

Eigenvalue Signal Processing for Phased-Array Weather Radar Polarimetry: Removing the Bias Induced by Antenna Coherent Cross-Channel Coupling

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Abstract—We present a novel digital signal processing procedure, named Eigenvalue Signal Processing (henceforth ESP), patented by the first author with Brookhaven Science Associates in 2013. The method enables the removal of antenna coherent cross-channel coupling in weather radar measurements at LDR mode and ATSR mode. In this work we focus on the LDR mode and consider reflectivity at horizontal transmit (Z_H), linear depolarization ratio at horizontal transmit (LDR_H) and degree of polarization at horizontal transmit (DOP_H). The eigenvalue signal processing method is substantiated by an experiment carried out in November 2012 with a parabolic reflector C-band weather radar located at the Selex Systems Integration (SI) facilities in Neuss, Germany. The experiment consists of the comparison of weather radar measurements taken 1.5 minutes apart in two hardware configurations, namely with cross-coupling on (cc_on) and cross-coupling off (cc_off). It is experimentally demonstrated that eigenvalue-derived variables are invariant with respect to antenna cross-channel coupling. This property had to be expected, since the eigenvalues of the Coherency matrix are SU(2) invariant.

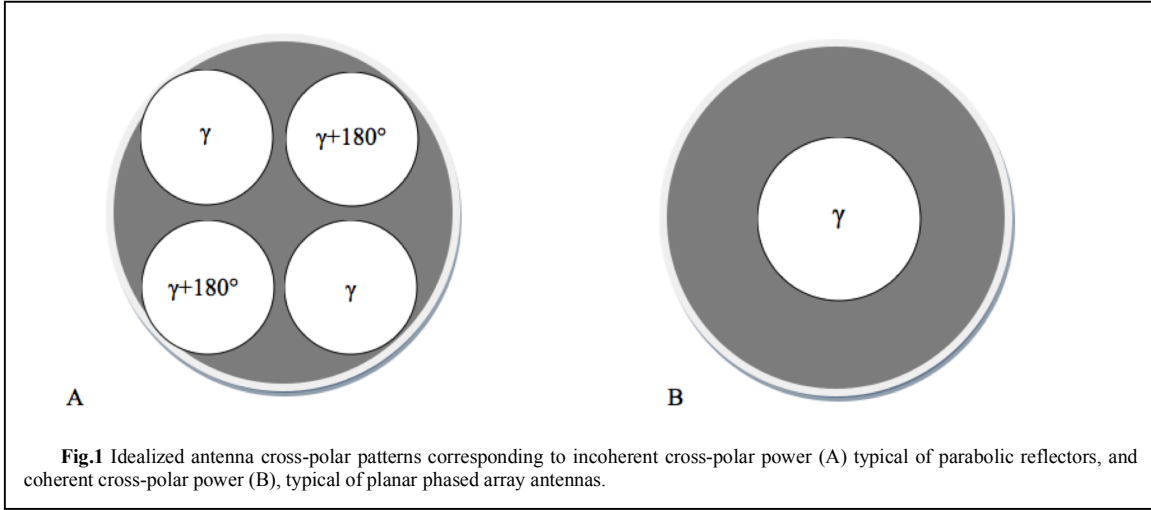
Keywords—Coherency matrix, Covariance matrix, polarimetric phased array weather radar, eigenvalues.

I. INTRODUCTION

The development of polarimetric phased array weather radars is critical for the MPAR (multi-function phased array radar) program. The major technological challenge in phased array weather radar polarimetry is attaining an acceptable cross-polar isolation between the H and V channels of the radar system. The present paper proposes eigenvalue signal processing as a solution for the problem of antenna cross-polarization isolation. It is potentially suitable for implementation in polarimetric phased array antennas, but also in conventional parabolic reflectors, whenever the antenna cross-polarization isolation is sub-optimal. Eigenvalue Signal Processing is applicable when the radar operates at either LDR mode or ATSR mode (Alternate Transmission Simultaneous

Receive) but not at STSR mode (Simultaneous Transmission Simultaneous Receive). In general, ESP is applicable at orthogonal modes (LDR, ATSR modes) that is, when the receive polarization basis is orthogonal with respect to the transmit polarization, but not at hybrid modes (e.g. STSR mode). LDR mode corresponds to horizontal polarization transmit, and simultaneous reception of H and V; ATSR mode corresponds to H transmit and simultaneous H and V receive, followed by V transmit and simultaneous H and V receive. Also, ESP assumes target reflection symmetry, that is, it assumes that the meteorological scatterers in question do have a canting angle close to 0° . This assumption is customarily made in weather radar observations and also underpins the use of the STSR mode (implemented in the US NEXRAD network and in most operational weather radar systems at S, C and X bands) which, in presence of canted scatterers, would yield biased estimates of the polarimetric variables. Besides orthogonal polarization bases and scatterers' reflection symmetry, no other assumptions are needed for the application of ESP.

