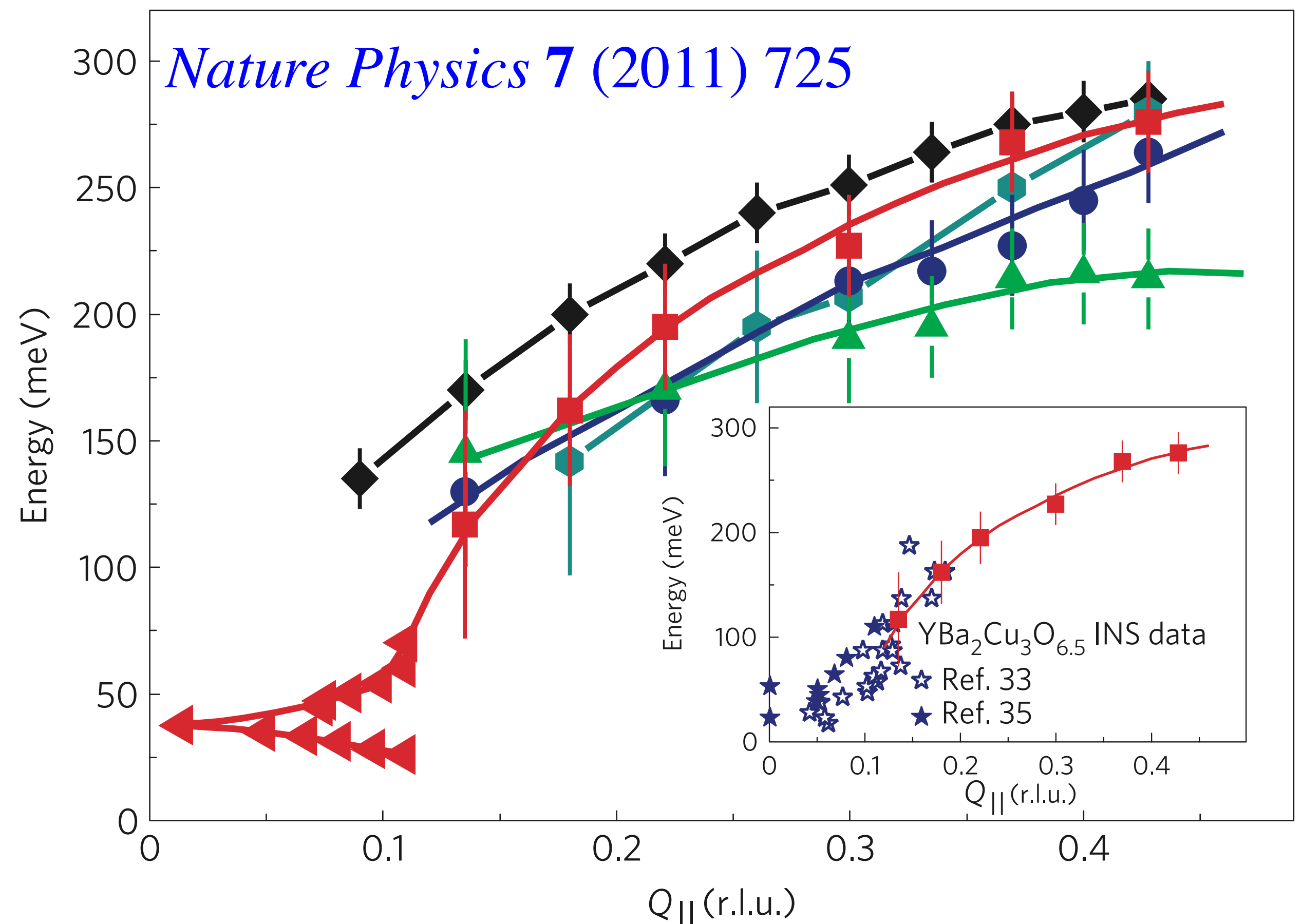


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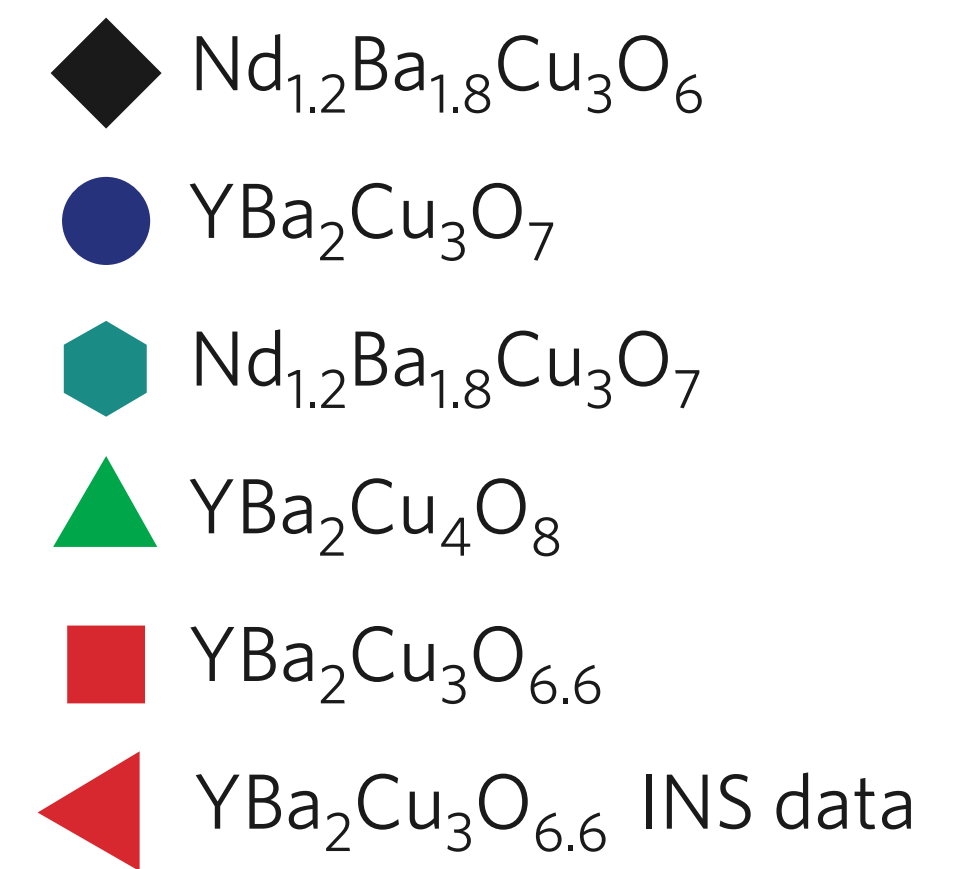


- Difficult to have intense paramagnons from a small Fermi surface.
- Spin waves only present at low energies in the presence of antiferromagnetic order

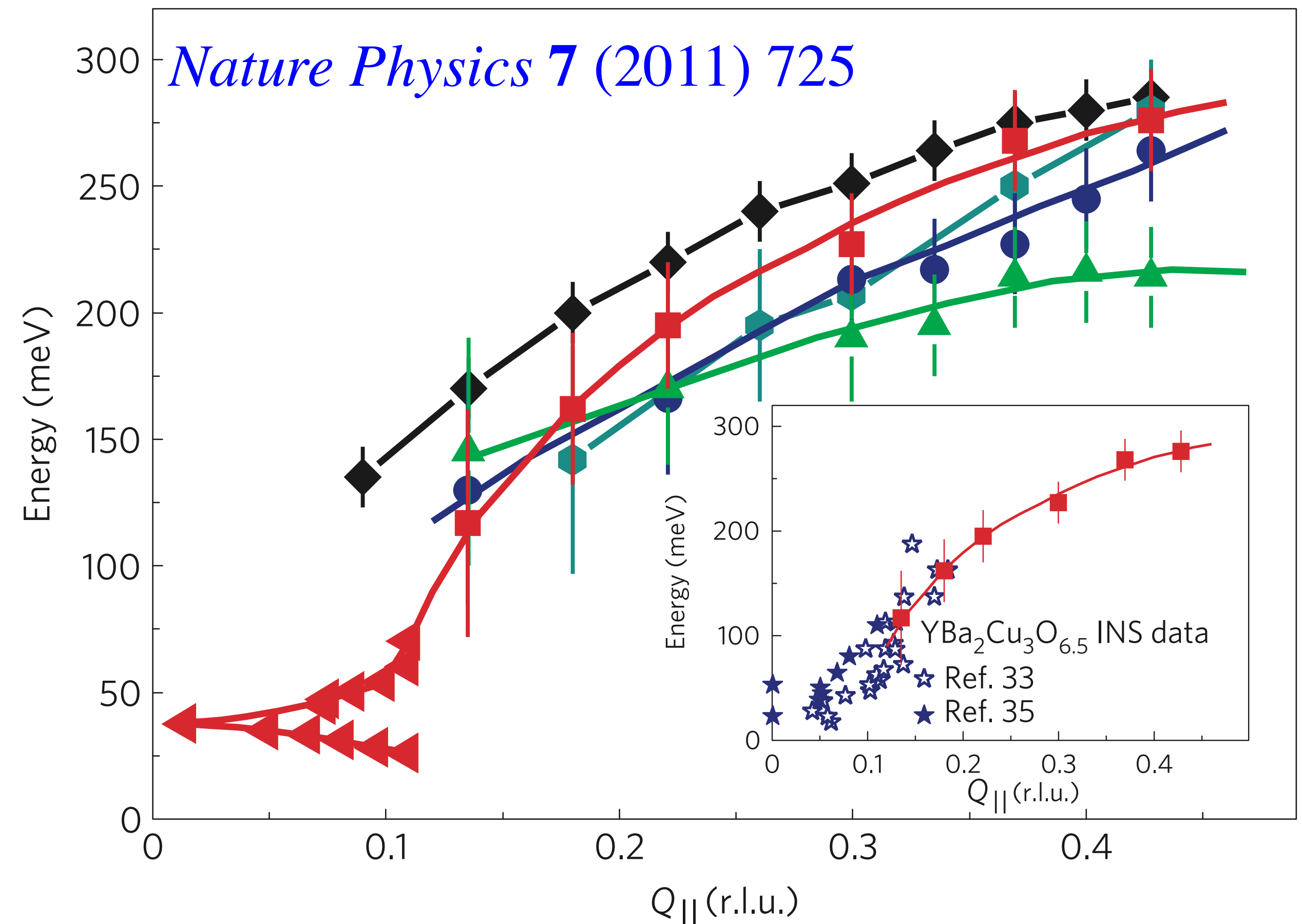


Intense paramagnon excitations in a large family of high-temperature superconductors

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- Difficult to have intense paramagnons from a small Fermi surface.
- Spin waves only present at low energies in the presence of antiferromagnetic order
- Most natural interpretation is a spinon continuum (I claim)...



Pseudogap puzzles:

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- B. RIXS: Intense paramagnons
- C. Quantum oscillations
- D. Velocities of Bogoliubov quasiparticles
in the d-wave superconductor

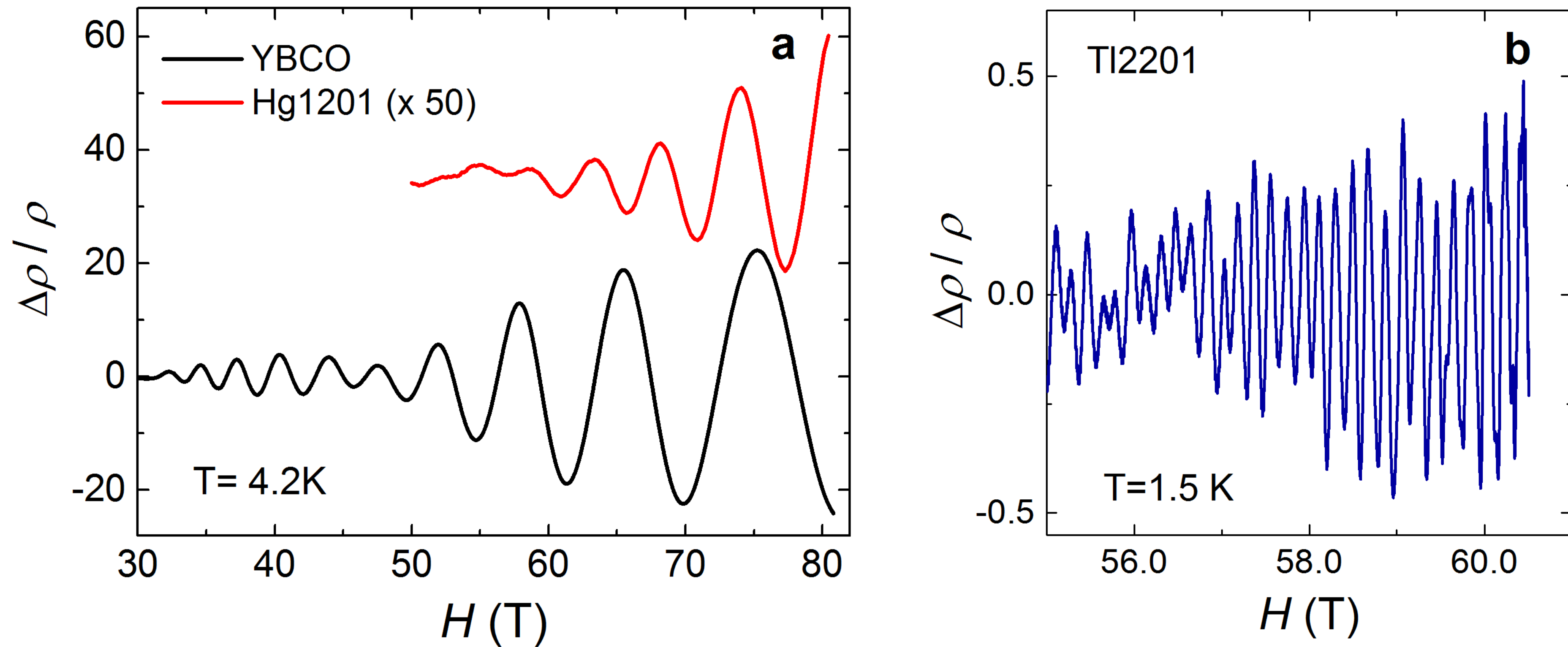


Figure 4

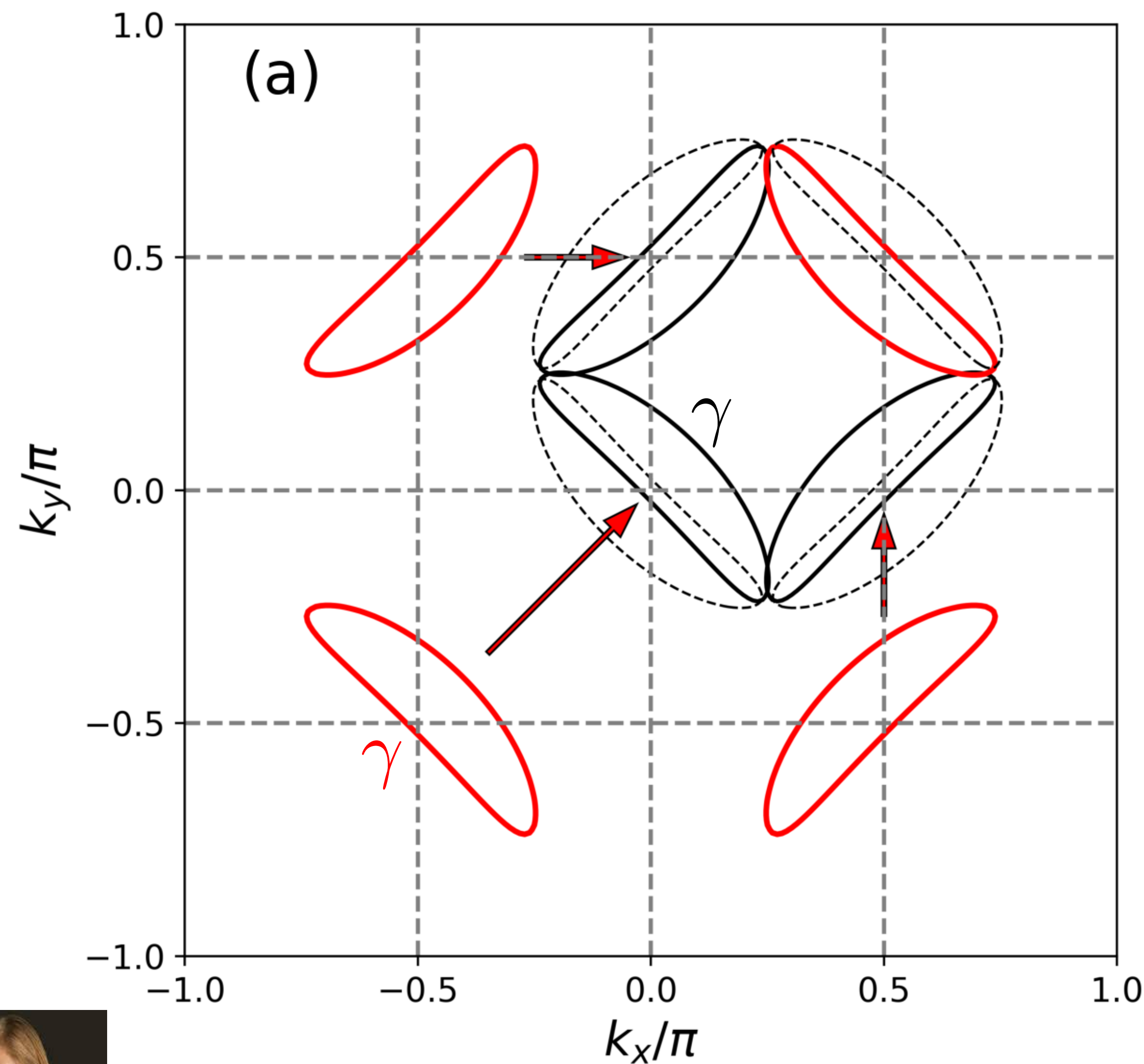
Quantum oscillations in cuprates. **a)** Underdoped YBCO ($p = 0.11$, $T_c = 62$ K) (black (22)) and Hg1201 ($p \simeq 0.1$, $T_c = 72$ K) (red (33), $\times 50$). **b)** Overdoped Tl2201 ($p \simeq 0.3$, $T_c \approx 10$ K) (28).

22. Vignolle B, et al. 2013. *C. R. Physique* 14:39–52

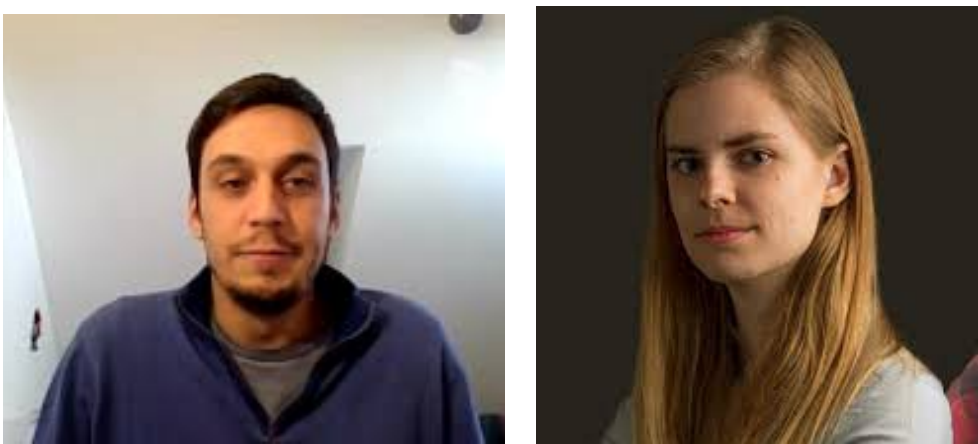
28. Vignolle B, et al. 2008. *Nature* 455:952–955

