Cycloids by design2

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A method was previously used to generate the Gaussian cycloid **[1]**, wherein the ordinate function was the 'bell' arc while the abscissa its indefinite integral.

This method is employed here using a variety of well-known arcs as ordinates. To simplify matters computationally, given a sampled ordinate **y**, its indefinite integral – in OCTAVE code – is just **x=r*cumsum(y)**, where **r** is the sampling rate. Cycloid areas and arc-lengths while formally

$$A = \int y(t)dx(t)$$

and

$$L = \int \sqrt{x'(t)^2 + y'(t)^2}dt$$

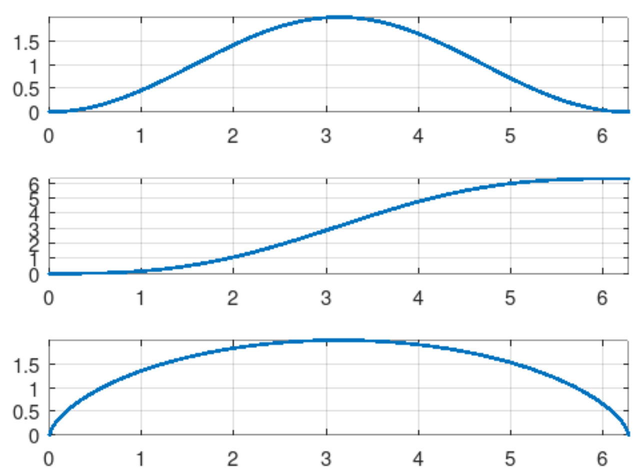
become (again in OCTAVE)

A = max(cumsum(y*[0 diff(x)]')) and **L = max(cumsum(sqrt(diff(x).^2+diff(y).^2))**

In the dozen exemplars below, the tiered results show the ordinate, abscissa, and cycloid in that order. Sampling bounds and scaling variations ensured that a full arch is obtained every time, and the rectangle enclosing each cycloid has dimensions $2\pi \times 2 = 4\pi$ (as in the classical cycloid) thus allowing comparison of area and arc-length estimates on an equal scale; these estimates are bracketed below each plot.

Raised cosine arch (classical cycloid)

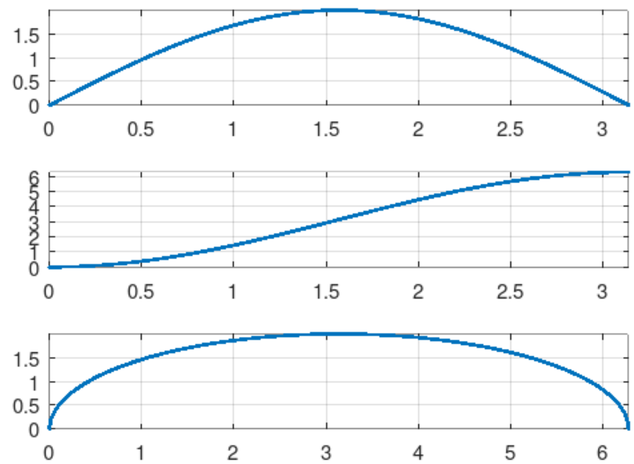
```
r=.001;,t=0:r:2*pi;,y=1-cos(t);,x=r*cumsum(y);
```



[9.4248 8.0000]

Elliptical arch

**r=.001;,t=0:r:pi;,y=2*sin(t);,
x=r*cumsum(y);,x=2*pi*x/max(x);**

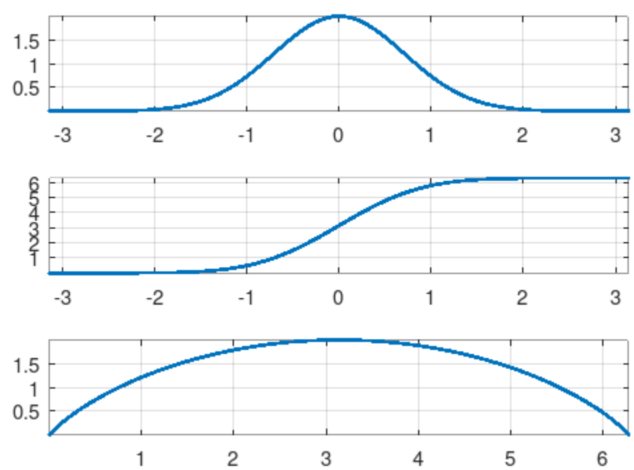


[9.8696   8.1751]

The elliptical cycloid has the largest area and longest arc-length of all cycloids here.

