Accuracy and feasibility of simulating fluid-elastic structure interaction in regular geometries

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Executive Summary

The objective of this report is to assess the feasibility of conducting industry-relevant fluid-elastic structure interaction simulations to aid in the design of civil helicopters operating under, e.g., hovering and forward flight conditions using the open-source Widely Applicable Lattice Boltzmann solver from ERLAngen (waLBerla)[2] [3] [4] Computational Fluid Dynamics (CFD) software package coupled with aeromechanics solvers. Specifically, this report aims to demonstrate the accuracy of the obtained simulation results, identifies bottlenecks in the waLBerla software stack relevant to the intended use-cases, both in terms of software architecture as well as lattice-Boltzmann (LB) method/Bounday Condition (BC) implementations, and presents mitigation strategies.

In the build-up to simulate elastic rotor blades, simulations are first conducted for single and tandem rigid rotor blade configurations using a coupled virtual blade model, namely the Actuator Line Model (ALM) [5], available within Leonardo's proprietary code AeroX, and the official waLBerla[2] distribution, hereafter referred to as the AeroX ALM – waLBerla (ALM-WB) coupled software. In this coupled software, which is developed as part of Deliverable 3.1, the mechanical forces arising in the rotors are computed by the ALM and are subsequently injected into the incompressible Lattice Boltzmann Method (LBM) computations via an appropriate LBM forcing scheme. The numerical simulations aim to model the aerodynamics of rotor blades in the simplifying situation wherein flow interactions with solid boundaries, such as the helicopter fuselage or a landing/hovering surface, are not included. Specifically, Large Eddy Simulations (LES) are conducted for high Reynolds number turbulent flows using the Cumulant LBM collision scheme[6], the Smagorinsky (SM) turbulence model [7, 8] and the Guo forcing scheme [9] for the blade configurations under consideration. The simulation results of the hovering and forward flight conditions are validated against experimental wind-tunnel measurements [1] and yield excellent correlations, thus demonstrating the utility of the ALM-WB coupled software.

Next, in order to establish the utility of the the waLBerla software stack to model more realistic situations wherein the blade-generated flow field interacts with solid bodies such as the helicopter fuselage or a landing/hovering surface, a simpler use case of LES of highly turbulent external flows over an Ahmed Body (AB) [10] has been undertaken. This widely studied problem models the turbulent flow over a bluff body whose shape mimics that of terrestrial vehicles and allows for investigating a solver's ability to handle curved boundaries along with flow-bluff body-ground interactions. For this study, we use the optimized recursive regularized collision developed on the computationally efficient D3Q19 lattice [11] and the Shear Improved SMagorinsky (SISM) turbulence model [12] previously implemented in a modified version of the waLBerla code[3] as part of Deliverable 2.4. The simulations reveal the limitations of waLBerla in terms of the accuracy of available no-slip boundary condition implementations, and that of the employed multigrid meshing strategy. An evaluation of the most optimal mitigation strategy that is compatible with the existing software framework and the project goals is currently underway.

Lastly, for extending the capabilities of the ALM-WB coupled software to simulate practical rotor blade configurations characterized by high rotor-tip Mach numbers, waLBerla must be extended to include LB techniques that model thermal and compressible flows. The LB literature largely prescribes two approaches for modelling thermal and compressible flows: the inclusion of non-local corrections on standard lattices or adoption of higher order lattices. Note that standard lattices, which prescribe lattice velocities in the first Brillouin region, are characterized by first nearest neighbour interactions that allow for designing computationally efficient data access patterns and require only a single ghost node layer for parallel communications. With this backdrop, the first strategy of incorporating non-local computations into waLBerla, which necessarily require strided data accesses, would severely compromise computational performance especially on multi-grid meshes. The alternative of using higher-order lattices, i.e., stencils having lattice velocities lying outside the first Brillouin zone, not only require strided data accesses, but more importantly also require additional ghost node layers for parallel communication, and would would therefore require an overhaul of the underlying waLBerla software stack. Thus it can be appreciated that existing LB methods for thermal and compressible flows are not viable solutions vis-à-vis integration with waLBerla. In this context, the very recently proposed Onsager-Regularized (OReg) LB scheme [13, 14, 15, 16] could present a viable theoretical and computational alternative to simulate thermal and compressible flows on standard lattices without having to incorporate non-local corrections. Here, we present results from preliminary investigations made to assess the ability of the OReg scheme to overcome the need for including non-local corrections on standard lattices that are used in waLBerla.