33. Barisic N, et al. 2013. *Nat. Phys.* 9:761–764

FL*

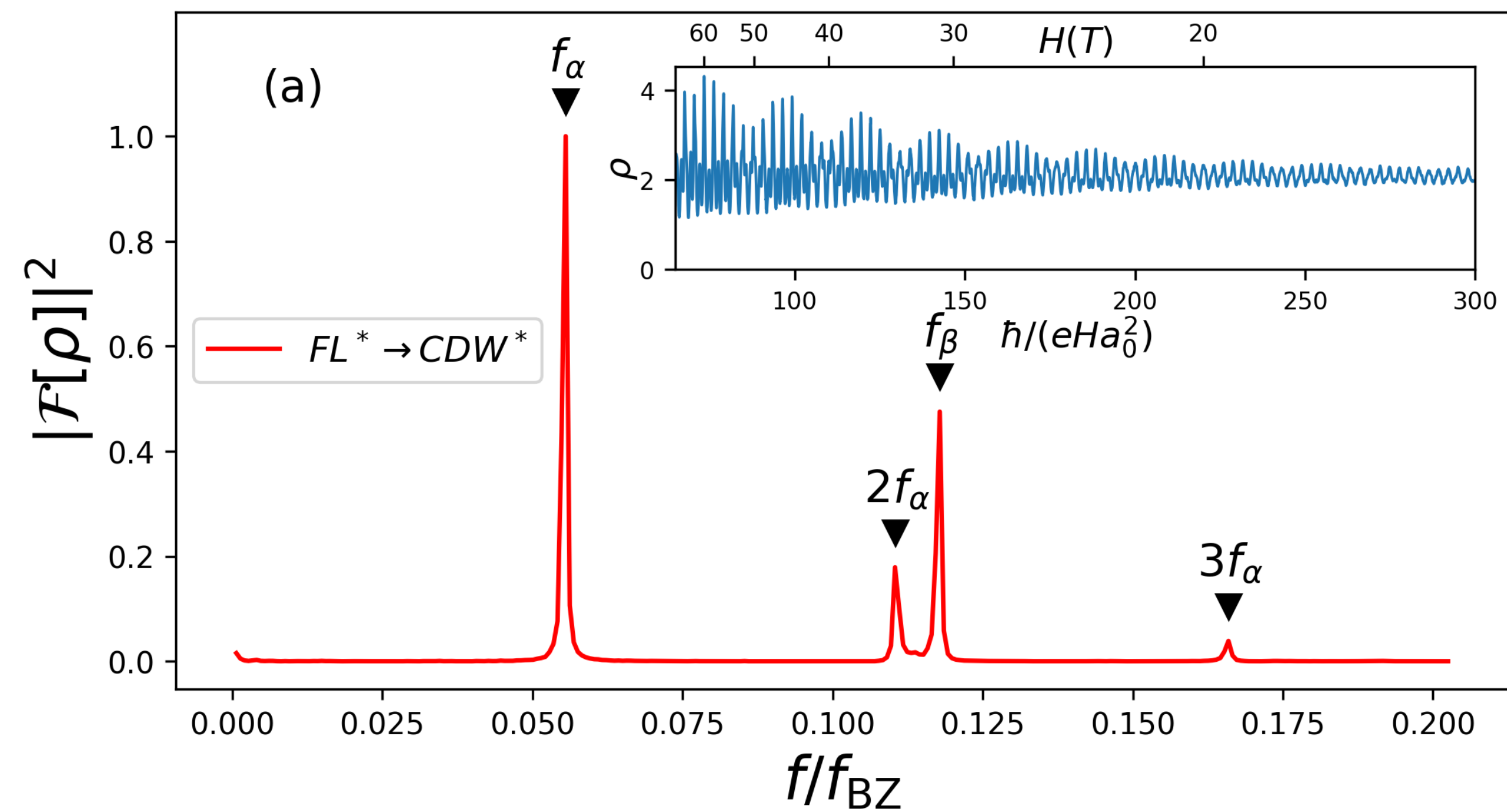
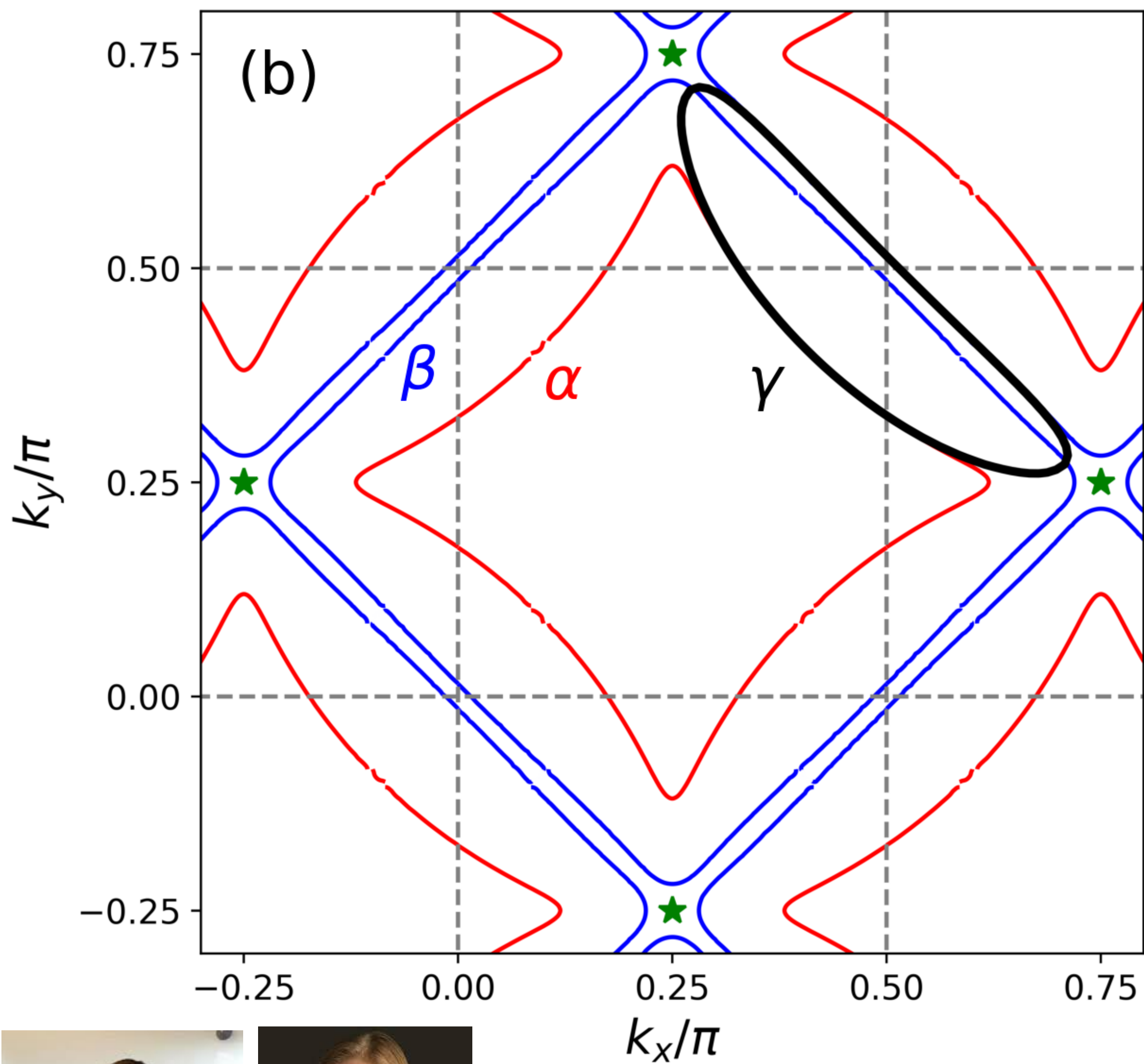


N. Harrison and S. Sebastian
electron pocket
(PRL **106**, 226402 (2011))



Pietro Bonetti, Maine Christos and S.S. (BCS), arXiv:2405.08817

$FL^* \rightarrow CDW^*$

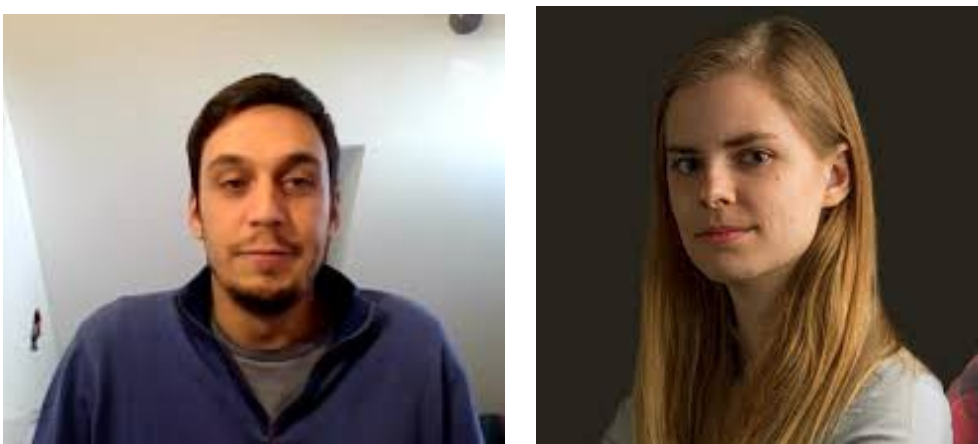


Computation does not account for spinons.

α and β pockets

show clear quantum oscillations.

Long Zhang and Jia-Wei Mei,
EPL **114**, 47008 (2016)

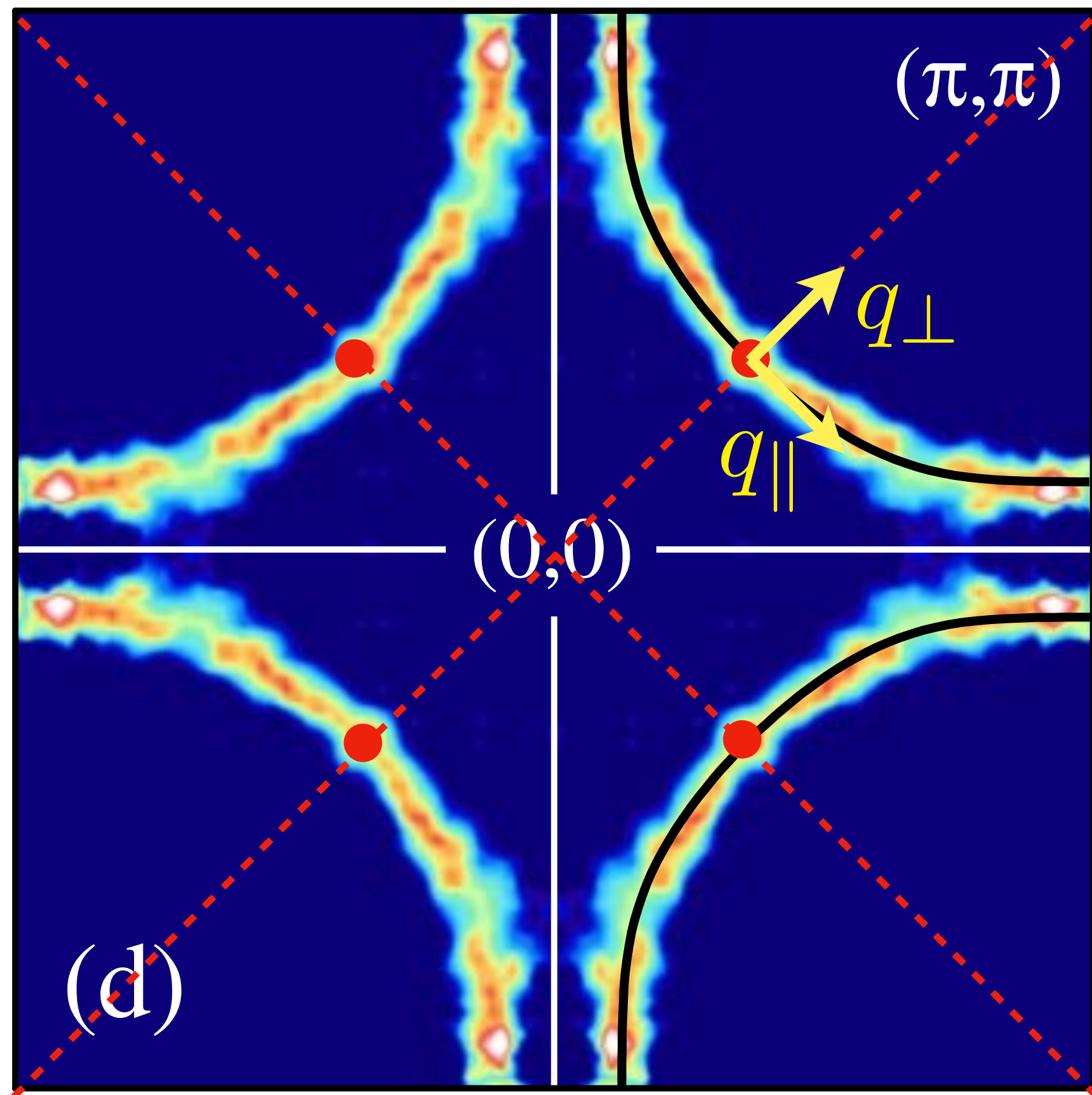


Pietro Bonetti, Maine Christos and S.S. (BCS), arXiv:2405.08817

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- D. Velocities of Bogoliubov quasiparticles
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FL \rightarrow dSC



BCS/Bogoliubov quasiparticles
in a d -wave superconductor

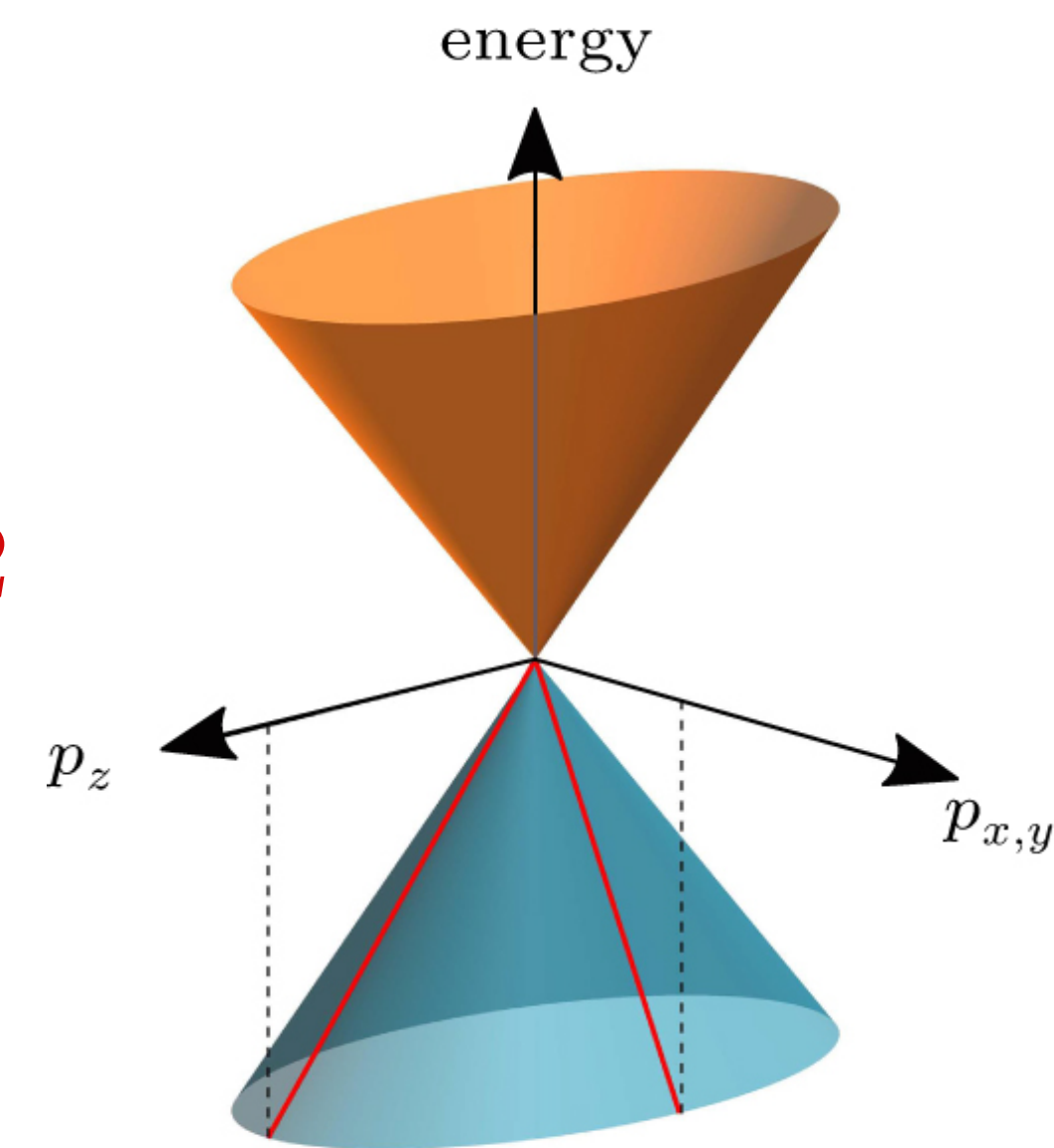
$$E_{\mathbf{k}} = \left(\varepsilon_{\mathbf{k}}^2 + \Delta_{\mathbf{k}}^2 \right)^{1/2}$$

$$\Delta_{\mathbf{k}} = \Delta_0 (\cos k_x - \cos k_y)$$

4 nodal points where

$$E_{\mathbf{k}_0 + \mathbf{q}} = \left(v_F^2 q_{\perp}^2 + v_{\Delta}^2 q_{\parallel}^2 \right)^{1/2}$$

with $v_F \gg v_{\Delta}$.



Early work described the emergence of nodal d -wave superconductivity from a spin liquid with spinons with a Dirac dispersion: in this approach, the spinons of the spin liquid are directly transformed into the nodal Bogoliubov quasiparticles of the d -wave superconductor.

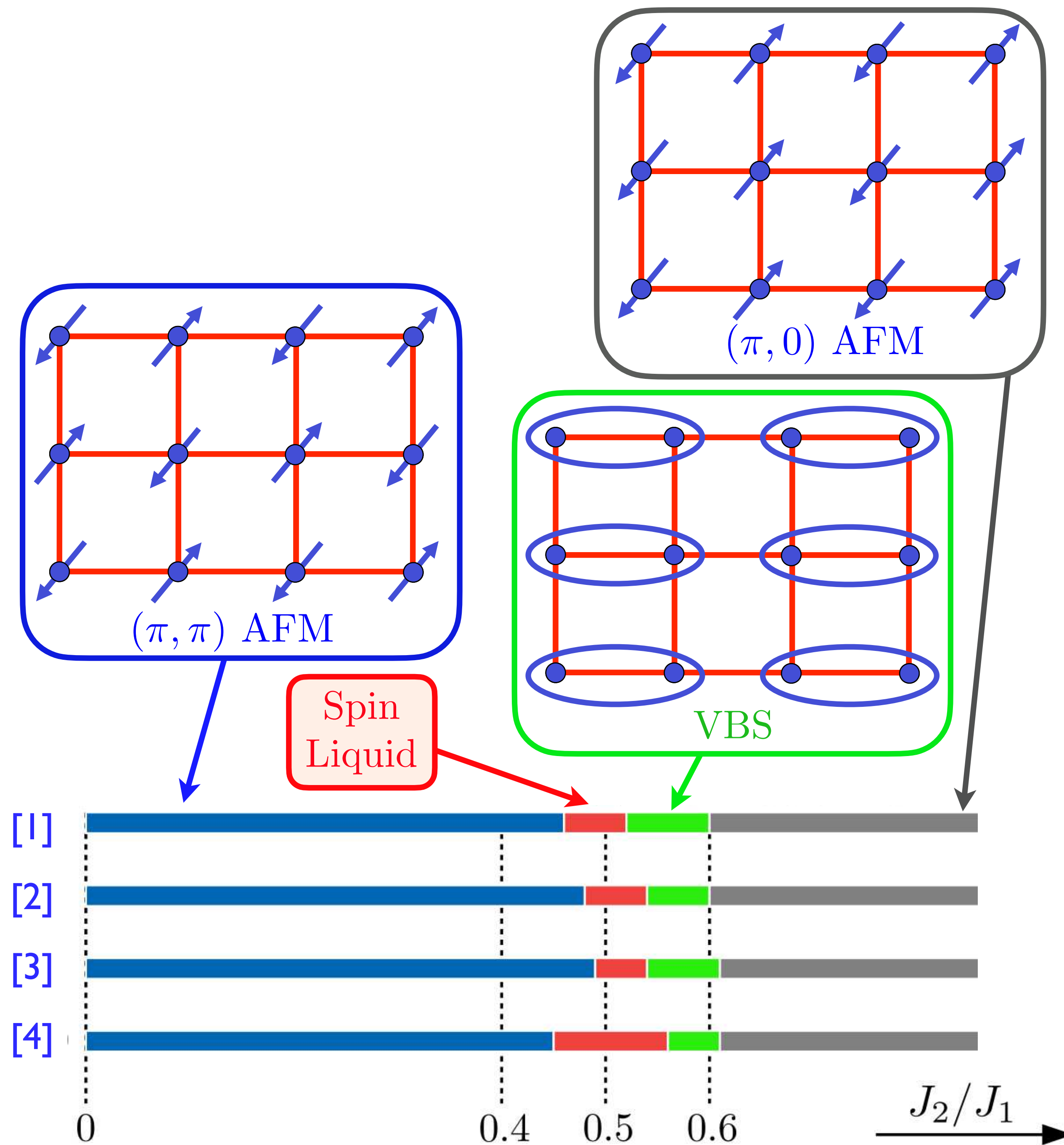
F.C. Zhang, C. Gros, T.M. Rice, H. Shiba, Superconductor Sci. Tech. **1** (1988) 36

D. A. Ivanov, T. Senthil, PRB **66**, 115111 (2002).

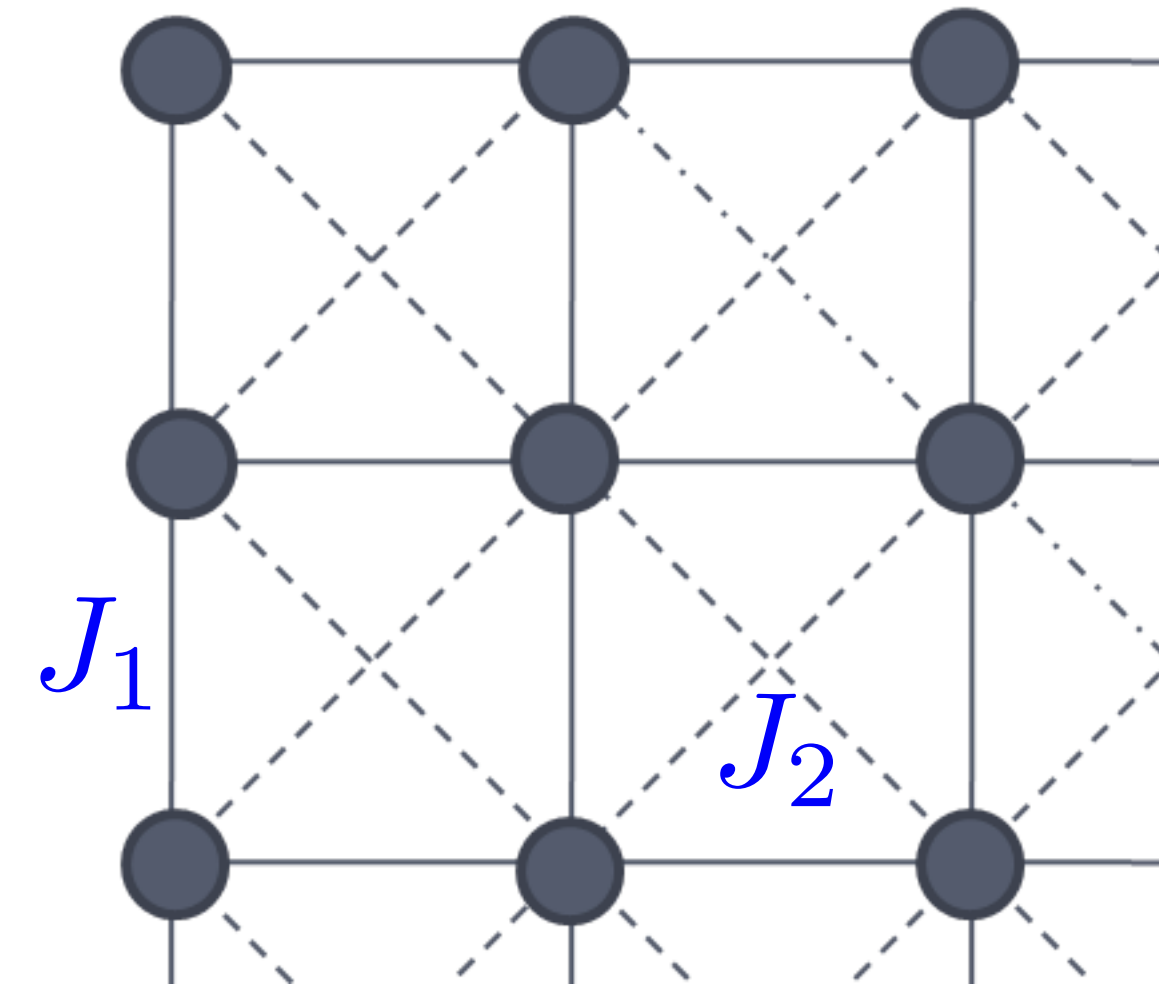
P. A. Lee, N. Nagaosa, X.-G. Wen, RMP **78**, 17 (2006)

This leads to a d -wave superconductor with $v_F \approx v_\Delta$ at low doping, in contrast to observations.

