A NONLINEAR ANALYTICAL PROCEDURE FOR ELECTROMAGNETIC TRANSIENTS IN FERROMAGNETIC SHIELDS WITH AN EXPONENTIAL PERMEABILITY MODEL

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ABSTRACT

Electrically conductive, ferromagnetic shielding materials exhibit nonlinear behavior (including saturation) under intense pulsed electromagnetic field conditions such as those associated with lightning, electromagnetic pulse (EMP), and electrostatic discharge (ESD). Previously, an analytical procedure was developed to characterize the nonlinear electric field transients induced at the inner surface of long, thin-walled, cylindrical, electrically conductive, ferromagnetic shields by axially-directed short-duration surface current pulses on the outer surface. For a general relative differential permeability function $\mu_{rd}(H)$, it was shown that the peak value of the electric field transient can be expressed in terms of an effective permeability $\mu_E(\beta)$ and the time at which the peak occurs can be expressed in terms of effective permeability $\mu_T(\beta)$, where the applied pulse parameter $\beta \equiv Q_o/\sigma d^2$ is a fundamental combination of the <u>nonmagnetic</u> problem parameters (the charge per unit circumference, Q_o, transported along the cylinder during the pulse; the electrical conductivity, σ, of the material; and the wall thickness, d, of the cylinder).

For a simple exponential relative differential permeability model $\mu_{rd}(H)=\mu_{ri}\, exp(-H/H_1)$, the analytical procedure can be extended significantly. An applied pulse parameter $\zeta \equiv Q_o/\sigma d^2 B_s$ has been identified that includes the saturation magnetization B_s, which is a <u>magnetic</u> problem parameter. Furthermore, the effective permeabilities $\mu_E(\zeta)$ can be expressed as $\mu_E(\zeta)=\mu_{ri}\Omega_E(\zeta)$, where $\Omega_E(\zeta)$ is a fundamental quantity that depends only on ζ. Similarly, $\mu_T(\zeta)$ can be expressed as $\mu_T(\zeta)=\mu_{ri}\Omega_T(\zeta)$, where $\Omega_E(\zeta)$ is a fundamental quantity that depends only on ζ. Results of numerical calculations for $\Omega_E(\zeta)$ and $\Omega_T(\zeta)$ are presented.

INTRODUCTION

The intense electromagnetic transients associated with electromagnetic pulse (EMP), electrostatic discharge (ESD), and lightning can disrupt or damage sensitive electronic equipment. Such equipment and associated lines are protected by electrically conductive shields that are designed to attenuate the electromagnetic fields to tolerable levels.

Ferromagnetic materials offer potential advantages as shields due to their high magnetic permeabilities; however, there has been very limited guidance concerning the nonlinear behavior that such materials exhibit under intense pulsed electromagnetic field conditions. For ferromagnetic materials the flux density $B(H)$ varies with the magnetic field intensity H. The relative differential magnetic permeability, $\mu_{rd}(H)$, is defined as

$$\mu_{rd}(H) \equiv \frac{1}{\mu_o}\frac{dB(H)}{dH} \tag{1}$$

where $\mu_o = 4\pi x 10^{-7}$ henry/meter is the permeability of free space. For the cases considered here, hysteresis is neglected and $\mu_{rd}(H)$ is assumed to be a single-valued, reversible function of H. The analysis of pulsed electromagnetic field interactions with ferromagnetic shields is complicated by the variation of $\mu_{rd}(H)$ with H, which makes the phenomenon nonlinear.

This paper describes an extension of an analytical procedure that can be used to assess the nonlinear effects of a field-dependent magnetic permeability on the performance of long, thin-walled, cylindrical, electrically conductive, ferromagnetic shields that are subjected to short duration surface current pulses (Figure 1). This analytical procedure has application to modelling long shielded cables and electrical conduits that may be subjected to intense surface current pulses.

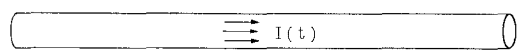

Figure 1. A long, thin-walled cylinder subjected to an axially directed surface current pulse.