[2]https://walberla.net/
[3]https://github.com/multixscale/RR-BGK_SISM_waLBerla/

1 Introduction

1.1 Scope of the deliverable

This deliverable aims to provide notes on the ability of the LB-based waLBerla software stack to aid in the design and development of rotorcrafts such as civil helicopters. Specifically, we describe the successes in using an ALM-WB coupled software for modelling flows generated by rigid helicopter blades using the ALM [5] virtual blade model without including effects of interactions with solid surfaces such as the helicopter fuselage or a hovering/landing surface under the assumption of incompressibility. Thereafter, to assess the utility of the ALM-WB coupled software to include interactions of solid surfaces with the blade-generated flow fields, we present our findings on waLBerla's ability to model the external turbulent flow over a bluff body representing a terrrestrial ground vehicle. Lastly, we discuss the challenges associated with extending waLBerla, and consequently the ALM-WB coupled software stack, to model realistic compressible flow conditions associated with the intended use-case, followed by results of initial investigations made towards establishing the viability of extending the recently proposed OReg LB scheme to directly integrate within the existing waLBerla framework.

1.2 Report outline

In Section 2, we outline the results of investigations made on using LB methods already available in waLBerla for a simple helicopter rotor blade use-case for which experimental wind tunnel results are available for validation. The primary objective here is to evaluate waLBerla's capability of conducting multi-grid, GPU accelerated, fluid-structure interaction simulations of rigid rotor blades in the absence of solid boundary effects using the coupled ALM-WB software. Thereafter, in Section 3, we assess waLBerla's ability to model boundary effects on turbulent fluid flows through the simpler turbulent flow past an AB problem. In Section 4, we present preliminary investigations made to develop compressible flow LB schemes for describing near-tip compressibility effects which adhere to the software design philosophy and retain the computational efficiency offered by waLBerla. Lastly, Section 5 concludes the report and presents an outlook to the future work.

2 Incompressible Actuator Line - Lattice Boltzmann simulations of single and tandem helicopter rotor blades under different operating conditions

The capacity to characterize the interactions between the flow fields produced by rotor blades and the surrounding environment is essential in the design of rotary wing aircrafts such as helicopters. Such characterizations require accurate modelling not only of the turbulent fluid dynamics and rotating blade geometries, but also the associated fluid-structure interactions. However, full blade resolved simulations are prohibitively expensive in terms of the required computational resources due to multiple time and length scales associated with the problem as well as the need for dynamic meshes. Consequently such studies (see, e.g., [17]) typically make use of simplifications such as employing the Unsteady Reynolds Averaged Navier-Stokes (URANS) equations coupled with turbulence models for describing the flow physics and virtual blade models such as the ALM for describing the forces generated by the rotor.

Here, we numerically characterize, through the coupled ALM-WB software, the capability of out-of-the-box waLBerla methods to conduct GPU accelerated multi-grid simulations of single and tandem rotor blades in the hovering and forward flight conditions, comparing these with wind tunnel experiments [1].

In the single rotor case, the hovering configuration (Fig. 1a), comprises of measurements of the thrust produced at different power inputs for various blade pitch angles, whereas for the forward flight condition (Fig.1b), the power required to impose a fixed thrust at various advance ratios, i.e., $u_\infty/(2\omega R)$ where u_∞ is the freestream velocity, ω is the rotational speed (rpm) and R the rotor radius, along with resulting blade control angles are available for validation. The two overlapping tandem rotors (Fig. 1c), which have similar operating conditions as the single rotor forward flight case, report the power consumption and blade control angles for both front and rear rotors. All the rotors considered have a rotor radius of 2.32m.