In general, the unwanted cross-polar power radiated by the antenna can be split in two components: the incoherent cross-polar power, and the coherent cross-polar power. The incoherent cross-polar power appears as a quad of perfectly symmetric offset lobes, and is produced by the natural geometry of the electric field lines on the radiating surface of the antenna. The quad of offset cross-polar lobes is present in parabolic reflectors as well as in microstrip patch antennas. When a cloud of spheres is illuminated (or, more generally, any target with reflection symmetry), such quad of offset lobes produces backscattered cross-polar power that is uncorrelated with the backscattered copolar power, and the cross-polar correlation coefficient (ρ_{vh}) is equal to zero. The bias induced by incoherent cross-polar power cannot be removed by eigenvalue signal processing. Luckily however, its contribution to the total bias is small. The coherent cross-polar power appears whenever the 4 offset lobes are unbalanced (as it is



often the case in real parabolic reflectors, where the offset cross-polar lobes display different amplitudes and are not perfectly symmetric), or whenever cross-polar power is radiated axially, that is, along the bore sight of the antenna. Such coaxial cross-polar power is generated by a number of sources. In the case of parabolic reflectors, it can be generated by imperfections in the reflector surface, feed-horn misalignment, scattering from the feed support struts. In the case of a planar phased array scanning off the horizontal and vertical planes, it is generated by the misalignment of the radiated field lines with respect to the local horizontal. The coherent cross-polar power significantly increases the cross-polar correlation coefficient, but the bias it introduces in the polarimetric variables can be removed by eigenvalue signal processing. The contribution of coherent cross-polar power to the bias is dominant, and its removal guarantees an excellent performance of the antenna. Eigenvalue Signal Processing relies on the fact that the eigenvalues of the Coherency matrix J are $SU(2)$ invariant and therefore any coupling in the receive polarization basis does not affect the eigenvalue-derived variables. A rigorous analytical proof of the robustness with respect to cross-channel coupling is provided in [1] for the degree of polarization at horizontal transmit DOP_H (indicated with p_H in the formulae) and can easily be extended to all eigenvalue-derived variables treated in the present paper. From a more intuitive perspective, the diagonalization of the Coherency matrix can be seen as an alignment between the scatterers (assumed to be vertically aligned because of reflection symmetry imposed by the Earth's gravity field, and therefore characterized by an intrinsic cross-polar correlation coefficient ρ_{xh} equal to zero) and the antenna height spinor (assumed to be slightly tilted from the vertical because of coherent cross-channel coupling and therefore inducing a positive non-zero cross-polar correlation coefficient ρ_{xh} when spherical scatterers like light rain or small ice crystals are illuminated). The antenna/target alignment produced by the diagonalization of the 2×2 Coherency matrix has to be intended as a rotation in the $SU(2)$ sense, the Special Unitary group of 2×2 matrices.

II. EIGENVALUE-DERIVED VARIABLES

A. LDR mode

We refer to Section II of reference [1]. Weather radars at LDR mode measure the Coherency matrix at horizontal polarization transmit (J_H), a matrix with 4 degrees of freedom. Note that this matrix is the upper left 2×2 minor of the backscatter covariance matrix.

$$J_H = \begin{bmatrix} \langle |s_{hh}|^2 \rangle & \langle s_{hh}^* s_{vh} \rangle \\ \langle s_{hh} s_{vh}^* \rangle & \langle |s_{vh}|^2 \rangle \end{bmatrix} \quad (1)$$

From the Coherency matrix, radar variables are evaluated. From the two degrees of freedom on the diagonal, we can extract reflectivity (Z_H) and linear depolarization ratio (LDR_H)

$$Z_H \propto \langle |s_{hh}|^2 \rangle \quad (2)$$

$$LDR_H = \frac{\langle |s_{vh}|^2 \rangle}{\langle |s_{hh}|^2 \rangle} \quad (3)$$

Reflectivity is proportional to the power backscattered at horizontal polarization, the linear depolarization ratio is representative of the target-induced coupling between copolar (horizontal) and cross-polar (vertical) channels. The two degrees of freedom on the off-diagonal term are captured by the cross-polar correlation coefficient (ρ_{xh}) and the cross-polar phase ψ_{xh} (propagation Φ_{xh} plus back scatter δ_{xh}).

$$\rho_{xh} = \frac{|\langle s_{hh}^* s_{vh} \rangle|}{\sqrt{\langle |s_{hh}|^2 \rangle \langle |s_{vh}|^2 \rangle}} \quad (4)$$

$$\psi_{xh} = \Phi_{xh} + \delta_{xh} = \arg[\langle s_{hh}^* s_{vh} \rangle] \quad (5)$$

The cross-polar correlation coefficient ranges between 0 and 1, and is a normalized measure of the correlation between copolar and cross-polar signals. The cross-correlation coefficient departs from zero if and only if the target departs from reflection symmetry. Besides canted hydrometeors, ground clutter, aircrafts or other man-made objects can appear with

intrinsic positive, non-zero ρ_{ch} . The Coherency matrix can be diagonalized to yield its eigenvalues

$$\mathbf{J}_H = \mathbf{U} \begin{bmatrix} \lambda_{H1} & 0 \\ 0 & \lambda_{H2} \end{bmatrix} \mathbf{U}^{-1} \quad (6)$$

The first two eigenvalue-derived variables are the trace (corresponding to total backscattered power) and the degree of polarization at horizontal transmit (corresponding to the ratio of completely polarized power to total power).

$$\text{Tr} \mathbf{J}_H = \lambda_{H1} + \lambda_{H2} \quad (7)$$

$$p_H = \frac{\lambda_{H1} - \lambda_{H2}}{\lambda_{H1} + \lambda_{H2}} = \sqrt{1 - \frac{4 \det \mathbf{J}_H}{[\text{Tr} \mathbf{J}_H]^2}} \quad (8)$$

