linearized system and, therefore, by the stability of the matrix

$$\begin{aligned} & -g\frac{r_1^2}{2}C(c_{\rm av},\,d_{\rm av})A_1 \\ &= -g\frac{r_1^2}{2}\frac{\mathrm{Im}[\hat{P}(j\omega_1)]}{\mathrm{Im}[\hat{M}(j\omega_1)]} \\ &\quad\cdot\begin{pmatrix}\mathrm{Re}[\hat{M}(j\omega_1)] & (\mathrm{Re}[\hat{M}(j\omega_1)])^2-(\mathrm{Im}[\hat{M}(j\omega_1)])^2 \\ |\hat{M}(j\omega_1)|^2 & |\hat{M}(j\omega_1)|^2\,\mathrm{Re}[\hat{M}(j\omega_1)]\end{pmatrix}. \end{aligned} \tag{A2.1}$$

Again, it is easy to check that this system is stable under the conditions of the theorem. Now, let us return to the assumption that $C(c_{\rm av},\,d_{\rm av})$ is nonsingular. C is singular if and only if $c_{\rm av}=0$. Assuming that a tuned value with $c_{\rm av}^+=0$ exists, it would have to satisfy

$$\begin{aligned} d_{\rm av}^{+2}|\hat{P}(j\omega_1)|^2\,\mathrm{Re}[\hat{M}(j\omega_1)] - d_{\rm av}^+\,\mathrm{Re}[\hat{P}(j\omega_1)\hat{M}(j\omega_1)] \\ - d_{\rm av}^+\,\mathrm{Re}[\hat{P}(j\omega_1)\hat{M}^*(j\omega_1)] \\ + \mathrm{Re}[\hat{M}(j\omega_1)] = 0 \end{aligned} \tag{A2.2}$$

and therefore $d_{\rm av}^{+2}|\hat{P}(j\omega_1)|^2 - 2d_{\rm av}^+\ \mathrm{Re}[\hat{P}(j\omega_1)]+1=0$. However, this equation has no real solution if $\mathrm{Im}[\hat{P}(j\omega_1)]\neq 0$.

For arbitrary n, the equation for the tuned value is quite more complicated. If $c_{\rm av}^+\neq 0$, $C(c_{\rm av},\,d_{\rm av})$ is nonsingular. Take $n=2$ for simplicity. Then, $c_{\rm av}^+$, $d_{\rm av}^+$ satisfy

$$r_1^2\Delta_2(d_{\rm av}^+)\left(A_1\begin{pmatrix}c_{\rm av}^+\\ d_{\rm av}^+\end{pmatrix}-b_1\right)+r_2^2\Delta_1(d_{\rm av}^+)\left(A_2\begin{pmatrix}c_{\rm av}^+\\ d_{\rm av}^+\end{pmatrix}-b_2\right)=0. \tag{A2.3}$$

This is a set of two polynomial equations in $c_{\rm av}^+$, $d_{\rm av}^+$. The maximum power of $d_{\rm av}^+$ is 3 and of $c_{\rm av}^+$ is 1. The coefficient of $c_{\rm av}^+$ is a polynomial in $d_{\rm av}^+$ of degree 2. Therefore, $c_{\rm av}^+$ can be eliminated to yield an equation of degree 5 for $d_{\rm av}^+$, which has at most five real solutions. $c_{\rm av}^+$ is then determined uniquely from $d_{\rm av}^+$. The procedure is easily extended for arbitrary n, indicating that there are at most $4n-3$ tuned values.

Proof of Theorem 6.3: The proof is similar to the proof of Theorem 6.2. The matrix of the linearized system is now

$$-g\frac{r_1^2}{2}C(c_{\rm av},\,d_{\rm av})A_1 = -g\frac{r_1^2}{2}c_{\rm av}^2|\hat{P}(j\omega_1)|^2 \cdot\begin{pmatrix}1 & \mathrm{Re}[\hat{M}(j\omega_1)] \\ \mathrm{Re}[\hat{M}(j\omega_1)] & |\hat{M}|^2\end{pmatrix} \tag{A3.1}$$

which is always stable. The fact that the matrices A_i are symmetric and positive definite implies the additional results. In particular, the equation for $d_{\rm av}^+$ is again a polynomial of order $4n-3$, but its leading coefficient is guaranteed to be nonzero. Since the order is odd, there must exist at least one real tuned value. The bound on the tuned values also follows from the positive definiteness of the matrices A_i.

