

A Domain Decomposition Scheme to Solve Integral Equations Using Equivalent Surfaces

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I. Introduction

The domain decomposition method has been used in many electromagnetic solvers to accelerate computation and provide a natural interface to parallel computing [1]. The finite element (FEM) and finite difference (FD) methods have been used as the solvers of each subdomain [2, 3]. In this paper, a domain decomposition scheme based on the equivalence theorem and the method of moments (MOM) is introduced. The unknowns on every subscatterer are transferred to the unknowns on its surrounding equivalent surface [4]. With this scheme, both the number of unknowns and the memory usage are reduced.

II. Formulations

1. Equivalence Theorem

The equivalence theorem, also known as Huygens' theorem, was conjectured by Huygens and made rigorous by other scientists [5] that the field off a closed surface can be determined by the tangential components of the fields on the surface. This may be derived from Maxwell's equations. The electric field may be written [6]

$$\begin{aligned}\mathbf{E}(\mathbf{r}) &= \nabla \times \oint_S dS' g(\mathbf{r}-\mathbf{r}') \mathbf{n}(\mathbf{r}') \times \mathbf{E}_s(\mathbf{r}') - \frac{1}{i\omega\epsilon} \nabla \times \nabla \times \oint_S dS' g(\mathbf{r}-\mathbf{r}') \mathbf{n}(\mathbf{r}') \times \mathbf{H}_s(\mathbf{r}') \\ &= \mathbf{K}_{EM}^S(\mathbf{r}, \mathbf{r}') \mathbf{M}_S(\mathbf{r}') + \mathbf{L}_{EJ}^S(\mathbf{r}, \mathbf{r}') \mathbf{J}_S(\mathbf{r}'),\end{aligned}\quad (1)$$

where $\mathbf{J}_S = \mathbf{n} \times \mathbf{H}_S$, $\mathbf{M}_S = -\mathbf{n} \times \mathbf{E}_S$ and $g(\mathbf{r}-\mathbf{r}')$ is the Green's function in the embedding medium. The formula for magnetic fields can be derived from (1) using duality principle:

$$\begin{aligned}\mathbf{H}(\mathbf{r}) &= \nabla \times \oint_S dS' g(\mathbf{r}-\mathbf{r}') \mathbf{n}(\mathbf{r}') \times \mathbf{H}_s(\mathbf{r}') + \frac{1}{i\omega\mu} \nabla \times \nabla \times \oint_S dS' g(\mathbf{r}-\mathbf{r}') \mathbf{n}(\mathbf{r}') \times \mathbf{E}_s(\mathbf{r}') \\ &= -\mathbf{K}_{HJ}^S(\mathbf{r}, \mathbf{r}') \mathbf{J}_S(\mathbf{r}') + \mathbf{L}_{HM}^S(\mathbf{r}, \mathbf{r}') \mathbf{M}_S(\mathbf{r}'),\end{aligned}\quad (2)$$

Eqs. (1) and (2) provide a way to decompose the whole solution domain into several subdomains using the equivalent current on the surface of the subdomains.

2. Electric Field Integral Equation

The electric field integral equation (EFIE) can be used to solve for the electric current distribution on a perfect electric conductor (PEC). It can be derived from the equivalence theorem,

$$-\mathbf{t} \cdot \mathbf{E}^{inc} = \mathbf{t} \cdot \mathbf{L}_{EJ}^S \mathbf{J}_S. \quad (3)$$

Using subdomain methods such as FEM or MOM, we can derive the matrix equation $[\bar{\mathbf{Z}}_{EE}^S] \cdot [\mathbf{J}_S] = -[\mathbf{E}^{inc}]$. The current coefficient on each basis function can be solved from this equation. Once the surface current is known, the scattered field can be computed as

$$\mathbf{E}^{sca} = \mathbf{L}_{EJ}^S \cdot \mathbf{J}_S. \quad (4)$$

3. Using Equivalent Surfaces to Solve the One-Object Scattering Problem

The procedure of solving the one object problem can be divided into three steps: outside-in propagation, solving for the current on the object and inside-out propagation as shown in (b), (c) and (d) in Fig. 1. The incident electric and magnetic currents on the equivalence surface are first computed from \mathbf{E}^{inc} and \mathbf{H}^{inc} . Substituting the incident field with the incident currents will generate the same incident field inside the surface by

$$\mathbf{E}^{inc} = \mathbf{K}_{EM}^S \mathbf{M}_S^{inc} + \mathbf{L}_{EJ}^S \mathbf{J}_S^{inc} \quad (5)$$

and null field outside. Since the currents generate field only propagating inside, this step is called outside-in propagation. In the next step, the electric currents on the object are solved given the incident wave on its surface. Once this current on the object is known, the equivalent electric and magnetic currents on the surface can be computed, which will generate null field inside and the scattered field outside. Therefore, we call these currents scattered currents and this step is defined as inside-out propagation. These three steps can be written in matrix form as the following