These variables can also be expressed in terms of the entries of the Coherency matrix \mathbf{J}_H as:

$$\text{Tr} \mathbf{J}_H = \langle |s_{hh}|^2 \rangle + \langle |s_{vh}|^2 \rangle \quad (9)$$

$$p_H = \sqrt{1 - \frac{4[\langle |s_{hh}|^2 \rangle \langle |s_{vh}|^2 \rangle - |\langle s_{hh} s_{vh}^* \rangle|^2]}{[\langle |s_{hh}|^2 \rangle + \langle |s_{vh}|^2 \rangle]^2}} \quad (10)$$

The degree of polarization is related to LDR_H and ρ_{ch} by a fundamental identity in radar polarimetry, obtainable by simple algebraic manipulation of the formula (10).

$$(1 - p_H^2) = \frac{4 \text{LDR}_H}{[1 + \text{LDR}_H]^2} (1 - \rho_{\text{ch}}^2) \quad (11)$$

The diagonalization of the Coherency matrix nulls the cross-polar correlation coefficient ρ_{ch} , and formula (11) can be simplified to

$$p_H = \frac{1 - \text{LDR}_H}{1 + \text{LDR}_H} \quad (12)$$

From formula (12), simple algebraic manipulation permits to define the first eigenvalue derived variable: LDR_{H_ESP}

$$\text{LDR}_{H_ESP} \equiv \frac{\lambda_{H2}}{\lambda_{H1}} \quad (13)$$

For an ideal antenna with no coherent cross-channel coupling (that is, an antenna that yields $\rho_{\text{ch}}=0$ when scatterers with reflection symmetry are illuminated), LDR_{H_ESP} is numerically identical to LDR . In presence of coupling, LDR is positively biased, whereas LDR_{H_ESP} is not significantly biased.

The next step is the realization that the trace of the Coherency matrix is invariant for unitary transformations, and therefore we have that

$$\text{Trace}_H = \langle |s_{hh}|^2 \rangle + \langle |s_{vh}|^2 \rangle = Z_H [1 + \text{LDR}_H] = \lambda_{H1} + \lambda_{H2} \quad (14)$$

This leads to the identification of the largest eigenvalue of the Coherency matrix (λ_{H1}) as the copolar (main) power received

at LDR mode by an “aligned” antenna. This observation defines the second ESP variable:

$$Z_{H_ESP} \equiv \lambda_{H1} \quad (15)$$

The development above suggests a meaning for the eigenvalues of the Coherency matrix, that is, the largest and the smallest eigenvalues correspond to the copolar and cross-polar powers respectively, measured by an antenna whose antenna height spinor is perfectly aligned with the principal axes of the illuminated scatterer. The eigenvalues correspond to estimates of copolar and cross-polar powers that are unbiased by antenna cross-polarization coupling.

B. ATSR mode

If the weather radar operates at ATSR mode, the development above can also be applied to the Coherency matrix at vertical polarization transmit \mathbf{J}_V .

$$\mathbf{J}_V = \begin{bmatrix} \langle |s_{vv}|^2 \rangle & \langle s_{vv} s_{hv}^* \rangle \\ \langle s_{vv} s_{hv}^* \rangle & \langle |s_{hv}|^2 \rangle \end{bmatrix} \quad (16)$$

and the reflectivity at vertical transmit and the linear depolarization ratio at vertical transmit

$$Z_V \propto \langle |s_{vv}|^2 \rangle \quad (17)$$

$$\text{LDR}_V = \frac{\langle |s_{hv}|^2 \rangle}{\langle |s_{vv}|^2 \rangle} \quad (18)$$

can be corrected for the bias induced by antenna coherent cross-polarization simply by replacing them with their eigenvalue-derived counterparts

$$Z_{V_ESP} \equiv \lambda_{V1} \quad (19)$$

$$\text{LDR}_{V_ESP} \equiv \frac{\lambda_{V2}}{\lambda_{V1}} \quad (20)$$

where λ_{V1} and λ_{V2} are respectively the largest and smallest eigenvalues of the Coherency matrix at vertical polarization transmit. The last eigenvalue-derived variable is the bias-corrected replacement for differential reflectivity Z_{DR} , that can be obtained as

$$Z_{\text{DR_ESP}} \equiv \frac{Z_{H_ESP}}{Z_{V_ESP}} \equiv \frac{\lambda_{H1}}{\lambda_{V1}} \quad (21)$$