MATHEMATICAL FORMULATION

The problem involves long, thin-walled, cylinders that are subject to surface current pulses, $I_o(t)$, that are directed axially along the outer surface ($r = r_o$). This leads to a transverse electromagnetic field (TEM) condition at the outer surface. The magnetic field at the outer surface is

$$H(r_o,t) = \frac{I_o(t)}{2\pi r_o} \tag{2}$$

For surface current pulses that are of sufficiently short duration, it has been observed numerically that a nonlinear impulse response exists that depends on Q_o, the charge per unit length of circumference (coulombs/meter) that is transported along the outer surface during the applied pulse.

A planar approximation to the cylindrical problem considers a sheet of thickness, d, and electrical conductivity, σ. The problem is described mathematically by the nonlinear partial differential equation [1] [2]

$$\frac{\partial^2 H}{\partial x^2} = \sigma \mu_o \mu_{rd}(H)\frac{\partial H}{\partial t} \tag{3}$$

subject to the initial condition

$$H(x,0) = 0 , \tag{4}$$

an impulse applied field on the outer surface ($x = 0$)

$$H(0,t) = Q_o \delta(t) \, , \tag{5}$$

and a vanishing field at the inner surface (x = d)

$$H(d,t) = 0 \, . \tag{6}$$

Although the magnetic field intensity vanishes at the inner surface, the electric field response at the inner surface (x = d) is determined from the partial derivative of H evaluated at the inner surface

$$E(d,t) = -\frac{1}{\sigma}\frac{\partial H(x,t)}{\partial x}\bigg|_{x=d} \tag{7}$$

It is this transient electric field response that is of interest.

GENERAL DIMENSIONLESS FORMULATION

In [1] and [2] it was shown that for a general relative differential permeability function $\mu_{rd}(H)$, the problem described by Eqs. 3-6 can be put into a dimensionless form involving the solution of the partial differential equation

$$\frac{\partial^2 \Psi}{\partial \xi^2} = \mu_{rd}(\beta \Psi/\mu_o)\frac{\partial \Psi}{\partial \tau} \tag{8}$$

subject to the initial condition ($\tau = 0$)

$$\Psi(\xi,0;\beta) = 0 \, , \tag{9}$$

a unit impulse on the outer surface ($\xi = 0$)

$$\Psi(0,\tau;\beta) = \delta(\tau) \, , \tag{10}$$

and a vanishing field at the inner surface ($\xi = 1$)

$$\Psi(1,\tau;\beta) = 0 \, , \tag{11}$$

where ξ is a dimensionless spatial variable

$$\xi \equiv \frac{x}{d} \, , \tag{12}$$

τ is a dimensionless temporal variable

$$\tau \equiv \frac{t}{\mu_o \sigma d^2} \, , \tag{13}$$

and Ψ is a dimensionless field variable

$$\Psi(\xi,\tau;\beta) \equiv \frac{H(x,t)}{\left(\dfrac{Q_o}{\mu_o \sigma d^2}\right)} \, . \tag{14}$$

The applied pulse parameter, β, is a fundamental combination of all of the nonmagnetic parameters

$$\beta \equiv \frac{Q_o}{\sigma d^2} \tag{15}$$

The nomenclature $\Psi(\xi,\tau;\beta)$ is used to denote that the solution depends on the parameter β as well as the independent variables ξ and τ. Note that the relative differential permeability in terms of Ψ is obtained by a simple substitution of

$$H = \frac{Q_o}{\mu_o \sigma d^2}\Psi = \frac{\beta}{\mu_o}\Psi \tag{16}$$

into $\mu_{rd}(H)$, which yields

$$\mu_{rd}(H) = \mu_{rd}\left(\beta \Psi/\mu_o\right) \, . \tag{17}$$

Although the problem for the magnetic field intensity, H, is in dimensionless form, the electric field, E, is not in a dimensionless form. From Eq. 7, an expression for the electric field at the inner surface ($x = d$) is

$$\frac{E(d,t)}{\left(\dfrac{Q_o}{\mu_o \sigma^2 d^3}\right)} = -\frac{\partial \Psi(\xi,\tau;\beta)}{\partial \xi}\bigg|_{\xi=1} \tag{18}$$

where the right hand side is dimensionless. The relationship given by Eq. 18 holds for a general $\mu_{rd}(H)$ and provides a useful insight into the manner in which the nonmagnetic parameters enter into the nonlinear solution.

