

# Free and forced obliquities of the Galilean satellites of Jupiter

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## Abstract

The obliquity, or angular separation between orbit normal and spin pole, is an important parameter for the geodynamics of most Solar System bodies. Tidal dissipation has driven the obliquities of the Galilean satellites of Jupiter to small, but non-zero values. We present estimates of the free and forced obliquities of these satellites using a simple secular variation model for the orbits, and spin pole precession rate estimates based on gravity field parameters derived from Galileo spacecraft encounters. The free obliquity values are not well constrained by observations, but are presumed to be very small. The forced obliquity variations depend only on the orbital variations and the spin pole precession rate parameters, which are quite well known. These variations are large enough to influence spatial and temporal patterns of tidal dissipation and tidal stress.

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## 1. Introduction

The Galilean satellites of Jupiter exhibit considerable evidence of the influence of tidal dissipation. One consequence of that process is that the spin poles of these bodies (Io, Europa, Ganymede, and Callisto) are nearly coincident with their respective orbit normals. In fact, most previous analyses have assumed these obliquities to be identically zero. The objective of the present investigation is to continue an examination into the question of the orientations of the spin poles of Galilean satellites, relative to their respective orbit poles.

It is well known that the obliquities of these bodies are small, and for many purposes it is quite adequate to treat them as zero. However, there are still incentives to examine the situation further. First is that it is impossible for the obliquities to be identically zero, or even constant. Second is that finite obliquities will change the spatial and temporal pattern, and the total amount, of tidal dissipation and tidal stress within these bodies. Ignoring these variations could

lead to incorrect models of several important geodynamic processes.

As will be demonstrated subsequently, it is impossible for the obliquities to be identically zero since the orbit poles are inclined to the equator plane of Jupiter and are precessing. If the obliquity were momentarily zero, the precessional torque would vanish and the spin pole would be unable to follow the orbit pole. Further, it is not even possible for the obliquities to be constant since the orbit precession occurs at non-uniform rates. If the spin poles were able to precess rapidly enough to track the motion of the orbit poles, then the obliquities could be quite small. However, the spin precession rates of the Galilean satellites, other than Io, are small compared to most of the rates associated with the orbit precession, and the obliquity variations are expected to be comparable to the variations in orbital inclination. These orbital inclinations to Jupiter's equator plane are all quite small, but they are much larger than the obliquities which would result if tidal dissipation had driven the spin to a generalized Cassini state appropriate to a uniform precession of the orbit.

To the extent that the obliquities are non-zero, there will be interesting consequences for the spatial and temporal pat-

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terns of tidal stress and tidal dissipation within the bodies. If the obliquity and orbital eccentricity are both zero, then the tide raising body will always be on the equator, and at fixed distance. If the orbital eccentricity is non-zero, then the tidal amplitude will vary over the orbit, and the location of maximum tidal displacement will librate in longitude. If the obliquity is non-zero, then the sub-jovian point will also librate in latitude, and the patterns of dissipation and stress from that motion will add to the contribution from longitudinal librations. The eccentric and oblique patterns of stress and dissipation will change relative phase as the orbit precesses. This will lead to temporal changes in the global average rate and spatial pattern of tidal stress and dissipation.

The precession of the spin pole of a rotating body can be modeled by equating the rate of change of spin angular momentum to the applied gravitational torque. For a rapidly rotating body, this torque balance can be written in the form (Ward, 1973, 1992; Kinoshita, 1977; Bills, 1990; Hilton, 1991)

$$\frac{d\hat{s}}{dt} = \frac{\alpha}{(1-e^2)^{3/2}} (\hat{n} \cdot \hat{s}) (\hat{s} \times \hat{n}), \quad (1)$$

where  $\hat{s}$  and  $\hat{n}$  are unit vectors along the spin pole and orbit normal respectively,  $e$  is the orbital eccentricity, and  $\alpha$  is a scalar rate parameter which depends on the principal moments of inertia  $A < B < C$ , the spin rate  $\sigma$ , and the orbital mean motion  $n$ , via

$$\alpha = \frac{3}{2} \left( \frac{C - (A + B)/2}{C} \right) \frac{n^2}{\sigma}. \quad (2)$$

The situation for a synchronous rotator differs significantly if the orbital eccentricity and obliquity are large. However, as will be shown below, when the eccentricity and obliquity are small enough, the rapid rotator and synchronous rotator forms of the precession equation converge. The Galilean satellites have small enough eccentricities and obliquities that the trajectories of their spin pole motion are well represented by the equations above.

In order to examine the obliquity variations of these four satellites of Jupiter, we will need estimates of several quantities. First, we need estimates of the spin precession rate parameter  $\alpha$  for each of the bodies. Second, we need a model of how the orbit varies with time. Explicitly, we need representations of the long period variations in eccentricity and orientation of the orbit normal. In order to obtain an explicit solution to the differential equation of spin precession, we need an initial condition on the spin pole. That would appear to be the most challenging part of the analysis, from an observational perspective, in that present observations are not sufficiently accurate. However, we will see that the spin pole trajectory can be written in a form which separates the influence of initial conditions from the forced response of the changing orbit. If dissipative effects are included, we obtain a solution which has no long-term memory and thus does not require any initial conditions. Finally, the obliquity  $\varepsilon$  is

obtained directly from the relationship

$$\cos \varepsilon = \hat{n} \cdot \hat{s}. \quad (3)$$

We can already anticipate that rapid variations in the orbit normal will map directly into obliquity variations, since the spin pole will not be able to respond quickly enough. Conversely, slow variations in the orbit pole will not contribute to obliquity oscillations since the spin pole can follow them. The most interesting case is one in which the orbit pole and spin pole rates of precession are comparable, as it can lead to resonant amplification of the spin pole motion. Our linear solution for forced spin pole motion will make that resonant amplification effect quite clear.

The remainder of this paper is divided into 5 sections. In Section 2 we examine the torque balance for synchronously rotating triaxial bodies, and develop approximations to the spin pole precession equation. In Section 3 we estimate the rate parameters  $\alpha$  which specify how fast the spin poles of the Galilean satellites will respond to a unit torque. In Section 4 we explore solutions to the spin precession equation for the Galilean satellites, for a generic orbital model. In Section 5 we develop a series of simple analytic models of the secular orbital evolution of the Galilean satellites, and then adopt the published model of Lieske (1998) for further analysis. In Section 6 we briefly summarize the results and discuss implications.

## 2. Precessional torque balance

In this section we will examine the influence of rotation rate on spin pole precession. The most familiar form of the precessional torque balance, as represented in Eqs. (1) and (2), is only strictly valid in the case of a rapid rotator. In that situation, the solar gravitational torque can be averaged over the spin period, holding the orbital position fixed, and then separately averaged over the orbit. For a resonant rotator, departures from axial symmetry modify the torque balance, and this must be properly accounted for. We will see, however, that the proper torque balance equation for the Galilean satellites can be written in a form which is quite similar to that for a rapid rotator.

The precession of the spin pole of a planet or satellite is modeled by equating the change in spin angular momentum to the applied torque. The instantaneous gravitational torque acting on a triaxial body, due to a distant point mass, can be written in the form

$$T = \frac{3Gm_s}{r^3} (\hat{u} \times I \cdot \hat{u}) \quad (4)$$

in which  $G$  is the gravitational constant,  $m_s$  is the source mass,  $r$  is the distance from the rotator to the source,  $I$  is the inertia tensor of the triaxial body, and  $\hat{u}$  is a unit vector oriented toward the source, as seen from the center of the rotator. This formulation yields both short period torques, which give rise to nutations and librations, and long period

torques which cause the precession we are mainly interested in. For recent discussions of the short period effects, see [Wu et al. \(2001\)](#) and [Williams et al. \(2001\)](#).

If the triaxial body and point source are in a binary orbit, and the torques are averaged over the rotation period and orbital period of the body, we can write the precession equation in the form

$$\frac{d\hat{s}}{dt} = \frac{3}{2} \left( \frac{n^2}{\sigma} \right) (\alpha^* (\hat{n} \cdot \hat{s}) + \beta^*) (\hat{s} \times \hat{n}), \quad (5)$$

where  $\hat{n}$  and  $\hat{s}$  are unit vectors along the orbit pole and spin pole, respectively,  $n$  and  $\sigma$  are the angular rates of mean orbital motion and rotation of the triaxial body, and  $\alpha^*$  and  $\beta^*$  are functions of the orbital eccentricity  $e$  and the principal moments of inertia ( $A < B < C$ ). The particular forms taken by the dimensionless parameters  $\alpha^*$  and  $\beta^*$  depend on the relative rates of rotational and orbital motion, a point to which we will return momentarily.