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The Effect of Time Delay and Discrete Control on the Contact Stability of Simple Position Controllers

John Fiala and Ronald Lumia

***Abstract*— By analysis of the driving-point admittance, it is shown how time delays and discrete control can create instabilities for a simple position controller in contact with the environment. The lowest frequency of contact instability due to time delay or sampling is determined analytically. It is shown how mechanical compliance between the motor and point of contact can eliminate these instabilities. To achieve the best relative stability when contacting arbitrary environments, the mechanical/control design of manipulators should maintain a critical relationship between the frequency of the compliant mode and a frequency associated with contact instability.**

I. Introduction

The simple proportional-derivative (PD) controllers used for controlling most robots show a remarkable robustness in a number of tasks, including those which involve contact with the environment. Recently, some authors have noted that the time delays and sampling in these controllers should have a detrimental effect on stability during contact with certain environments. Goldenberg and Clark [1] describe

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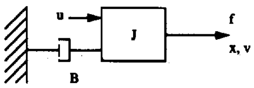

Fig. 1. Damped mass system.

a manipulator which exhibits instability during contact as a function of the load inertia and the environmental stiffness. Colgate and Hogan [2], [3] provide a theoretical explanation for the phenomenon, and suggest the use of the system admittance and the concept of passive physical equivalents to predict the regions of instability. A more complete treatment of the approach is described by Colgate [4]. The use of the passive physical equivalent concept has lead to a concurrent engineering approach to wrist design where mechanical design and controller design are approached simultaneously [5]. The concept of passivity was also used by Anderson and Spong [6] in the design of robot impedance controllers where a model of the environment is a fundamental part of the entire system model. Anderson and Spong also describe some of the effects of time delay on system stability in their analysis of force reflecting teleoperated systems [7].

It is interesting to note that such instabilities are not often observed in practice. The authors in [4], [8] conclude that inherent mechanical compliance ultimately stabilizes these systems. The idea that the robustness of simple position controllers in contact is due to mechanical compliance is analyzed in detail in this paper.

II. Ideal Proportional-Derivative Position Controller

First, consider a force generator (motor) u acting on a damped mass, as shown in Fig. 1. An ideal proportional-derivative position control law

$$u = K_p(x_d - x) - K_v v \tag{1}$$

with K_p, $K_v > 0$ would produce a stable system, both in free space and in contact, since it has a passive physical equivalent [4]. That is, the same control could be achieved by attaching a physical spring and damper to the mass, and physical systems are always passive, or energy dissipating. Alternatively, one could examine the passivity of the driving-point admittance of the mechanical system. If the driving-point admittance does not have more than 90° of phase shift, then there is a passive physical equivalent, and the system will be stable in contact with any passive environment [3], [4], [8]. To investigate stability, Newman [8] showed that it is only necessary to look at the admittance due to feedback. The admittance due to feedforward can be ignored. Consequently, for stability, the driving-point admittance of (1) is

$$Y(s) = \frac{-v(s)}{f(s)} = \frac{s}{(Js+B)s + (K_p + K_v s)}. \tag{2}$$

The phase of $Y(j\omega)$ for this system is between 90° and $-90°$ for all $\omega > 0$, and the system is stable in contact with arbitrary environments.

III. Effect of Time Delay

It is well known that time delay adversely affects the stability of control systems. For the controller (1) with a time delay T, the driving-point admittance is

$$Y(s) = \frac{s}{(Js+B)s + [K_p + K_v s]e^{-Ts}}. \tag{3}$$

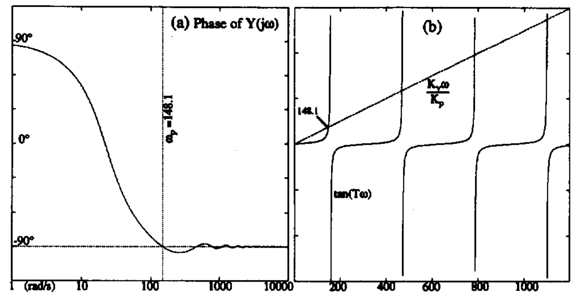

Fig. 2. Effect of time delay on the passivity of the damped mass system.