GENERAL EFFECTIVE PERMEABILITIES

Although the entire electric field transient, $E(d,t)$, is sometimes of interest, the quantities of primary interest are the peak value, E_{peak}, and the time, t_{peak} at which the peak value occurs. In the ratio involving the electric field in Eq. 18, the right hand side is a dimensionless quantity that depends on the applied pulse parameter β. In [1] and [2] it was observed that E_{peak} for the linear solution for a constant permeability also contains the factor $Q_o/(\mu_o \sigma^2 d^3)$; consequently, if the peak value for the electric field response is referenced to that for the linear solution, then the result is dimensionless and depends on the parameter β. It is convenient to express the result in terms of effective permeabilities.

An effective permeability was defined in [2] as the value of a constant relative permeability needed in the linear solution to achieve the same values for E_{peak} and t_{peak}. The effective permeability for the peak value of the electric field, $\mu_E(\beta)$, is defined as

$$\mu_E(\beta) \equiv 5.922053727\frac{Q_o}{\mu_o E_{peak} \sigma^2 d^3} \tag{19}$$

The effective permeability, $\mu_E(\beta)$, is the value of the constant relative permeability that would yield the same value for E_{peak} from the linear solution. The effective permeability for the time at which the peak value occurs, $\mu_T(\beta)$, is defined as

$$\mu_T(\beta) \equiv 10.898979\frac{t_{peak}}{\mu_o \sigma d^2} \tag{20}$$

The effective permeability, $\mu_T(\beta)$, is the value of the constant relative permeability that would yield the same result for t_{peak} from the linear solution. The transformations given by Eqs. 19 and 20 can be used to transform numerical results for a particular case into effective permeabilities that are of more general use.

NONLINEAR ELECTRIC FIELD TRANSIENTS

The peak transient electric field induced at the inner surface, E_{peak}, can be expressed in terms of the effective permeability $\mu_E(\beta)$

$$E_{peak} = 5.922053727\frac{Q_o}{\mu_o \mu_E(\beta)\sigma^2 d^3} \tag{21}$$

and the time of the peak, t_{peak}, in terms of the effective permeability $\mu_T(\beta)$

$$t_{peak} = 0.091751715\,\mu_o \mu_T(\beta)\sigma d^2 \tag{22}$$

The nonlinearity of the problem is contained in the effective permeabilities $\mu_E(\beta)$ and $\mu_T(\beta)$.

DIFFERENTIAL PERMEABILITY MODEL

Consider the simple magnetization curve

$$B(H) = B_s\left[1 - \exp\left(-\frac{H}{H_1}\right)\right] \tag{23}$$

where B_s is the saturation magnetization and H_1 is a characteristic value for the curve. Substitution of Eq. 23 into the definition given in Eq. 1 yields a relative differential permeability model of the form

$$\mu_{rd}(H) = \frac{B_s}{\mu_o H_1}\exp\left(-\frac{H}{H_1}\right), \tag{24}$$

or

$$\mu_{rd}(H) = \mu_{ri}\exp\left(-\frac{H}{H_1}\right), \tag{25}$$

where

$$\mu_{ri} \equiv \frac{B_s}{\mu_o H_1}, \tag{26}$$

is the initial value of the permeability

$$\mu_{rd}(0) = \mu_{ri}. \tag{27}$$

This simple exponential relative differential permeability model contains two parameters that can be used to model the relative differential permeability. Only two of the parameters B_s, H_1, and μ_{ri} can be specified independently; consequently, not all values of $\mu_{rd}(0)$, H_1, and B_s can be accommodated. As will be seen later, μ_{ri} determines the results for low amplitude pulses, and B_s determines the transition to saturation; consequently, μ_{ri} and B_s are regarded as the two most important parameters for the present case.