Recent insights on
square lattice
spin liquids

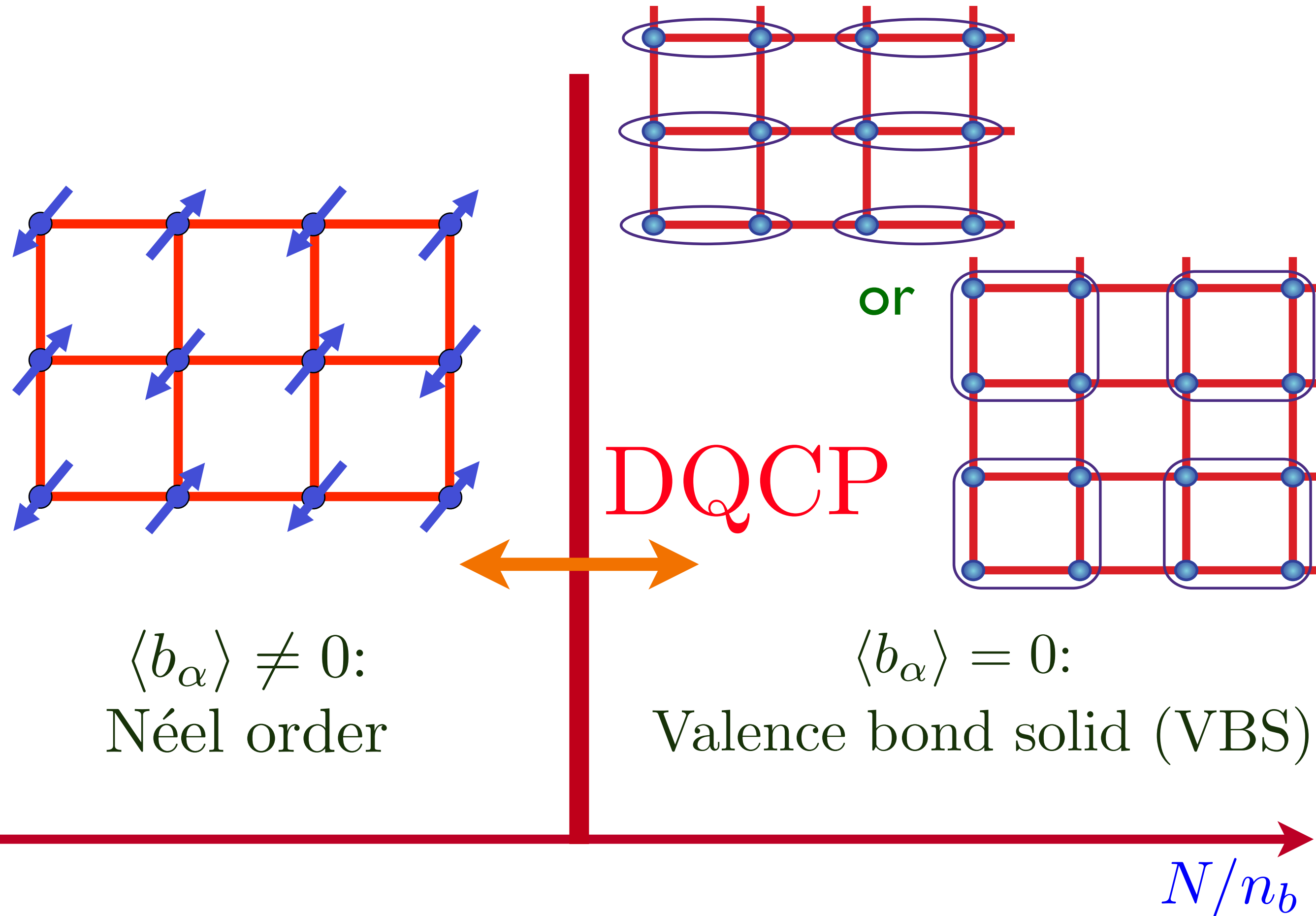


$$H = J_1 \sum_{\langle i,j \rangle} \mathbf{S}_i \cdot \mathbf{S}_j + J_2 \sum_{\langle\langle i,j \rangle\rangle} \mathbf{S}_i \cdot \mathbf{S}_j$$



1. L. Wang and A. W. Sandvik, *Phys. Rev. Lett.* **121**, 107202 (2018)
2. F. Ferrari and F. Becca, *Phys. Rev. B* **102**, 014417 (2020)
3. Y. Nomura and M. Imada, *Phys. Rev. X* **11**, 031034 (2021)
4. W.-Y. Liu, S.-S. Gong, Y.-B. Li, D. Poilblanc, W.-Q. Chen, and Z.-C. Gu, *Science Bulletin* **67**, 1034 (2022)

Insulating $S=1/2$ antiferromagnet



$$H = \sum_{i < j} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j$$

Schwinger bosons

$$\mathbf{S}_i = \frac{1}{2} b_{i\alpha}^\dagger \boldsymbol{\sigma}_{\alpha\beta} b_{i\beta}, \quad \sum_{\alpha=1}^{N=2} b_{i\alpha}^\dagger b_{i\alpha} = n_b = 2S$$

Mean-field spin liquid
with gapped bosonic spinons.

Low energy \mathbb{CP}^1 U(1) gauge theory

$$z_\alpha \sim b_{A\alpha} + \varepsilon_{\alpha\beta} b_{B\beta}^\dagger$$

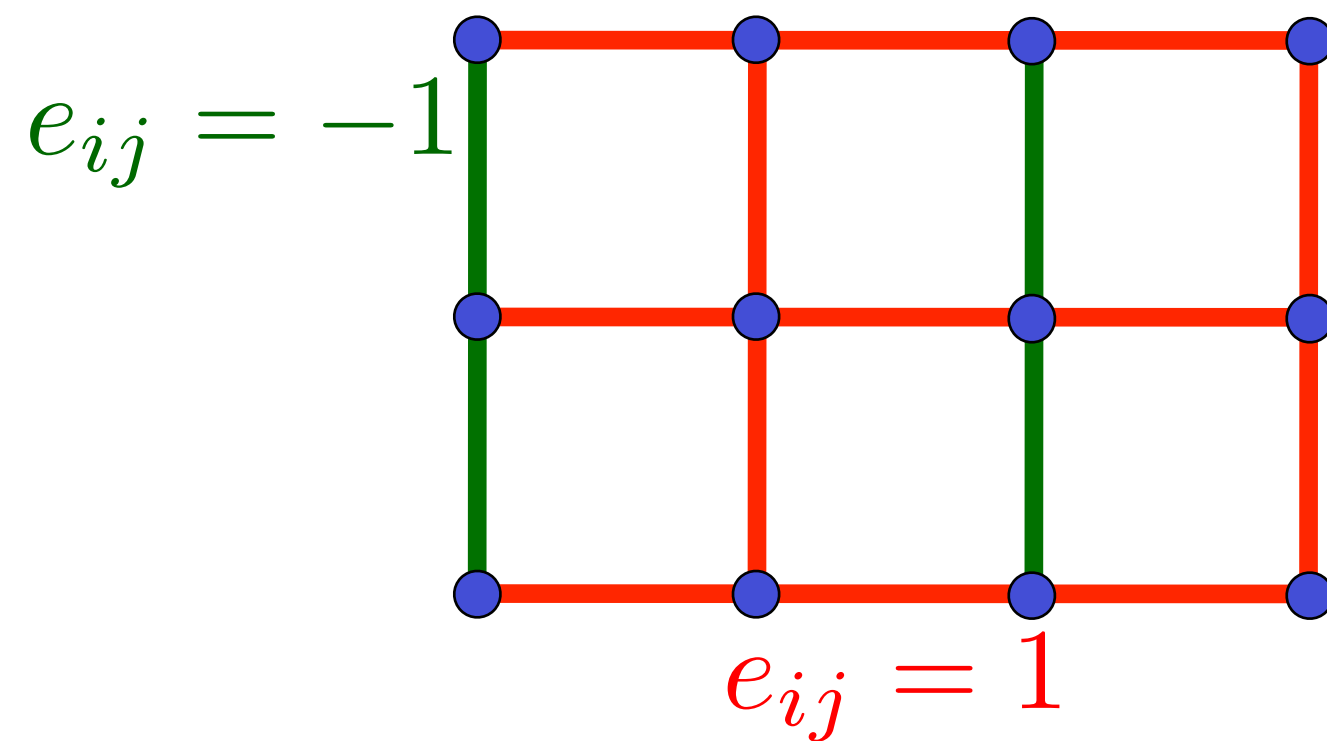
$$\mathcal{L} = |(\partial_\mu - ia_\mu) z_\alpha|^2 + s |z_\alpha|^2 + u |z_\alpha|^4 + \mathcal{L}_{\text{monopole}}$$

N. Read and S. Sachdev, Phys. Rev. Lett. **62**, 1694 (1989)

N. Read and S. Sachdev, Phys. Rev. B **42**, 4568 (1990)

Insulating $S=1/2$ antiferromagnet

π -flux Spin liquid



$$H = \sum_{i < j} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j$$

Schwinger fermions

$$\mathbf{S}_i = \frac{1}{2} f_{i\alpha}^\dagger \boldsymbol{\sigma}_{\alpha\beta} f_{i\beta}, \quad \sum_{\alpha=\uparrow,\downarrow} f_{i\alpha}^\dagger f_{i\alpha} = 1$$

π -flux mean-field theory
with gapless spinons at 2 Dirac points.

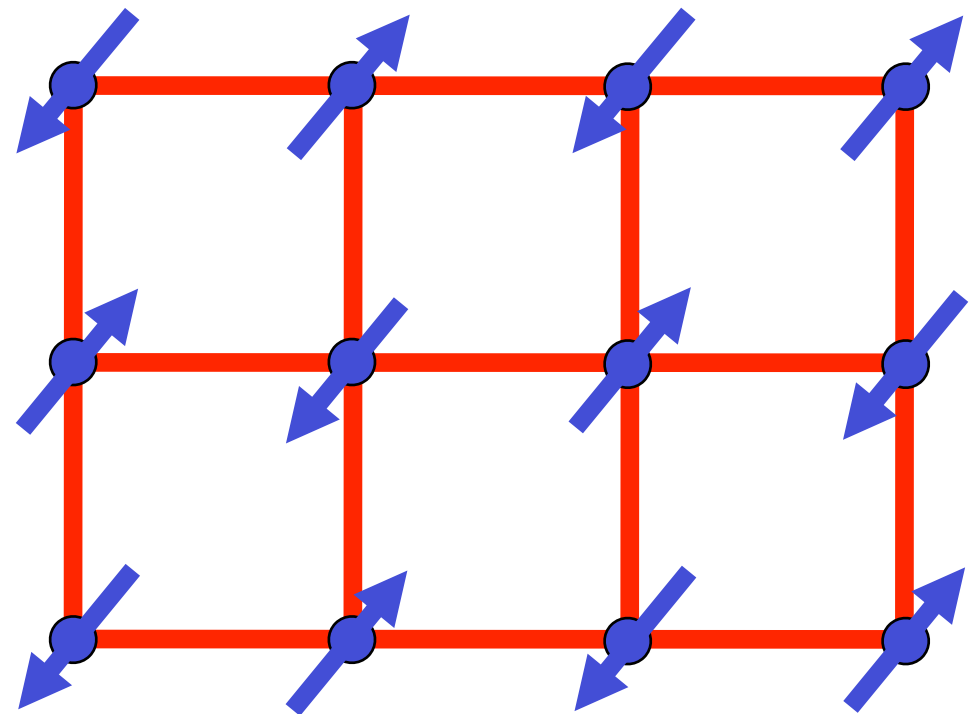
I. Affleck and J.B. Marston, PRB **37**, 3774 (1988)



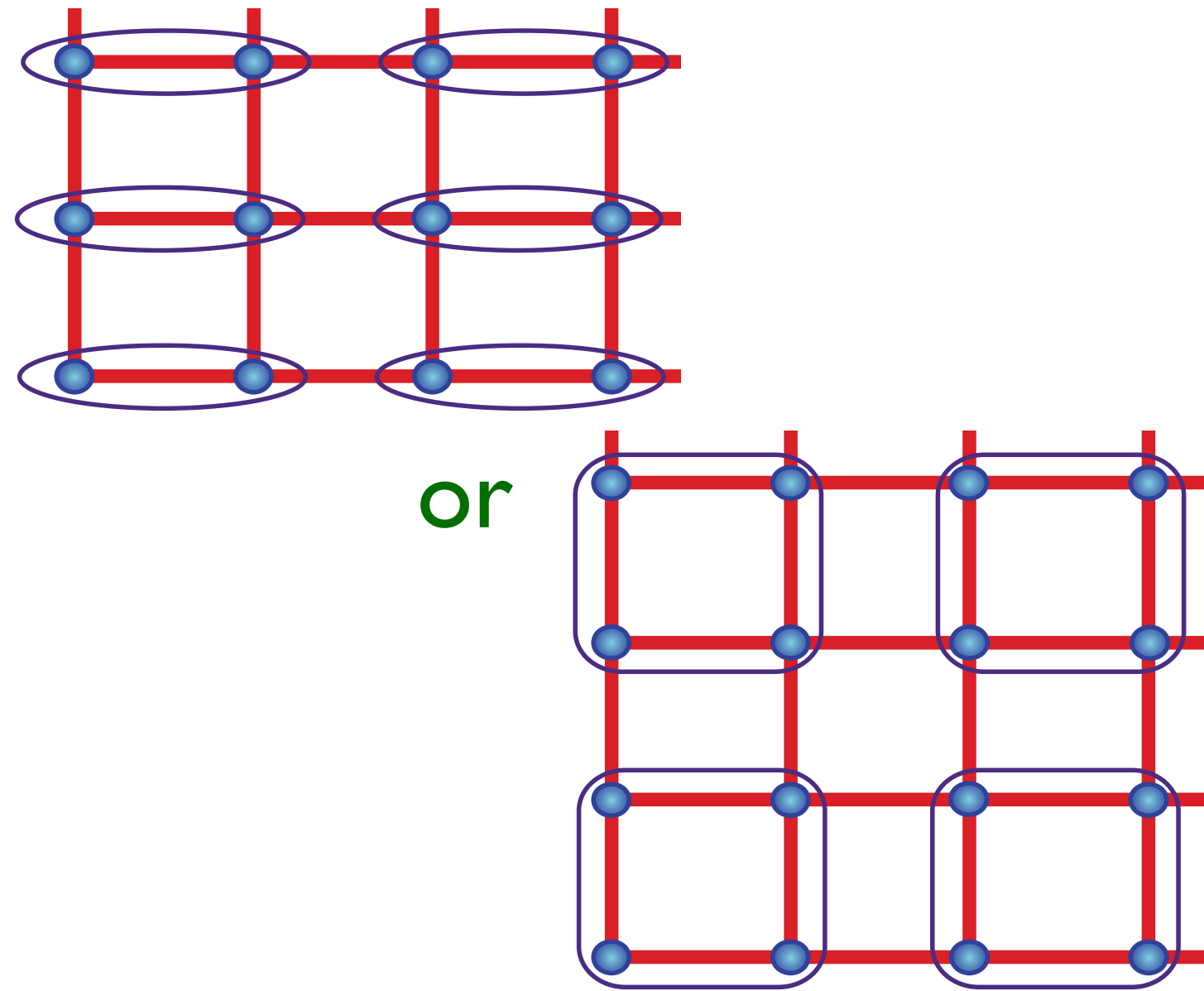
$$H_f = iJ \sum_{\langle ij \rangle} e_{ij} \left(f_{i\alpha}^\dagger f_{j\alpha} - f_{j\alpha}^\dagger f_{i\alpha} \right), \quad \varepsilon_{\mathbf{k}} = 2J \sqrt{\sin^2(k_x) + \sin^2(k_y)}$$

SU(2) QCD with $N_f = 2$ massless fermions; $\mathcal{L} = i\bar{\Psi}_s \gamma_\mu D_\mu \Psi_s$.

Insulating $S=1/2$ antiferromagnet



Néel order



Valence bond solid (VBS)

$$H = \sum_{i < j} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j$$

Schwinger fermions

$$\mathbf{S}_i = \frac{1}{2} f_{i\alpha}^\dagger \boldsymbol{\sigma}_{\alpha\beta} f_{i\beta}, \quad \sum_{\alpha=\uparrow,\downarrow} f_{i\alpha}^\dagger f_{i\alpha} = 1$$

π -flux mean-field theory
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I. Affleck and J.B. Marston, PRB **37**, 3774 (1988)



$$H_f = iJ \sum_{\langle ij \rangle} e_{ij} \left(f_{i\alpha}^\dagger f_{j\alpha} - f_{j\alpha}^\dagger f_{i\alpha} \right), \quad \varepsilon_{\mathbf{k}} = 2J \sqrt{\sin^2(k_x) + \sin^2(k_y)}$$

SU(2) QCD with $N_f = 2$ massless fermions; $\mathcal{L} = i\bar{\Psi}_s \gamma_\mu D_\mu \Psi_s$.
Confining instability to precisely the Néel and VBS orders of \mathbb{CP}^1 theory.

Bosonic spinons:
 \mathbb{CP}^1 U(1) gauge theory

Nearly-
critical
 $S=1/2$
square
lattice anti-
ferromagnet

$SU(2)_N$ gauge theory of $N_f = 2$
fundamental, massless, Dirac fermions.

Obtained from a saddle-point of
fermionic spinons moving in π -flux.

