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ON THE DESIGN OF DISSIPATIVE LQG-TYPE CONTROLLERS

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ABSTRACT

In this paper, the design of dissipative linear-quadratic-Gaussian-type compensators for positive real plants is considered. It is shown that, if the noise covariance matrices (used as weighting matrices) satisfy certain conditions, the compensator has a strictly positive real transfer function matrix. The stability of the resulting closed-loop system is guaranteed regardless of modeling errors as long as the plant remains positive real. In view of this property, the controller is expected to be useful for vibration suppression in large, flexible space structures.

INTRODUCTION

The linear-quadratic-Gaussian (LQG) controller has attained considerable maturity since its inception in the fifties and sixties, and has come to be generally regarded as one of the standard controller design methods. One attribute of LQG-type compensators is that, although they guarantee closed-loop stability, the compensator itself is not necessarily stable [1]. It would be of interest to characterize the class of *stable* LQG-type compensators. Going one step further, if the LQG compensator is restricted to be not only *stable*, but also *dissipative*, this would define an important subclass. Since we will be dealing with linear, time-invariant systems, we will consider that such systems are dissipative if, and only if, the corresponding transfer function is strictly positive real (SPR) (see Appendix). The importance of such compensators is that they would not only be dissipative, but would also be optimal with respect to an LQG performance function. One reason for considering dissipative compensators is that, when used to control positive-real (PR) plants, they offer excellent robustness to modeling errors. That is, the stability is guaranteed despite modeling errors as long as the plant is PR. An important application of dissipative compensators would be for vibration suppression in large, flexible space structures (LFSS), which are characterized by significant unmodeled dynamics and parameter errors. The linearized elastic-mode dynamics of LFSS with compatible collocated actuators and sensors are PR systems regardless of the unmodeled dynamics or parameter uncertainties [2,3], and can, therefore, be robustly stabilized by an SPR compensator. A method for designing a dissipative compensator for LFSS was given in [5]. However, the compensator obtained was not optimal.

The objective of this paper is to investigate the conditions under which an LQG-type compensator is SPR, so that one can simultaneously have high performance and robustness to unmodeled dynamics.

DISSIPATIVE OPTIMAL CONTROLLERS

We consider a minimal realization of a PR system expressed as:

$$\dot{x} = Ax + Bu + v \tag{1a}$$

$$y = Cx + w \tag{1b}$$

where v and w are white, zero-mean Gaussian noises. Since the system (1) is PR, we assume, without loss of generality (see Appendix), that the following equations hold for some matrix $Q_A \geq 0$:

$$A + A^T = -Q_A \leq 0 \tag{2}$$

and

$$B = C^T \tag{3}$$

Conditions (2) and (3) are equivalent to the Kalman-Yacubovich Positive Real Lemma [5]. The LQG compensator for the system in (1)-(3) is given by (see[6]):

$$u = -u' \tag{4}$$

$$\dot{\hat{x}} = [\,A - BR^{-1}B^TP_c - P_fBR_w^{-1}B^T]\hat{x} + P_fBR_w^{-1}\,y \tag{5}$$

$$u' = R^{-1}B^TP_c\hat{x} \tag{6}$$

where $P_c = P_c^T > 0$ and $P_f = P_f^T > 0$ are the LQ-regulator and the Kalman-Bucy filter Riccati matrices, which satisfy the algebraic Riccati equations:

$$P_cA + A^TP_c - P_cBR^{-1}B^TP_c + Q = 0 \tag{7}$$

$$P_fA^T + AP_f - P_fBR_w^{-1}B^TP_f + Q_v = 0 \tag{8}$$

where Q and R are the usual weighting matrices for the state and input, and Q_v and R_w are the covariance matrices of v and w. It is assumed that Q > 0, and $(A,Q_v^{1/2})$ is observable.

The main result is stated as follows.

Theorem - Consider the PR system in (1)-(3) and the LQG-type controller in (4)-(8). If Q, R, Q_v and R_w are such that

$$Q_v = Q_A + BR^{-1}B^T \tag{9}$$

$$R_w = R \tag{10}$$

and

$$Q - BR^{-1}B^T \triangleq Q_B > 0 \tag{11}$$

then the controller in (5)-(6) (described by the transfer function from y to u') is SPR.

Proof:

Introducing equations (2), (9) and (10) into (8) it becomes clear that $P_f = I$ is a solution to equation (8). From equation (7) it follows

$$\begin{aligned} & P_c(A-BR^{-1}B^TP_c-BR^{-1}B^T) + (A-BR^{-1}B^TP_c-BR^{-1}B^T)^TP_c \\ & = -Q - P_cBR^{-1}B^TP_c - P_cBR^{-1}B^T - BR^{-1}B^TP_c \\ & = -Q - (P_c+I)BR^{-1}B^T(P_c+I) + BR^{-1}B^T \\ & = -Q_B - (P_c+I)BR^{-1}B^T(P_c+I) < 0 \end{aligned}$$

where Q_B is defined in (11). In view of equations (3), (10), and the above, it follows that the controller in equations (5) and (6) is strictly positive real (see [7]).■

The above result states that, if the weighting matrices for the regulator and the filter are chosen in a certain manner, the resulting LQG-type compensator is SPR. However, it should be noted that this compensator would not be optimal with respect to actual noise covariance matrices. The noise covariance matrices are used herein merely as compensator design parameters, and have no statistical meaning.

Condition (11) is equivalent to introducing an additional term $y^TR^{-1}y$ in the LQ performance index (since $Q = Q_B + CR^{-1}C^T$) and is not particularly restrictive.

CONCLUDING REMARKS

The problem of designing linear-quadratic-Gaussian type compensators for positive real systems was considered. Sufficient conditions were obtained for the compensator to be strictly positive real. The resulting feedback configuration is guaranteed to be stable despite unmodeled plant dynamics and parameter inaccuracies, as long as the plant is positive real. One application of such compensators would be for controlling elastic motion of large flexible space structures using collocated actuators and sensors. Further research is needed for extending these results for controlling rigid-body modes (in addition to the elastic modes) of flexible spacecraft.

APPENDIX

H(s) is defined to be PR if it satisfies the following conditions:

(i) H(s) has no poles in Re s > 0 and the poles of H(s) on the imaginary axis are simple and the associated residues are nonnegative definite.

(ii) For any real ω for which $j\omega$ is not a pole of H(s),

$$H(j\omega) + H^T(-j\omega) \geq 0$$

H(s) is termed SPR if $H(s-\mu)$ is PR for some real $\mu > 0$.

Consider a positive real system expressed as:

$$\dot{z} = Dz + Fu \tag{A.1}$$

$$y = Gz \tag{A.2}$$

Then, there exist matrices P>0 and L such that [5]

$$PD + D^TP = -LL^T \tag{A.3}$$

$$PF = G^T \tag{A.4}$$

Define $x = P^{1/2}z$, where $P^{1/2}$ is a symmetric square root of P. Introducing this definition in (A.1) and (A.2) we obtain (1.a) and (1.b) with $A=P^{1/2}DP^{-1/2}$, $B=P^{1/2}F$ and $C=GP^{-1/2}$. Multiplying (A.3) on the left and on the right by $P^{-1/2}$ we obtain (2) with $Q_A = P^{-1/2}LL^TP^{-1/2}$. Multiplying (A.4) on the left by $P^{-1/2}$ we obtain (3) (also see[6]).■

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