The fidelity of any analytical results to physical reality depends on the ability of the relative differential permeability function $\mu_{rd}(H)$ to model the actual relative differential permeability of a material. The simple magnetization curve given by Eq. 23 exhibits saturation; however, it does not exhibit the appropriate limiting behavior for large H. For large H, the above magnetization curve exhibits the limiting behavior $B(H) \to B_s$ rather than the physical limiting behavior $B(H) \to B_s + \mu_o H$.

Typically, as the value of H is increased from zero, relative differential permeabilities start at some initial value $\mu_{rd}(0)$, increase to some maximum value $\mu_{rd}(H_m)$ at some value $H = H_m$, and then decrease to the permeability of free space ($\mu_{rd}(\infty) = 1$) as the material undergoes saturation. The initial value of the relative differential permeability, $\mu_{rd}(0)$, is important for small applied pulse levels. Physically, the relative differential permeability approaches unity for large values of H such that $\mu_{rd}(\infty) = 1$; therefore, it should be noted that the model does not exhibit the appropriate limiting behavior for large H where $\mu_{rd}(\infty)=0$ rather than $\mu_{rd}(\infty)=1$. The saturated value of the relative differential permeability, $\mu_{rd}(\infty)$, is important for large applied pulse levels.

Although it does not exhibit a realistic limiting behavior, the simple two-parameter exponential relative differential permeability representation allows one to model the saturation process and thereby gain a fundamental insight into the nonlinear process for the active part of the differential permeability.

ALTERNATE DIMENSIONLESS FORMULATION

The correlation for a general relative permeability function can be extended to a more fundamental level for the exponential permeability model given by Eq. 25. The problem consisting of the partial differential equation

$$\frac{\partial^2 H}{\partial x^2} = \sigma\mu_o\mu_{ri}\exp\left(-\frac{H}{H_1}\right)\frac{\partial H}{\partial t} \tag{28}$$

subject to the conditions given by Eqs.4-6 can be put into a dimensionless form involving the solution of the partial differential equation

$$\frac{\partial^2 \Phi}{\partial \xi^2} = \exp(-\zeta\Phi)\frac{\partial \Phi}{\partial \eta} \tag{29}$$

subject to the initial condition ($\eta = 0$)

$$\Phi(\xi,0;\zeta) = 0, \tag{30}$$

a unit impulse on the outer surface ($\xi = 0$)

$$\Phi(0,\eta;\zeta) = \delta(\eta), \tag{31}$$

and a vanishing field at the inner surface ($\xi = 1$)

$$\Phi(1,\eta;\zeta) = 0 \tag{32}$$

where ξ is the dimensionless spatial variable given in Eq. 12, η is a new dimensionless temporal variable

$$\eta \equiv \frac{\tau}{\mu_{ri}} = \frac{t}{\mu_o\mu_{ri}\sigma d^2}, \tag{33}$$

and Φ is a new dimensionless field variable

$$\Phi(\xi,\eta;\zeta) \equiv \mu_{ri}\Psi(\xi,\tau;\beta) = \frac{H(x,t)}{\left(\dfrac{Q_o}{\mu_o\mu_{ri}\sigma d^2}\right)}. \tag{34}$$

The applied pulse parameter, ζ,

$$\zeta \equiv \frac{Q_o}{\sigma d^2 B_s} = \frac{\beta}{B_s} \tag{35}$$

is a fundamental combination of all of the nonmagnetic parameters and the magnetic parameter B_s. The nomenclature $\Phi(\xi,\tau;\zeta)$ is used to denote that the solution depends on the applied pulse parameter ζ as well as the independent variables ξ and η. Note that the relative differential permeability in terms of Φ is obtained by a simple substitution of

$$H = \frac{Q_o}{\mu_o \mu_{ri} \sigma d^2} \Phi = H_1 \zeta \Phi \tag{36}$$

into $\mu_{rd}(H)$, which yields

$$\mu_{ri} \exp\left(-\frac{H}{H_1}\right) = \mu_{ri} \exp\left(-\zeta \Phi\right) . \tag{37}$$

As in the general dimensionless formulation, the problem for the magnetic field intensity, H, is in dimensionless form; however, the electric field, E, is not dimensionless. An expression for the electric field at the inner surface ($x = d$) is

$$\frac{E(d,t)}{\left(\frac{Q_o}{\mu_o \mu_{ri} \sigma^2 d^3}\right)} = -\left.\frac{\partial \Phi(\xi,\eta;\zeta)}{\partial \xi}\right|_{\xi = 1} \tag{38}$$

where the right hand side is dimensionless. The relationship given by Eq. 38 holds for an exponential $\mu_{rd}(H)$ and provides a useful insight into the manner in which the problem parameters enter into the nonlinear solution.