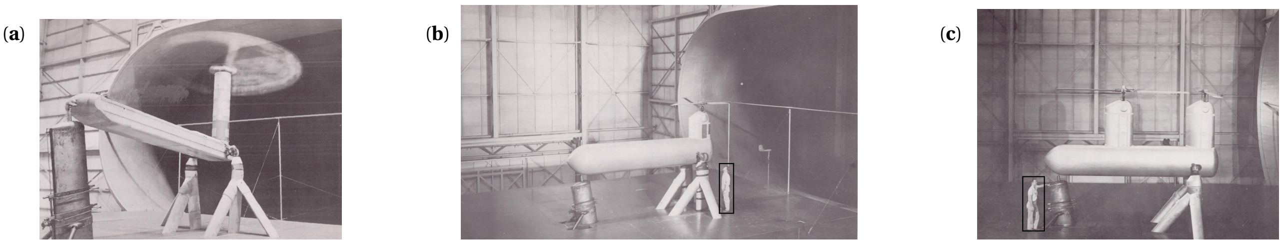

Figure 1: Experimental setups for the single rotor case in (a) the hovering condition, (b) the forward flight condition and (c) the tandem rotor case in the forward flight condition. (Images reproduced from [1]).

All waLBerla simulations are conducted with the Cumulant Collision Model, the Smagorinski turbulence Model and the Guo forcing scheme, employ *four* levels of grid refinement, prescribe the grid spacing in the finest level as Δx = 0.039m and employ a time step of Δt = 0.6654μs. It can be seen in the validation plots provided below that the simulation results obtained using waLBerla correlate well with the experimental data.

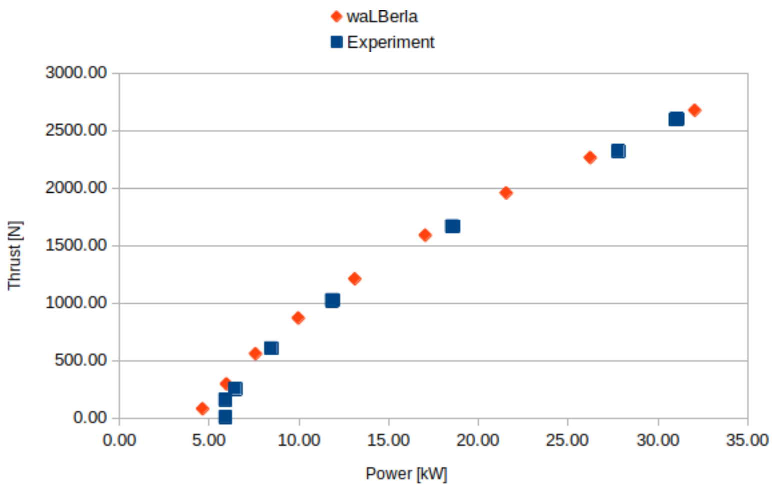

Figure 2: Validation of ALM-WB for the single rotor in hovering condition.