The real part of $Y(j\omega)$ is found in the equation displayed at the bottom of the page. The positive real condition, Real$\{Y(j\omega)\} \geq 0$, can be used to test the passivity of the admittance [4]. As also derived in [4], this yields that the system presents a passive admittance as long as

$$K_v\omega\cos(T\omega) - K_p\sin(T\omega) + B\omega \geq 0 \tag{4}$$

which indicates that the system may exhibit passivity violations for frequencies above a certain value. These violations are characterized by phase in excess of $-90°$ over specific frequency intervals, which means that the system will exhibit instability in contact with springs of certain stiffnesses. Fig. 2(a) shows the phase of $Y(j\omega)$ for $J = 1$, $B = 0$, $T = 0.01$, $K_p = 360$, and $K_v = 27$. (Any consistent set of units may be chosen for these numbers.)

Without friction, the frequencies delimiting the passivity violations are the intersections of the line $K_v\omega/K_p$ with $\tan(T\omega)$, as depicted in Fig. 2(b). The frequency of the first such intersection is the frequency ω_p at which passivity is first violated. The presence of friction may stabilize the system. The amount of damping required to completely stabilize the contact instability can be used as a measure of relative stability. For the system of (3), damping can stabilize the system only at unreasonable levels. Suppose substantial friction of $B = K_v$ exists. Then (4) can be written

$$\frac{K_v\omega}{K_p} \geq \frac{\sin(T\omega)}{1+\cos(T\omega)} = \tan\left(\frac{T\omega}{2}\right) \tag{5}$$

which indicates that contact instabilities still exist, but for stiffer environments. Note also that as B increases, so does ω_p. Only when $B \gg K_v$ can the system be completely stabilized. Such high levels of friction clearly limit performance.

For most robots, there is some compliance between the motor and the point of contact [9]. The presence of this compliance may help stabilize the system during contact. The damped mass model can be modified to include a compliant transmission modeled as a damped spring [4], as shown in Fig. 3. To simplify initially, let $B_t = 0$. Computing Real$\{Y(j\omega)\} = N(\omega)/D(\omega)$ yields

$$N(\omega) = 16K_t^2 K_v\omega^2\cos(T\omega) - 16K_t^2 K_p\omega\sin(T\omega) + 16K_t^2 B\omega^2. \tag{6}$$

$$\frac{K_v\omega^2\cos(T\omega) - K_p\omega\sin(T\omega) + B\omega^2}{[-J\omega^2\sin(T\omega) + B\omega\cos(T\omega) + K_v\omega]^2 + [-B\omega\sin(T\omega) - J\omega^2\cos(T\omega) + K_p]^2}.$$

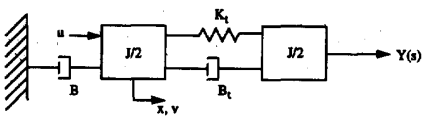

Fig. 3. System with compliance.

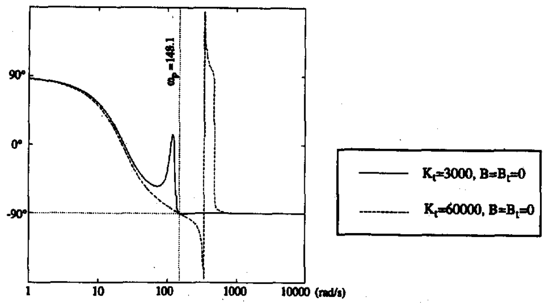

Fig. 4. Effect of time delay on phase of driving-point admittance of system with compliance.

Since $D(\omega) > 0$ for $\omega > 0$, as before, the system will be stable in contact with arbitrary passive environments provided $N(\omega) \geq 0$. This produces condition (4) again, and apparently the system with compliance will suffer the same type of passivity violations as before. However, the effect of the compliance is to improve the relative stability of the system by requiring less damping at certain frequencies.

The undamped natural frequencies of the system are

$$\omega_{n1} = \sqrt{\frac{2K_t}{J}}, \qquad \omega_{n2} = \sqrt{\frac{4K_t}{J}}.$$

At contact frequencies below ω_{n1}, the masses move together with the spring unstretched [10], such that the transmission damping has little effect. At these frequencies, the situation of (4) applies, which requires significant motor-side friction B to stabilize. Above ω_{n2}, the masses move out of phase, and the system may be stabilized by a smaller amount of friction B_t in the transmission. In order to have a system with the best relative stability, i.e., which can be stabilized by the least amount of friction, the requirement is

$$\omega_r < \omega_p \tag{7}$$

where

$$\omega_r \equiv \sqrt{\frac{4K_t}{J}}. \tag{8}$$

This relation can be achieved by decreasing the time delay, which increases ω_p, or by decreasing the stiffness of the transmission K_t. Although the presence of friction will also affect these values, a system design maintaining $\omega_r < \omega_p$, assuming no friction, will retain good relative stability in the actual system.