From Eq. 19 and Eq. 38 it can be deduced that for the simple exponential permeability model, $\mu_E(\zeta)$ can be expressed as

$$\mu_E(\zeta) = \mu_{ri} \Omega_E(\zeta) \tag{39}$$

where $\Omega_E(\zeta)$ is a function of ζ only. Similarly, from Eq. 20 and Eq. 38 it can be deduced that $\mu_T(\zeta)$ can be expressed as

$$\mu_T(\zeta) = \mu_{ri} \Omega_T(\zeta) \tag{40}$$

where $\Omega_T(\zeta)$ is a function of ζ only.

Once the fundamental quantities $\Omega_E(\zeta)$ and $\Omega_T(\zeta)$ have been evaluated numerically for one set of parameters, they can be easily scaled to other permeabilities in the class. From Eq. 21, the peak transient electric field induced at the inner surface, E_{peak}, can be expressed in terms of $\Omega_E(\zeta)$

$$E_{peak} = 5.9220537270 \frac{Q_o}{\mu_o \mu_{ri} \Omega_E(\zeta) \sigma^2 d^3} , \tag{41}$$

and from Eq. 22 the time of the peak, t_{peak}, can be expressed in terms of $\mu_T(\zeta)$

$$t_{peak} = 0.091751715 \mu_o \mu_{ri} \Omega_T(\zeta) \sigma d^2 . \tag{42}$$

It can be observed that normalizing $\mu_E(\zeta)$ with respect to the μ_{ri} yields

$$\frac{\mu_E(\zeta)}{\mu_{ri}} = \Omega_E(\zeta) \tag{43}$$

where $\Omega_E(\zeta)$ is a function of ζ only. Similarly, normalizing $\mu_T(\zeta)$ with respect to μ_{ri} yields

$$\frac{\mu_T(\zeta)}{\mu_{ri}} = \Omega_T(\zeta) \tag{44}$$

where $\Omega_T(\zeta)$ is a function of ζ only. It is important to note that H_1 does not enter into either $\Omega_E(\zeta)$ or $\Omega_T(\zeta)$; consequently, there is only one curve for $\Omega_E(\zeta)$ and only one curve for $\Omega_T(\zeta)$. Thus, $\Omega_E(\zeta)$ and $\Omega_T(\zeta)$ are fundamental quantities that characterize the nonlinearity of the problem with an exponential relative differential permeability model.

RESULTS OF NUMERICAL CALCULATIONS

In general, the curves $\Omega_E(\zeta)$ and $\Omega_T(\zeta)$ that correspond to the exponential differential permeability representation $\mu_{rd}(H)$ must be evaluated numerically. That is, at least one set of values for E_{peak} and t_{peak} must be determined for each value of ζ of interest; however, once the calculation has been accomplished, the nonlinearity of the problem has been characterized. An implicit finite difference formulation using backward time differencing has been used to perform numerical characterizations using the analytical procedure outlined in [2].

For the numerical calculations, applied pulses of the following form were used

$$H(0,t) = A\left[\exp\left(-\frac{t}{t_f}\right) - \exp\left(-\frac{t}{t_r}\right)\right] \tag{45}$$

where A is the amplitude factor (amperes/meter), t_r is a time constant (seconds) that is associated with the risetime of the applied pulse, and t_f is a time constant (seconds) that is associated with the falltime of the applied pulse. For such pulses, Q_o (coulombs/meter) is given by

$$Q_o = A\left(t_f - t_r\right) . \tag{46}$$

For sufficiently small t_r and t_f, the applied pulse has the effect of an impulse with magnitude Q_o and can be used to represent the condition of Eq 5.