Several features of this formulation deserve comment. All but the terms within the first set of parentheses are dimensionless. The direction of the precessional motion is dependent only on the two unit vectors  $\hat{n}$  and  $\hat{s}$ , and is perpendicular to both of them, due to the  $\hat{s} \times \hat{n}$  term. The orbital mean motion  $n$  is related to source strength  $Gm_s$  and orbital semimajor axis  $a$  via Kepler's third law

$$a^3 n^2 = Gm_s (1 + \nu), \quad (6)$$

where the mass ratio is

$$\nu = \frac{m}{m_s} \quad (7)$$

and  $m$  is the mass of the rotator. For small mass ratios ( $\nu \ll 1$ ) we can make the approximation

$$\frac{Gm_s}{a^3} = n^2 \quad (8)$$

which was employed in writing Eq. (5).

In averaging the torques, we need to write functions of orbital radius  $r$  and orbital true anomaly  $f$  in terms of orbital mean anomaly  $M$ , which varies linearly with time. A convenient format for such expansions was introduced by [Cayley \(1861\)](#). He tabulated expansion coefficients for functions of the form

$$\left( \frac{r}{a} \right)^p \cos(qf) = \sum_{j=0}^{\infty} \mathcal{C}_j^{p,q}[e] \cos(jM), \quad (9)$$

$$\left( \frac{r}{a} \right)^p \sin(qf) = \sum_{j=0}^{\infty} \mathcal{S}_j^{p,q}[e] \sin(jM), \quad (10)$$

where  $p$  and  $q$  are integers, and the coefficients  $\mathcal{C}_j^{p,q}$  and  $\mathcal{S}_j^{p,q}$  are functions of orbital eccentricity  $e$ . Explicitly, those coefficients are given by the integrals

$$\mathcal{C}_j^{p,q}[e] = \frac{1}{2\pi} \int_0^{2\pi} \left( \frac{r}{a} \right)^p \cos(qf) \cos(jM) dM, \quad (11)$$

$$\mathcal{S}_j^{p,q}[e] = \frac{1}{2\pi} \int_0^{2\pi} \left( \frac{r}{a} \right)^p \sin(qf) \sin(jM) dM. \quad (12)$$

The evaluation of these integrals, though rather tedious for Cayley, is now readily implemented via recurrence relations ([Hughes, 1981](#); [Vakhidov, 2001](#)).

If the rotation angle of the axis of least inertia of the triaxial body is

$$s = \tau + bM, \quad (13)$$

where  $\tau$  is the angle, measured from the ascending node of the orbit on the equator plane at periape, and  $b$  is a half integer, then we will need three coefficients:  $\mathcal{C}_0^{-3,0}[e]$ ,  $\mathcal{C}_{2b}^{-3,2b}[e]$ , and  $\mathcal{S}_{2b}^{-3,2b}[e]$ . The first of these has a simple closed-form expression

$$\mathcal{C}_0^{-3,0}[e] = (1 - e^2)^{-3/2}. \quad (14)$$

The others are given in terms of Taylor series expansions, with different forms for each value of the spin-orbit rate ratio  $b$ .

The most familiar form of the precessional equation is that which is applicable to rapid rotators, such as Earth or Mars. In that case, the torques can be averaged over the spin period, holding the orbital position fixed, and then separately averaged over the orbital position angle. In that case the dimensionless parameters  $\alpha^*$  and  $\beta^*$  are given by

$$\alpha^* c = J_2 \mathcal{C}_0^{-3,0}[e] = J_2 \left( 1 + \frac{3}{2} e^2 + \dots \right), \quad (15)$$

$$\beta^* = 0, \quad (16)$$

where  $J_2$  is the degree two zonal harmonic coefficient of the gravitational potential of the rotator, which is related to the principal moments ( $A$ ,  $B$ ,  $C$ ), mass  $m$  and mean radius  $R$  of the body via

$$J_2 m R^2 = C - \left( \frac{A + B}{2} \right), \quad (17)$$

and  $c$  is the dimensionless polar moment of inertia

$$c = \frac{C}{m R^2}. \quad (18)$$

For a synchronous rotator, in which  $b = 1$ , the torque averaging is still a simple calculation, but is somewhat more tedious. After adjusting the phase angle  $\tau$  so that the mean torque about the spin axis vanishes (in order to maintain synchronous rotation), the result can be written as

$$\alpha^* c = J_2 \mathcal{C}_0^{-3,0}[e] + C_{2,2} \mathcal{C}_2^{-3,2}[e], \quad (19)$$

$$\beta^* c = -C_{2,2} \mathcal{C}_2^{-3,2}[e], \quad (20)$$

where  $C_{2,2}$  is a harmonic coefficient of degree two and order two in the potential of the rotator, and is given by

$$C_{2,2} m R^2 = \left( \frac{B - A}{4} \right). \quad (21)$$

Note that the rapid rotator has no term proportional to the difference in equatorial moments, as the spin averaging is equivalent to setting  $A = B$ . If we truncate the Cayley coefficient expansions at second degree in eccentricity, we have for the synchronous case

$$\alpha^* c = J_2 \left( 1 + \frac{3}{2} e^2 \right) + C_{2,2} \left( 1 - \frac{5}{2} e^2 \right), \quad (22)$$

$$\beta^* c = -C_{2,2} \left( 1 - \frac{5}{2} e^2 \right). \quad (23)$$

If the obliquity is small enough that

$$\hat{n} \cdot \hat{s} \simeq 1 \quad (24)$$

then the precession formula can be written as

$$\frac{d\hat{s}}{dt} = \frac{3}{2} \left( \frac{n^2}{\sigma} \right) Q[e] (\hat{s} \times \hat{n}) \quad (25)$$

with

$$Q[e] = (\alpha^* + \beta^*). \quad (26)$$

We will see below that this small angle approximation is very well justified for the Galilean satellites.

If the Taylor series expansion in orbital eccentricity  $e$  is truncated at degree two, we can write

$$\frac{d\hat{s}}{dt} = (\alpha_0 + \alpha_2 e^2) (\hat{s} \times \hat{n}). \quad (27)$$

The rapid rotator version of this formula can be written with

$$\alpha_0 = \frac{3}{2} \left( \frac{J_2}{c} \right) \left( \frac{n}{\sigma} \right) n, \quad (28)$$

$$\alpha_2 = \frac{9}{4} \left( \frac{J_2}{c} \right) \left( \frac{n}{\sigma} \right) n = \frac{3}{2} \alpha_0. \quad (29)$$

The corresponding form for synchronous rotators, with  $\sigma = n$ , is

$$\alpha_0 = \frac{3}{2} \left( \frac{J_2}{c} \right) n, \quad (30)$$

$$\alpha_2 = \frac{9}{4} \left( \frac{J_2}{c} \right) n. \quad (31)$$

We thus see that, keeping only terms of first order in obliquity and second order in eccentricity, the resonant rotator and rapid rotator versions of the precession equation are virtually identical in form. Analyses of the dynamical evolution of the Moon or other resonant rotators into low obliquity configurations (Ward, 1975a; Peale, 1969; Jankowski et al., 1989; Gladman et al., 1996) are obligated to consider the higher order terms, but for treatment of the present situation, this simpler form is quite adequate.

### 3. Spin precession rates

In this section we estimate the spin pole precession rate parameters  $\alpha$  for the Galilean satellites. The formula from

which they are estimated was given above, in Eq. (1). It depends on the orbital mean motion  $n$ , which is very well known, and on the difference  $C - (A + B)/2$  between the polar moment and the average of the two equatorial moments of inertia, which are not nearly as well known. We first comment on how the current the moment estimates were obtained, and then produce estimates of the spin rate parameters, with corresponding error estimates.

The degree two component of the gravitational potential of a satellite, at a point specified by radius  $r$ , latitude  $\theta$ , and longitude  $\phi$ , can be written in the form

$$\Phi_2 = \frac{GM}{r^3} \left( -J_2 P_{2,0}[\mu] + (C_{2,1} \cos \phi + S_{2,1} \sin \phi) P_{2,1}[\mu] + (C_{2,2} \cos 2\phi + S_{2,2} \sin 2\phi) P_{2,2}[\mu] \right), \quad (32)$$

where  $G$  is the gravitational constant,  $M$  is the mass of the body, latitudinal position is parameterized by

$$\mu = \sin \theta \quad (33)$$

and  $P_{l,m}[\mu]$  is an associated Legendre function of degree  $l$  and order  $m$ . Alternatively, the quadrupole component of the potential can be written, via MacCullagh's formula, in the form

$$\Phi_2 = \frac{3}{2} \frac{G}{r^3} (J - I), \quad (34)$$

where the mean moment of inertia is

$$I = \frac{A + B + C}{3} \quad (35)$$

and the moment of inertia  $J$  about an axis along the unit vector  $u$  is

$$J = \hat{u}^t \cdot \begin{pmatrix} A & 0 & 0 \\ 0 & B & 0 \\ 0 & 0 & C \end{pmatrix} \cdot \hat{u} \quad (36)$$

and the unit vector itself is

$$\hat{u}(\theta, \phi) = \{ \cos \theta \cos \phi, \cos \theta \sin \phi, \sin \theta \}^t. \quad (37)$$

There are 5 spherical harmonic coefficients and 6 independent terms in the inertia tensor.