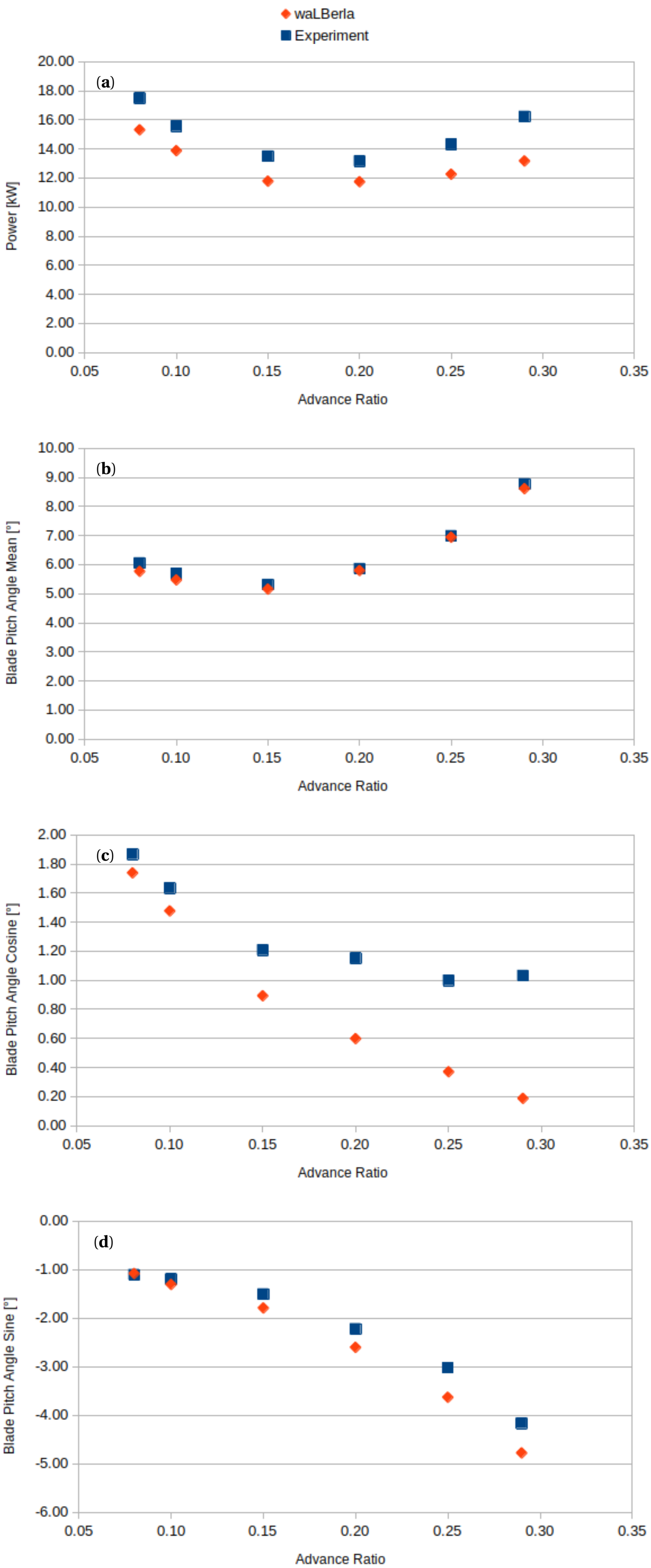

Figure 3: Validation of ALM-WB for the single rotor in the forward flight condition.