$$\begin{bmatrix} \mathbf{M}_S^{sca} \\ \mathbf{J}_S^{sca} \end{bmatrix} = \underbrace{\begin{bmatrix} -\hat{\mathbf{n}} \times \bar{\mathbf{L}}_{EJ}^S \\ \hat{\mathbf{n}} \times \bar{\mathbf{K}}_{HJ}^S \end{bmatrix}}_{\text{Inside-Out}} \cdot \underbrace{\left[\bar{\mathbf{Z}}_{EE}^S \right]^{-1}}_{\text{CurrentSolver}} \cdot \underbrace{\begin{bmatrix} \bar{\mathbf{K}}_{EM}^S & \bar{\mathbf{L}}_{EJ}^S \end{bmatrix}}_{\text{Outside-In}} \cdot \begin{bmatrix} \mathbf{M}_S^{inc} \\ \mathbf{J}_S^{inc} \end{bmatrix} = [\bar{\mathbf{S}}] \cdot \begin{bmatrix} \mathbf{M}_S^{inc} \\ \mathbf{J}_S^{inc} \end{bmatrix} \quad (6)$$

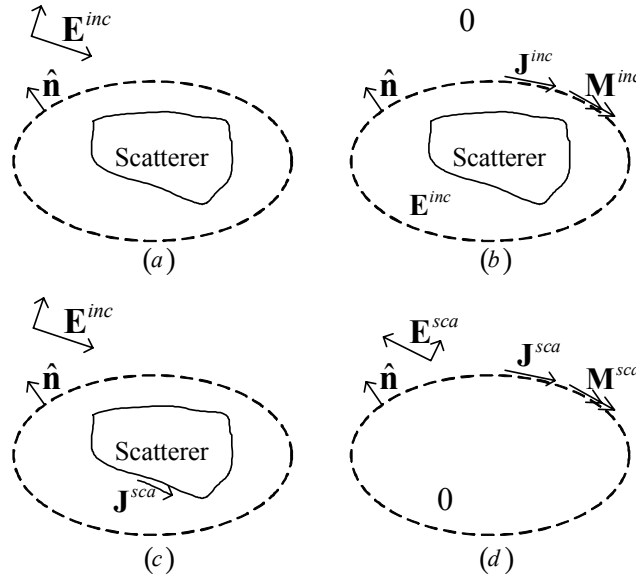


Figure 1 An example of using equivalence surface: plane wave scattered by a PEC object

With the above equation, the scattered currents were computed given the incident currents on the equivalent surface. Hence, we call $\bar{\mathbf{S}}$ matrix the scattering matrix. It is also seen that the unknowns on the scatterer are transferred to the unknowns on the equivalent surface with the information of the scatterer embedded in the $\bar{\mathbf{S}}$ matrix. To compute the field outside, we only need to know the scattered current on the surface. This method has advantages in analyzing scatterers with fine structures, which have to be modeled with many unknowns. Because these fine structures mainly contribute to near-field interactions, but are not important for the far field, the unknown density on the

equivalent surface can be much smaller than the one on the scatterer without losing accuracy.

3. Solving Multi-Object Scattering Problems with Equivalent Surfaces.

This equivalent surface scheme shows its advantages especially in solving the multi-object scattering problem. By representing the scatterers with equivalent surfaces, interactions between two objects are substituted with interactions between two equivalent surfaces. The translation operator is used to compute this interaction [6]. For simplicity, the equation for three scatterers is shown below, in which scatterers 2 and 3 are enclosed with equivalent surfaces:

$$\begin{aligned}
& [\bar{\mathbf{L}}_{11}^S] \cdot [\mathbf{J}_1^S] + [\bar{\mathbf{T}}_{12}^{PH}] \cdot \begin{bmatrix} \mathbf{J}_2^{sca} \\ \mathbf{M}_2^{sca} \end{bmatrix} + [\bar{\mathbf{T}}_{13}^{PH}] \cdot \begin{bmatrix} \mathbf{J}_3^{sca} \\ \mathbf{M}_3^{sca} \end{bmatrix} = -[\mathbf{E}_1^{inc}] \\
& -[\bar{\mathbf{S}}_{22}] \cdot [\bar{\mathbf{T}}_{21}^{HP}] \cdot [\mathbf{J}_1^S] + \begin{bmatrix} \mathbf{J}_2^{sca} \\ \mathbf{M}_2^{sca} \end{bmatrix} - [\bar{\mathbf{S}}_{22}] \cdot [\bar{\mathbf{T}}_{23}^{HH}] \cdot \begin{bmatrix} \mathbf{J}_3^{sca} \\ \mathbf{M}_3^{sca} \end{bmatrix} = [\bar{\mathbf{S}}_{22}] \cdot \begin{bmatrix} \mathbf{J}_2^{inc} \\ \mathbf{M}_2^{inc} \end{bmatrix} \\
& -[\bar{\mathbf{S}}_{33}] \cdot [\bar{\mathbf{T}}_{31}^{HP}] \cdot [\mathbf{J}_1^S] - [\bar{\mathbf{S}}_{33}] \cdot [\bar{\mathbf{T}}_{32}^{HH}] \cdot \begin{bmatrix} \mathbf{J}_2^{sca} \\ \mathbf{M}_2^{sca} \end{bmatrix} + \begin{bmatrix} \mathbf{J}_3^{sca} \\ \mathbf{M}_3^{sca} \end{bmatrix} = [\bar{\mathbf{S}}_{33}] \cdot \begin{bmatrix} \mathbf{J}_3^{inc} \\ \mathbf{M}_3^{inc} \end{bmatrix}, \tag{7}
\end{aligned}$$