$SO(5)$ non-linear σ -model
of Néel/VBS orders
with $k = 1$ WZW term

Bosonic spinons:
 \mathbb{CP}^1 U(1) gauge theory

Nearly-
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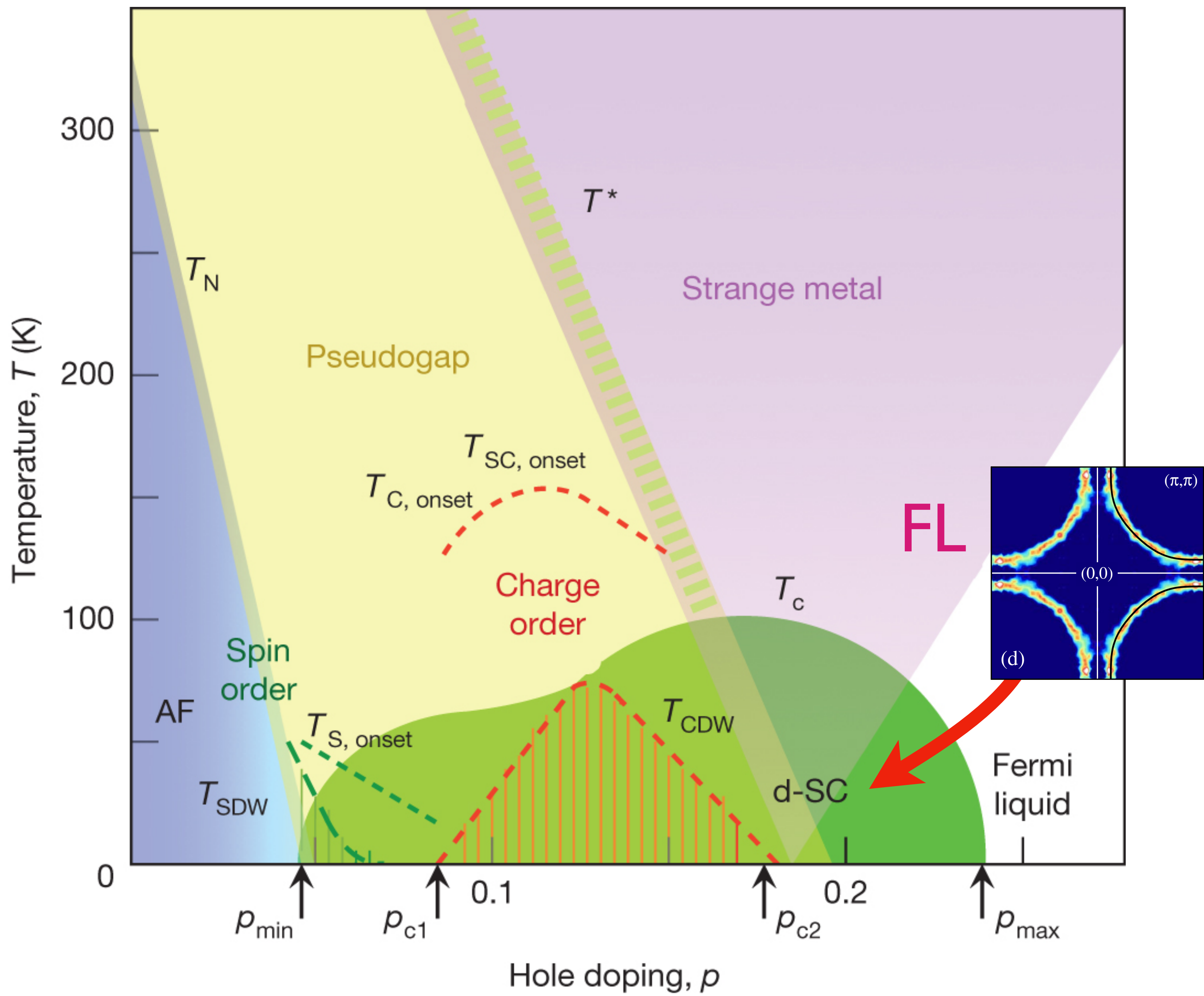
$SO(5)$ non-linear σ -model
of Néel/VBS orders
with $k = 1$ WZW term

Many numerical works show that deconfined critical theory applies over a substantial length scale, but ultimately confines at the longest distances.

Zheng Zhou, Liangdong Hu, Wei Zhu, and Yin-Chen He, PRX **14**, 021044 (2024); S. M. Chester and N. Su, PRL **132**, 111601 (2024).
B.-B. Chen, X. Zhang, Y. Wang, K. Sun, and Z. Y. Meng, arXiv:2307.05307;
J. Takahashi, H. Shao, B. Zhao, W. Guo, and A. W. Sandvik, arXiv:2405.06607.

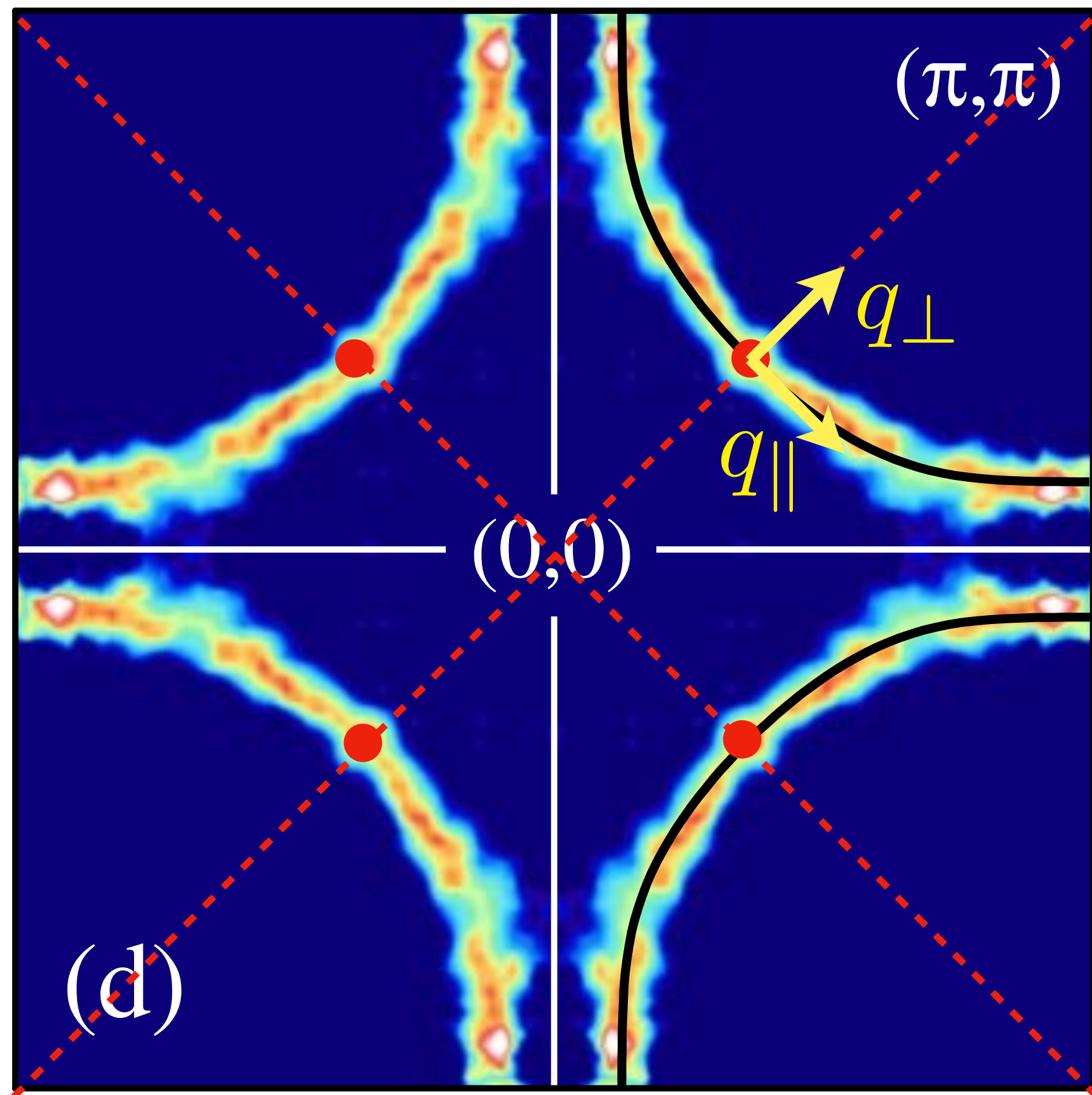
Pseudogap answers:

- A. Photoemission: Fermi arcs
- B. RIXS: Intense paramagnons
- C. Quantum oscillations
- D. Velocities of Bogoliubov quasiparticles
in the d-wave superconductor



BCS-type theory of *d*-wave
superconductivity
(and charge order)
induced by
antiferromagnetic spin
fluctuations.

FL \rightarrow dSC



BCS/Bogoliubov quasiparticles
in a d -wave superconductor

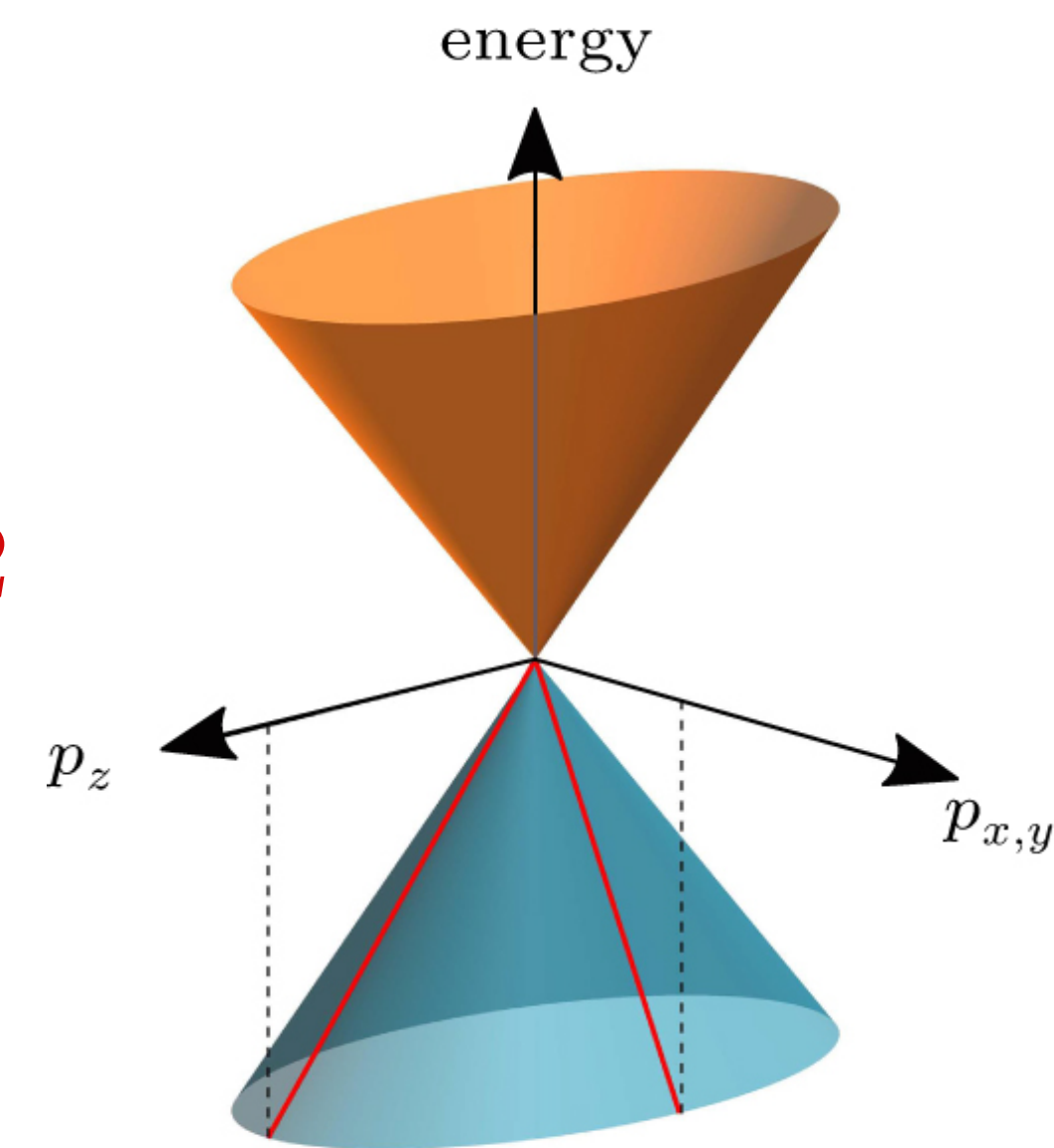
$$E_{\mathbf{k}} = \left(\varepsilon_{\mathbf{k}}^2 + \Delta_{\mathbf{k}}^2 \right)^{1/2}$$

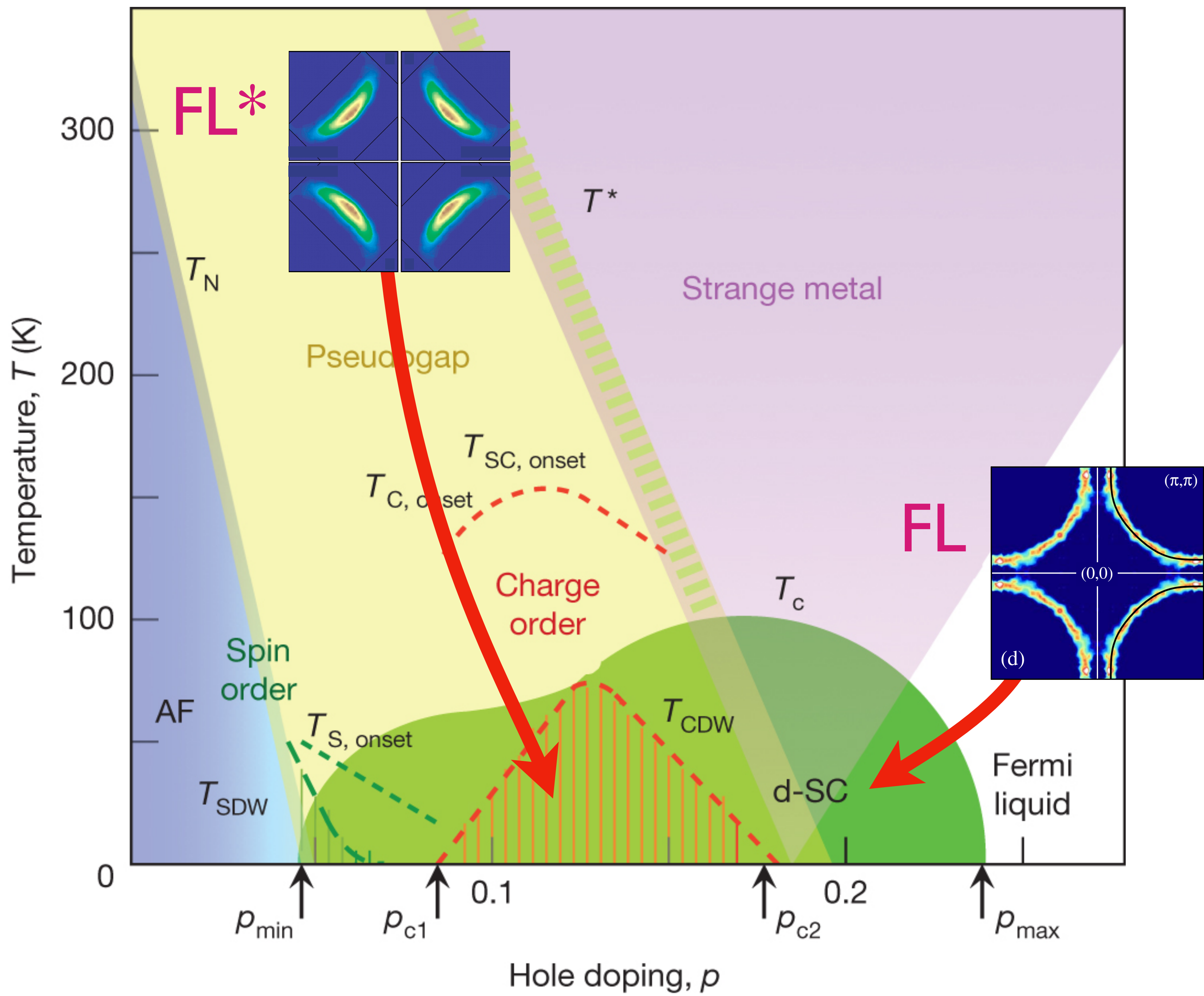
$$\Delta_{\mathbf{k}} = \Delta_0 (\cos k_x - \cos k_y)$$

4 nodal points where

$$E_{\mathbf{k}_0 + \mathbf{q}} = \left(v_F^2 q_{\perp}^2 + v_{\Delta}^2 q_{\parallel}^2 \right)^{1/2}$$

with $v_F \gg v_{\Delta}$.

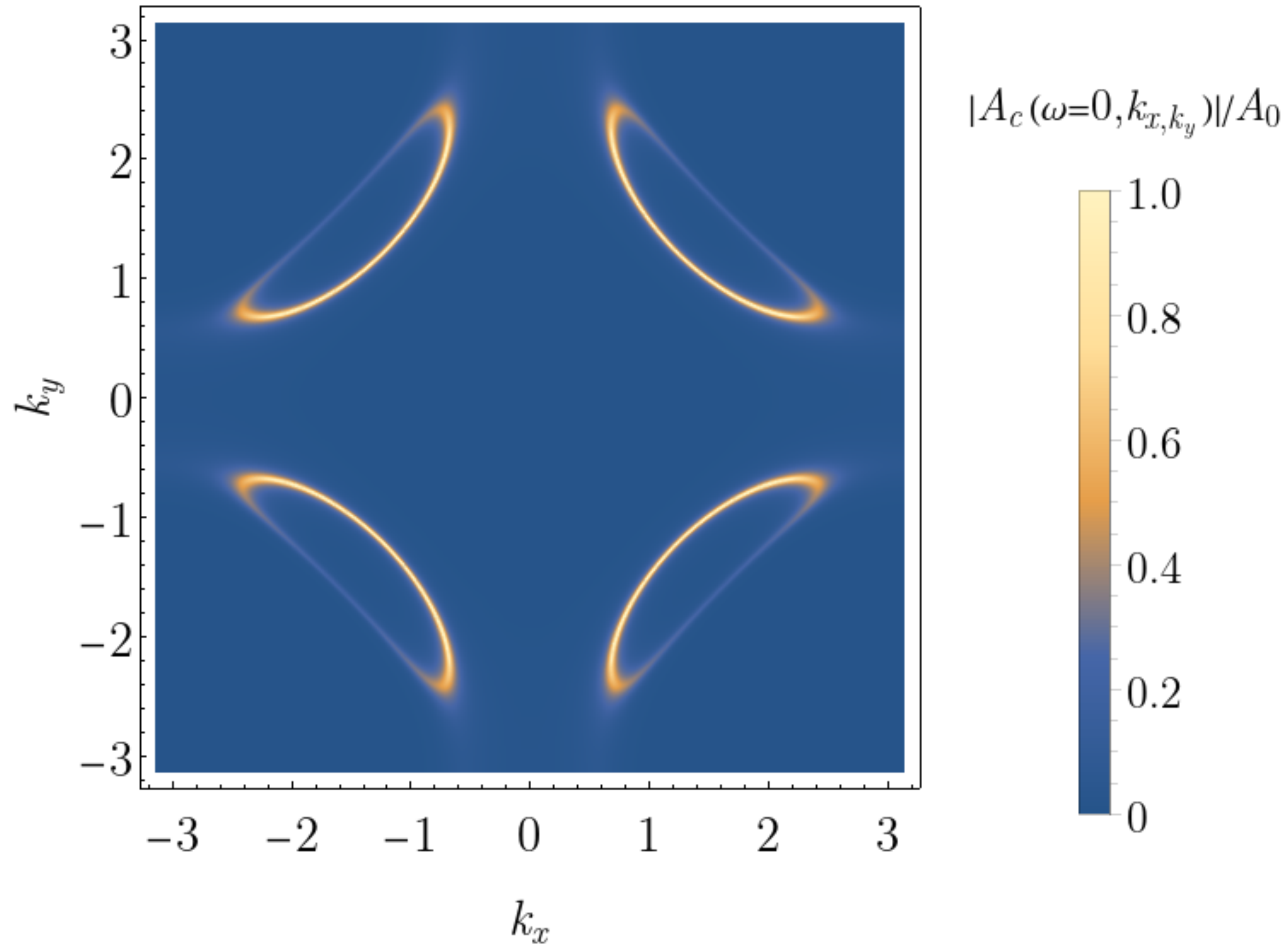




Obtain *d*-wave superconductor and charge order from a theory of *confinement* instabilities of FL*.

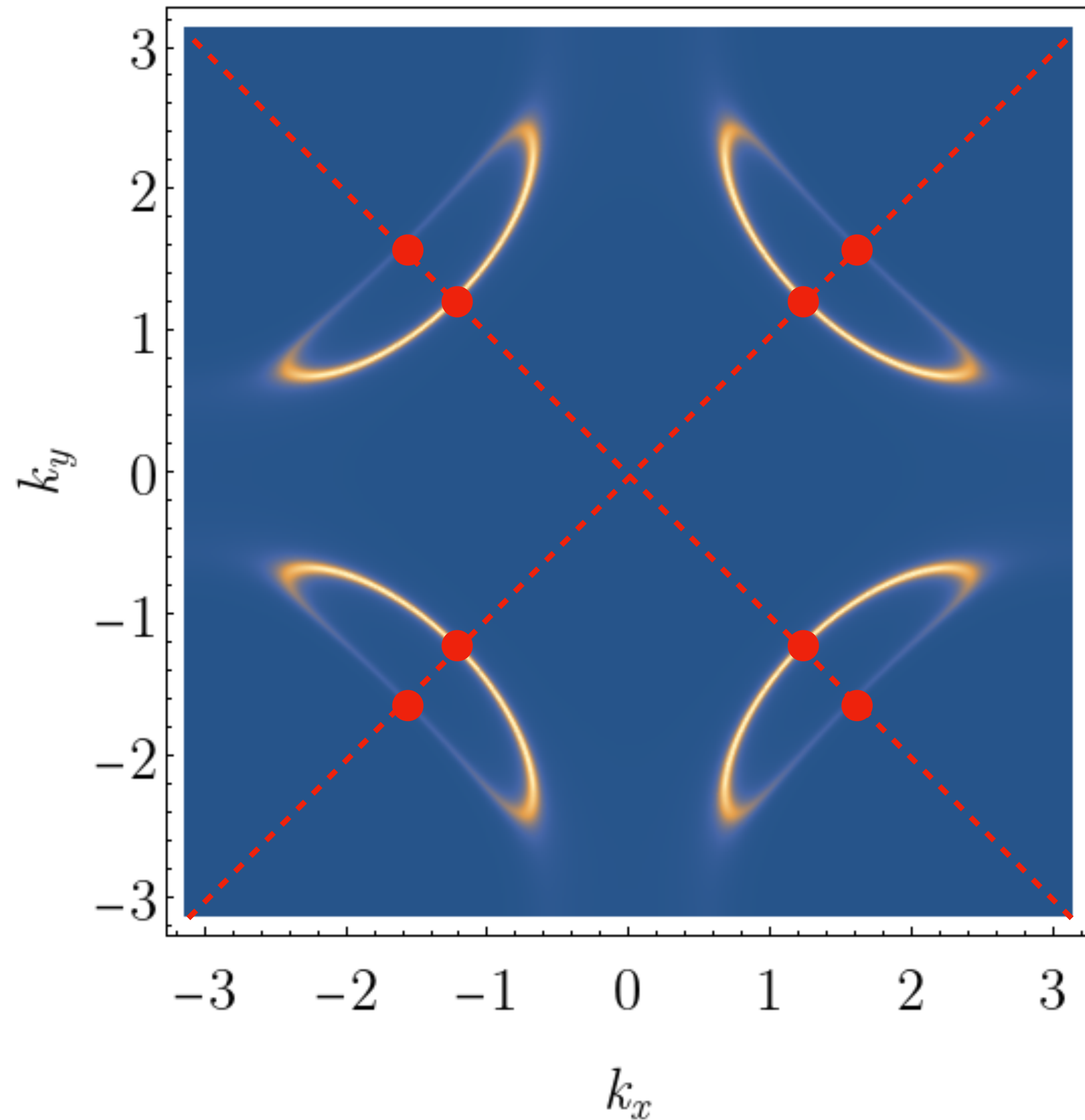
The resulting low T ordered states should be adiabatically connected to the corresponding states obtained from instabilities of FL.