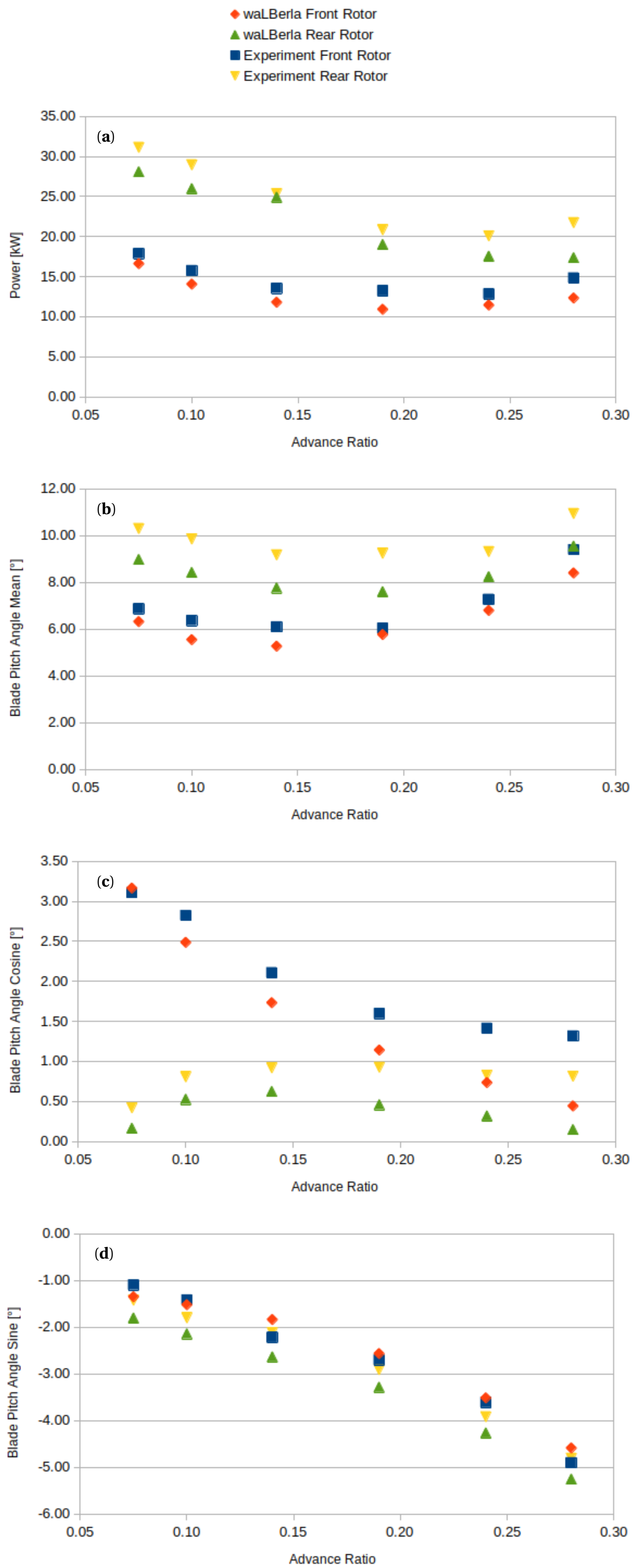

Figure 4: Validation of ALM-WB for the tandem rotor in the forward flight condition.

3 External flow past an Ahmed Body

In order to assess the behaviour of waLBerla in the presence of geometries with solid walls, we simulate a well-known test-case to benchmark numerical methods in CFD namely the flow past an Ahmed Body (AB)[10] which comprises a simplified geometry representative of ground vehicles. Despite its simple geometry, the flow physics associated with the AB problem are non-trivial and consequently accurate numerical modelling requires an accurate description of the flow close to the solid walls. As such, LES simulations of turbulent, wall-bounded flows, require special attention in the near-wall region failing which an extremely fine near-wall spatial resolution is required. Indeed, within the LB framework, studies performed on AB with commericial tools adopt wall functions with adverse pressure gradient corrections [18] (in PowerFlow), or mixed no-slip/equilibrium wall function boundary conditions [19] (in ProLB). Within waLBerla, the only available description to simulate stationary no-slip walls is via the bounce-back approach which, for curved surfaces, are found to be inaccurate even with extremely fine near-wall spatial discretizations due to the “staircase” approximation arising from the use of cartesian grids. We point out that such inaccuracies were not observed in the turbulent flow past Simple Frigate Shape 2 (SFS2) model considered in Deliverable 2.4 since the ship geometry comprises surfaces parallel to LB grid planes where the bounce-back method exhibits second-order accuracy.

For the AB problem, the simulated flow is characterized by a Reynolds (Re) number of 2.784×10^6 and a free-stream velocity of 40 m/s. The AB has a characteristic length of L = 1.044 m and is place at a distance of 4L and 10L from the inlet and outlet of the computational domain respectively. The slant angle of the AB model at the trailing upright is fixed to 25 degrees. The inlet boundary is modelled via the bounce-back boundary condition with Ladd correction[20][21] and the outlet is represented via the extrapolation-based outflow condition [6]. At this stage, we highlight that, strictly speaking, the lower domain boundary, which represents the ground, is a no-slip boundary. However, currently waLBerla does not seem to support bounce-back boundary conditions to be imposed on the multi-grid boundaries. Consequently, both the upper and lower domain boundaries are modelled through a free-slip boundary representation.

Figure 5: Meshing strategy chosen in the calculation of the flow past the Ahmed Body. Different colours correspond to different levels of refinement. The region coloured in red corresponds to the finest-sampling region, with a lattice spacing $\Delta x = 1.3$ mm.

Similarly to the resolution achieved by the commercial software ProLB in Ref.[19], the computational domain is meshed through *six* grid-refinement regions (see Figure 5) resulting in a cartesian mesh of about 120 million cells, reaching a spatial sampling of about 1.3 mm close to the AB wall. We point out that this sampling is not dense enough to properly resolve the boundary layer, as necessary in wall resolved LES calculations: such sampling requires the construction of a mesh with a much higher number of refinement levels, which is currently a topic under study.