The theory exposed in this Section permits to remove the bias induced by antenna coherent cross-polarization coupling in power-like weather radar variables, specifically reflectivity Z , Linear Depolarization Ratio LDR and Differential Reflectivity Z_{DR} . The elegance of eigenvalue signal processing resides in the fact that bias correction does not require to know the amount of cross-coupling, since the latter is intrinsically measured by the cross-polar correlation coefficients ρ_{ch} and ρ_{cv} and the diagonalization process of the Coherency matrix automatically brings them to zero. This feature is particularly attractive since the concept of graceful degradation, generally

referring to the deterioration of the sensitivity of phased array radars, can in this context be applied also to the polarimetric purity of the phased array antenna. It may have been noted that ESP does not provide replacements for variables derived from the 1,3 term of the covariance matrix, specifically the copolar correlation coefficient ρ_{hv} and the specific differential phase KDP. At ATSR mode however, both the copolar correlation coefficient ρ_{hv} and the specific differential phase KDP are not significantly affected by antenna cross-channel coupling [4]. For completeness' sake, we mention that an eigenvalue-derived proxy for the copolar correlation coefficient ρ_{hv} is scattering entropy H [5]. This case however is profoundly different: contrary to all variables treated in the present paper, which are derived from the eigenvalues of the Coherency matrices at H and V transmit (upper left and lower right minors of the Covariance matrix), scattering entropy is derived from the eigenvalues of the 3x3 Covariance matrix. Entropy enjoys all the desirable properties of eigenvalue-derived variables, but it is not an exact replacement for the copolar correlation coefficient, that is, the numerical values assumed by the two variables for the same target are different even when acquired by an ideal antenna. On the other hand, all variables presented in this paper are exact replacements, that is, the eigenvalue variables derived from the Coherency matrix do correspond exactly to the unbiased standard polarimetric radar variables. Entropy H and the copolar correlation coefficient ρ_{hv} do have a similar physical meaning and generally do display the same discrimination capabilities [5]. Entropy is a measure of the diversity of scattering matrices that form the Covariance matrix, the copolar correlation coefficient is a measure of the diversity of H to V axis ratios of the scatterers.

An analytical proof of the robustness of Coherency-matrix eigenvalue-derived variables can be obtained by tedious but simple algebraic computations as shown in [1] for the degree of polarization at horizontal transmit. Analytical modeling of cross-channel coupling is important, since it permits a rigorous proof of the robustness of eigenvalue-derived variables with respect to possible hardware options/radar architectures. In general, the action of a coupled antenna on the polarimetric measurements can be modeled as a congruence transformation:

$$\mathbf{S}' = \mathbf{F}^t \mathbf{S} \mathbf{F} = \begin{bmatrix} F_{hh} & F_x \\ F_x & F_{vv} \end{bmatrix} \begin{bmatrix} S_{hh} & S_{hv} \\ S_{vh} & S_{vv} \end{bmatrix} \begin{bmatrix} F_{hh} & F_x \\ F_x & F_{vv} \end{bmatrix} \quad (22)$$

From this general form, many special cases can be derived, that take into account the specific hardware configuration under test. For example, for phased-array antennas, the cross-polar power coming from misprojection can be modeled as an asymmetric distortion matrix [3].

$$\begin{aligned} \mathbf{S}' &= \mathbf{F}^t \mathbf{S} \mathbf{F} \\ &= \begin{bmatrix} \cos \phi & -\cos \theta \sin \phi \\ 0 & \sin \theta \end{bmatrix} \begin{bmatrix} S_{hh} & S_{hv} \\ S_{vh} & S_{vv} \end{bmatrix} \begin{bmatrix} \cos \phi & 0 \\ -\cos \theta \sin \phi & \sin \theta \end{bmatrix} \end{aligned} \quad (23)$$

TABLE I. ESP VARIABLES

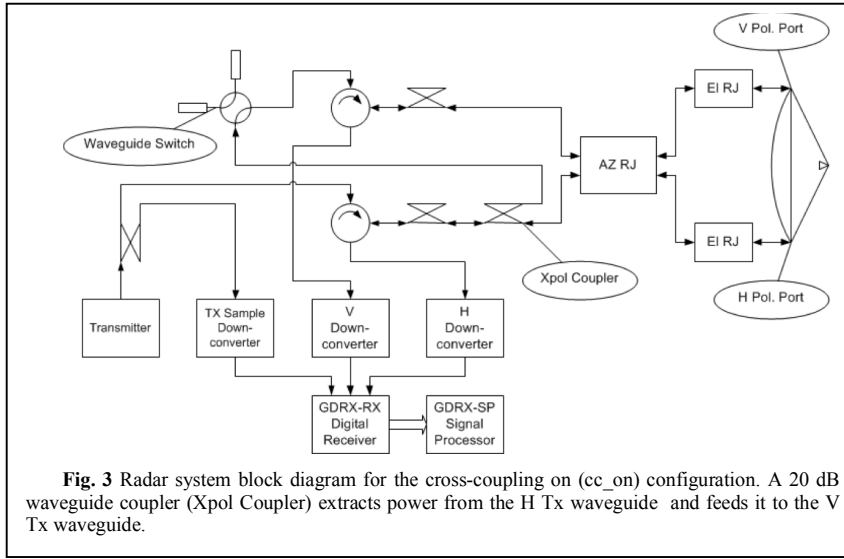
| Name | Standard | ESP variables |
|--|-------------|----------------|
| Reflectivity at horizontal transmit | Z_H | Z_{H_ESP} |
| Reflectivity at vertical transmit | Z_V | Z_{V_ESP} |
| Linear Depolariz. Ratio at horizontal tx | LDR_H | LDR_{H_ESP} |
| Linear Depolariz. Ratio at vertical tx | LDR_V | LDR_{V_ESP} |
| Differential Reflectivity | Z_{DR} | Z_{DR_ESP} |
| Copolar correlation coefficient | ρ_{hv} | - |
| Specific Differential phase | KDP | - |

Fig. 2 At ATSR mode, eigenvalue-derived variables (far-right column) provide cross-coupling robust replacements for most standard radar meteorological polarimetric variables (central column). The copolar correlation coefficient and the specific differential phase do not have eigenvalue-derived counterparts, however, at ATSR mode, they are not sensitive to antenna cross-channel coupling [4]. Even though not commonly used in radar meteorology, the degree of polarization and the trace of the coherency matrix (not listed in the table) are also eigenvalue-derived variables, and automatically enjoy the property of being robust against antenna cross-channel coupling.