Estimates of the degree two gravity fields of the Galilean satellites have been obtained from the perturbations experienced by the Galileo spacecraft during numerous close encounters (Anderson et al., 1996a, 1996b, 1998a, 1998b). It is difficult to estimate the full degree two gravity field from these incomplete samples of the surface field, but the situation has been improved by the insight (Hubbard and Anderson, 1978) that the expected degree two components will be dominated by tidal deformation and will have a relatively simple spatial pattern.

The expected pattern is that the potential will be a superposition of a linear response to the rotational potential, which is symmetric about the rotation axis, and a linear response to the tidal potential, which is symmetric about the mean satellite-primary line. As a result of this assumed symmetry, only two of the five spherical harmonic coefficients

of degree two will be non-zero, and they will be linearly related. That is, we expect to find

$$J_2 = \frac{10}{3}C_{2,2} \quad (38)$$

and

$$C_{2,2} = \frac{k_s}{4}q, \quad (39)$$

where  $k_s$  is the proportionality constant, or secular Love number, and  $q$  is a parameter which characterizes the relative strength of the rotational tendency for oblate mass distribution versus the gravitational tendency toward spherical symmetry. For a body with mass  $M$ , radius  $R$ , and rotation rate  $\sigma$ , that ratio is

$$q = \frac{\sigma^2 R^3}{GM}. \quad (40)$$

Using this template, the solution algorithm effectively reduces the gravity field to 2 unknown parameters, the total mass of the satellite and the scale factor  $k_s$  for the degree two pattern. Estimates of the polar moment of inertia are also obtained, under the assumption of hydrostatic balance, from the Darwin–Radau relation

$$c \equiv \frac{C}{MR^2} = \frac{2}{3} \left( 1 - \frac{2}{5} \sqrt{\frac{4 - k_s}{1 + k_s}} \right). \quad (41)$$

For a homogeneous fluid body, the secular Love number is  $k_s = 3/2$  and the normalized polar moment is  $c = 2/5$ .

In terms of the spherical harmonic coefficients of the potential, we can now write the spin pole precession rate parameter as

$$\alpha = \frac{3}{2} \left( \frac{J_2}{c} \right) n. \quad (42)$$

We note that this parameter is expected to be a rapidly decreasing function of distance from Jupiter. As written above it appears to have only a linear dependence on mean motion  $n$ . However, if we combine the expected tidal values of the other parameters, we can rewrite it in the form

$$\alpha = \frac{45}{64} \left( \frac{5 + 2\delta k_s}{\pi G \rho} \right) n^3, \quad (43)$$

where we have expressed the satellite mass in terms of radius  $R$  and mean density  $\rho$ , and have written the secular Love number in terms of departure from the homogeneous value

$$k_s = \frac{3}{2} + \delta k_s. \quad (44)$$

Written this way, the factor in parentheses will be nearly the same for all of the Galilean satellites, and the spin pole precession rate will be proportional to  $n^3$ , or from Kepler's third law, proportional to  $a^{-9/2}$ . The rate of spin precession for the close satellites will be much greater than for the distant ones.

Table 1 contains values of the relevant input parameters and estimates of the resulting spin pole precession rate parameters for each of the Galilean satellites. Note that the relative accuracy of the spin precession rate of Io is roughly 0.5% and all the others are roughly 5%. This is due to the better relative accuracy of the gravity field parameters of Io, which is mainly a result of larger departures from spherical symmetry.

#### 4. Spin response to orbit variations

In this section we explore the response of the spin poles of the Galilean satellites to generic variations in orbital parameters. Specifically, we note that the orbital eccentricities are all quite small ( $e \lesssim 0.01$ ) and that the spin pole evolution depends on eccentricity only via a term

$$(1 - e^2)^{-3/2} = 1 + 3/2e^2 + 5/8e^4 + \dots \quad (45)$$

We will thus ignore eccentricity in our further analysis. The most important driver of spin pole evolution is the rate and amplitude of variations in orientation of the orbit normal. The torque from Jupiter causes the spin poles of the Galilean satellites to precess about their instantaneous orbit normals. If the orbit normals were fixed in orientation, the spin pole trajectories would just be circular cones centered on the respective orbit poles. However, as the orbit poles themselves precess, the spin pole trajectories become more convoluted.

For now, we consider generic variation in the orientation of the orbit normal, and examine how the spin pole responds. The simplest approach to constructing spin pole trajectories would appear to be direct numerical integration of the equations of motion. In that case, initial conditions would be required. However, since the present spin pole positions of the Galilean satellites are not well determined, other than to indicate that they nearly coincide with their respective orbit poles, this presents a challenge. If we were to pursue the numerical integration route, we would need to examine how the derived trajectories varied as the initial conditions were allowed to span a plausible range of values.

More insight can be obtained by constructing a first order analytic model for the spin precession. It will not yield as

Table 1  
Spin precession rate parameters

Body	$n$ (deg/day)	$J_2$ ( $10^{-6}$ )	$C/MR^2$	$\alpha$ (deg/day)
Io	203.48895	$1845.9 \pm 4.2$	$0.3769 \pm 0.0004$	$1.495 \pm 0.005$
Europa	101.37472	$435.5 \pm 8.2$	$0.346 \pm 0.002$	$(1.91 \pm 0.05) \times 10^{-1}$
Ganymede	50.317608	$127.4 \pm 2.7$	$0.311 \pm 0.003$	$(3.09 \pm 0.05) \times 10^{-2}$
Callisto	21.571071	$32.7 \pm 0.8$	$0.355 \pm 0.004$	$(3.08 \pm 0.11) \times 10^{-3}$

accurate a representation of the high frequency variations as could be obtained by numerical integration, but it does relax our requirement for precise initial conditions. Similar linear analyses of spin pole precession have been constructed previously, in the context of studying obliquity variations of the Earth (Miskovitch, 1931; Sharaf and Boudnikova, 1967; Vernekar, 1972; Berger, 1976), Mars (Ward, 1973, 1992), and Venus (Ward and deCampi, 1979; Yoder, 1979, 1995, 1997).

The first step in that process is to represent the unit vectors  $\hat{s}$  and  $\hat{n}$ , which point along the spin pole and orbit pole, in terms of complex scalars  $S$  and  $N$ , by projecting each of them onto the invariable plane. In the present context, that will be approximated by Jupiter's equator plane. That is, we are ignoring the slow precession of Jupiter's spin pole, since it is much slower than the Galilean satellite precession rates. If we also ignore the variations in satellite orbital eccentricity values, the governing equation for spin pole precession can now be written in the simple linear form

$$\frac{dS}{dt} = -i\alpha(N - S). \quad (46)$$

If the orbit pole evolution is represented via the series

$$N[t] = \sum_j n_j \exp[i(f_j t + \gamma_j)] \quad (47)$$

then the corresponding solution for the spin pole can be written simply as

$$S[t] = S_{\text{free}} + S_{\text{forced}}, \quad (48)$$

where the free pole motion, which depends only on the initial condition is

$$S_{\text{free}} = S[0] \exp(i\alpha t) \quad (49)$$

and the forced motion is

$$S_{\text{forced}} = \sum_j s_j [\exp[i f_j t] - \exp[i\alpha t]] \exp[i\gamma_j] \quad (50)$$

with amplitudes given by

$$s_j = \left( \frac{\alpha}{\alpha + f_j} \right) n_j. \quad (51)$$

Each term in the series describing the orbit pole has a corresponding term in the forced spin pole series. The spin rate parameter  $\alpha$  is positive, and all of the orbit pole rates  $f_j$  are negative. If one of the sums  $\alpha + f_j$  is close to zero, then the corresponding amplitude in the spin trajectory will be amplified.

Dissipation can be easily introduced by simply making the spin precession parameter complex:  $\alpha \rightarrow \alpha + i\beta$ . When included this way, the dissipation completely damps the free term and somewhat modifies the forced terms. Assuming that the damping term is small, the resulting model for damped forced spin evolution takes the form

$$S[t] = \sum_j s_j (\exp[i f_j t]) \exp[i\gamma_j]. \quad (52)$$

The second of the terms in square brackets in the original equation for forced response is removed by dissipation. To obtain this expression, we allow a finite value of  $\beta$ , take the limit as  $t \rightarrow \infty$ , and then set  $\beta$  back to zero. It is evident that the orbit pole and spin pole trajectories are characterized by identical frequencies and phases, but different amplitudes.

This solution can be viewed as a rough generalization of the Cassini state for the case of non-uniform orbit precession. In the case of a single orbit precession frequency, the expected end-state for dissipative spin evolution is a special situation in which the obliquity has adjusted to a value at which the system maintains a constant relative geometry. That is, the spin pole and orbit pole remain co-planar with the invariable pole as the spin pole precesses about the orbit pole and the orbit pole precesses about the invariable pole (Colombo, 1966; Peale, 1969; Ward, 1975b; Henrard and Murigande, 1987).