We adopt a Recursively-Regularized-BGK collision realized on the D3Q19 lattice [11] coupled with a Shear-Improved-Smagorinsky [12] model, having a Smagorinsky coefficient $C_s = 0.125$, to treat unresolved subgrid scales. The employed equilibrium distribution function takes the third order polynomial form [11]. The simulation is performed on a single Graphical Processing Unit (GPU) node comprising 4 Nvidia-A100 GPUs for a physical time of 6.0 seconds simulated over 84907 time steps corresponding to $\Delta t = 70.665\mu$s. The overall wall clock time is approximately 10 hours. The average velocity field needed for validation is computed by sampling the instantaneous velocities in the last second of simulation

In Figure 6 we present the time averaged stream-wise component of the velocity (normalized by the freestream velocity) in the center plane of the AB, while in Figure 7 and 8 we respectively display the velocity components u_x and u_z along specific lines in the wake, compared with the experimental results of Lienhart *et al.*[2]. From the planar velocity profile, we notice the presence of spurious noisy oscillations on the front surface of the body (these spurious oscillations are highlighted in the inlet of Figure 6). These artefacts are likely due to the (first order) staircase approximation of the front curved wall of the AB due to the use of bounce-back to model solid walls, which introduce numerical noise in turbulent flow conditions. Despite not being shown, a further confirmation of this interpretation comes from the fact that these oscillations increase in intensity if a coarser near-wall sampling is used (i.e. a less accurate description of the real geometry is considered).

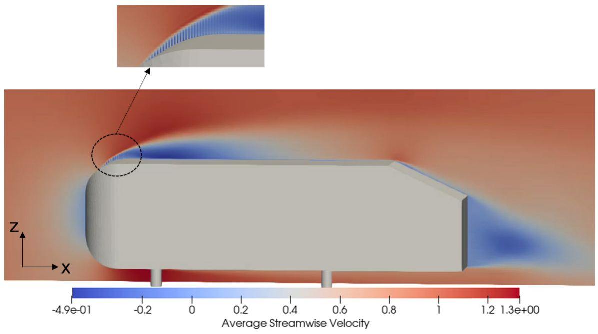

Figure 6: Time-averaged streamwise component of the velocity field around the Ahmed Body at Re = $2.784 \cdot 10^6$. The velocity field is shown in the symmetry plane of the Ahmed body, here defined as y = 0. The inlet underlines the appearance of spurious numerical noise in proximity of the front surface of the body.

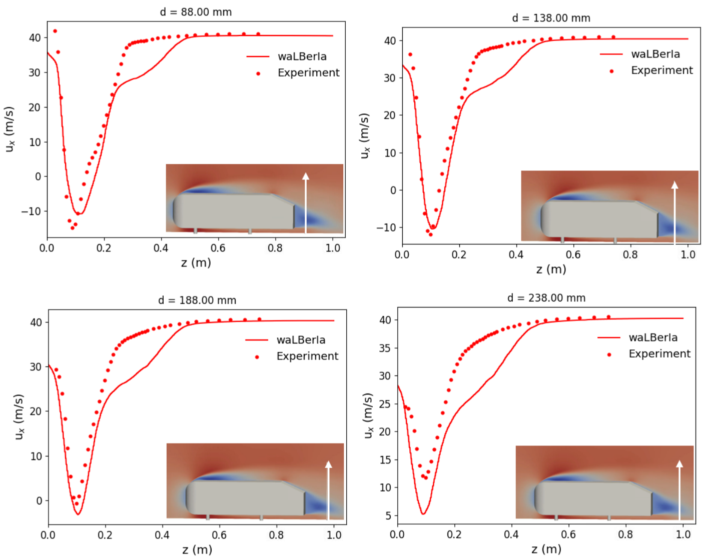

Figure 7: Comparison of the streamwise component of the velocity probed at different spatial locations (denoted by the white vertical arrow) in the wake of the Ahmed body against experimental data [2]

Note that while these oscillations are not observed on the object rear slant, the absence of near wall sampling, proper wall modelling on both the upper surface and the inclined surface of the AB, coupled with the improper lower domain boundary description decrease the accuracy of the predicted flow in the wake. This is clear looking at the comparison with experimental data presented in Figure 7 and 8 which yield quantitative errors larger than 15/20 %.