FL*

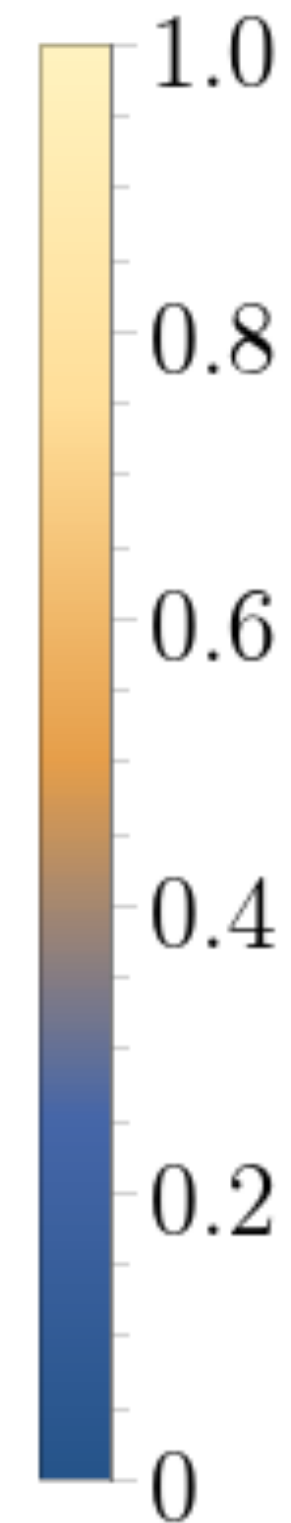


E. Mascot, A. Nikolaenko, M. Tikhanovskaya, Ya-Hui Zhang,
D. K. Morr, and S. S., PRB **105**, 075146 (2022)

FL* → dSC*



$|A_c(\omega=0, k_x, k_y)|/A_0$

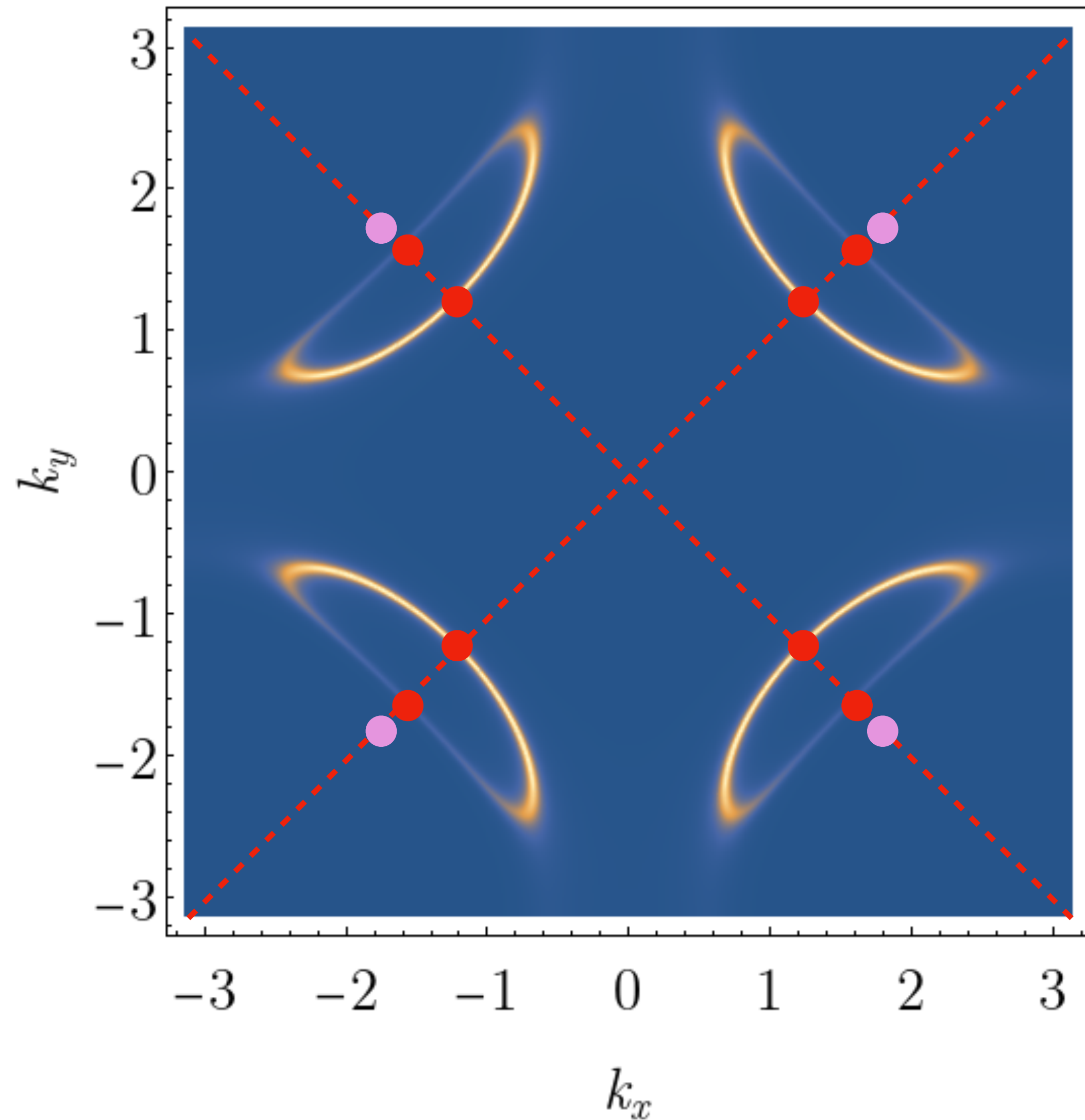


$$E_{\mathbf{k}} = (\varepsilon_{\mathbf{k}}^2 + \Delta_{\mathbf{k}}^2)^{1/2}$$

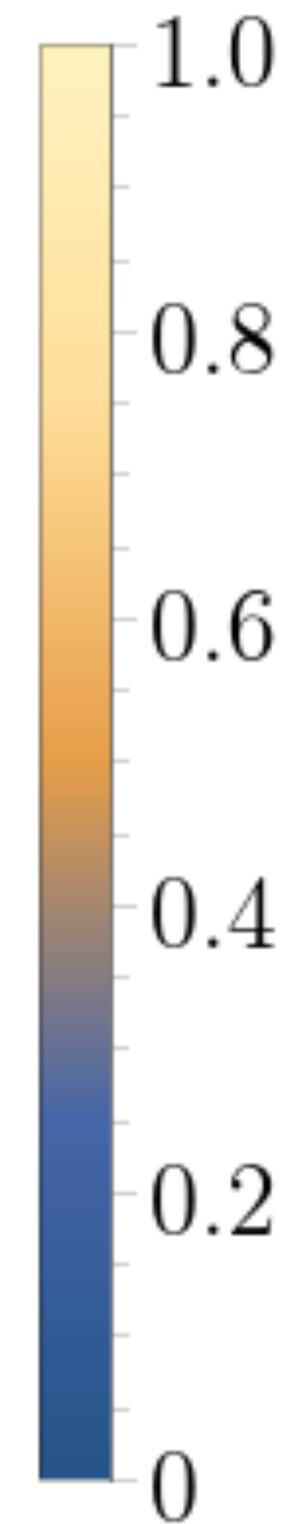
$$\Delta_{\mathbf{k}} = \Delta_0 (\cos k_x - \cos k_y)$$

Adding *d*-wave pairing
to the hole pockets
leads to 8 nodal points???

$FL^* \rightarrow dSC^*$



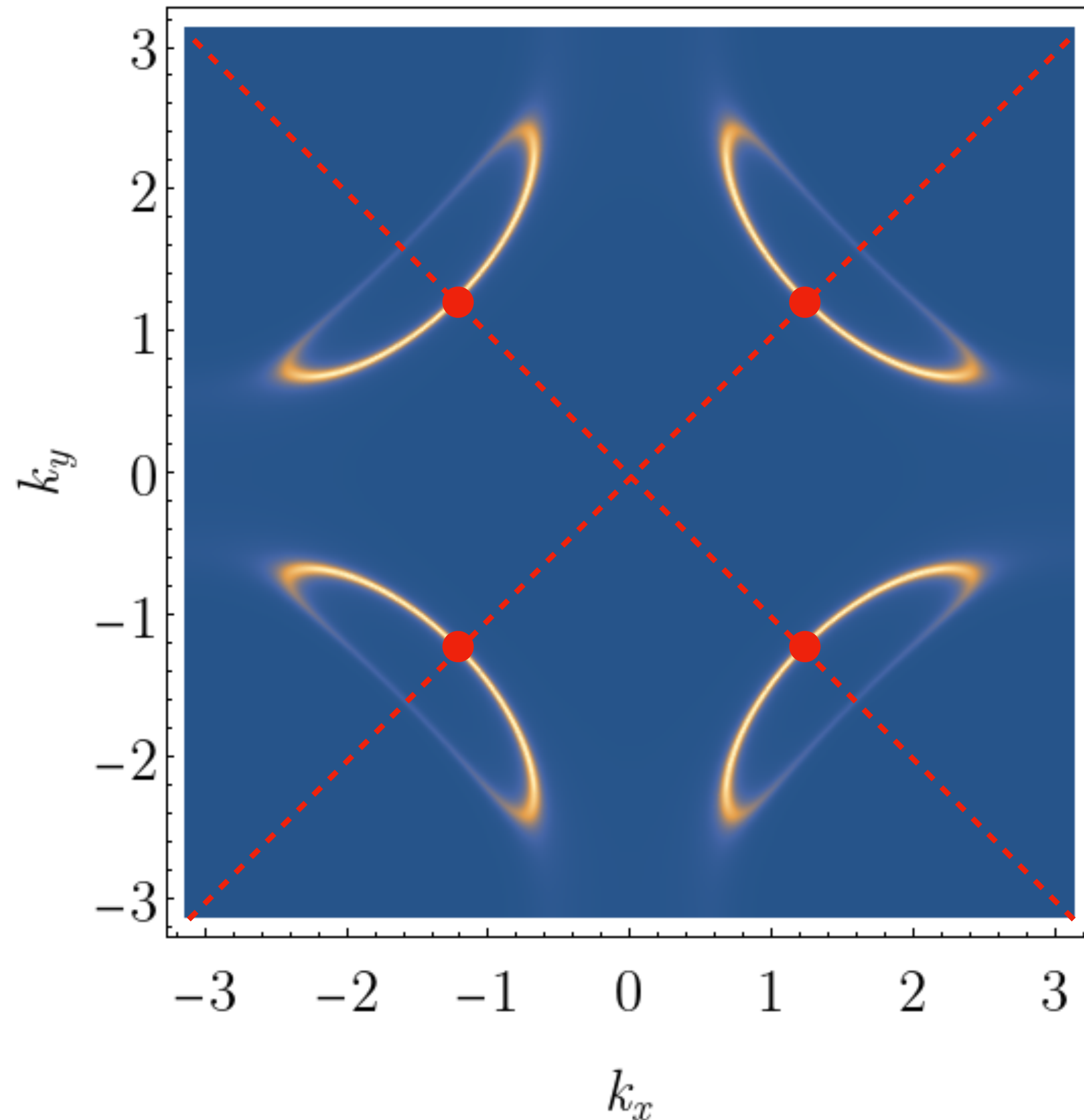
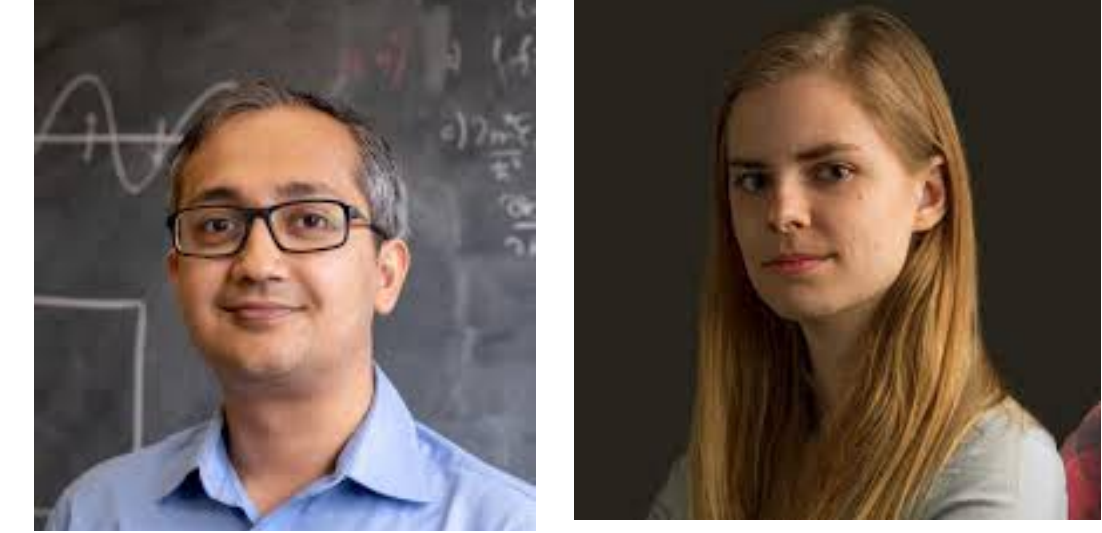
$|A_c(\omega=0, k_x, k_y)|/A_0$



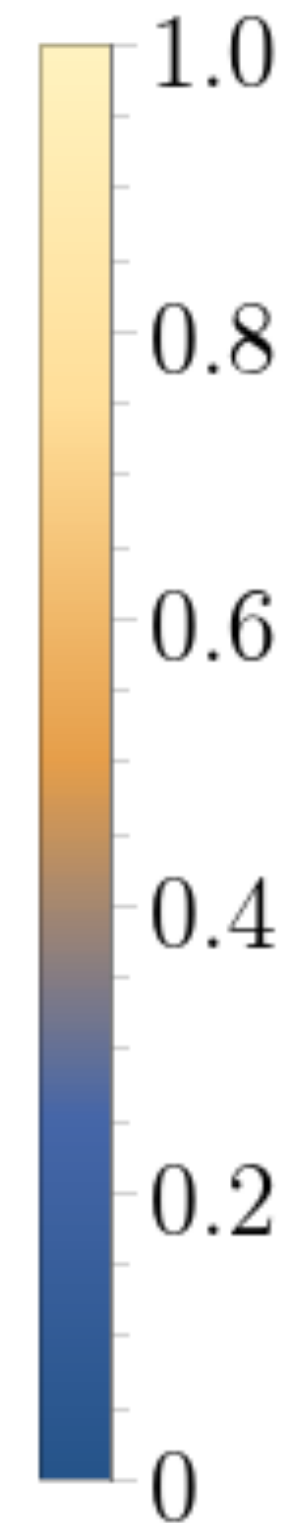
8 nodal points of
Bogoliubov quasiparticles
from the Fermi pockets
and
4 nodal points of
fermionic Dirac spinons from
the π -flux spin liquid

$$FL^* \rightarrow dSC$$

Shubhayu Chatterjee and S. Sachdev,
PRB **94**, 205117 (2016)
Maine Christos and S. Sachdev,
npj Quantum Materials **9**, 4 (2024)



$$|A_c(\omega=0, k_x, k_y)|/A_0$$



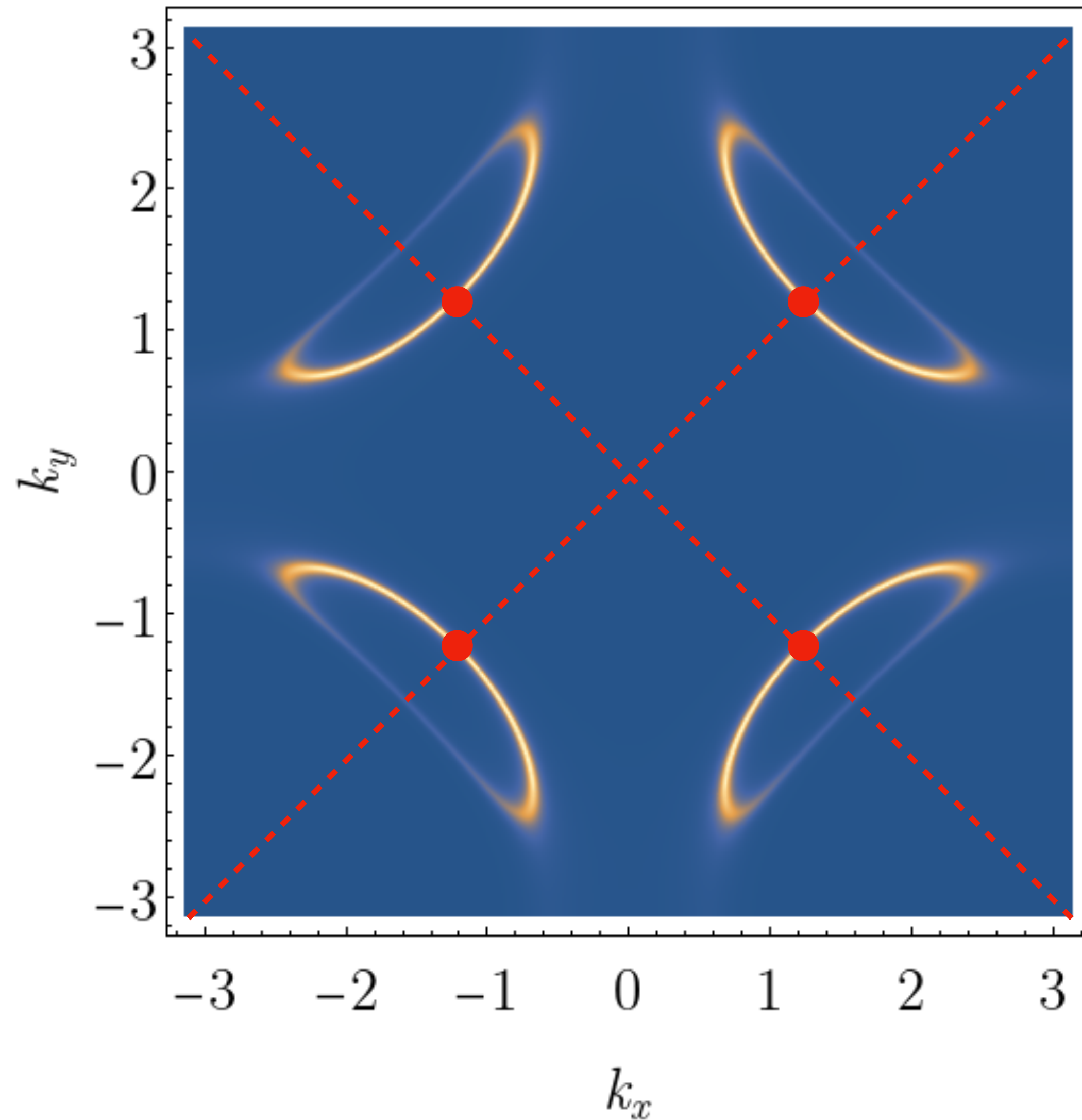
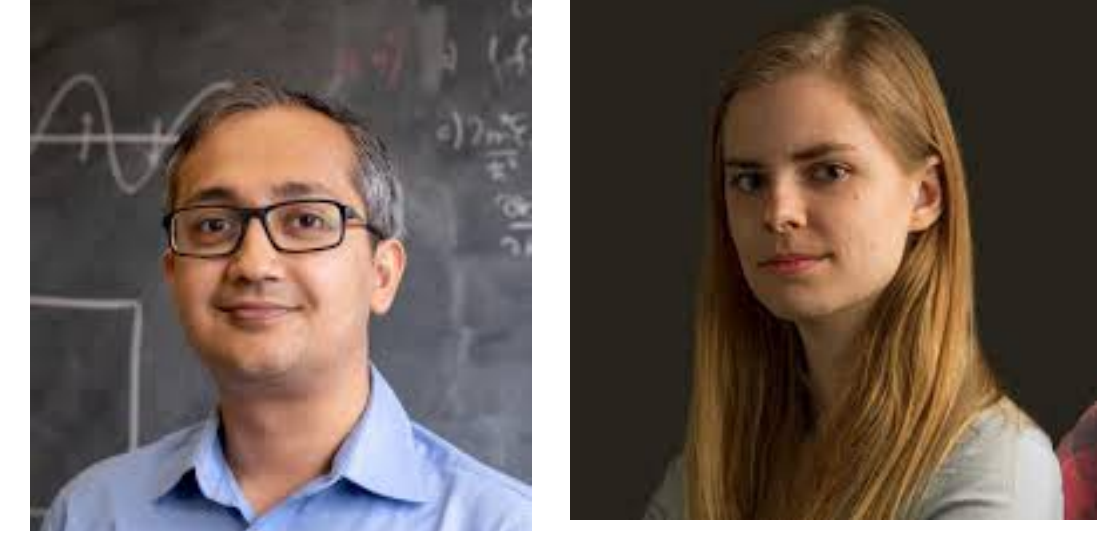
d-wave superconductor obtained
by condensing charge- e , $SU(2)$
fundamental boson B .