If the orbit pole precession is not steady, no such coplanar configuration is attainable. However, the motions of the orbit and spin poles can achieve a mode-by-mode equivalent of the Cassini state. The solution above is such that each mode of the orbit pole precession, with amplitude  $n_j$ , rate  $f_j$ , and phase  $\gamma_j$ , has a corresponding mode of spin pole precession with rate and phase identical to the orbit mode values, and with an amplitude proportional to the orbit amplitude. The constant of proportionality is just the ratio  $\alpha/(\alpha + f_j)$  of the spin precession rate to the relative spin-orbit precession rate.

Since the series representing the orbit pole and spin pole are similar in form, it is not surprising that angular separation between spin pole and orbit pole has a simple expression

$$\Delta S[t] \equiv S[t] - N[t] = \sum_j \Delta s_j \exp[i(f_j t + \gamma_j)]. \quad (53)$$

The amplitude of each term is just the difference in amplitudes of the spin and orbit solutions:

$$\Delta s_j = s_j - n_j = \left( \frac{\alpha}{\alpha + f_j} - 1 \right) n_j = \left( \frac{-f_j}{\alpha + f_j} \right) n_j. \quad (54)$$

The magnitude of the phasor generated this way is the obliquity. It has the same frequencies as the orbital inclination, but different amplitudes.

The spectral admittance, or ratio of obliquity to inclination, at frequency  $f$  is just

$$\frac{\Delta s}{n} = \frac{-f}{\alpha + f}. \quad (55)$$

Written this way, it is clear that if any of the forcing frequencies are close to  $-\alpha$ , the corresponding obliquity amplitude will be large due to resonant amplification. It incorrectly implies an infinite response at the resonant frequency. To properly model the behavior in the immediate vicinity of the resonance a finite dissipation term needs to be retained. However, it will emerge that the present configuration of the Galilean satellites is such that none of the orbital periods are close enough to the spin precession periods to cause any difficulties with the linear theory.

## 5. Secular orbit model

In this section we develop a simple model of the secular variations in the orbital parameters of the Galilean satellites. In a previous section we saw that the spin pole precession rate parameters range from  $3 \times 10^{-3}$  to 1.5 degrees per day. This implies that any orbital variations which occur at substantially higher rates will not influence the spin pole and will contribute directly to the obliquity variations. Our primary interest here is in developing a simple orbital model which captures the essence of the variations which occur on time scales comparable to the spin precession rate, as those most significantly influence obliquity.

Mutual interactions between the satellites lead to very high frequency perturbations in their orbits, with some rather significant oscillations occurring with periods of only a few days (Musotto et al., 2002; Lieske, 1998; Sampson, 1921). These we will completely ignore. On time scales of several years, the distance and direction from Jupiter to the Sun and Saturn vary, and this will influence the satellite orbits. On very long time scales, the orbit of Jupiter varies and the equator plane of Jupiter precesses. Both of these processes are important for a general model of the satellite orbits but will be neglected in the present analysis, as they will contribute very little to obliquity variations.

The orbital motion of an isolated pair of spherically symmetric bodies is very simple. Each of them follows a Keplerian ellipse about their center of mass, and the trajectories can be described by 6 parameters:  $\{a, e, I, \Omega, \varpi, M\}$ , with 5 of them constant, and one of them ( $M$ ) changing at a constant rate. In this notation,  $a$  is the semimajor axis,  $e$  is the eccentricity,  $I$  is the inclination,  $\Omega$  is the longitude of the node,  $\varpi$  is the longitude of periaipse, and  $M$  is the mean anomaly. The situation for the Galilean satellites is much more complex. Each of them receives perturbations from the non-spherical mass distribution of Jupiter, the presence of the other satellites, and the distant effects from the Sun and other planets. Even in a perturbed orbit, we can still represent the instantaneous position and velocity in terms of the 6 orbital elements, but rather than have 5 of them constant, all of them will vary somewhat. We seek orbital models which represent variations in the orbital parameter pairs  $\{e, \varpi\}$  and  $\{I, \Omega\}$  which take place on time scales long compared to the unperturbed orbital period.

In order to develop an accurate model of the slow orbital variations, we would need to consider 4 primary contributions. First is the contribution from the oblate figure of Jupiter. Second is the secular interactions between the orbits, in which each body is replaced by a hoop of mass obtained via the time averaged position of the unperturbed orbital motion. Third is the influence of solar torques. Fourth is the resonant interaction between the pairs of satellites. There are 2:1 mean motion resonances between Io–Europa, and Europa–Ganymede, and a 7:3 mean motion resonance between Ganymede and Callisto.

Table 2  
Jupiter’s contribution to nodal and apsidal rates

Body	$a$ (km)	$d\varpi/dt$ (deg/day)	$d\Omega/dt$ (deg/day)
Io	421761	$1.4626 \times 10^{-1}$	$-1.4336 \times 10^{-1}$
Europa	671044	$2.8783 \times 10^{-2}$	$-2.8213 \times 10^{-2}$
Ganymede	1070370	$5.6151 \times 10^{-3}$	$-5.5039 \times 10^{-3}$
Callisto	1882600	$7.7815 \times 10^{-4}$	$-7.6274 \times 10^{-4}$

### 5.1. Jupiter oblateness

The influence of the oblate figure of Jupiter on the orbits of the satellites is quite simple to model. If we consider the first two even zonal harmonics of Jupiter,  $J_2$  and  $J_4$ , their contribution to orbital evolution can be approximated by the expressions (Greenberg, 1981)

$$\frac{d\varpi}{dt} = +n \left( \frac{3}{2} J_2 \zeta^{-2} - \left( \frac{9}{8} J_2^2 + \frac{15}{4} J_4 \right) \zeta^{-4} \right), \quad (56)$$

$$\frac{d\Omega}{dt} = -n \left( \frac{3}{2} J_2 \zeta^{-2} - \left( \frac{27}{8} J_2^2 + \frac{15}{4} J_4 \right) \zeta^{-4} \right), \quad (57)$$

where

$$\zeta = \frac{a}{R_J} \quad (58)$$

is the orbital semimajor axis normalized by the radius of Jupiter. The nodal lines of the orbits regress ( $d\Omega/dt < 0$ ) and the apsidal lines advance ( $d\varpi/dt > 0$ ). The rates are very similar, but the nodal and apsidal motions are in opposite directions. At this level of approximation, all the other orbital elements remain unchanged. As estimates for the zonal coefficients and radius of Jupiter, we use the values (Lieske, 1998)

$$\begin{aligned} R_J &= 71420 \text{ km}, \\ J_2 &= 1.48485 \times 10^{-2}, \\ J_4 &= -8.107 \times 10^{-4}. \end{aligned} \quad (59)$$

Note that, in this situation, the mean motion  $n$  and semimajor axis  $a$  are no longer functions only of the Jupiter monopole moment, via Kepler’s third law. Instead, they are related via

$$n^2 = \frac{1}{a} \frac{\partial \Phi_J}{\partial r} = \frac{Gm_J}{a^3} \left( 1 + \frac{3}{2} J_2 \zeta^{-2} - \frac{15}{8} J_4 \zeta^{-4} \right), \quad (60)$$

where  $\Phi_J$  is the gravitational potential of Jupiter. This makes relatively small changes in the semimajor axes. It amounts to of a few parts in  $10^4$  at Io and parts in  $10^5$  for the others.

The resulting apsidal and nodal rates, due to the figure of Jupiter alone, are listed in Table 2. If the oblateness contributions were dominant, then the satellite orbits would precess at uniform rates, and it would be reasonable to expect the satellite spin poles to have been driven to Cassini states (Colombo, 1966; Peale, 1969; Ward, 1975b), which means that they would precess about their respective orbit poles in such a way as to remain coplanar with the orbit pole and the spin pole of Jupiter. That was the basic assumption of the analysis by Bills and Ray (2000). However, as we will see

in the next section, mutual perturbations between pairs of satellites make the orbital motions rather unsteady.

## 5.2. Mutual orbit perturbations

In this section we consider the secular orbital perturbations due to pair-wise interactions between the satellites. For the secular variation analysis, the satellites are no longer treated as isolated point masses, but are instead averaged over their unperturbed orbital trajectories. We thus consider the response of gravitating and rotating mass hoops to their respective torques. In order to obtain a representation of the coupled behavior of this system, we first write a set of differential equations which reflect the perturbations each satellite experiences from its neighbors, and then solve this system of equations.