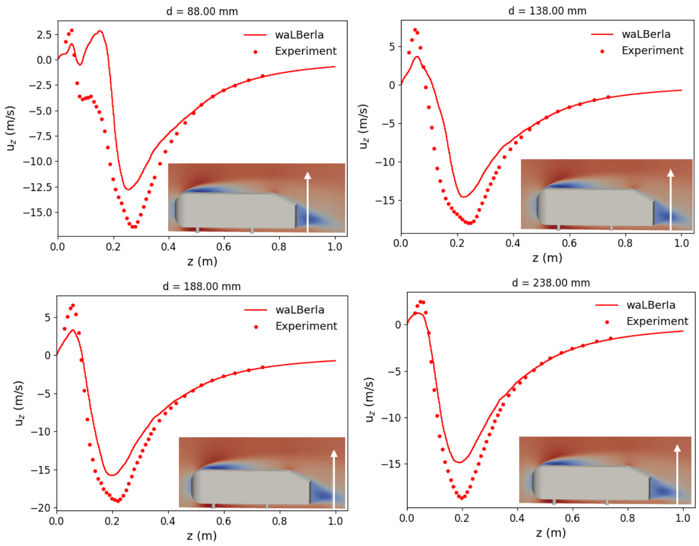

Figure 8: Same as Figure 7, but showing the transverse (u_z) component of the time-averaged velocity.

4 Viability of compressible flow simulations within `waLBerla`

The waLBerla software stack provides a variety of methods to perform incompressible and isothermal LB flow simulations on standard 2D and 3D lattices such as the D2Q9 and D3Q15/19/27 lattices in which velocities lie in the first Brillouin region. However, it is well known that, due to insufficient lattice symmetry, the dynamics obtained on these standard lattices can only approximate the isothermal flows Navier-Stokes equations in the low Mach number limit [22]. To be more specific, for thermal and compressible flow situations, the macroscopic dynamics recovered by LB schemes on standard lattices introduce appreciable spurious errors in both the momentum and energy conservation equations. It is noteworthy that this restriction is not severe for incompressible, isothermal flows in which case the weakly compressible nature of LB schemes on the D3Q27 lattice can be overcome by choosing simulation parameters that ensure the speed of sound in the numerical simulation coincides with the physical speed of sound [23]. However, since accurate descriptions of flow field near the rotor tips require compressible flow treatments due to large near-tip Mach numbers, these inherent limitations provide methodological challenges for conducting LB simulations of thermal and compressible flows on standard lattices and consequently within waLBerla.

Early efforts attempted to eliminate these spurious errors on standard lattices by augmenting the second order equilibrium distribution with $O(u^3)$ terms terms and have been successfully applied for a variety of applications [24, 25, 26, 27, 28, 29]. However, on standard lattices the associated errors cannot be completely eliminated irrespective of how the equilibrium state is represented. Indeed, the aforementioned attempt only corrects the off-diagonal spurious contributions in the stress tensor [30]. Consequently, for single-relaxation time schemes, significant efforts have been made to incorporate corrections in the form of force terms in the lattice Boltzmann equation either by direct non-local evaluations [31, 32, 33, 34] or through indirect non-local corrections to the equilibrium distribution representation [35]. In the context of waLBerla, these non-local computations would introduce significant computational overheads and thereby affect the overall scalability of the solver. Another approach to solve compressible flows in the LB framework is to use multispeed or higher order lattices [36, 37, 38, 39, 40]. However, in addition to increasing the computational cost due to larger number of lattice links and having small operating temperature ranges, the implementation of multispeed lattices into waLBerla would also require redesigning the underlying communication patterns associated with the streaming operation. Thus, it can be appreciated that the currently available LB methods for thermal and compressible flows cannot be easily implemented into waLBerla. In this context, the newly developed OReg LB scheme [14] could potentially provide the necessary solution.

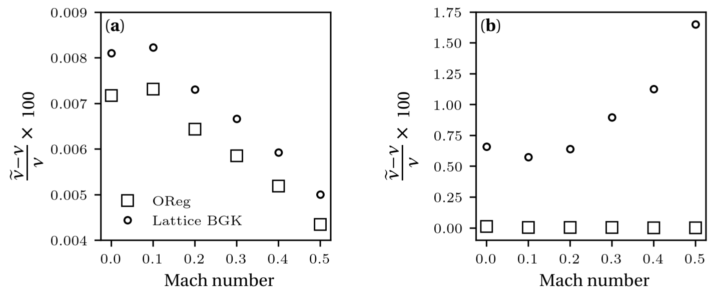

Figure 9: Percent relative error of the numerically computed viscosity $\tilde{\nu}$ to the imposed viscosity ν for different Mach numbers with the Onsager-Regularized scheme and the Lattice-BGK model for a decaying shear wave in the (a) axis-aligned, and (b) $\pi/4$-rotated configurations.