III. EIGENVALUE SIGNAL PROCESSING EXPERIMENT

Eigenvalue Signal Processing was tested at LDR mode for Z_{H_ESP} , LDR_{H_ESP} and DOP_H in an experiment conducted on November 10th 2012 at Selex SI facilities in Neuss, Nordrhein-Westfalen, Germany at around 16:20 local time, when ground temperature was +11°C. The parabolic reflector C-band radar acquired a PPI at 1.5° elevation in a weather event consisting of light stratiform rain, with a melting band visible as a low LDR ring around the radar at about 50 km distance. The radar was operated at LDR mode, in two different configurations indicated with *cc_on* (red curve in the plots) and *cc_off* (blue curve in the plots). The *cc_on* acquisition was taken between 16:18:21.122 and 16:19:40.826 CET (Central European Time), whereas the *cc_off* acquisition was taken between 16:19:40.826 and 16:21:00.643 CET. The two acquisitions are spaced in time by about 1.5 minutes, and it can reasonably be assumed that the illuminated scatterers are the same. In the *cc_off* acquisition, the radar was operated in its standard configuration, whereas in the *cc_on* acquisition, the detrimental effects of a suboptimal antenna were simulated by disconnecting the V transmit waveguide and by injecting into the Tx port of the V circulator a signal sample extracted from the H transmit channel via a 20 dB coupler.

The test data were generated with a modified METEOR 600CLP10 polarimetric Doppler weather radar system, that can implement both the STSR (hybrid) mode and the LDR mode: http://www.gematronik.com/fileadmin/media/pdf/ProductDatasheet/METEOR_600C-635C_engl_1907154815.pdf



By default, the radar is a hybrid design, that is, the transmitted pulse is split and fed with equal powers to the H and V waveguides, and reception occurs simultaneously in the H and V channels (STSR mode). In order to implement the LDR mode, a remotely controlled mechanical waveguide switch redirects the full transmit power to the horizontal (H) polarization channel while the system still receives on both channels. In the following, we refer to this LDR mode hardware configuration as cross-coupling off (cc_off). To simulate antenna coupling, the system was modified as indicated in Fig. 3: the transmitter is directly connected to the H polarization transmit channel and to increase the cross-polarization of the system, a waveguide coupler (xpol coupler in Fig. 3) is inserted into the channel; the pulse extracted is then injected into the vertical (V) polarization channel via the waveguide switch. In the following, we refer to this hardware configuration as cross-coupling on (cc_on). This allows a quick on- and off-switching of the transmit cross-polarization. The difference in attenuation between the H and V antenna waveguide runs was H/V=0.75 dB. The coupling loss of the xpol signal was 22.4 dB. The described set-up simulates coherent (coaxial) cross-polar power on transmission only, since the receive antenna is still acceptably isolated and can be modelled with the following matrix multiplication

$$\mathbf{S}' = \mathbf{S} \mathbf{F} = \begin{bmatrix} S_{hh} & S_{hv} \\ S_{vh} & S_{vv} \end{bmatrix} \begin{bmatrix} F_{hh} & 0 \\ F_{vh} & F_{vv} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad (24)$$

Depending on the exact hardware source of cross-polar power, the mathematical models may differ; the proposed experiment validates the robustness of eigenvalue signal processing with respect to coaxial cross-polarization on transmit. In the acquired PPIs considerable blockage occurs in the western sector; interference appears as radial lines and arcs throughout the rest of the PPI disc. Radials for the analysis were chosen at azimuth angles of 352° and 123°, where the ray only goes through light rain and the melting band, but avoids more complex scattering situations like ground clutter (visible at around 25 km from the radar in the NE sector) and

electromagnetic interference. In the following we present plots corresponding to radials at 352° azimuth, where light rain is present from 10 to 40 km and where wet aggregates (melting band) are present from 45 to 60 km. In the panels from A to F of Fig. 4 are polarimetric variables at LDR mode; red curves refer to variables obtained in the coupled configuration (cc_on), blue curves refer to variables obtained in the standard LDR mode configuration (cc_off). It can be observed how the coupled configuration (in red) affects variables that are NOT derived from the eigenvalues, like copolar reflectivity Z_H (in this case the bias is small), cross-polar reflectivity Z_V , linear depolarization ratio (LDR) and cross-polar correlation coefficient (ρ_{xh}). Eigenvalue-derived variables, specifically LDR_{ESP} and DOP_H , are not biased by antenna cross-

channel coupling.

IV. CONCLUSIONS

Eigenvalue Signal Processing (ESP) is a novel digital signal processing procedure that enables the removal of antenna cross-coupling bias in polarimetric weather radar variables at LDR mode and ATSR mode. It provides unbiased estimates of Z, LDR and ZDR.

