QUBO Formulation of Entropy Maximization

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Abstract

In this brief note, the authors present an approach to formulating the maximization of entropy (Shannon) as an approximate QUBO problem.

Consider a simple triune p.m.f. : $\{1-b-c, b, c\}$. For this p.m.f.,

$$H(X) = -(1-b-c)\log(1-b-c) - b\log(b) - c\log(c). \tag{1}$$

We next allow three binary decimal digits of precision for each of b and c. This yields a maximization in 6 variables, each of which can take values in $\{0,1\}$. We make use of the following approximations:

$$\log(b) \approx \frac{b-1}{0.693147} - \frac{(b-1)^2}{1.3863}. \tag{2}$$

A similar approximation works for the logarithm of c and of $(1-b-c)$. In terms of the three binary decimal digits of precision we can write $H(X)$ as

$$-(1-2^{-1}b0-2^{-2}b1-2^{-3}b2-2^{-1}c0-2^{-2}c1-2^{-3}c2)\log(1-2^{-1}b0-2^{-2}b1-2^{-3}b2-2^{-1}c0-2^{-2}c1-2^{-3}c2) \tag{3}$$

$$-(2^{-1}b0+2^{-2}b1+2^{-3}b2)\log(2^{-1}b0+2^{-2}b1+2^{-3}b2) \tag{4}$$

$$-(2^{-1}c0+2^{-2}c1+2^{-3}c2)\log(2^{-1}c0+2^{-2}c1+2^{-3}c2). \tag{5}$$

In the same vein, we can expand b^2 as

$$2^{-6}b2+2^{-3}b2b0+2^{-4}b2b1+2^{-2}b0+2^{-4}b1+2^{-2}b0b1. \tag{6}$$

However, when we come to b^3 and other cubic terms, we will encounter triune terms like $b0b1b2$ in the expansion. Each such term can be replaced by a new variable. For example, we can replace $b0b1b2$ by y and add the constraints: $y \leq b0$, $y \leq b1$, $y \leq b2$ and $y \geq b0+b1+b2-2$ to ensure equivalence while complying with the QUBO format requirements[1]. In future work, we will extend this methodology to capacity calculation. The results will then be compared with the Blahut-Arimoto approach results.

[1] Credit goes to an AI model for suggesting this.