Gaussian arch

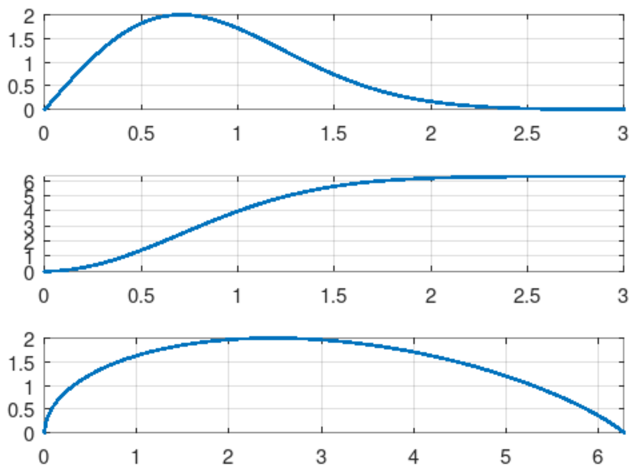
**r=.001;,t=-pi:r:pi;,y=2*exp(-t.^2);,
x=r*cumsum(y);,x=2*pi*x/max(x);**



[8.8858   7.8253]

<u>Rayleigh arch</u>

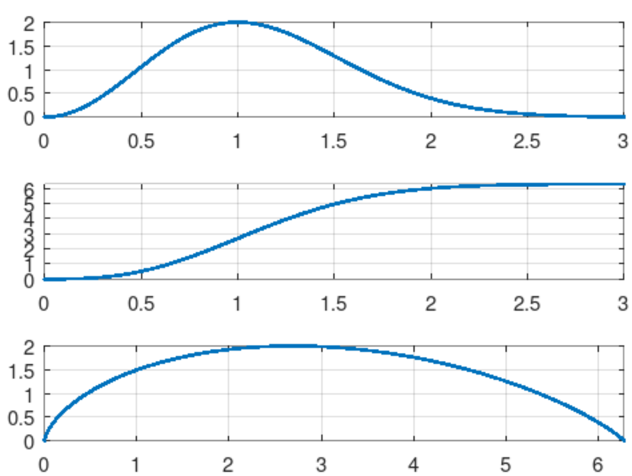
```
r=.001;,t=0:r:3;,y=t.*exp(-t.^2);,y=2*y/max(y);,
x=r*cumsum(y);,x=2*pi*x/max(x);
```



[9.1818   7.9958]

<u>Maxwell-Boltzmann arch</u>

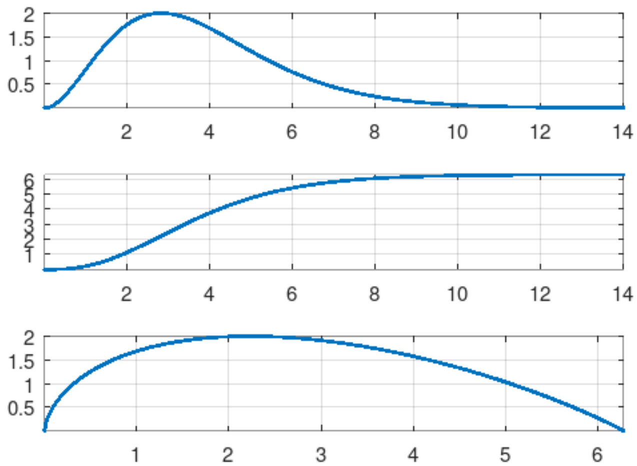
```
r=.001;,t=0:r:3;,y=(t.^2).*exp(-t.^2);,y=2*y/max(y);,
x=r*cumsum(y);,x=2*pi*x/max(x);
```