The degree of polarization at horizontal transmit DOP_H (or vertical transmit DOP_V) and the trace of the Coherency matrix are also eigenvalue-derived, and automatically enjoy the robustness with respect to antenna cross-channel coupling.

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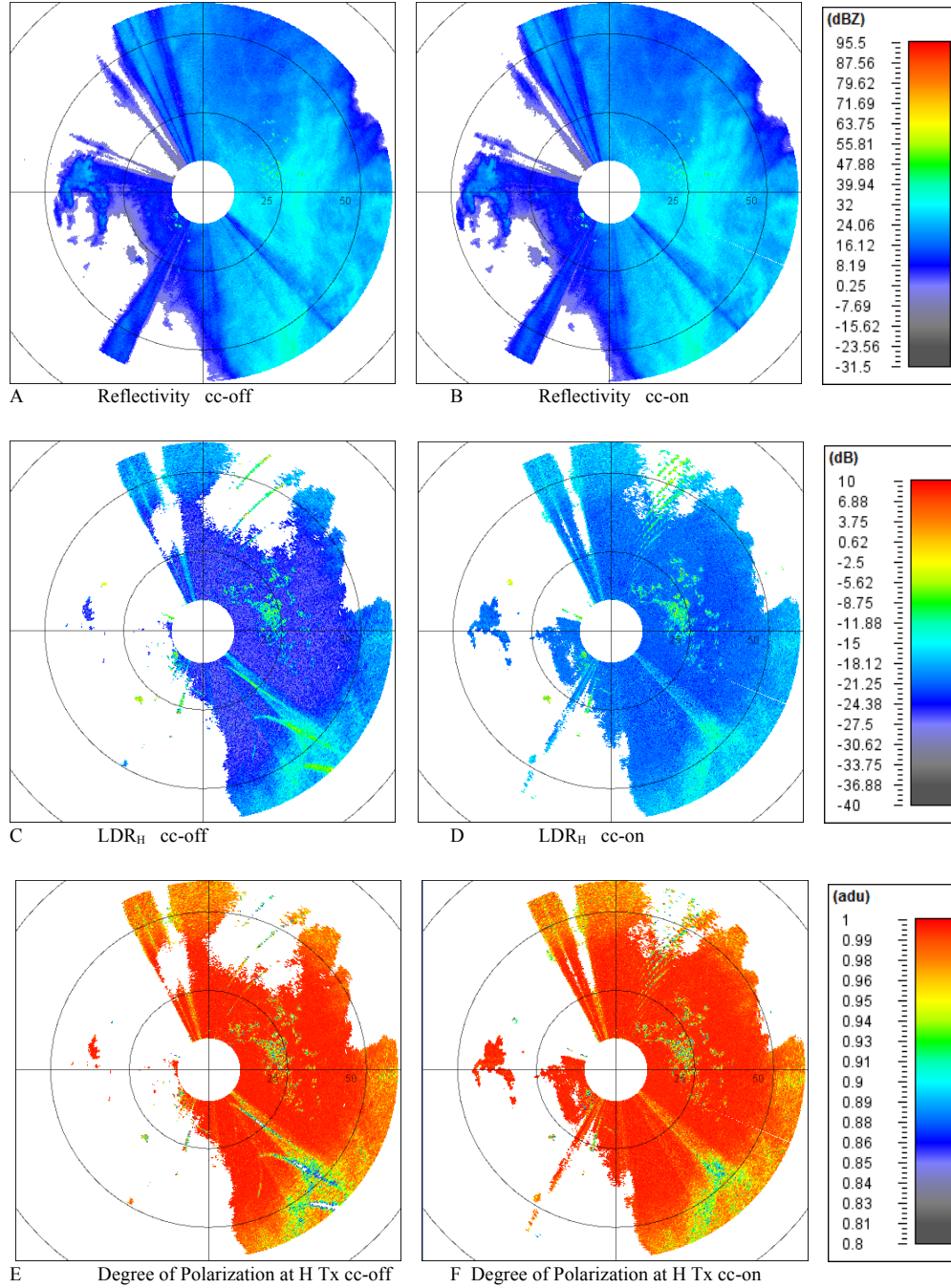
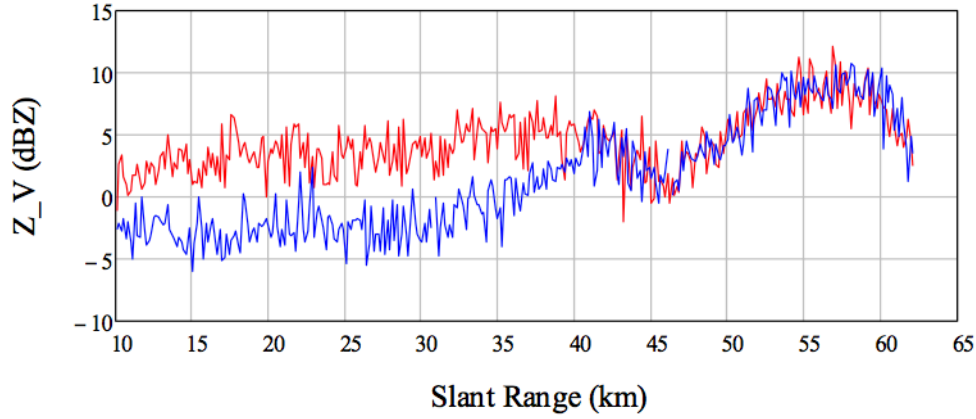
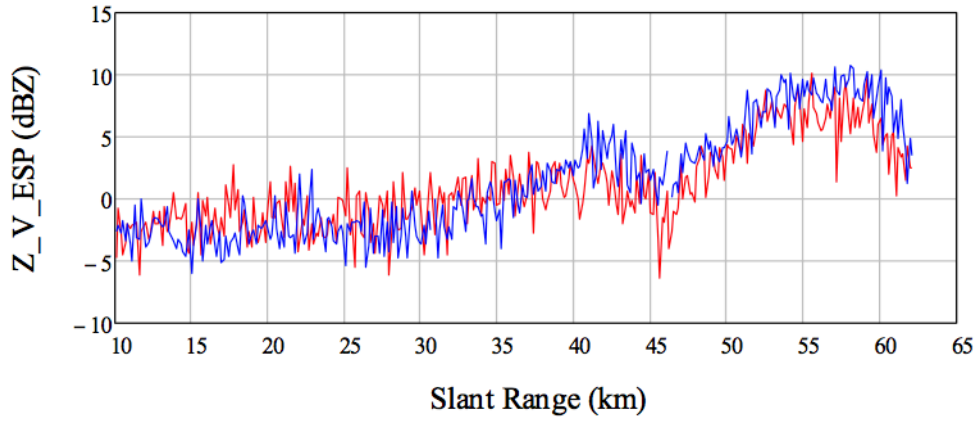