It would, of course, be possible to obtain solutions via numerical integration of the averaged equations of motion. However, it is much more instructive to obtain a simple analytic solution. If the perturbations are restricted to pair-wise interactions, and the expansion is limited to first order terms in the masses of the perturbing bodies, it is quite simple to obtain a solution in which the eccentricity and inclination oscillations are decoupled. In that solution, we will estimate the normal modes of oscillation of the coupled perturbations. There will be as many modes in the solution as there are satellites. The frequencies of oscillation depend only on the masses and semimajor axes of the interacting bodies. The amplitudes and phases of the oscillations are set by the initial conditions.

It will be convenient to use a new set of variables to describe the orbits. For each satellite  $j$ , we define:

$$\begin{aligned} h_j &= e_j \sin \varpi_j, \\ k_j &= e_j \cos \varpi_j, \end{aligned} \quad (61)$$

and

$$\begin{aligned} p_j &= I_j \sin \Omega_j, \\ q_j &= I_j \cos \Omega_j. \end{aligned} \quad (62)$$

In terms of these variables, the secular part of the disturbing function can be written as (Dermott and Nicholson, 1986; Murray and Dermott, 1999)

$$\mathcal{R}_j = n_j a_j^2 (\mathcal{A}_j + \mathcal{B}_j), \quad (63)$$

where  $\mathcal{A}$  and  $\mathcal{B}$  are separate matrices which account for the eccentricity and inclination effects, respectively. They have explicit forms

$$\mathcal{A}_j = \frac{1}{2} A_{jj} (h_j^2 + k_j^2) + A_{jk} (h_j h_k + k_j k_k), \quad (64)$$

$$\mathcal{B}_j = \frac{1}{2} B_{jj} (p_j^2 + q_j^2) + B_{jk} (p_j p_k + q_j q_k) \quad (65)$$

and the individual matrix elements are

$$A_{jj} = + \frac{n_j}{4} \sum_{k \neq j} \frac{m_k}{m_c + m_j} \sigma_{jk} \tau_{jk} b[3/2, 1; \sigma_{jk}], \quad (66)$$

$$A_{jk} = - \frac{n_j}{4} \frac{m_k}{m_c + m_j} \sigma_{jk} \tau_{jk} b[3/2, 2; \sigma_{jk}], \quad (67)$$

and

$$B_{jj} = - \frac{n_j}{4} \sum_{k \neq j} \frac{m_k}{m_c + m_j} \sigma_{jk} \tau_{jk} b[3/2, 1; \sigma_{jk}], \quad (68)$$

$$B_{jk} = + \frac{n_j}{4} \frac{m_k}{m_c + m_j} \sigma_{jk} \tau_{jk} b[3/2, 2; \sigma_{jk}], \quad (69)$$

where the masses of the satellites are  $m_j$  and the mass of the central body is  $m_c$ . The ratios of the semimajor axes are expressed via (Dermott and Nicholson, 1986)

$$\sigma_{jk} = \begin{cases} a_k/a_j & \text{if } a_j > a_k, \\ a_j/a_k & \text{otherwise,} \end{cases} \quad (70)$$

and

$$\tau_{jk} = \begin{cases} 1 & \text{if } a_j > a_k, \\ a_j/a_k & \text{otherwise.} \end{cases} \quad (71)$$

The Laplace coefficients are defined by the relationship

$$\frac{1}{2} b[s, r; x] = \frac{1}{2\pi} \int_0^{2\pi} \frac{\cos r\phi}{(1 - 2x \cos \phi + x^2)^s} d\phi, \quad (72)$$

where  $s$  is a half integer.

In terms of the disturbing function  $\mathcal{R}_j$ , the perturbation equations for satellite  $j$  can be written as

$$\frac{dh_j}{dt} = + \frac{1}{n_j a_j^2} \frac{\partial \mathcal{R}_j}{\partial k_j}, \quad \frac{dk_j}{dt} = - \frac{1}{n_j a_j^2} \frac{\partial \mathcal{R}_j}{\partial h_j}, \quad (73)$$

$$\frac{dp_j}{dt} = + \frac{1}{n_j a_j^2} \frac{\partial \mathcal{R}_j}{\partial q_j}, \quad \frac{dq_j}{dt} = - \frac{1}{n_j a_j^2} \frac{\partial \mathcal{R}_j}{\partial p_j}. \quad (74)$$

The solutions to these differential equations are readily obtained as a superposition of normal modes

$$\begin{aligned} h_j(t) &= \sum_{i=1}^4 e_{ji} \sin(g_i t + \gamma_i), \\ k_j(t) &= \sum_{i=1}^4 e_{ji} \cos(g_i t + \gamma_i), \end{aligned} \quad (75)$$

and

$$\begin{aligned} p_j(t) &= \sum_{i=1}^4 n_{ji} \sin(f_i t + \varphi_i), \\ q_j(t) &= \sum_{i=1}^4 n_{ji} \cos(f_i t + \varphi_i). \end{aligned} \quad (76)$$

As written here, the index  $j$  identifies the satellite and the index  $i$  corresponds to the mode of oscillation. The frequencies  $g_i$  and  $f_i$  are eigenvalues of the matrices  $\mathcal{A}$  and  $\mathcal{B}$ , respectively. Likewise, the amplitudes  $e_{ji}$  and  $n_{ji}$  are corresponding eigenvectors. The phases  $\gamma_i$  and  $\varphi_i$ , and suitable scaling of the eigenvectors are all determined from the initial conditions.

Table 3  
Orbital frequencies without oblateness

Mode	$f_i$ (deg/day)	$g_i$ (deg/day)
1	$-9.396 \times 10^{-3}$	$8.725 \times 10^{-3}$
2	$-3.365 \times 10^{-3}$	$3.156 \times 10^{-3}$
3	$-1.335 \times 10^{-3}$	$1.584 \times 10^{-3}$
4	0	$6.298 \times 10^{-4}$

Table 4  
Orbital frequencies with oblateness

Mode	$f_i$ (deg/day)	$g_i$ (deg/day)
1	$-1.34001 \times 10^{-1}$	$1.34043 \times 10^{-1}$
2	$-3.31993 \times 10^{-2}$	$3.31784 \times 10^{-2}$
3	$-6.93302 \times 10^{-3}$	$6.94117 \times 10^{-3}$
4	$-1.56144 \times 10^{-3}$	$1.62174 \times 10^{-3}$

Table 3 lists the eigenvalues for the Galilean satellite system secular variation model.

Note that in this solution, as in the case of oblateness perturbations alone, the nodal lines regress ( $f_i \leq 0$ ) and the apsidal lines advance ( $g_i > 0$ ). However, for the inclination solution, one of the frequencies is zero. This corresponds to the fact that, in the absence of oblateness effects from the central body, there is a degeneracy in that the choice of a reference plane for the orbital inclinations is arbitrary.

Comparing these rates with those from the oblate figure of Jupiter, it is clear that the oblateness effect cannot be ignored. It is a simple matter to include the secular effects of an oblate primary and the mutual orbital perturbations of the satellites. In fact, all that is required is adding some extra terms to the diagonals of the matrices  $\mathcal{A}$  and  $\mathcal{B}$ . The extra terms are (Dermott and Nicholson, 1986; Malhotra et al., 1989; Murray and Dermott, 1999)

$$\Delta \mathcal{A}_j = +\frac{1}{2}n_j^2 a_j^2 \left( \frac{3}{2}J_2 \zeta_j^{-2} - \left( \frac{9}{8}J_2^2 + \frac{15}{4}J_4 \right) \zeta_j^{-4} \right) e_j^2, \quad (77)$$

$$\Delta \mathcal{B}_j = -\frac{1}{2}n_j^2 a_j^2 \left( \frac{3}{2}J_2 \zeta_j^{-2} - \left( \frac{27}{8}J_2^2 + \frac{15}{4}J_4 \right) \zeta_j^{-4} \right) \times \sin^2 I_j. \quad (78)$$

With those additions, the orbital frequencies are quite different, as may be seen in Table 4.

Note that the modal frequencies are now close to the apsidal and nodal precession rates computed initially for the four satellites, using the oblateness of Jupiter alone. However, these are not individual satellite responses, but frequencies of the coupled modes of oscillation of the entire system. Note also that the oblateness of Jupiter has removed the degeneracy of the inclination system, as there is now a preferred orientation of the orbit planes.

### 5.3. Solar torques

In this section we consider the influence of solar torques on the orbital motions of the Galilean satellite system. The

solar torque is a relatively small perturbation to the effects already considered, but it is easy to incorporate. In the spirit of secular perturbation analysis, we consider the Sun to be a nearly circular ring of mass lying in Jupiter's orbit plane. It is thus equidistant from each of the satellites, on average, and is nearly, but not quite, in the reference equatorial plane. The secular effect of the solar torque is thus very similar to that of the oblate figure of Jupiter, but with two minor changes. The main similarity is that both torques cause the satellite orbit nodal lines to regress and apsidal lines to advance. The differences are related to the distance and direction of the source of torque. The average distance to the Sun is the same for all the satellites, so that the solar effect on each of them is rather similar, in marked contrast to the influence of Jupiter's oblate figure. The solar torque, acting alone, would make the satellite orbit poles precess about Jupiter's orbit pole, whereas the oblate figure of Jupiter would make them precess about Jupiter's spin pole.