For a given value of ζ, Eqs. 43 and 44 indicate that $\Omega_E(\zeta)$ and $\Omega_T(\zeta)$ should be invariant. Table 1 shows the results of numerical calculations for selected cases with ζ=9x10^{-3} for which the nonlinearity of the problem is beginning to manifest itself. The problem parameters (σ, d, H_1, B_s, μ_{ri}, A, t_r, and t_f) were varied over orders of magnitude, and the resulting values for E_{peak} and t_{peak} also vary over orders of magnitude. Although the effective permeabilities $\mu_E(\zeta)$ and $\mu_T(\zeta)$ vary over orders of magnitude, it can be observed that $\Omega_E(\zeta)$ and $\Omega_T(\zeta)$ are very nearly constant. In each case extensive numerical experiments were performed to verify the scaling in Eqs. 43 and 44; however, in some cases very precise numerical calculations (using extremely small spatial intervals and extremely small time increments) were required to observe the scaling effect. The correlation given by Eqs. 39 and 40 allows the values of the fundamental quantities $\Omega_E(\zeta)$ and $\Omega_T(\zeta)$ determined from the calculations for one set of problem parameters (σ, d, H_1, B_s, μ_{ri}, A, t_r, and t_f) to be scaled to another set of problem parameters for which ζ has the same value.

For applied pulses with very small amplitudes, $H(x,t)$ undergoes only small excursions near zero; consequently, the exponential $\mu_{rd}(H)$ given in Eq. 25 behaves as a constant permeability

$$\mu_{rd}(H) = \mu_{ri} . \tag{47}$$

As discussed in [2], since there is a mathematical solution, this case provides a check on the accuracy of the numerical algorithm. For low level pulses, the numerical results were in excellent agreement with the analytical solution for the linear problem, and the value of the effective permeabilities for small ζ were in excellent agreement with the initial value for the relative differential permeability $\mu_{rd}(0)$. It was observed that $\Omega_E(\zeta)$ and $\Omega_T(\zeta)$ approach unity so that to an excellent approximation

$$\Omega_E(0) = \Omega_T(0) = 1 \ . \tag{48}$$

For high amplitude pulses, computational difficulties arise in the finite difference calculations. In order to circumvent the problems, a differential permeability

$$\mu_{rd}(H) = A_1 \exp\left(-\frac{H}{H_1}\right) + 1 \ , \tag{49}$$

was used with A_1=8999 so that $\mu_{rd}(0)$=9000. Inclusion of a nonzero limiting value for $\mu_{rd}(H)$ mitigates the computational problems. Calculated values for $\Omega_E(\zeta)$ and $\Omega_T(\zeta)$ are shown in Figure 2.

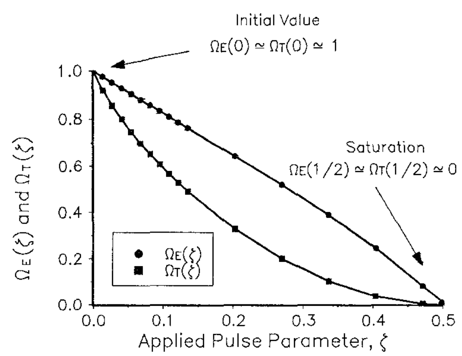

Figure 2. Fundamental quantities $\Omega_E(\zeta)$ and $\Omega_T(\zeta)$ for an exponential relative differential permeability model.

Near ζ=1/2 both $\Omega_E(\zeta)$ and $\Omega_T(\zeta)$ exhibit the onset of saturation. As discussed in [2], the limiting nonlinear theory for a step magnetization curve [3] - [5] predicts that saturation of the ferromagnetic shield occurs when

$$\zeta = \frac{1}{2} \ . \tag{50}$$

Although the numerical calculations are quite difficult near ζ=1/2, the numerical results are in close agreement with the prediction of the limiting nonlinear theory for a step magnetization curve, and to a good approximation it can be asserted that

$$\Omega_E(1/2) \simeq \Omega_T(1/2) \simeq 0 \ . \tag{51}$$

The above criterion closely predicts the onset of saturation for the differential permeability representation considered in this study; however, the fine structure of the behavior is <u>not</u> predicted by the limiting theory. Although $\Omega_E(\zeta)$ and $\Omega_T(\zeta)$ are very close for ζ=0 and for ζ=1/2, for intermediate pulses ($0<\zeta<1/2$), $\Omega_E(\zeta)$ and $\Omega_T(\zeta)$ are distinctly different. In particular, $\Omega_E(\zeta)$ does not exhibit the exponential behavior of $\mu_{rd}(H)$.