The B Higgs condensate allows
spinons and Bogoliubov
quasiparticles
to hybridize.

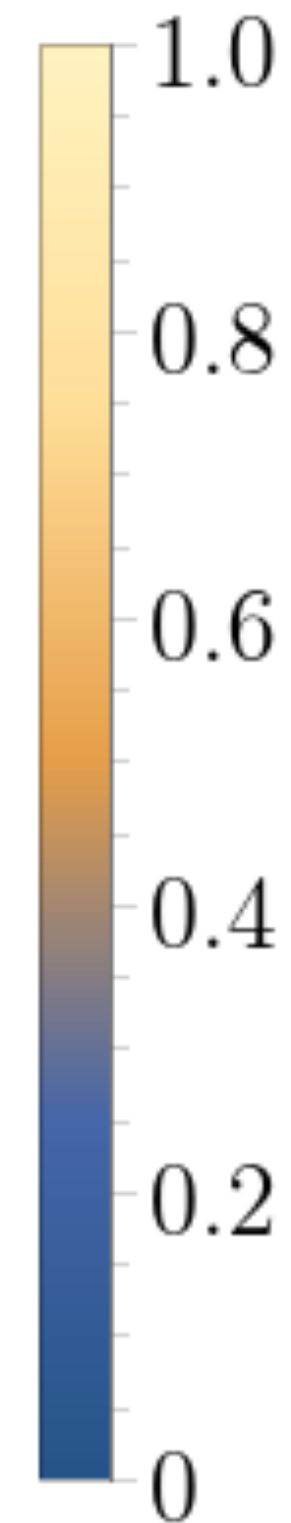
8 nodal points annihilate each
other, leaving 4 nodal points
with anisotropic velocities, just
as in a BCS *d*-wave state.

$$FL^* \rightarrow dSC$$

Shubhayu Chatterjee and S. Sachdev,
PRB **94**, 205117 (2016)
Maine Christos and S. Sachdev,
npj Quantum Materials **9**, 4 (2024)



$$|A_c(\omega=0, k_x, k_y)|/A_0$$

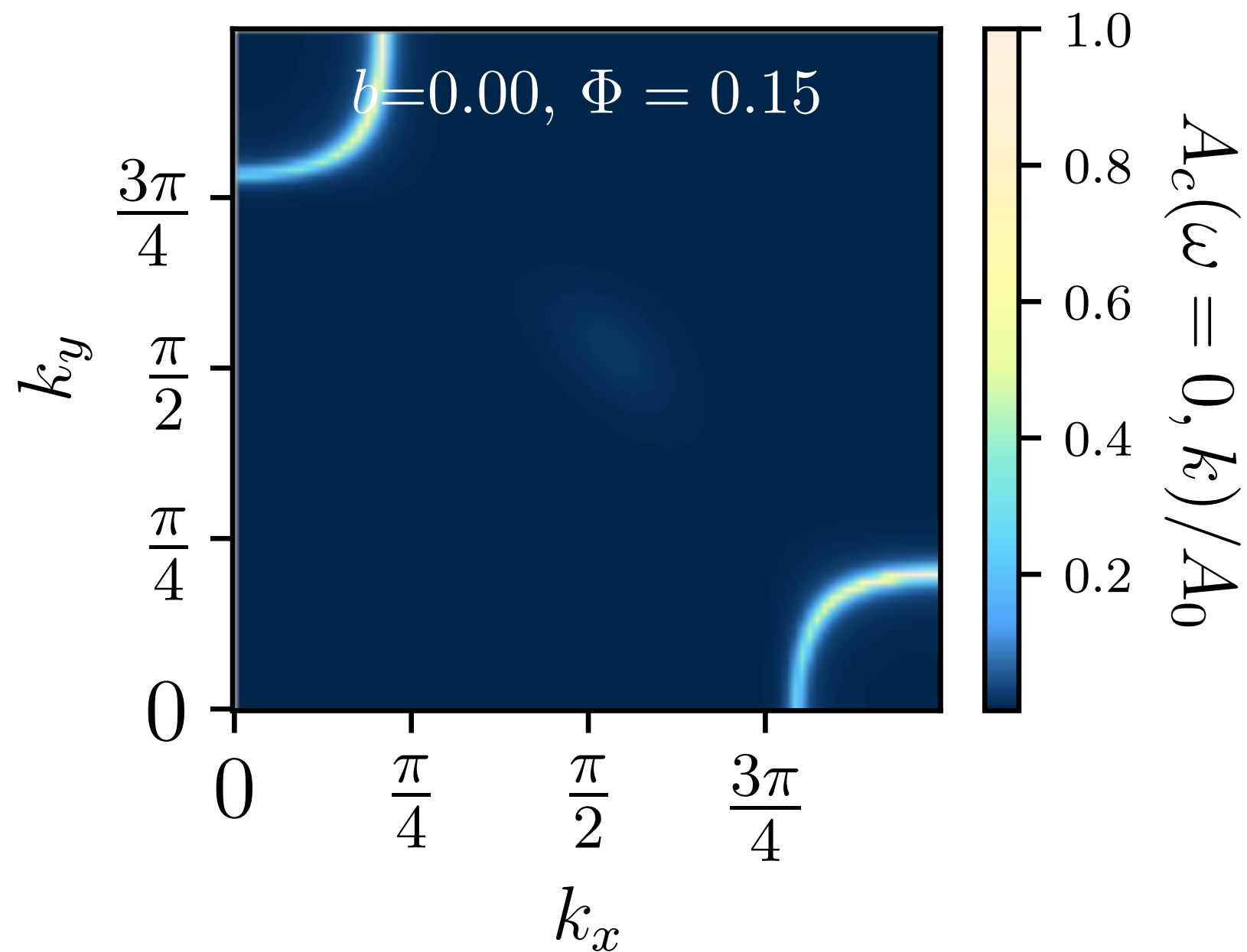


The spinons do *not* become the
Bogoliubov quasiparticles,
they *annihilate* the unwanted
Bogoliubov quasiparticles.
This leads to a *d*-wave superconductor
with 4 nodal Bogoliubov quasiparticles,
with $v_F \gg v_\Delta$,
consistent with observations.

Electron spectral density in electron-doped cuprates

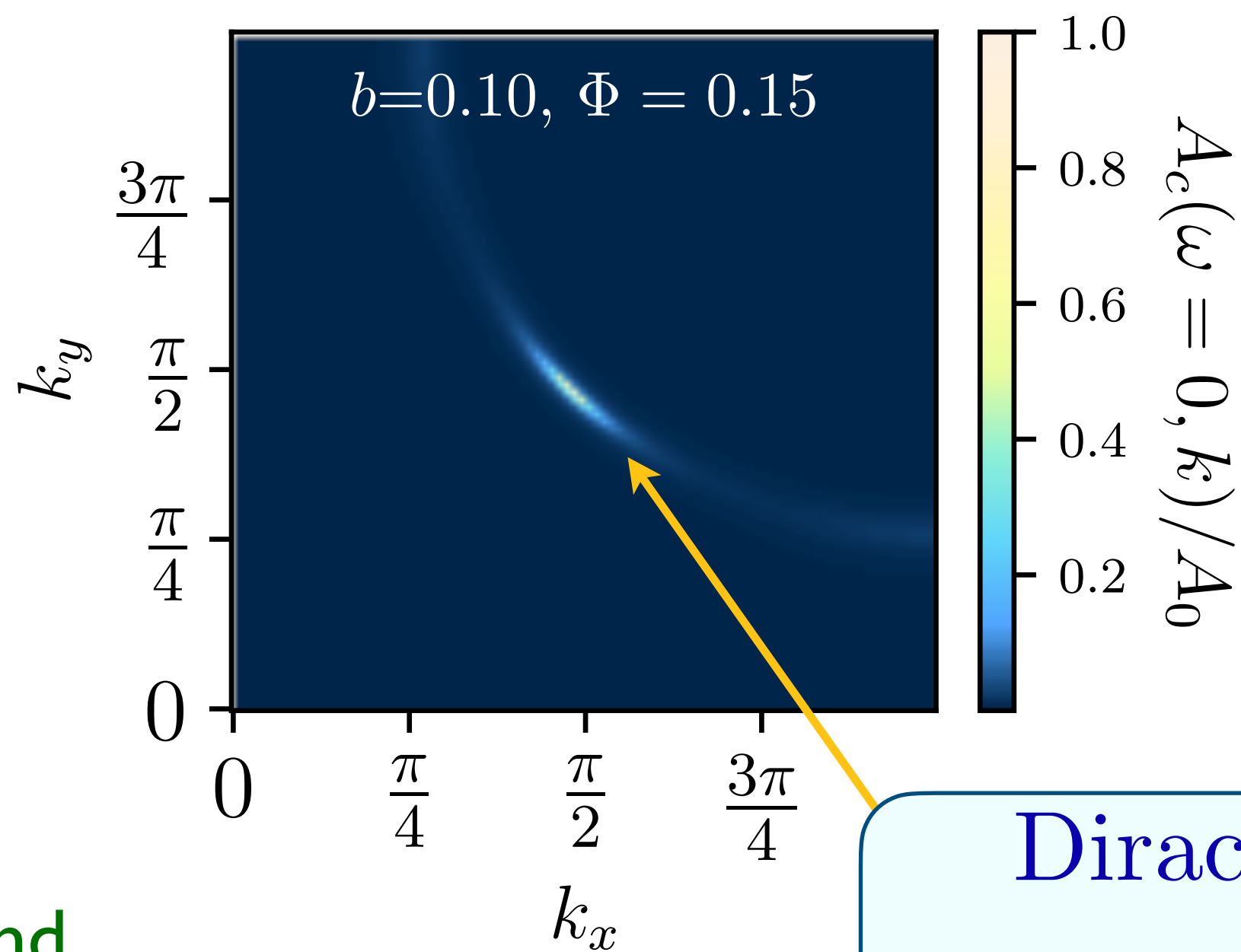


Maine Christos and
S.Sachdev, npj Quantum
Materials **9**, 4 (2024)



FL*

d-wave superconductor
obtained by condensing
charge-*e*, SU(2) fundamental
boson *B*.



dSC

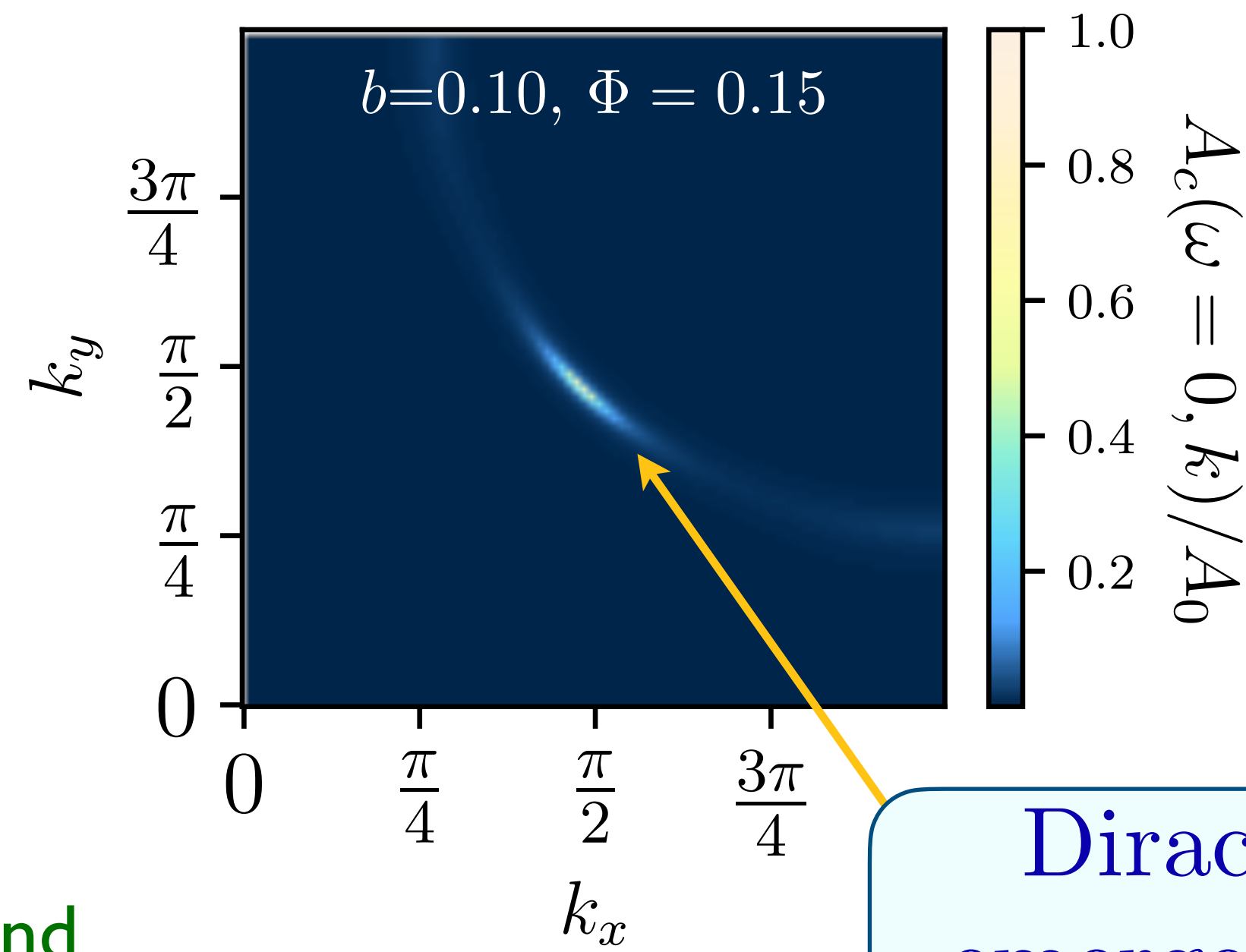
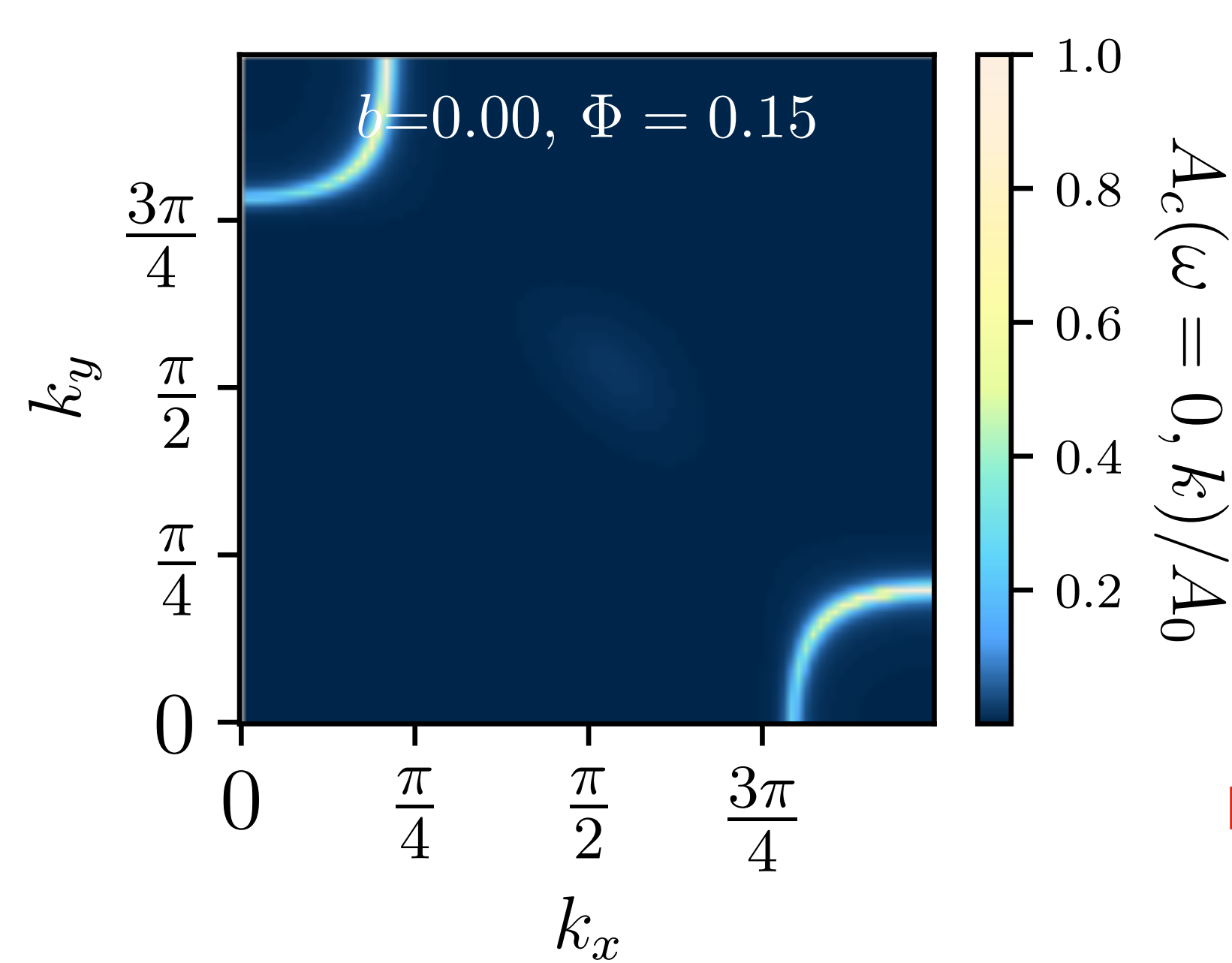
In the electron-doped case,
nodes of the *d*-wave
superconductor *are* remnants
of the spinons
of the π -flux state.

Dirac node
emerges inside
normal state gap

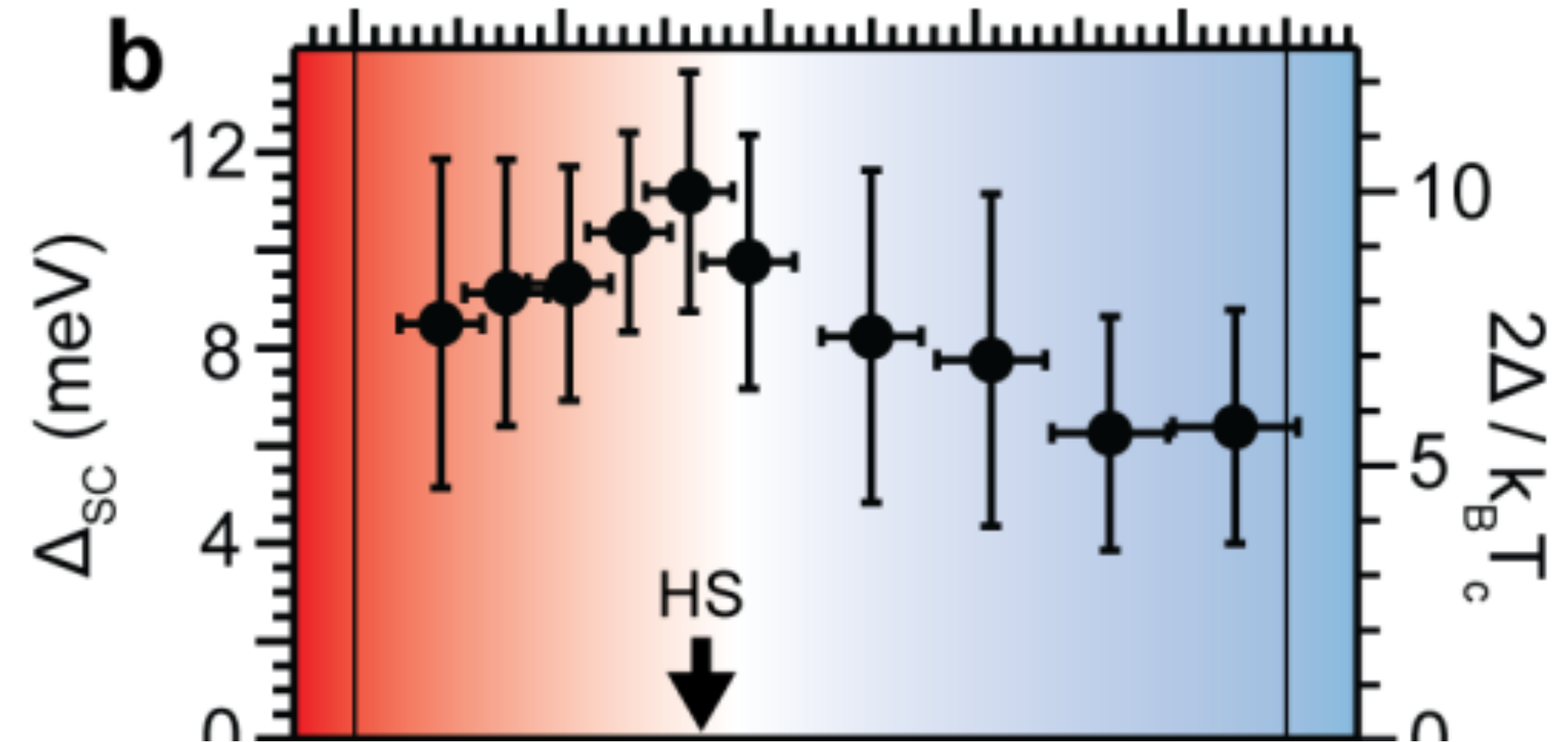
Electron spectral density in electron-doped cuprates