On very long time scales ( $\gtrsim 10^4$  years), the eccentricity and inclination of Jupiter's orbit both change due to interaction with the other planets. On even longer time scales, the orientation of Jupiter's spin pole also changes due to solar torques. In that process, the solar torques on the satellite orbits play an important role, as they provide a long lever arm for the solar torque, and they are quite firmly coupled to the equator of Jupiter. We will ignore those long period effects, and use the present values of obliquity and eccentricity. However, it is worth noting that changes in those parameters will influence the satellite orbits and spin trajectories on very long time scales.

In the absence of other effects, the solar torques would make the orbit planes precess at rates which are given by

$$\frac{d\Omega_i}{dt} = \frac{H}{n_i} \quad (79)$$

with

$$H = \frac{3}{4} \left( \frac{GM_s}{b_J^3} \right) \cos \varepsilon_J = 5.1958 \times 10^{-3} \text{ (deg/day)}^2, \quad (80)$$

where  $M_s$  is the solar mass,  $\varepsilon_J$  is Jupiter's obliquity, and the semiminor axis  $b_J$  of Jupiter's heliocentric orbit is related to the eccentricity  $e$  and semimajor axis  $a$  via

$$b^2 = a^2(1 - e^2). \quad (81)$$

The nodal rate contributions, computed this way, for the Galilean satellites are  $\{-33.45, -67.16, -135.3, \text{ and } -315.6\}$  arcsec/year, for Io, Europa, Ganymede, and Callisto, respectively. The corresponding apsidal rate contributions from solar torques have the same magnitude but opposite sign

$$\frac{d\varpi_i}{dt} = -\frac{d\Omega_i}{dt}. \quad (82)$$

To properly include these effects, we add the solar torque contributions to the diagonals of the matrices  $\mathcal{A}$  and  $\mathcal{B}$ , in much the same way as the oblateness effect of Jupiter was dealt with. The resulting nodal and apsidal rates are listed in Table 5.

Table 5  
Orbital frequencies with solar torque

Mode	$f_i$ (deg/day)	$g_i$ (deg/day)
1	$-1.34029 \times 10^{-1}$	$1.34071 \times 10^{-1}$
2	$-3.32623 \times 10^{-2}$	$3.32413 \times 10^{-2}$
3	$-7.04153 \times 10^{-3}$	$7.04831 \times 10^{-3}$
4	$-1.79897 \times 10^{-3}$	$1.86081 \times 10^{-3}$

#### 5.4. Resonant interactions

In this section we briefly consider the influence of mean motion resonances in the Galilean satellite system on the orbital motion. Resonances can profoundly influence the dynamics of orbital systems, and the Galilean satellites are the best known example of such a situation. Despite that circumstance, we will argue that the mean motion resonances, important as they are for understanding the variations of eccentricity and periaapse, can be ignored in developing a simple representation of the inclinations and nodes.

The mean motions of the inner three Galilean satellites very nearly correspond to successive ratios of 2:1. The actual values are

$$\begin{aligned} n_1 - 2n_2 &= 0.739506 \text{ deg/day,} \\ n_2 - 2n_3 &= 0.739506 \text{ deg/day,} \end{aligned} \quad (83)$$

or

$$(n_1 - 2n_2) - (n_2 - 2n_3) = n_1 - 3n_2 + 2n_3 = 0. \quad (84)$$

The corresponding mean longitudes satisfy the relationship (Lieske, 1998)

$$\lambda_1 - 3\lambda_2 - 2\lambda_3 = 180^\circ + 0.064^\circ \sin\left(\frac{t - t_0}{2071 \text{ day}}\right). \quad (85)$$

That is, the mean value is  $180^\circ$  and the angle librates with a small amplitude and rather long period. The small amplitude of libration is clearly related to tidal dissipation in the system (Peale et al., 1979; Yoder and Peale, 1981; Malhotra, 1991) though exactly how the dissipation in Jupiter is balanced against dissipation in the satellites remains controversial (Greenberg, 1987; Goldstein and Jacobs, 1995; Aksnes and Franklin, 2001; Ioannou and Lindzen, 1993a, 1993b, 1994; Peale and Lee, 2002).

In addition, there is also a near resonance between Ganymede and Callisto (Lieske, 1973)

$$3n_3 - 7n_4 = -0.04467 \text{ deg/day.}$$

Though this is a fourth order resonance, the commensurability is close enough that it too has significant influence on the orbits.

A proper treatment of the secular dynamics of a system with resonances is rather complicated. The averaging involved in the standard derivation of the secular disturbing function explicitly assumes that the orbital mean motions are not commensurate. Several recent analyses have considered the influence of near commensurabilities in mean motion

Table 6  
Orbital frequencies from Lieske (1998)

Mode	$f_i$ (deg/day)	$g_i$ (deg/day)
1	$-1.32806 \times 10^{-1}$	$1.61023 \times 10^{-1}$
2	$-3.26154 \times 10^{-2}$	$4.64564 \times 10^{-2}$
3	$-7.17678 \times 10^{-3}$	$7.12408 \times 10^{-3}$
4	$-1.76018 \times 10^{-3}$	$1.83939 \times 10^{-3}$

on the secular system (Malhotra et al., 1989; Apostolos and Dermott, 1997, 1999). The primary effect is a change in the frequencies of the apsidal oscillations. Since our primary objective, at present, is to obtain a simple representation of the motions of the orbit pole, at frequencies which will influence the satellite spin poles, we can safely neglect these resonant effects.

One way to assess the error incurred by our neglect of resonances is to compare the current orbit model with a model which does include resonances. The most accurate analytic model of the Galilean satellite system, at present, is that of Lieske (1998). It is based on the extensive development by Sampson (1921), and includes a very wide range of time scales, including perturbations from the Sun and Saturn, precession of the Jupiter equator plane. Table 6 lists the frequencies from Lieske's model that correspond to the secular analysis developed above. A comparison between them and the values listed in Table 5 reveals that the inclination frequencies  $f_j$  are quite close, but that the eccentricity frequencies  $g_j$  in the two models are rather different, with Lieske's values consistently larger.

#### 5.5. Synthetic secular model

In this section we abandon our attempt to develop a secular variation model ab initio, and simply extract the low frequency components from an existing analytic model. In the previous several sections we have made successive approximations to the actual behavior of the Galilean satellite system, and have achieved fair agreement with established theories, at least in terms of the inclinations and nodes. The performance of the eccentricities and periaapses is appreciably worse. As the spin pole behavior depends more strongly on the parameter  $I$  and  $\Omega$  than on  $e$  and  $\varpi$ , it might appear that we are close to success in that regard. However, the effort required to include the resonant terms, and thereby gain full agreement, is not warranted at present.

The primary objective of the current effort is to examine the behavior of the spin poles of these bodies, and an accurate orbital model already exists. We will simply extract the low frequency part of the Galilean satellite ephemeris of Lieske (1998), with constants due to Arlot (1982), as given by Rohde and Sinclair (1992). That process is not quite as simple as it might sound, for at least two reasons. First is that Lieske, following the earlier work of Sampson (1921), represents the orbits in a cylindrical coordinate system, and what we require are amplitudes, frequencies and phases for a Poisson series representation of the slow variations in the