CONCLUSION

An correlation has been developed that uses a combination of mathematical and numerical analysis to characterize the nonlinear electric field transients induced at the inner surface by axially-directed short-duration surface current pulses on the outer surface of long, thin-walled, cylindrical, electrically conductive, ferromagnetic shields with a simple exponential relative differential permeability model. An alternate dimensionless formulation was used to identify an applied pulse parameter ζ=$Q_o/(\sigma d^2 B_s)$, where the applied pulse parameter ζ is a fundamental combination of the nonmagnetic problem parameters (the charge per unit circumference, Q_o, transported along the cylinder during the pulse; the electrical conductivity, σ, of the material; the wall thickness, d, of the cylinder); and a magnetic problem parameter (the saturation magnetization, B_s, of the ferromagnetic material). The applied pulse parameter ζ incorporates the magnetic parameter, B_s, as well as the nonmagnetic parameters included in β for a general $\mu_{rd}(H)$. The applied pulse parameter ζ is a measure of the severity of an applied pulse relative to a given shield. The peak value of the electric field transient, E_{peak}, can be expressed in terms of a fundamental quantity $\Omega_E(\zeta)$, and the time at which the peak occurs, t_{peak}, can be expressed in terms of a fundamental quantity $\Omega_T(\zeta)$

The curves for $\Omega_E(\zeta)$ and $\Omega_T(\zeta)$ characterize the nonlinearity of the problem for the exponential relative differential permeability model. Once the fundamental quantities, $\Omega_E(\zeta)$ and $\Omega_T(\zeta)$, have been characterized numerically using one set of problem parameters (σ, d, B_s, μ_{ri}, A, t_r, and t_f) the results can be easily extended to another set of parameters. This correlation can achieve considerable savings in computational effort and cost.

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Fundamental Relations for Applied Pulse Parameter Value $\zeta = \frac{A(t_f - t_r)}{\sigma d^2 B_s} = 9\times10^{-3}$														
Shield Parameters					Applied Pulse Characteristics			Peak Electric Field Response		Effective Permeabilities		Fundamental Quantities		
d (m)	σ (S/m)	H_1 (A/m)	B_s (T)	μ_{ri}	A (A/m)	t_r (s)	t_f (s)	E_{peak} (V/m)	t_{peak} (s)	$\mu_E(\zeta)$	$\mu_T(\zeta)$	$\Omega_E(\zeta)$	$\Omega_T(\zeta)$
1×10^{-4}	1×10^{6}	$\frac{1}{4\pi}\times10^{8}$	1.0	0.1	1×10^{8}	1×10^{-13}	1×10^{-12}	4.30940×10^{3}	1.10931×10^{-10}	0.09842	0.09621	0.984	0.962
5×10^{-5}	2×10^{6}	$\frac{1}{4\pi}\times10^{7}$	2.0	2.0	5×10^{7}	2×10^{-13}	2×10^{-12}	4.30595×10^{2}	1.09332×10^{-9}	1.97000	1.89650	0.984	0.948
2×10^{-4}	5×10^{6}	$\frac{1}{4\pi}\times10^{9}$	0.1	10.0	4×10^{7}	5×10^{-13}	5×10^{-12}	4.23942×10^{-2}	2.30596×10^{-7}	10.00458	9.99995	1.000	1.000
1×10^{-3}	5×10^{5}	$\frac{1}{4\pi}\times10^{10}$	0.5	500.0	5×10^{6}	5×10^{-11}	5×10^{-10}	8.48182×10^{-2}	2.88244×10^{-5}	500.053	499.996	1.000	1.000

Table 1. Calculated values of the fundamental quantities $\Omega_E(\zeta)$ and $\Omega_T(\zeta)$ for cases with $\zeta=9\times10^{-3}$.