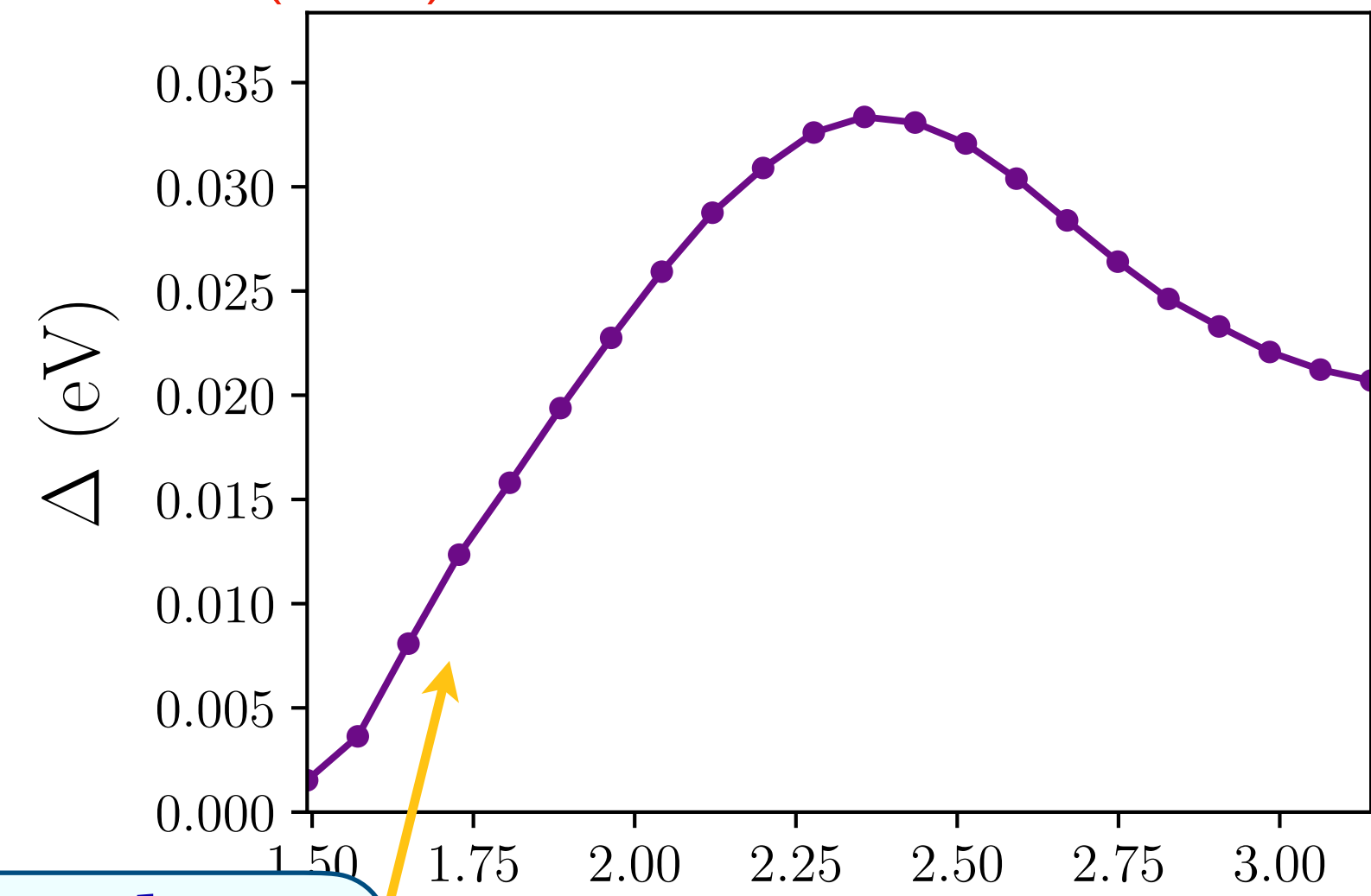
Maine Christos and S.Sachdev, npj Quantum Materials **9**, 4 (2024)



Dirac node emerges inside normal state gap



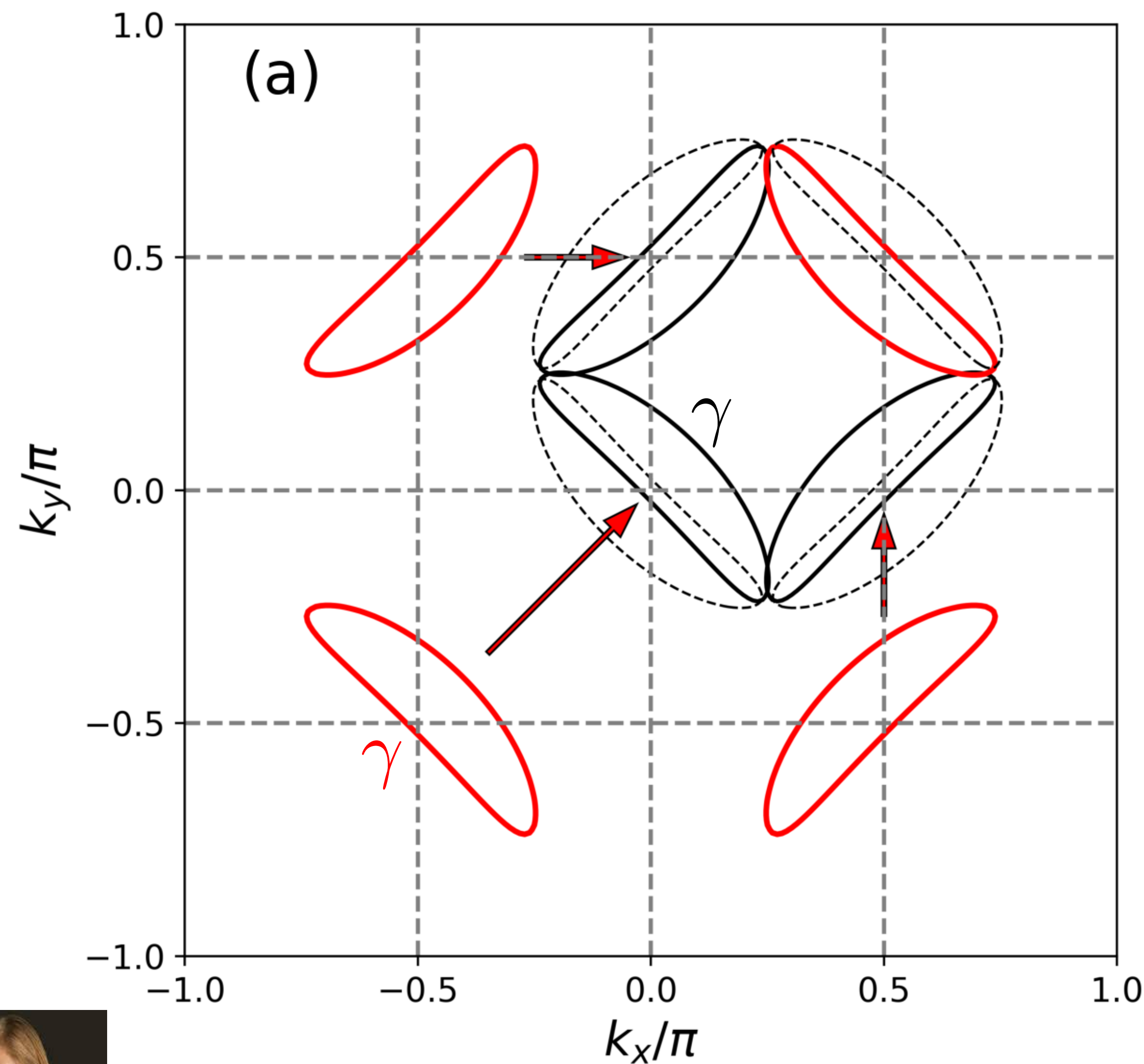
Bogoliubov Quasiparticle on the Gossamer Fermi Surface in Electron-Doped Cuprates, Ke-Jun Xu.....Z.-X. Shen; Nature Physics **19**, 1834 (2023)



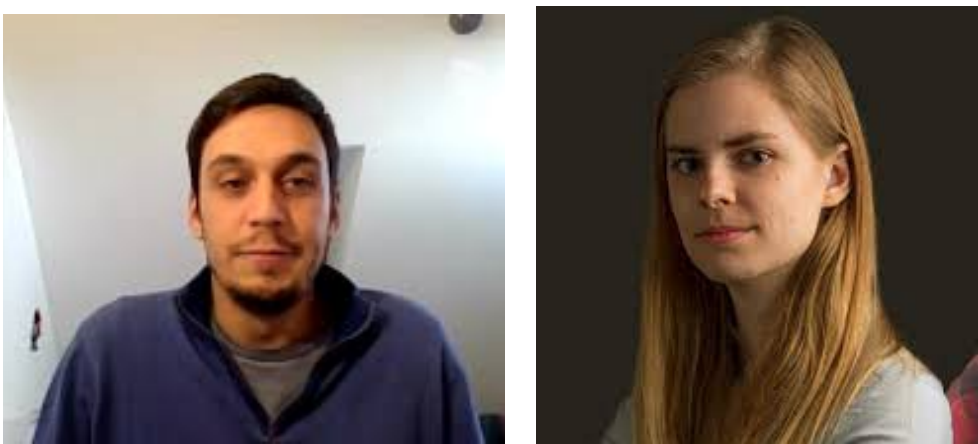
Pseudogap answers:

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FL*

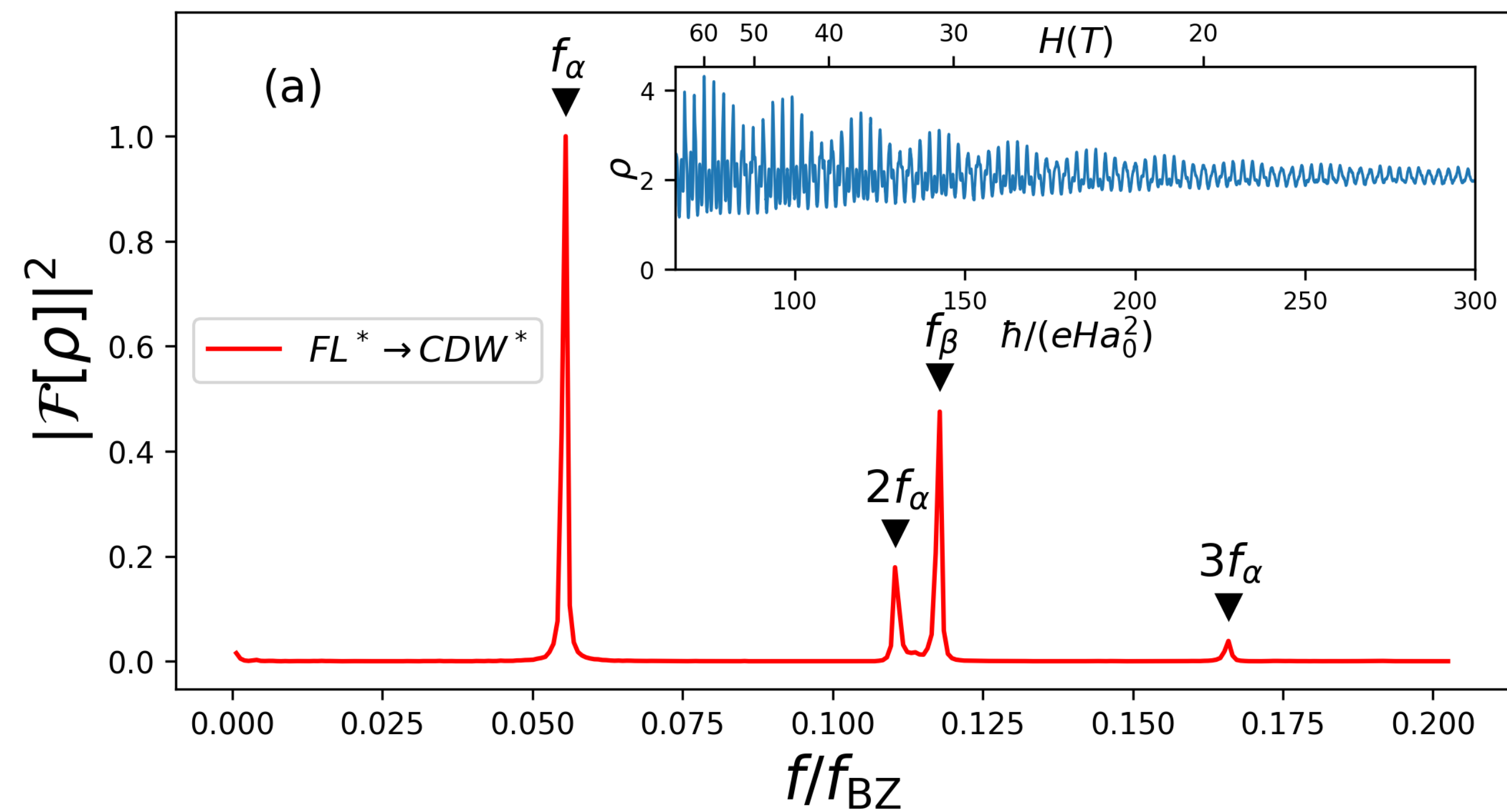
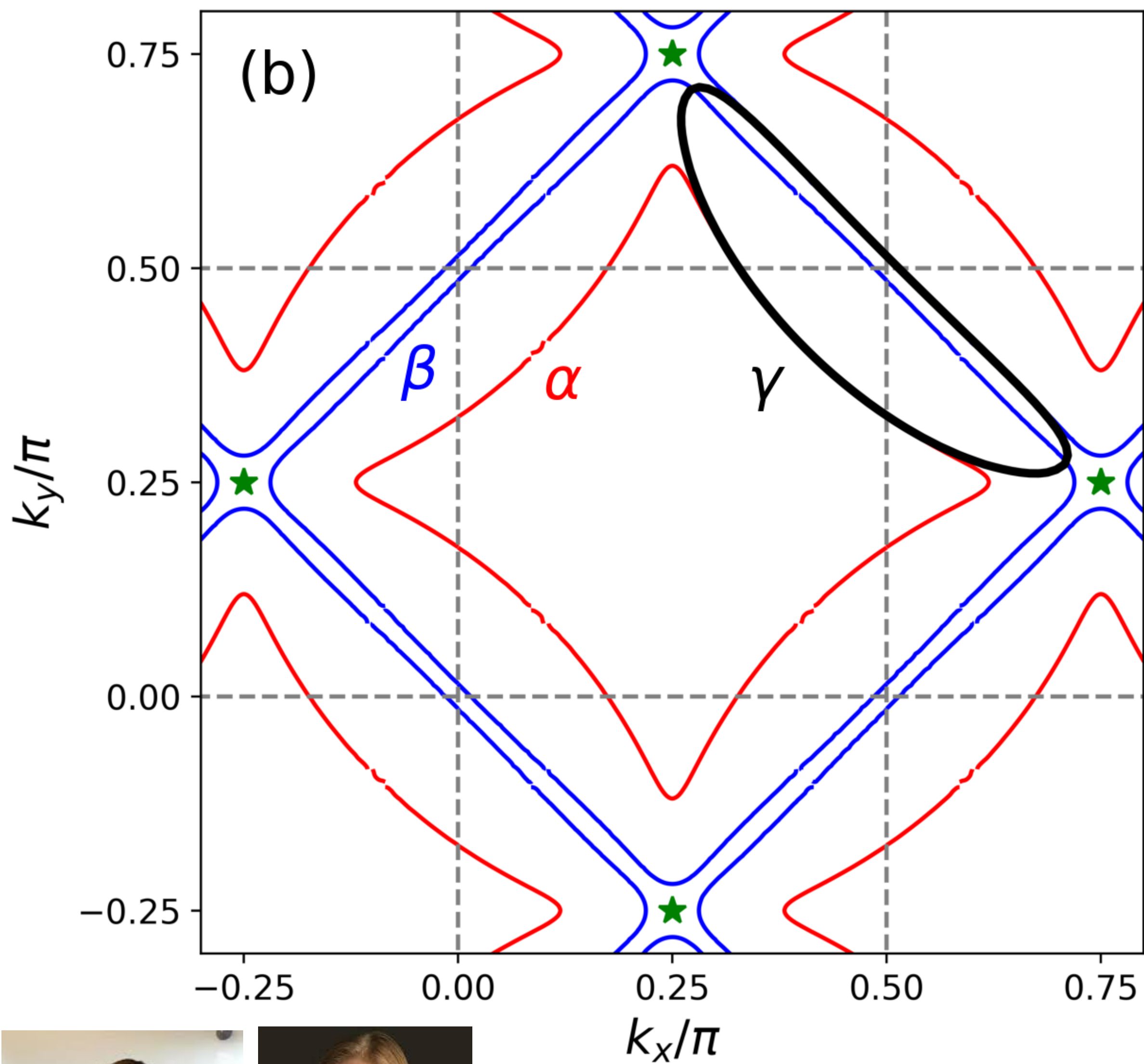


N. Harrison and S. Sebastian
electron pocket
(PRL **106**, 226402 (2011))



Pietro Bonetti, Maine Christos and S.S. (BCS), arXiv:2405.08817

$FL^* \rightarrow CDW^*$

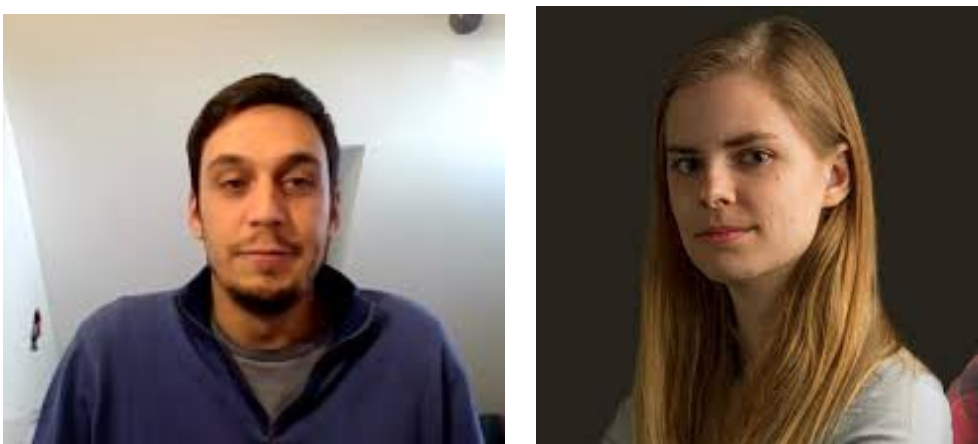


Computation does not account for spinons.

α and β pockets

show clear quantum oscillations.