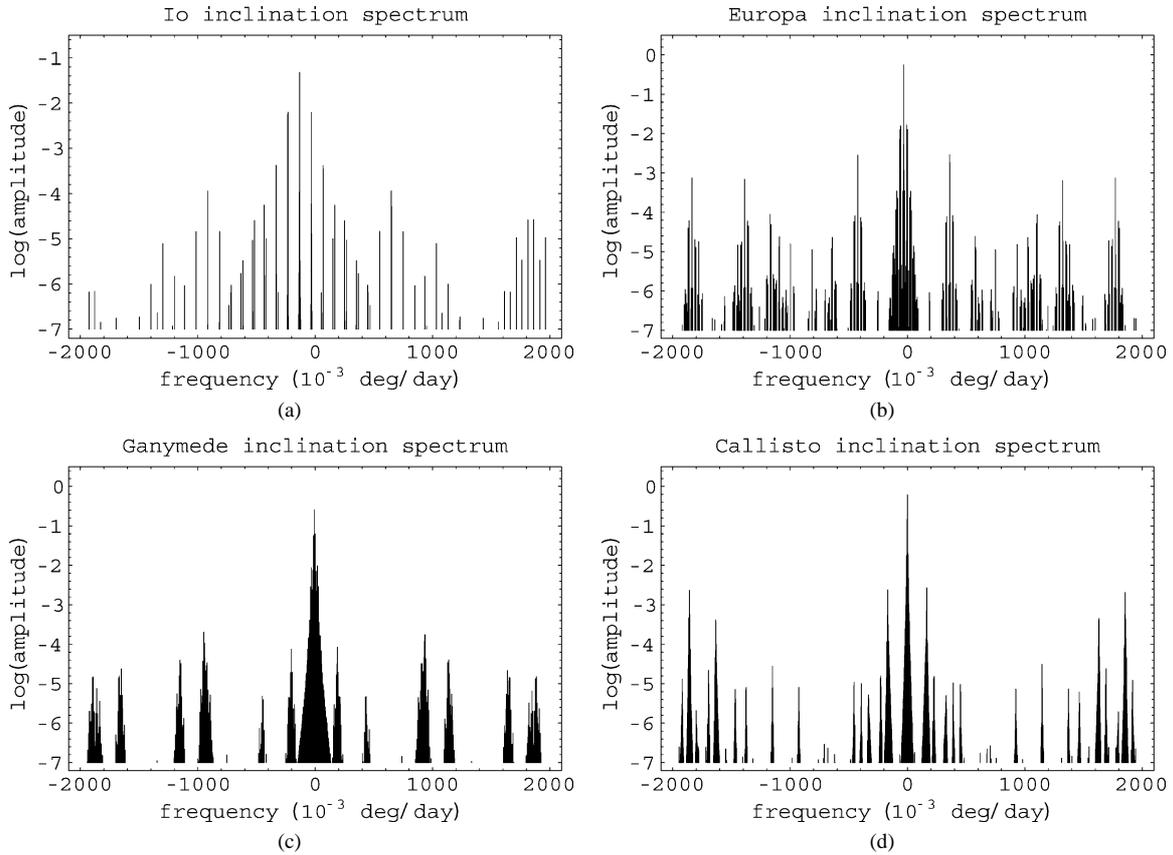


Fig. 1. (a) Inclination spectrum for Io. (b) Inclination spectrum for Europa. (c) Inclination spectrum for Ganymede. (d) Inclination spectrum for Callisto.

Keplerian element pairs  $(I, \Omega)$  and  $(e, \varpi)$ . Second is that the analytic formulation of Lieske's model involves not just trigonometric functions of time, but trigonometric functions of trigonometric functions of time.

The approach we use is similar to that employed by [Carpino et al. \(1987\)](#), [Nobili et al. \(1989\)](#) in extracting synthetic secular variation models from the results of numerical integration. The first step is to generate a list of Cartesian state vectors  $(x, y, z; dx/dt, dy/dt, dz/dt)$  for each satellite from Lieske's model. We used a time span of 6000 years, centered on the epoch of the ephemeris (JD = 3443000.5), with 0.25 year sampling. Next, each of these Cartesian state vectors is converted to a corresponding list of osculating Keplerian elements  $\{a, M, e, \varpi, I, \Omega\}$ . Finally, we estimate the amplitudes corresponding to each of the fundamental frequencies in a Poisson series representations of the coordinates  $\{h, k\}$  and  $\{p, q\}$ .

In a linear secular variation theory, there are as many frequencies as satellites. In higher order theories, many more frequencies appear. What had appeared to be isolated spectral lines, in the lowest order theory, now emerge as dense forests of multiply split lines. However, most of the side-band spacings are expected to be low order integer linear combinations of the frequencies which emerge in the linear theory.

[Figure 1](#) illustrates the inclination spectra of the Galilean satellites, as depicted in the model of [Lieske \(1998\)](#). For

each of the bodies, the dominant line in the inclination spectrum is at the corresponding secular frequency  $f_j$ . Some of the other important lines are also at secular frequencies, but many of them correspond to general terms in the disturbing function. The inclination spectra are very nearly symmetric in frequency about the dominant line.

[Figure 2](#) illustrates the obliquity spectra of the Galilean satellites, obtained from the inclination spectra via the linear mapping of [Eq. \(54\)](#). In contrast to the inclination spectra, which are quite symmetric, the obliquity spectra exhibit larger amplitudes near the frequency at which resonant amplification occurs.

[Figure 3](#) shows time series of the variations in the scalar quantities  $I$  and  $\varepsilon$ , for each of the satellites. The time spans illustrated are different for each satellite since the dominant frequencies decrease with increasing distance from Jupiter. Note that for Io and Europa, the obliquity values are considerably smaller than the inclination values. In both of these cases, the inclinations are nearly constant ( $(4.1 \pm 0.73) \times 10^{-2}$  degree for Io, and  $(4.68 \pm 0.23) \times 10^{-1}$  degree for Europa) and the spin pole precession rate parameter  $\alpha$  is large enough that the spin pole can easily keep pace with most of the motions of the orbit pole. As a result, the obliquity values are small and nearly constant ( $(4.05 \pm 0.76) \times 10^{-3}$  degree for Io, and  $(9.65 \pm 0.69) \times 10^{-2}$  degree for Europa).

For Ganymede and Callisto, the situation is somewhat more complex. For Ganymede, the range of inclination val-

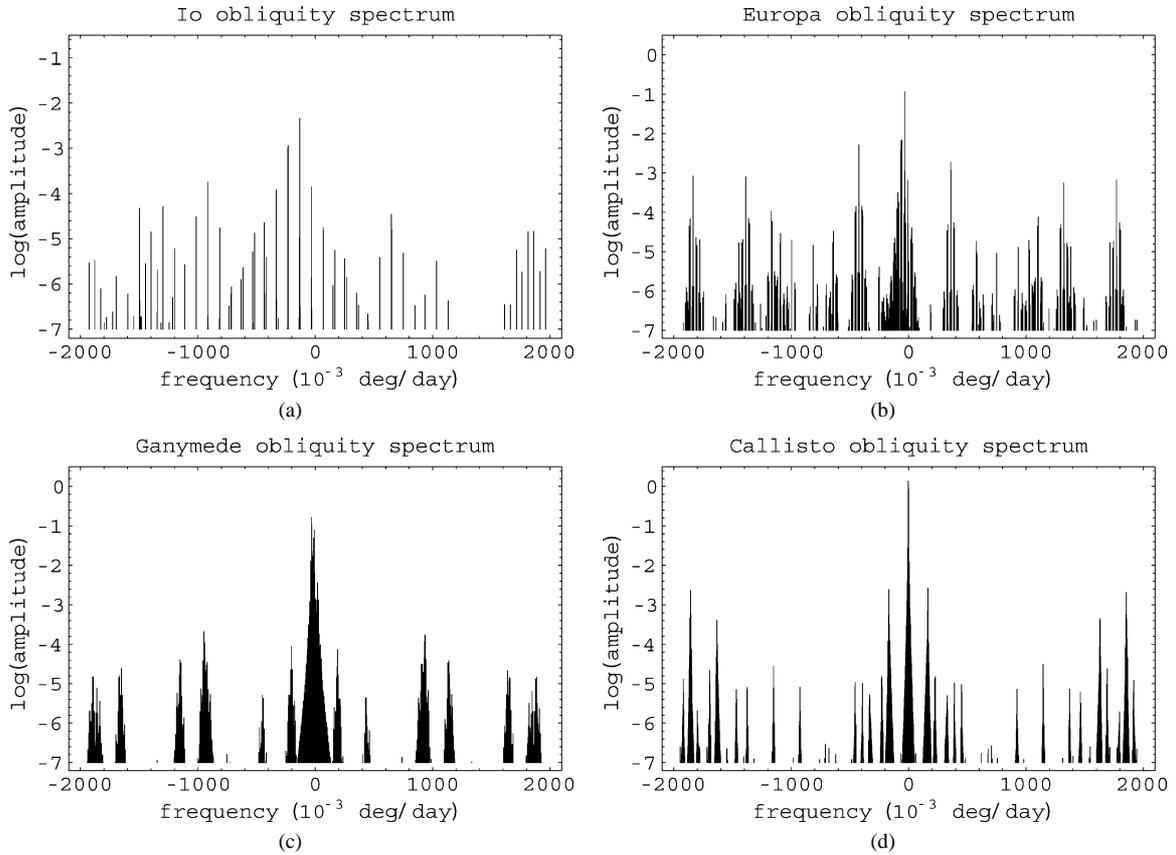


Fig. 2. (a) Obliquity spectrum for Io. (b) Obliquity spectrum for Europa. (c) Obliquity spectrum for Ganymede. (d) Obliquity spectrum for Callisto.