[9.0617   7.9089]

Planck arch

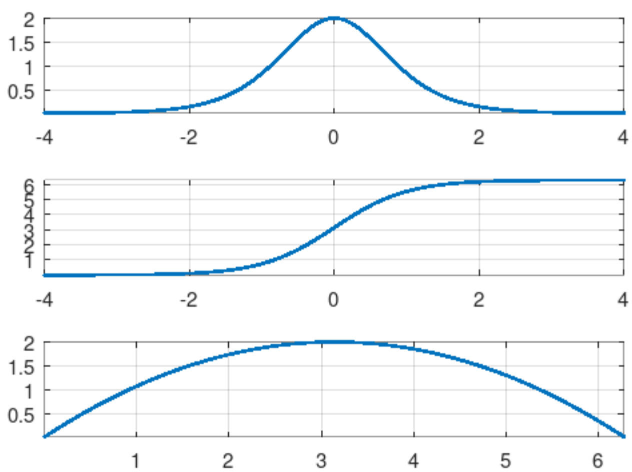
```
r=.001;,t=r:r:14;,y=(t.^3)./(exp(t)-1);,y=2*y/max(y);,
x=r*cumsum(y);,x=2*pi*x/max(x);
```



[8.8194   7.9439]

Soliton arch

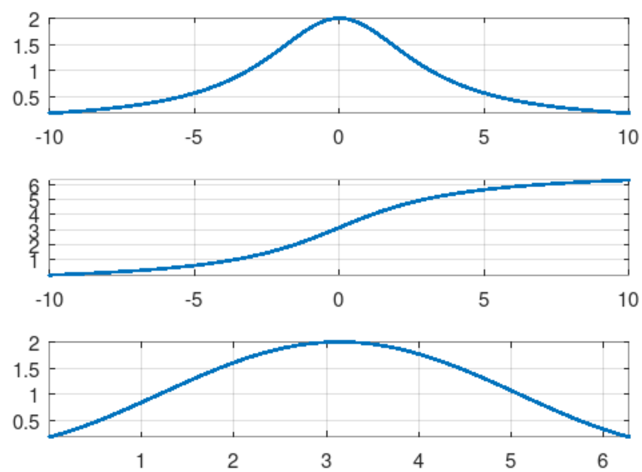
```
r=.001;,t=-4:r:4;,y=2*sech(t).^2;,
x=r*cumsum(y);,x=2*pi*x/max(x);
```



8.3832   7.7034]

__Cauchy arch__

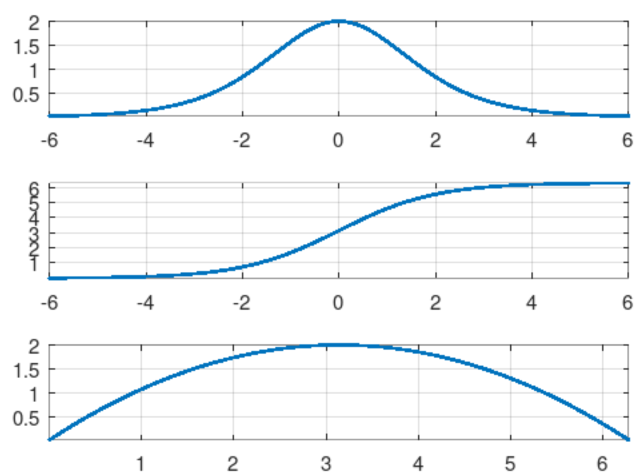
r=.001;,t=-10:r:10;,y=1./(pi^2+t.^2);,y=2*y/max(y);,
x=r*cumsum(y);,x=2*pi*x/max(x);



[7.7018   7.3718]