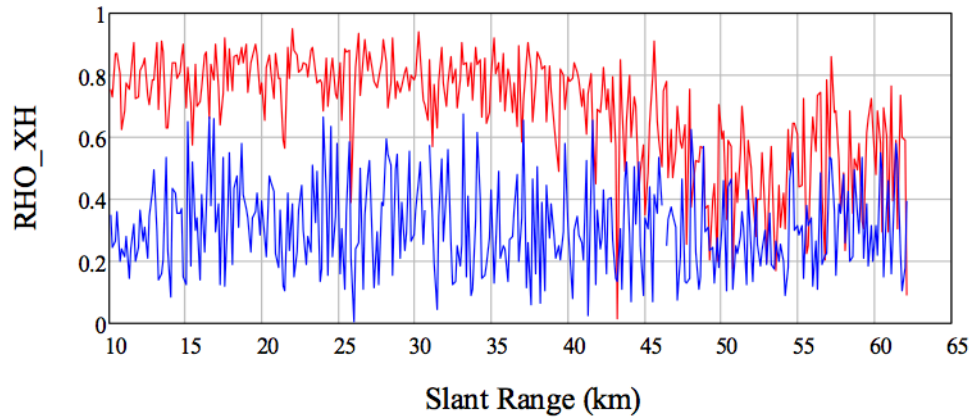
Fig. 4 Panels from A to F are PPIs at 1.5° elevation of polarimetric variables at LDR mode. The black circles indicate ranges of 25, 50 and 75 km respectively. The PPIs on the left (Panels A, C and E) correspond to data acquired in the cross-coupling off (cc-off) configuration; the PPIs on the right (Panels B, D and F) correspond to data acquired in the cross-coupling on (cc-on) configuration. Rain is present between 10 and 50 km from the radar, the melting band appears as a low LDR/DOP_H ring beyond 50 km. Beam blockage is present in the western quadrants, clutter is present mainly in the first quadrant at about 25 km range. Interference appears as low LDR/DOP_H lines/arcs and changes characteristics between the two acquisitions (spaced in time 1.5 minutes). Copolar reflectivity is not significantly affected by coupling. LDR is affected by coupling, and good isolation (panel C) enhances the contrast between rain and the melting band with respect to poor isolation (panel D). LDR_{ESP} (not reported for compactness) is identical to LDR cc-off (panel C) and does recover the unbiased LDR field as shown in Fig. 5E. Finally, DOP_H is invariant with respect to cross-channel coupling, as can be qualitatively assessed by panels E and F and further analyzed in Fig. 5F. For the quantitative analysis, we select a radial at 352° azimuth, where only rain and wet aggregates (melting band) are present.



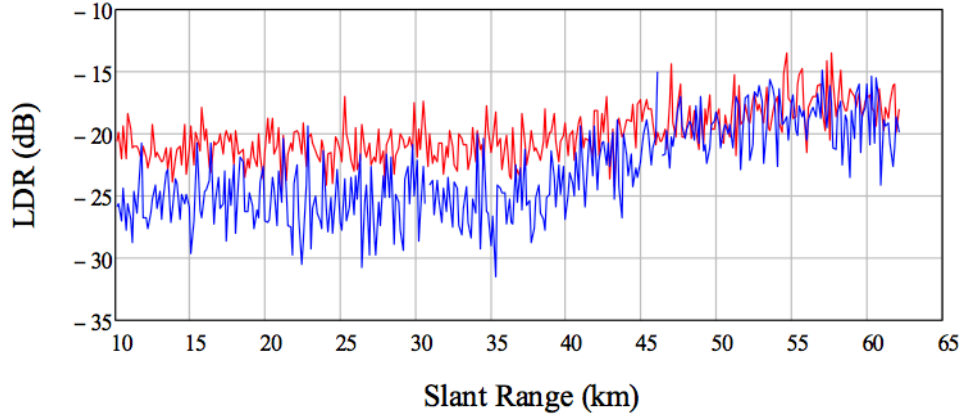
A Cross-polar Reflectivity Z_V (dBZ) for the cc-off (blue) and cc-on (red) configurations. Cross-channel coupling increases cross-polar reflectivity. This phenomenon is well visible in rain, between 10 and 40 km and less visible in the melting band (50-60 km). In general, for cross-polar reflectivity, $Z_{V_CC-OFF} \leq Z_{V_CC-ON}$