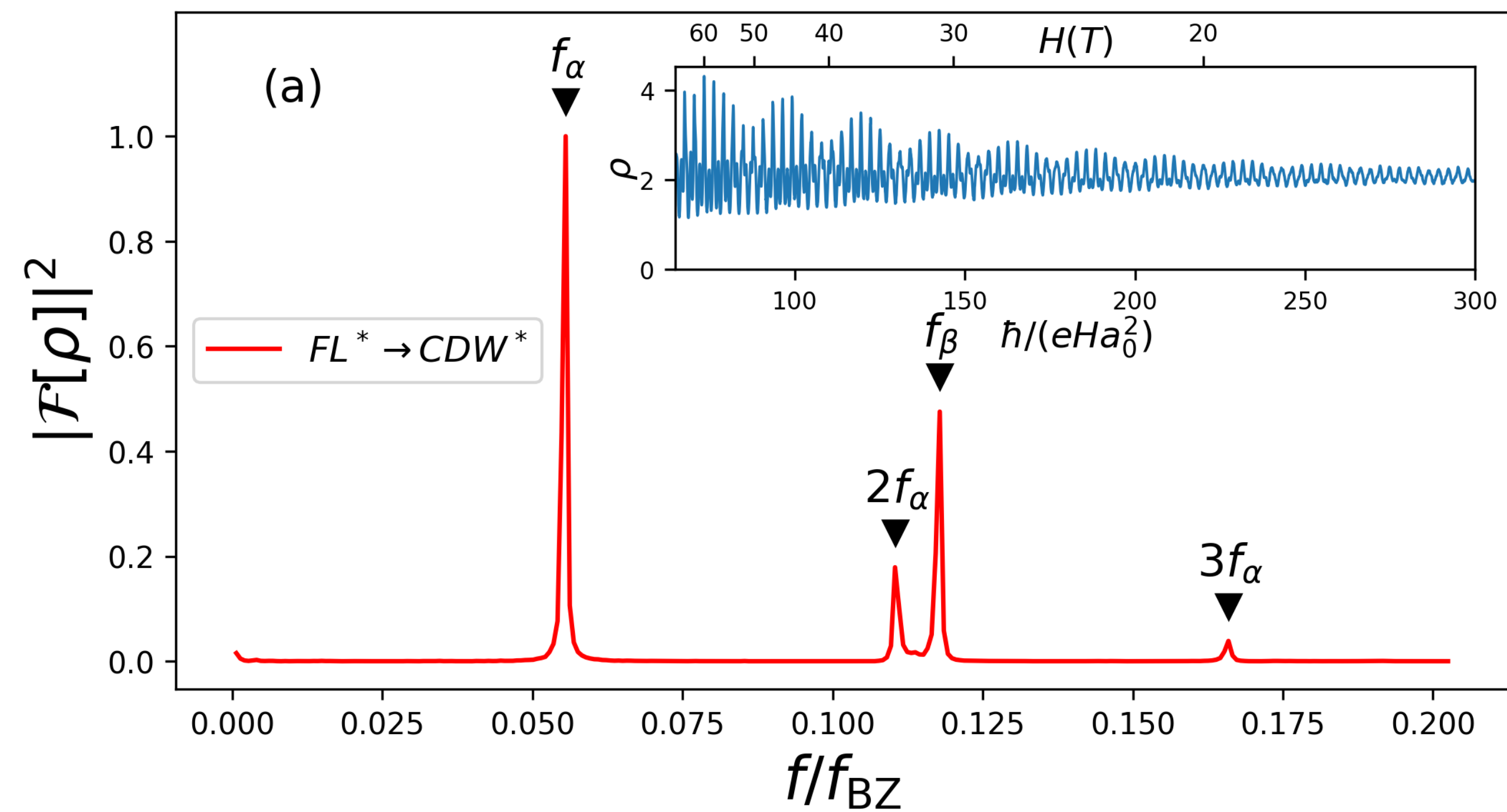
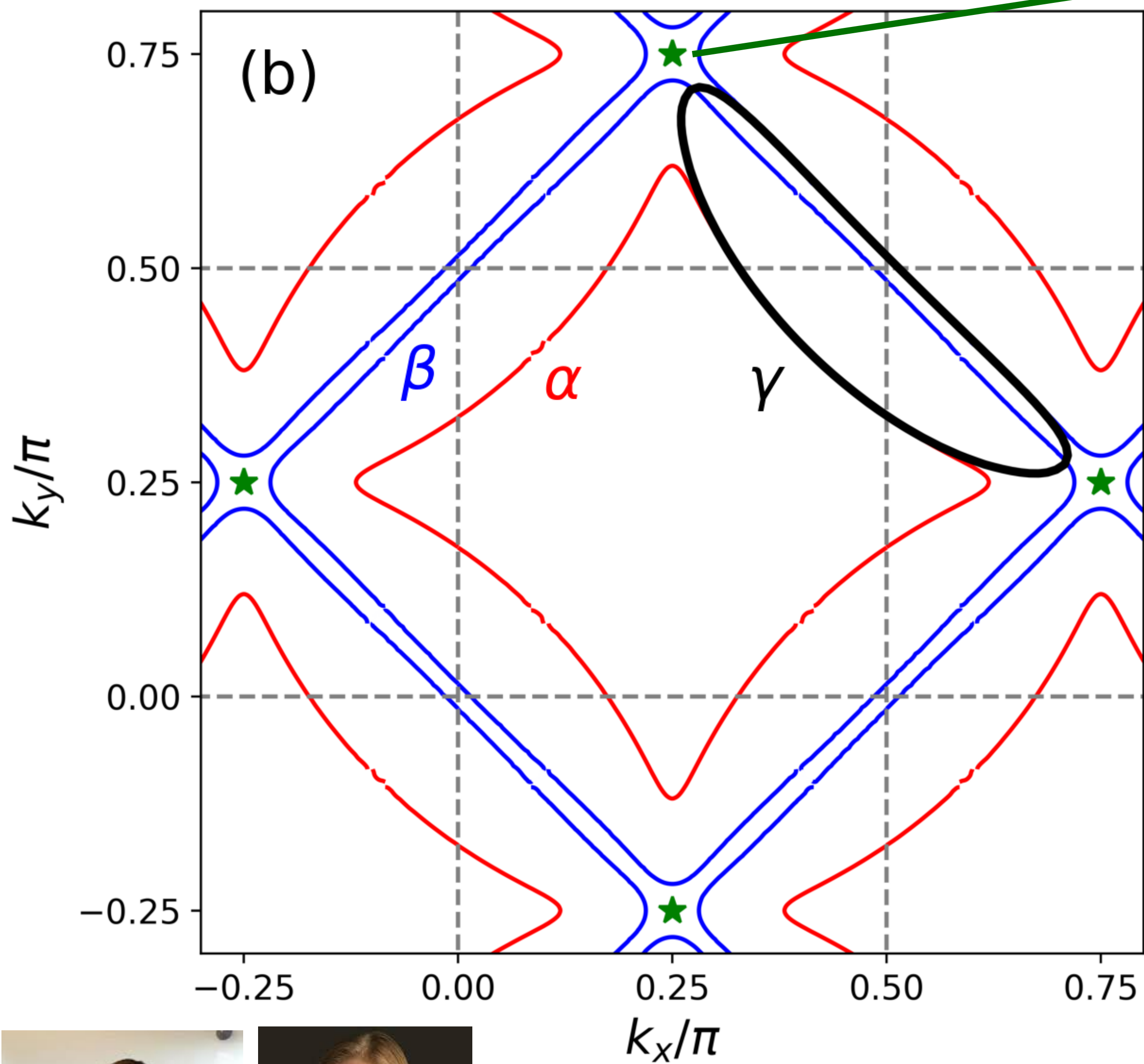
Long Zhang and Jia-Wei Mei,
EPL **114**, 47008 (2016)



Pietro Bonetti, Maine Christos and S.S. (BCS), arXiv:2405.08817

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Spinon

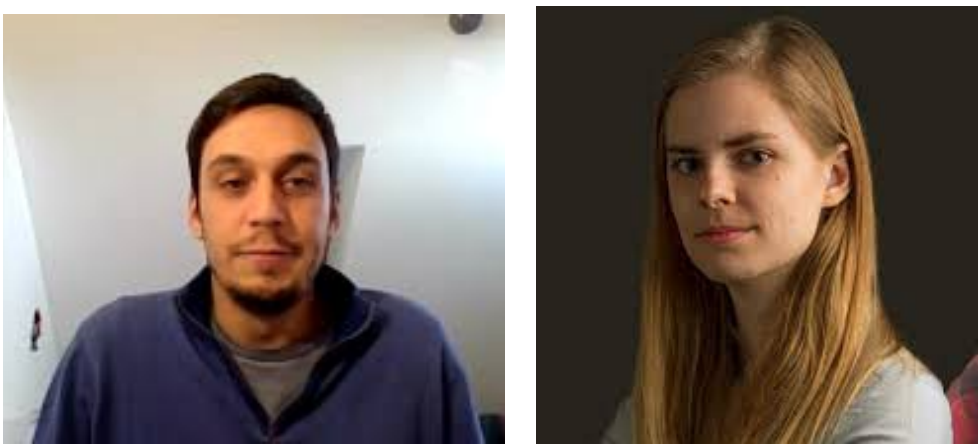


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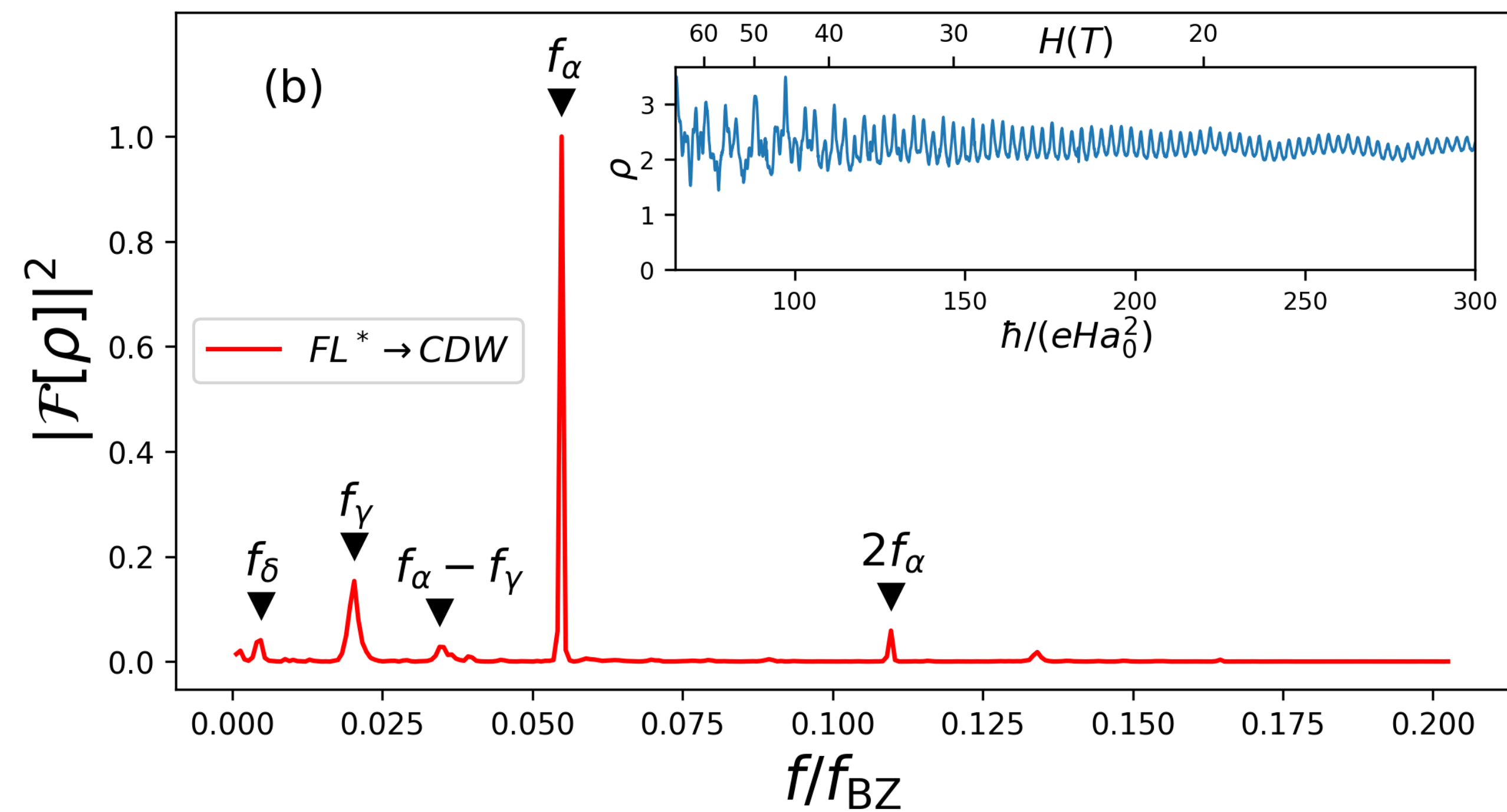
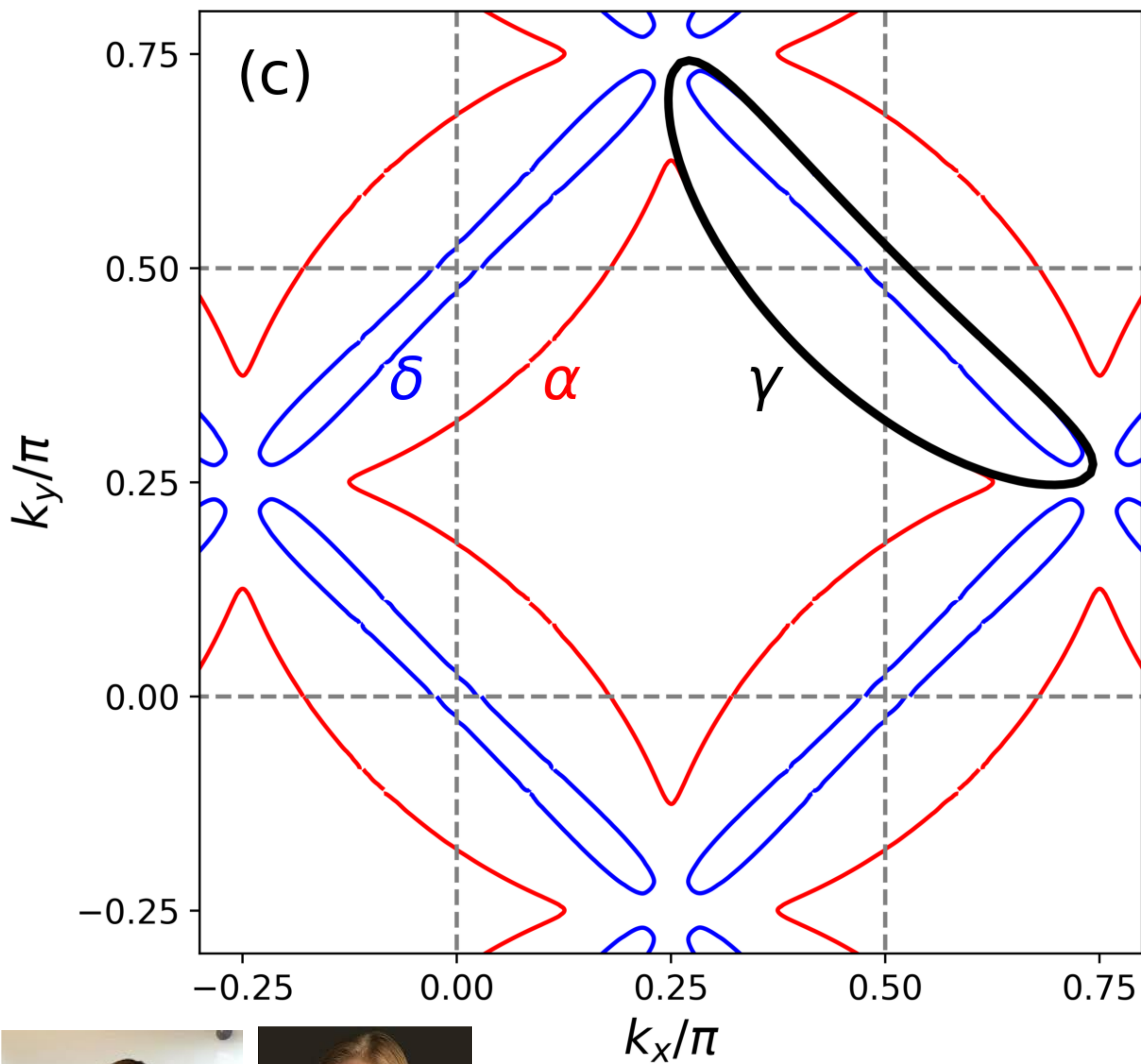
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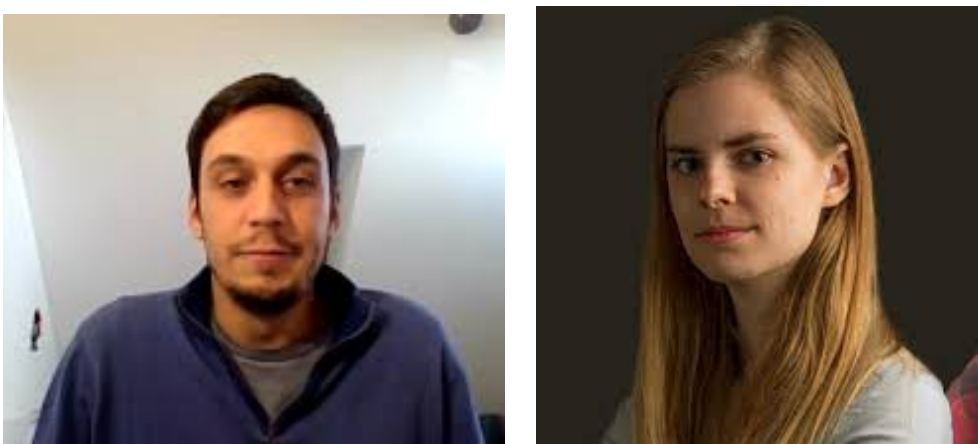


Pietro Bonetti, Maine Christos and S.S. (BCS), arXiv:2405.08817

$FL^* \rightarrow CDW$



Period 4 CDW obtained by condensing B .
Mixing between electrons and spinons removes β pocket.



Pietro Bonetti, Maine Christos and S.S. (BCS), arXiv:2405.08817

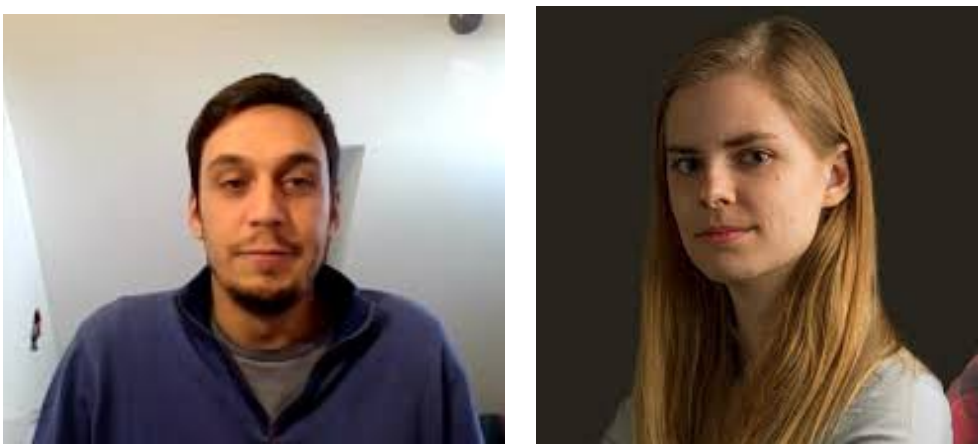
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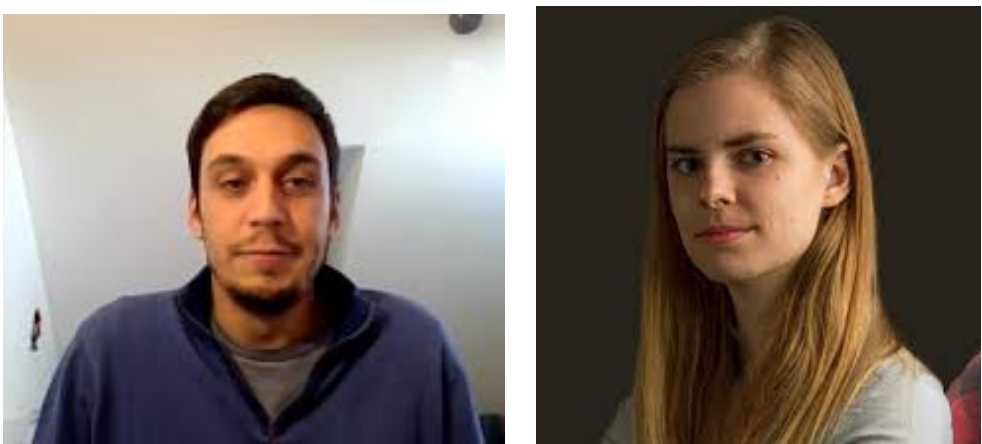
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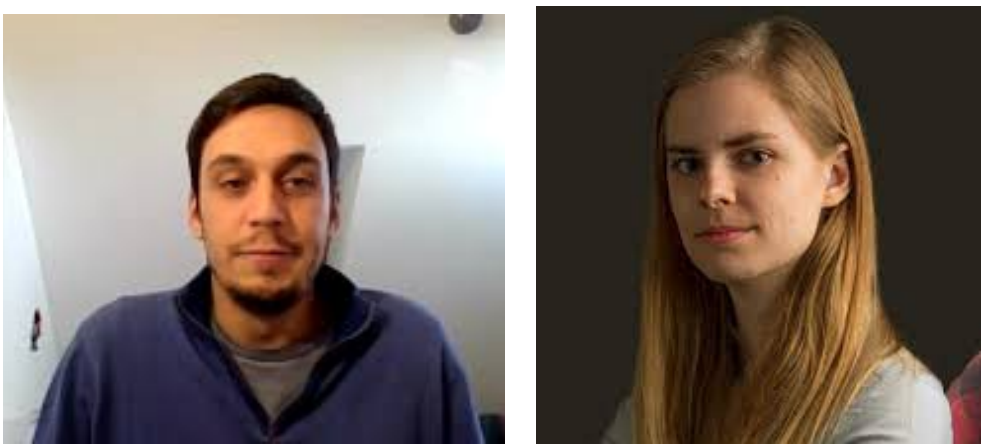
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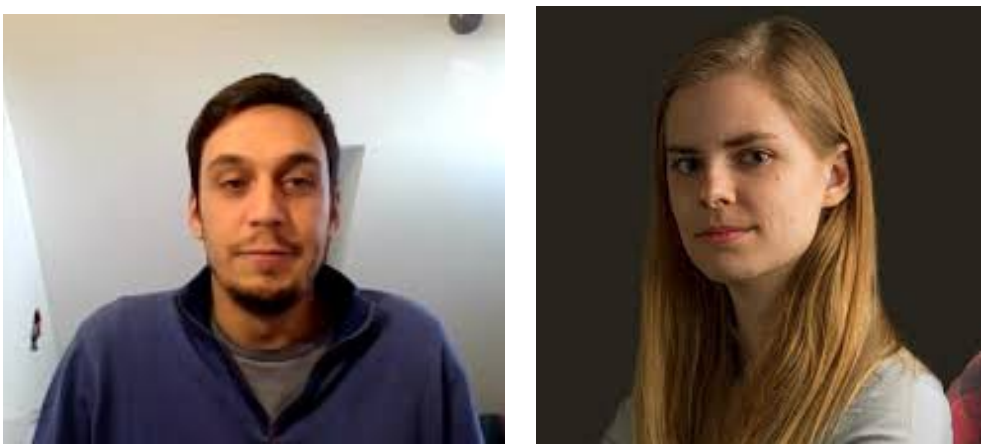


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These results imply that the free Dirac fermion theory of the spin liquid is not accurate for $S(\mathbf{q}, \omega)$, but is useful for fermiology. The bosonic \mathbb{CP}^1 theory of the *same* spin liquid should be a better starting point for $S(\mathbf{q}, \omega)$.



Pseudogap puzzles & answers:

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Spinons required by non-Luttinger Fermi surface in A
solve puzzles B, C, D!