__Logistic arch__

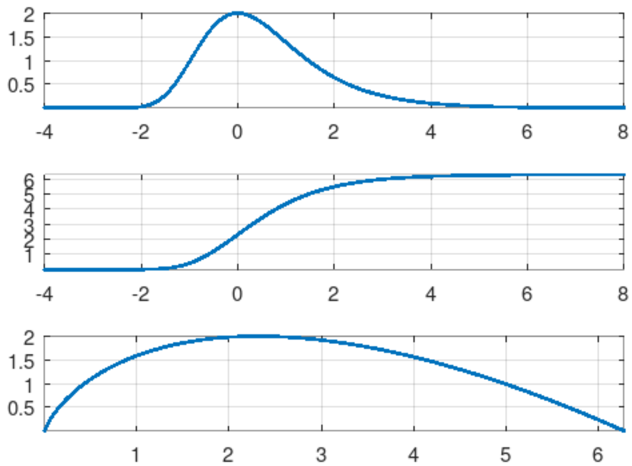
r=.001;,t=-6:r:6;,y=exp(-t)./((exp(-t)+1).^2);,y=2*y/max(y);,
x=r*cumsum(y);,x=2*pi*x/max(x);



[8.4189   7.6824]

Gumbel arch

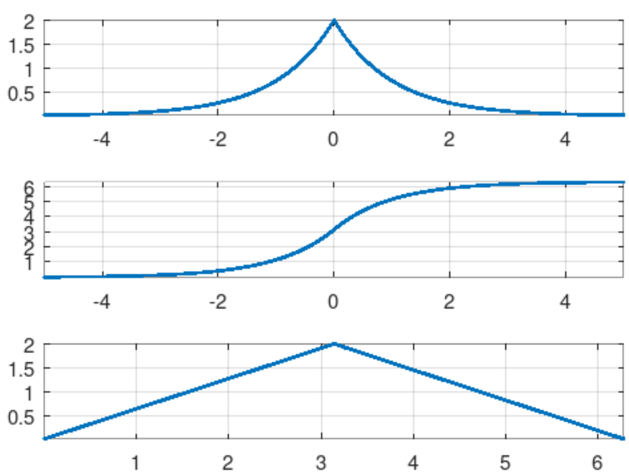
```
r=.001;,t=-4:r:8;,y=exp(-(t+exp(-t)));,y=2*y/max(y);,
x=r*cumsum(y);,x=2*pi*x/max(x);
```



[8.5426   7.8498]

Laplace arch

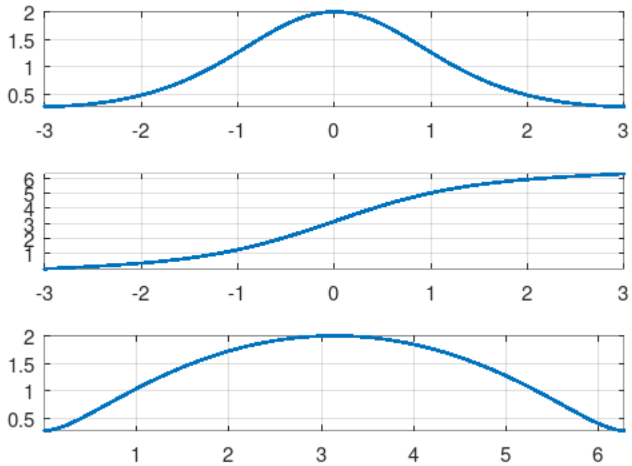
```
r=.001;,t=-5:r:5;,y=2*exp(-abs(t));,
x=r*cumsum(y);,x=2*pi*x/max(x);
```



[6.3255   7.4339]

Von Mises arch

```
r=.001;,t=-3:r:3;,y=exp(cos(t));,y=2*y/max(y);,
x=r*cumsum(y);,x=2*pi*x/max(x);
```



[8.4114 7.3448]

Dimensional results sorted on decreasing area are shown below.

ARCH	AREA	LENGTH
Elliptical	9.8696	8.1751
Cosine	9.4248	8.0000
Rayleigh	9.1818	7.9958
Maxwell	9.0617	7.9089
Gaussian	8.8858	7.8253
Planck	8.8194	7.9439
Gumbel	8.5426	7.8498
Logistic	8.4189	7.6824
Von Mises	8.4114	7.3448
Soliton	8.3832	7.7034
Cauchy	7.7018	7.3718
Laplace	6.3255	7.4339

[1] A. Cusmariu, 'A Gaussian cycloid2', https://zenodo.org/records/13824609, 2014