As an example, Fig. 4 shows the phase of $Y(j\omega)$ for the system of Fig. 3 with $J = 1$, controlled by (1) with a time delay of $T = 0.01$, and $K_p = 360$, $K_v = 27$. When $B = B_t = 0$, ω_p =148.1 rad/s, as obtained from the graph of Fig. 2(b). When $K_t = 3000$, $\omega_r < \omega_p$, and the system is made completely passive by transmission damping of $B_t = 0.55$. On the other hand, when $K_t = 60\,000$, the system requires $B_t > 100$ $(B = 0)$ or $B > 100$ $(B_t = 0)$ for passivity.

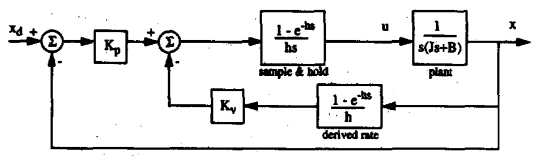

Fig. 5. Discrete controller with derived rate.

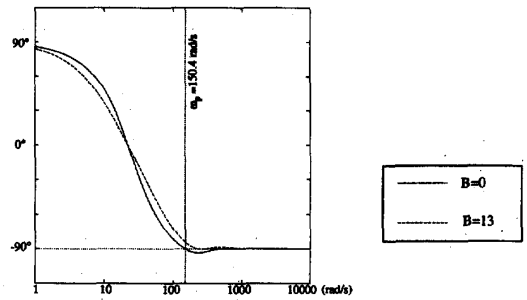

Fig. 6. Phase of the driving-point admittance of the derived rate controller.

IV. Effect of Discrete Control

Consider a discrete controller with derived rate, as depicted in Fig. 5, where the control law is written as

$$u = \frac{1 - e^{-hs}}{hs}\left(k_p(x_d - x) - \frac{1 - e^{-hs}}{hs}K_v v\right). \tag{9}$$

The positive real condition for the admittance yields

$$\begin{aligned} 2K_v \cos^2(h\omega) - (2K_v + hK_p)\cos(h\omega) \\ + (hK_p - Bh^2\omega^2) \leq 0 \qquad (h \neq 0) \end{aligned} \tag{10}$$

from which we obtain the stability relationship

$$\frac{(2K_v + hK_p) - \Gamma}{4K_v} \leq \cos(h\omega) \leq \frac{(2K_v + hK_p) + \Gamma}{4K_v} \tag{11}$$

$$\Gamma = \sqrt{8Bh^2\omega^2 K_v + (2K_v - hK_p)^2}.$$

There are multiple frequency intervals of phase in excess of $-90°$. The intervals above the first interval of passivity violation, however, have relatively little phase excess, as shown by the solid line in Fig. 6. The figure shows the phase for $h = 0.01$, $K_p = 360$, and $K_v = 27$. From (11) with $B = 0$, the first interval of passivity violation begins at the frequency

$$\omega_p = \frac{1}{h}\cos^{-1}\left(\frac{hK_p}{2K_v}\right). \tag{12}$$

The minimum damping B required for passivity can be obtained from (11) as well. Large amounts of damping may be required, depending on the gains and sampling rate chosen. For the example of Fig. 6, $B > 13$ is required to make the system completely passive. As with the time-delay problem, the controller of Fig. 5 has the same positive real condition (10) when a transmission compliance is inserted as in Fig. 3. Thus, as before, the presence of transmission compliance will improve the relative stability when $\omega_r < \omega_p$.

V. Effect of Discrete Control with Torque Loop

The effect of compliance external to the controller was examined in prior sections. Some modern manipulators, however, use drive transmissions surrounded by torque loops [11]. This approach places the major mechanical compliance of the device inside the controller.

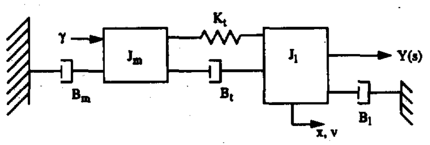

Fig. 7. System with compliance internal to control.