In order to assess the ability of the OReg scheme to the circumvent inclusion of spurious errors, we consider the decaying shear wave benchmark case [33, 30, 35]. In this benchmark, measurements are made of the numerical dissipation rate of a shear wave initialized in two configurations namely directions aligned or inclined to the horizontal axes. For the axis-aligned decaying shear wave, the case initialization is such that there are no spurious errors in the momentum equation; consequently in this setup the numerical dissipation rate is expected to be same as the theoretical rate for all considered methods. On the other hand, for the rotated case, existing schemes yield non-vanishing spurious errors in the momentum and energy equation and thus yield incorrect numerical dissipation rates that need to be corrected. The experiment being conducted here examines whether the OReg scheme yields the correct dissipation rate. For this experiment, we move away from the athermal LB formulation by using the energy conserving two dimensional guided equilibrium representation that has been derived only for the 2D D2Q9 lattice [33].

Here in Fig. 9(a), we plot the relative percent error between the numerically measured and the theoretically imposed viscosity, $\tilde{\nu}$ and ν respectively. It can be seen that the largest error with the Bhatnagar-Gross-Krook (BGK) and OReg collision model is approximately $O(10^{-5})$ implying that both the methods yield the correct result. However, it can been seen that the OReg method presents smaller deviations. Results presented in Fig. 9(b) demonstrate that the OReg scheme indeed yields the correct dissipation rate for the rotated wave case, thus providing empirical evidence that spurious errors associated with the anisotropy of standard lattices do not manifest in the momentum equation. A detailed manuscript describing these observations is currently under preparation [16].

5 Conclusions and Outlook

In this report we investigate the feasibility of simulating fluid-elastic structure interaction in regular geometries including non-steady turbulence phenomenon using the opensource waLBerla software. Since waLBerla does not currently support dynamic meshing strategies, blade resolved simulations of rotating rotor blades and deformable elastic rotor blades are found to be infeasible. However, through the use of virtual blade models, GPU accelerated multi-grid simulations for the case of rigid rotors in the absence of solid bodies demonstrate predictions of acceptable accuracy. A more thorough sensitivity analysis is, however, required to assess the robustness of the waLBerla for this use case. There after, investigations made on waLBerla's ability to model fluid flow descriptions for geometries with curved surface and available boundary treatments on the Ahmed body indicate possible future activities to increase the capabilities of waLBerla to appropriately treat complex wall bounded turbulent flows. A first step would require an implementation of resting-walls boundary conditions that offer improved geometric descriptions of curved surfaces, going beyond the staircase approximation associated with bounce-back currently available within waLBerla. Such an addition is, however, a non-trivial exercise as these advanced boundary conditions (see for example Ref. [41]) require velocity and density extrapolations from first and second neighbours to lattice nodes close to solid surfaces. To implement this approach, the current data-communication strategy of waLBerla should be modified so that data necessary to perform extrapolations are locally available at each time, after each stream-collide step on the boundary nodes. Currently efforts are being made to identify possible approaches that implement these methods within the waLBerla framework without compromising the numerical efficiency of the code. The successful implementation of efficient extrapolation strategies also opens avenues of efficient implementations of wall functions to model the boundary layer such as those proposed by Degrigny *et al.*[42] and Wilhelm *et al.*[43]. Lastly, in order to model compressibility effects that exist close to the rotor tips within the waLBerla framework, we find that additional methodological improvement to LB are necessary. In this context efforts to evaluate and characterize viable solutions along with their subsequent implementations into waLBerla are being made.

References

Acronyms used

HPC High Performance Computing
LB lattice-Boltzmann
LBM Lattice Boltzmann Method
CFD Computational Fluid Dynamics
Re Reynolds
LES Large Eddy Simulations
SM Smagorinsky
SISM Shear Improved SMagorinsky
GPU Graphical Processing Unit
SFS2 Simple Frigate Shape 2
ALM Actuator Line Model
ALM-WB AeroX ALM – waLBerla
BC Bounday Condition
OReg Onsager-Regularized
AB Ahmed Body
BGK Bhatnagar-Gross-Krook
URANS Unsteady Reynolds Averaged Navier-Stokes

Software mentioned

waLBerla Widely Applicable Lattice Boltzmann solver from ERLAngen

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