where $\bar{\mathbf{T}}_{ij}^{HH}$ is the translation matrix between equivalent surfaces and $\bar{\mathbf{T}}_{ij}^{HP}$ is the translation matrix from scatterer j to equivalent surface i . Similar equations can be derived for more than three objects. Furthermore, if the same subdomain exists repeatedly in the simulation, only one scattering matrix needs to be stored in the memory for these repeated elements. Hence, memory usage is greatly reduced.

III. Numerical Examples

The numerical example shown here is a 2×2 array of XM antennas. To reduce the size of this antenna, it was designed as a microstrip patch antenna with substrate permittivity above 20. This leads to a very dense mesh to model the XM antenna. The volume-surface integral equation with RWG basis and multiple delta gap excitations are used in the simulation. The total number of unknowns is 7780×4 , the unknown density is over 150 per free-space wavelength. Four equivalent surfaces are used to enclose each antenna, with only 1536 unknowns on each. The reduction of the number of unknowns is 80%. Moreover, only one $\bar{\mathbf{S}}$ matrix needs to be stored due to the symmetry of the structure. Therefore, this 2×2 antenna array can be simulated with the memory requirement of only one element. The electric and magnetic current distributions on the equivalent surfaces along with the radiation pattern are shown in Fig. 2.

IV. Conclusion

A domain decomposition scheme based on the equivalence theorem is introduced in this paper. The unknowns on the scatterers are transferred to unknowns on equivalence surfaces. This leads to a reduction in the number of unknowns and memory requirements.

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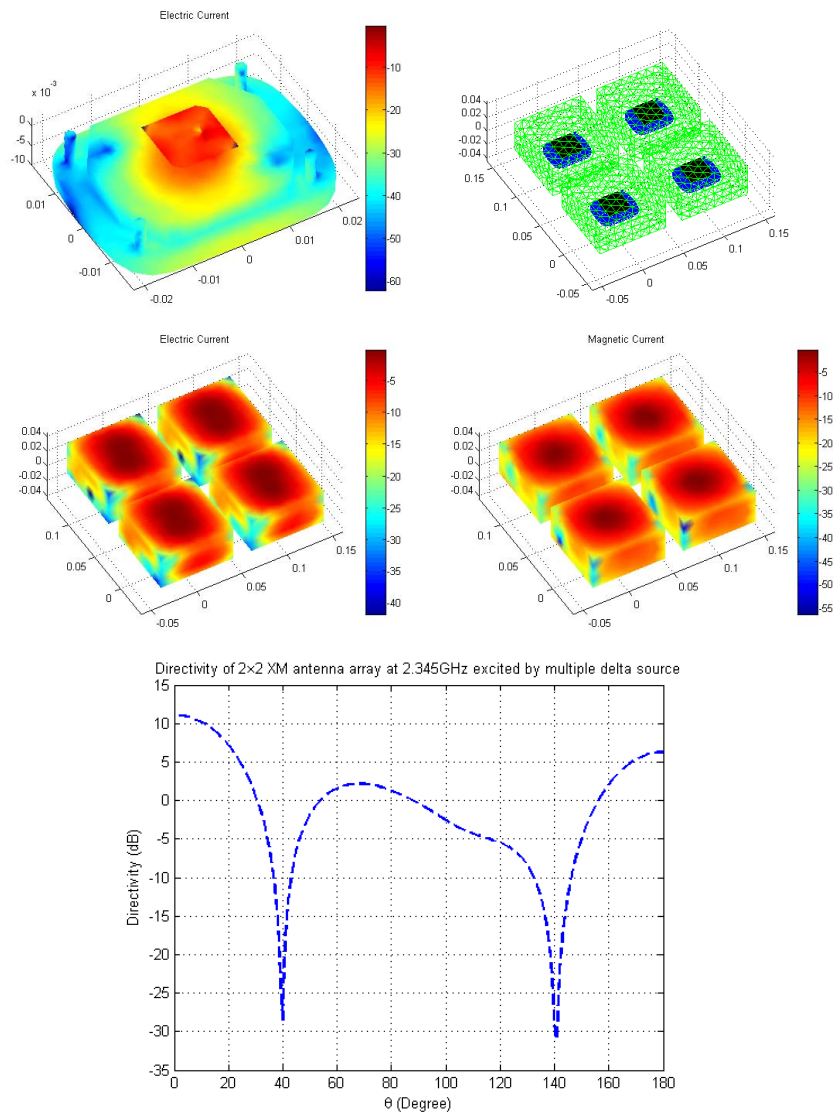


Figure 2 An example of simulating a 2 x 2 XM antenna array