To see how this affects stability, consider the model of Fig. 7. A spring representing the drive compliance has been inserted between a motor mass and a damped load. The position and velocity at the load are controlled using the equivalent of a torque loop, which attempts to control the force in the spring. The analog force control law is given by

$$\gamma = K(f_d - f) \tag{13}$$

where the desired force f_d is the input u from the position controller in Fig. 5, and f is the sensed force produced by the transmission spring K_t. The resonant frequency of the force loop is

$$\omega_r \equiv \sqrt{\frac{K_t(K+1)}{J_m}}. \tag{14}$$

Applying the positive real condition when there is no damping, $B_m = 0$, $B_t = 0$, and $B_l = 0$,

$$h^2 K K_t \omega^2 [J_m \omega^2 - (K+1)K_t][2K_v \cos(h\omega) - hK_p] \cdot [\cos(h\omega) - 1] \geq 0,$$

The frequency (12) is still a determining boundary of the first instability interval, but the actual frequency interval is given by

$$\omega_p \leq \omega \leq \omega_r \qquad \text{when}\,(\omega_p < \omega_r) \tag{15}$$

or

$$\omega_r \leq \omega \leq \omega_p \qquad \text{when}\,(\omega_r < \omega_p). \tag{16}$$

Thus, for the torque loop model, there is a prominent first region of contact instability. For the first case (15), the system may be passive in the upper part of the region provided that ω_r is sufficiently far from ω_p to hold the entire first instability interval starting at ω_p. In this case, the relative stability of the system in contact is similar to that of the damped mass system—a large amount of friction is needed to stabilize the system. Damping requirements will also be large when the system is unstable at ω_r, which can be a problem for (16) as well. This is due to the resonant frequency term

$$J_m \omega^2 - (K+1)K_t \tag{17}$$

in the positive real condition. For a given ω_r, this is most problematic when J_m is small, since J_m determines the steepness of (17) around ω_r.

Again, B_t is most effective in stabilizing the system for (16), rather than (15). In general, some damping is needed to stabilize the higher frequency nonpassive intervals. B_l will work as well as B_t for this. However, there is another way to stabilize the prominent first interval, provided J_m is not too small, as is illustrated in Fig. 8. The following base set of parameters is chosen for the figure: $B_m = 0$, $B_t = 0$, $B_l = 0$, $J_l = 1$, $J_m = 2$, $K_t = 12\,000$, $K_p = 360$, $K_v = 27$, $h = 0.01$. With $K = 7$, the situation is that of (15), and the phase of the driving-point admittance is shown as the solid line in Fig. 8. Only with damping of $B_m > 19$ and $B_l = 0.1$ can this admittance be made completely passive. An example of (16) can be obtained by using $K = 1$. The result is shown in Fig. 8 as the dashed line. Note that the passivity violation for this case is due to phase in excess of 90°, rather than −90°. Thus, the system

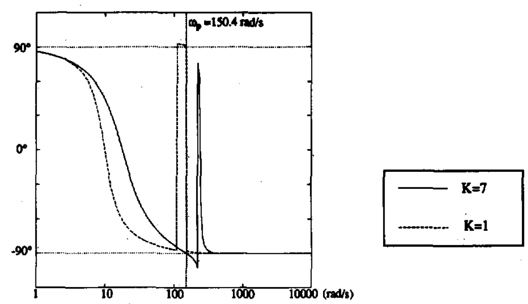

Fig. 8. Phase of the driving-point admittance of torque loop model controlled by derived rate controller.

can be made passive by the addition of phase lag in the PD loop. Since the controller will typically contain some phase lag due to computational delay, it is possible to design the system so that it is completely passive without excessive damping, provided $\omega_r < \omega_p$. For this example, lag due to a computational delay of $T = 0.0035$ and $B_l = 0.15$ makes the system stable in contact with arbitrary environments.

VI. Conclusions

Simple position controllers may exhibit contact instabilities, particularly when low sampling rates and long time delays are involved. Most commercial robot controllers are stable in contact because they are typically controlled with analog PD controllers. When commercial systems use discrete control, they use sampling rates as high as possible. This generally ensures that the frequencies of passivity violations are above the resonant mode of the mechanical structure, $\omega_p > \omega_r$. As shown in this paper, achieving this relationship for these two critical frequencies makes contact stability much easier to achieve.

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