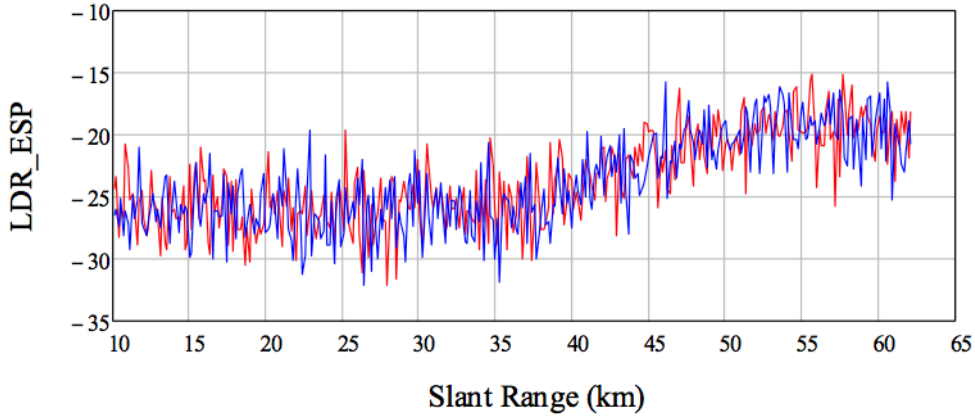
B Cross-polar Reflectivity cc-off Z_{V_CC-OFF} (blue) and cross-polar reflectivity eigenvalue-corrected obtained from the cc-on configuration Z_{V_ESP} (red). In general we have that $Z_{V_ESP} \leq Z_V$. Equality holds for an antenna with no coherent coupling.



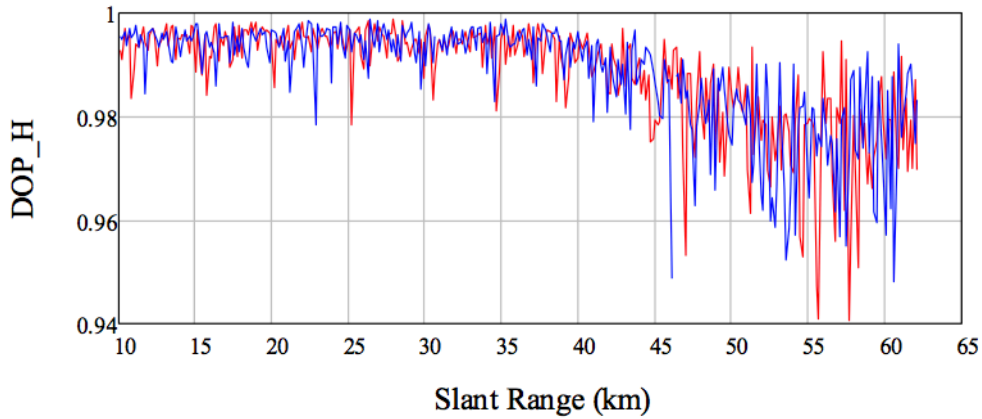
C Cross-polar Correlation coefficient ρ_{xh} . The cross-polar correlation coefficient is very sensitive to cross-channel coupling. Indeed, eigenvalue-signal processing hinges on the fact that the cross-polar correlation coefficient intrinsically measures the amount of coherent cross-channel coupling since, for scatterers with reflection symmetry, ρ_{xh} should be intrinsically 0. Deviations from 0 to positive values are attributed to antenna coherent cross-polar power.



D Linear Depolarization Ratio (dB). LDR is affected by antenna coupling as it is clearly illustrated by the red and blue curves, representing LDR with cc-on and cc-off respectively. The difference is blurred after 45 km, where more depolarizing scatterers (melting band) are present, and the effects of sub-optimal isolation become less visible. Lack of polarimetric purity decreases the polarimetric contrast and makes discrimination more difficult.



E Linear Depolarization Ratio (dB). Blue curve is cc-off (LDR in the standard configuration), red curve is LDR_{ESP} in cc-on configuration. Eigenvalue Signal Processing recovers the unbiased LDR and restores the dynamic range corresponding to the cc-off configuration. In general, $LDR_{ESP} \leq LDR$. Equality holds for an antenna with little or no coherent cross-channel coupling (as in this case, where the intrinsic ρ_{ch} of the antenna is only about 0.3).



F Degree Of Polarization at Horizontal transmit (DOP_H). The degree of polarization at horizontal transmit can be expressed in terms of the eigenvalues of the Coherency matrix, and is therefore invariant with respect to antenna cross-channel coupling.

Fig. 5 Radial plots for 352° azimuth. The radial goes through rain (10-40 km) and then crosses the melting band (>40 km). Red curves are derived from the cc-on configuration, Blue curves are derived from the cc-off configuration.