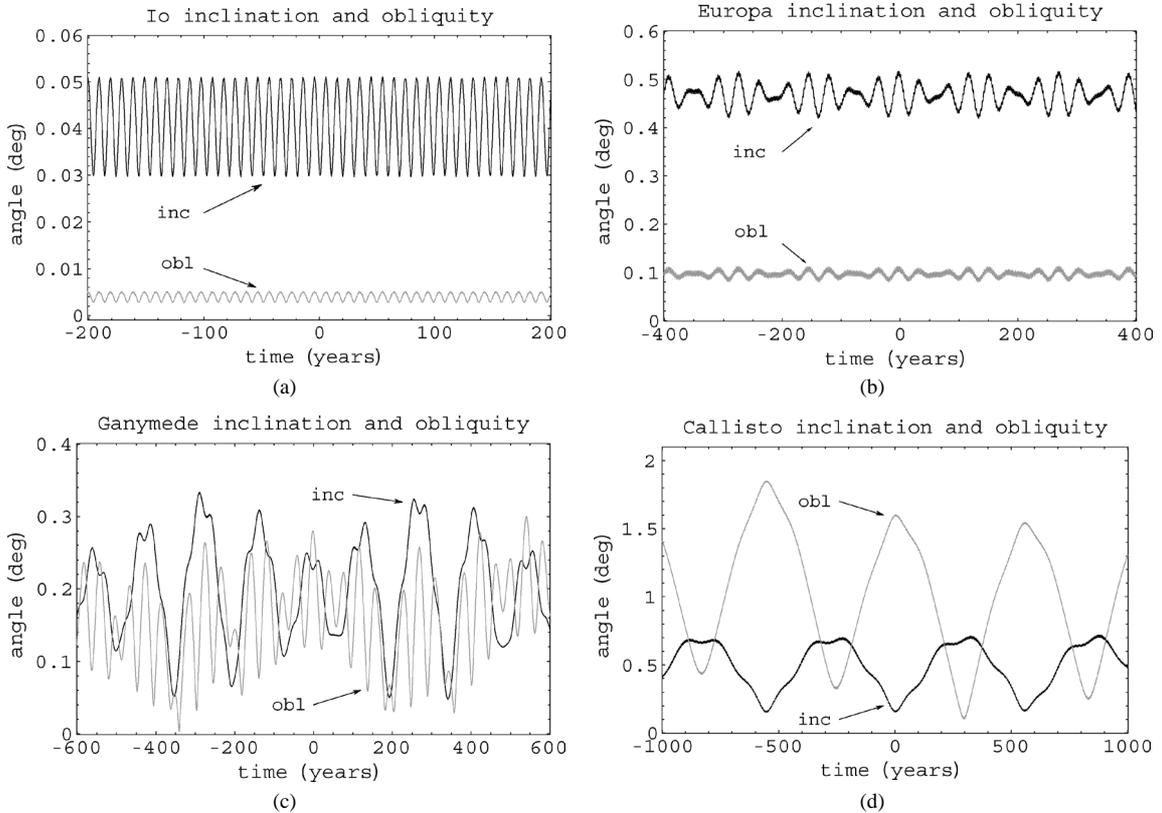


Fig. 3. (a) Obliquity time series for Io. (b) Obliquity time series for Europa. (c) Obliquity time series for Ganymede. (d) Obliquity time series for Callisto.

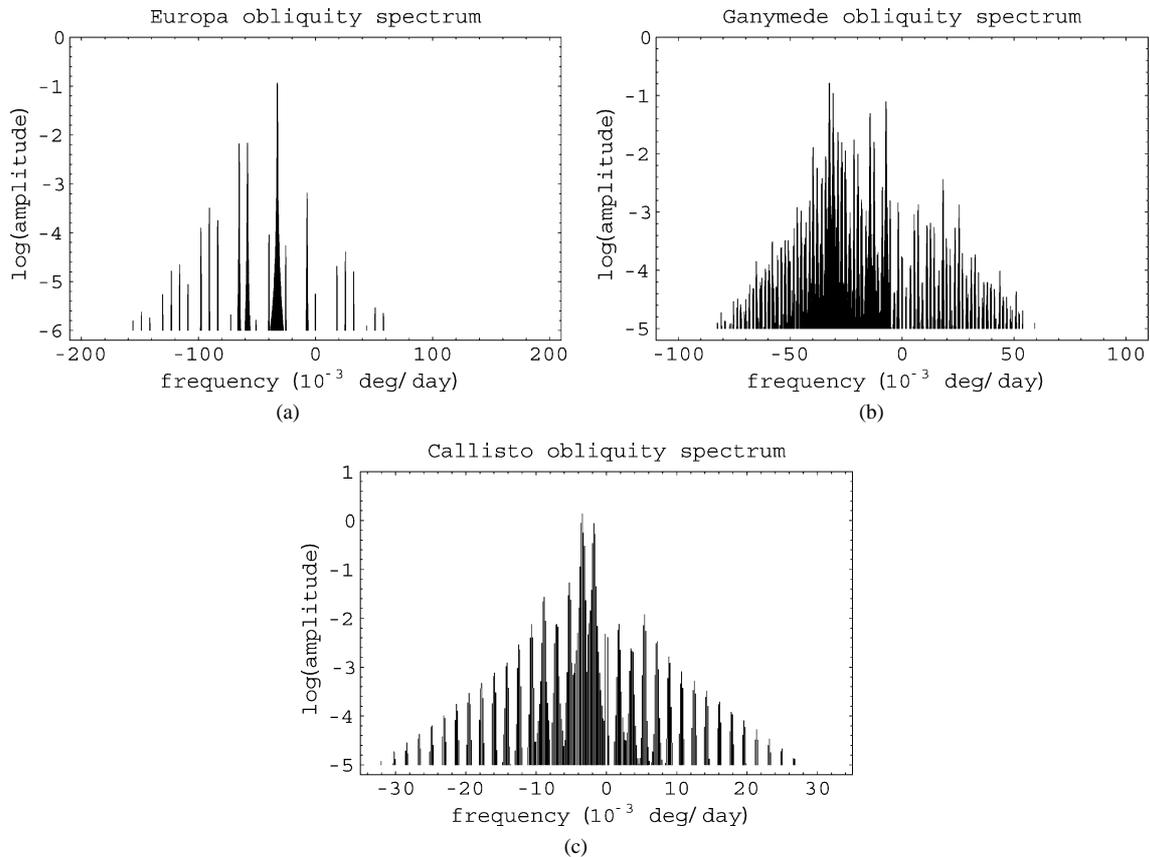


Fig. 4. (a) High resolution obliquity spectrum for Europa. (b) High resolution obliquity spectrum for Ganymede. (c) High resolution obliquity spectrum for Callisto.

ues is quite large  $(2.04 \pm 0.74) \times 10^{-1}$  degree and the obliquity values cover nearly an equal span  $(1.55 \pm 0.65) \times 10^{-1}$  degree, though with higher mean frequency of oscillation in obliquity than in inclination. The most extreme case is Callisto, where the inclination oscillates from  $0.15^\circ$  to  $0.70^\circ$  with a 580 year period, and the obliquity spans a range 3–4 times as large, with the same dominant period.

Figure 4 illustrates the asymmetry of the obliquity spectra of Europa, Ganymede, and Callisto, by focusing on a lower frequency range than is easily resolved in Fig. 2. For Io and Europa, the largest amplitude obliquity variation occurs at the same frequency as the largest amplitude inclination variation. However, for Ganymede and Callisto, the largest obliquity effect arises from resonant amplification of smaller terms in the inclination series.

## 6. Discussion

We have developed a simple model for the orbital and rotational precession trajectories of the Galilean satellites, and have shown that the forced obliquities of these bodies are non-zero. In fact, for all of these bodies, the forced obliquities are non-negligible in comparison to the forced eccentricity and inclination variations. For Io and Europa, due to their proximity to Jupiter, the spin pole precession

rates are fast enough that the obliquity variations are rather smaller than the inclination variations. For Ganymede and Callisto, the obliquity variations are actually comparable to the inclination variations.

We have deliberately used a very simple model of the orbital motion to illustrate the basic principles upon which the spin model is based. A better model of the spin pole motion could be rather easily produced by using a better model of the orbital motion. In particular, use of numerically integrated orbits, like that of Musotto et al. (2002), appears very promising. However, we anticipate that the basic conclusions would remain unchanged. That is, we expect that the forced obliquity of Io will be small, and for the other bodies the forced obliquities will be comparable to the orbital inclination variations.

Our analysis essentially assumes that tidal dissipation has driven the free obliquities of all four bodies to vanishingly small values. Sufficiently accurate monitoring of the spin pole orientations could conceivably reveal departures from this situation, as is the case for Venus (Yoder, 1997). In the Galilean satellite case, a likely source of excitation of free obliquities would be impacts of comets or asteroids (Peale, 1975, 1976).

Perhaps the most significant implication of our analysis is that it reveals an additional source of tidal stress and dissipation within the Galilean satellites. If the eccentricity and

obliquity were both zero, the tides raised by Jupiter on the satellites would be large but stationary. A finite orbital eccentricity causes the tidal bulge to librate in longitude, yielding time dependent stress and potentially significant heating. A finite obliquity causes the tidal bulge to librate in latitude. The global average rates of tidal heating from longitudinal and latitudinal librations both depend on the internal structure of the body, and do so in exactly the same way. The spatial patterns of heating from these sources are different, and the rates scale with  $e^2$  and  $\sin^2 \varepsilon$ , respectively.

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