MAGNETIZED PLASMA WITH FERROMAGNETIC GRAINS AS A VIABLE NEGATIVE REFRACTIVE INDEX MEDIUM

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ABSTRACT

The propagation of electromagnetic waves in a cold magnetized plasma with ferromagnetic grains (MPFG) in the high frequency domain is studied theoretically. The dispersion of MPFG which is controlled by the simultaneous characterization of the permittivity and permeability tensors. is investigated theoretically and numerically near the resonance frequency. It is found that MPFG becomes transparent for the waves that cannot propagate in conventional magnetized electron-ion plasma. The refractive index of the waves propagating parallel to the applied magnetic field is found to be negative for the extraordinary wave in certain frequency domain. The results obtained show that in a narrow band of the super-high-frequency range near the electron cyclotron frequency, MPFG possess all the known characteristics of negative refractive index media, which would make it as a viable alternative medium to demonstrate the known and predicted peculiar properties of media having negative index of refraction.

KEYWORDS

Magnetized Plasma, Ferromagnetic grains, Negative-Refractive Index, Left-handed media & Metamaterials.

1. INTRODUCTION

In his pioneering theoretical work, Veselago investigated the electrodynamics of media with simultaneous negative values of the permittivity ε and permeability μ. His analysis showed that for Maxwell's equations to be valid in such media, the corresponding index of refraction $n = \pm(\varepsilon\mu)^{1/2}$ must be negative, and referred the media as left-handed which is synonymous with negative refractive-index media (NRM). These media are expected to give rise to unprecedented characteristics to the propagation of EMWs, such as the reversal of the Doppler effect, the reversal of the Vavilov-Chernkov effect and anomalous (negative) refraction. [1]

Until recently, NRM and the associated peculiar properties are predicted through the application of Maxwell's equations but never observed experimentally because no natural materials had such properties [1]. However, in the late 1990s, following the proposal made by Pendry, et al. [2], Smith and co-workers reported the fabrication of "artificial" metamaterials composed of an array of conducting nonmagnetic split-ring resonators and continuous thin wires to make both the effective permittivity and permeability simultaneously negative so that EMW propagation is made possible in some special direction and special polarization at microwave frequencies [3, 4]. After this initial demonstration of the first metamaterial at microwave frequencies, various theoretical and experimental methods and materials that can be suitable to realize negative refractive index at higher microwaves and optical frequencies have been reported [5 - 10].
Metamaterials fabricated from chiral and anisotropic materials are widely used in an attempt to realize NRM. For anisotropic materials, the quantities ε and μ are tensors. In particular, for special types of anisotropic materials known as gyrotropic materials, when an external magnetic field is applied in a certain direction.

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The medium discussed in this paper consists of conventional magnetized electron-ion plasma with the addition of the ferrite grains that are embedded uniformly in the plasma. The MPFG medium is assumed to be a homogeneous medium which may be artificially fabricated from magnetized electron-ion plasma with the addition of ferrite grains. Hence. the MPFG is essentially a bi-anisotropic medium in a sense that both the permittivity and permeability are second-rank tensors. In principle, such type of "composite" medium with its dispersion properties controlled by the simultaneous specification of the permittivity and permeability tensors can easily be fabricated in laboratory by incorporating ferromagnetic grains in a magnetized electron-ion plasma. The resulting system, which is composed of the electric (magnetized plasma) and magnetic (ferrite grains) subsystems, is named as magnetized plasma with ferrite grains (MPFG). In such media the effective permittivity and permeability have the form $\varepsilon = \varepsilon_1 \pm \varepsilon_2$ and $\mu = \mu_1 \pm \mu_2$ where ε_1 and μ_1 are the diagonal components whereas ε_2 and μ_2 are of the off-diagonal components of the permittivity and the permeability tensors, respectively. In certain frequency range ε and μ may be simultaneously negative while the refractive index $n = \pm(\varepsilon\mu)^{1/2}$ is still real so that EMWs can propagate in the medium in the given frequency range. Moreover, the refractive index the MPFG may be negative for certain frequency domain in vicinity of the resonant frequencies.

The paper is organized as follows: In sections 2.1, the propagation of EMWs in a material medium is discussed. In section 2.2, the propagation of EMWs in magnetized plasma with ferrite grains is considered. The condition under which negative refractive index in the MPFG can be obtained is discussed in section 2.3. The results and discussion are presented in section 2.4. Finally, in section 3 concluding remarks are given.

2. PROPAGATION OF EMWS IN MPFG

2.1. The Dispersion Relation in Anisotropic Medium

The propagation of electromagnetic waves in a medium is described by the Maxwell's curl equations $\nabla\times\vec{E} = -(\partial\vec{B}/c\partial t)$ and $\nabla\times\vec{H} = (\partial\vec{D}/c\partial t)$, and the constitutive relations, $D = \varepsilon E$, and $B = \mu H$. For a plane EMW, whose electric and magnetic fields are proportional to $\exp[i(\vec{k}\cdot\vec{r} - \omega t)]$, Maxwell's equations may be rewritten as $\vec{B} = c(\vec{k}\times\vec{E})/\omega$ and $\vec{D} = -c(\vec{k}\times\vec{H})/\omega$. Making use of the definition $\vec{n} = (c/\omega)\vec{k}$, where its magnitude is equal to the refractive index the medium, manipulating these equations, we get the following system of equation:

$$(n^2\delta_{ij} - n_i n_j - \mu_{i\ell}\varepsilon_{\ell j})E_i = 0. \tag{1}$$

This equation has nontrivial solution if the determinant of the coefficients vanish. That is,

$$\left| n^2\delta_{ij} - n_i n_j - \mu_{i\ell}\varepsilon_{\ell j} \right| = 0, \tag{2}$$

which is the general dispersion relation.For an anisotropic medium the permittivity and the permeability in the constitutive relations are tensors. In particular, for a magnetized plasma where the direction of the external magnetic field acting on the plasma and the orientation of the magnetization of the ferrite grains both assumed to be in the z-direction, the permittivity, $\hat{\varepsilon}$, and permeability, $\hat{\mu}$, tensors will have the form [11]

$$\hat{\varepsilon} = \begin{pmatrix} \varepsilon_1 & i\varepsilon_2 & 0 \\ -i\varepsilon_2 & \varepsilon_1 & 0 \\ 0 & 0 & \varepsilon_3 \end{pmatrix}, \qquad \hat{\mu} = \begin{pmatrix} \mu_1 & i\mu_2 & 0 \\ -i\mu_2 & \mu_1 & 0 \\ 0 & 0 & \mu_3 \end{pmatrix}.$$

2.2. Permittivity and Permeability of MPFG

Consider magnetized electron-ion plasma with ferromagnetic grains (MPFG) in the presence of an external constant magnetic field $\mathbf{H}_0$ and relatively weak time-varying field $\mathbf{H}(t)$. The permittivity tensor of the plasma is well known and we take it according to the model of cold magnetized plasma. Choosing the z-axis along the external magnetic field, the nonzero components of the high frequency permittivity tensor components can be shown to have the form [7]:

$$\varepsilon_1 = 1 - \frac{\Omega_e^2}{\omega^2 - \omega_C^2}, \qquad \varepsilon_2 = \frac{\omega_C}{\omega}\frac{\Omega_e^2}{\omega^2 - \omega_C^2}, \qquad \varepsilon_3 = 1 - \frac{\Omega_e^2}{\omega^2}, \tag{3}$$

where $\Omega_e = \sqrt{(4\pi e^2 n_e)/m}$ and $\omega_C = eH_0/(mc)$ are the plasma frequency and the electron cyclotron frequency, respectively, n_e is the density number of the electrons, c is the speed of light in vacuum, and m is the mass of an electron.

In the presence of the external constant magnetic field $\boldsymbol{H_0}$ and the relatively weak time-varying field $\boldsymbol{H(t)}$, magnetic dipole moments are induced in the MPFG and form a magnetic subsystem. As a consequence, the subsystem possesses variable magnetization that results from the interaction of the magnetic dipoles with the variable electromagnetic field. The permeability of the system can be obtained by analyzing the motion of the magnetization vector of the ferrite grains in those fields. In particular, for spherical and homogeneously magnetized ferrite grains, the nonzero components of the permeability tensor of the grain's subsystem can be shown to have the form [8]:

$$\mu_1 = 1 - \xi\frac{\omega_M \omega_H}{\omega^2 - \omega_H^2}, \qquad \mu_2 = \xi\frac{\omega\omega_M}{\omega^2 - \omega_H^2}, \qquad \mu_3 = 1, \tag{4}$$

where $\omega_M = (16\pi^2 a^3 N_g \chi\omega_H)/3$ with a the grain's radius, N_g is the density of the grains, χ is the static magnetic susceptibility, and for the MPFG system we assumed $\omega_H = \omega_C$. These components of the permittivity and permeability tensors will be utilized in the next sections, to describe the propagation of electromagnetic waves and pulses in MPFG.

2.3. Negative Refractive Index in MPFG

Consider a monochromatic plane EMW propagating along the applied magnetic field $\mathbf{H_0}$ with the wave vector oriented along the z-axis, i.e., $\vec{k} = k\hat{z}$. Employing the dispersion relation (2), the refractive index of the MPFG takes the form

$$n_\pm^2 = (\varepsilon_1 \pm \varepsilon_2)(\mu_1 \pm \mu_2), \tag{5}$$

where ± represent ordinary and extraordinary waves, respectively. This result coincides with the known result for the magnetized electron-ion plasma, if we put $\mu_1 = 1$ and $\mu_2 = 0$. It is obvious that EMWs can propagate in a medium when $n^2 = \varepsilon\mu > 0$, whereas it becomes nontransparent when $n^2 = \varepsilon\mu < 0$; where ε and μ are the effective values.

For the ordinary wave, (5) with account of (3) and (4),we get

$$n_+^2 = \left[1 - \frac{\Omega_e^2}{\omega(\omega + \omega_C)}\right]\left[1 + \frac{\xi\omega_M}{\omega + \omega_C}\right], \tag{6}$$

and for the extraordinary wave, we obtain

$$n_-^2 = \left[1 - \frac{\Omega_e^2}{\omega(\omega - \omega_C)}\right]\left[1 - \frac{\xi\omega_M}{\omega - \omega_C}\right]. \tag{7}$$

From (6), we see that the ordinary wave propagates for frequencies $\omega > (-\omega_C + \sqrt{\omega_C^2 + 4\Omega_e^2})/2$. For frequencies less than this value, the effective permittivity $\varepsilon = \varepsilon_1 + \varepsilon_2$ of the MPFG is negative while the effective permeability $\mu = \mu_1 + \mu_2$ is still positive, so that the MPFG is nontransparent to the ordinary wave. For this wave the medium behaves only as ordinary (RHM) medium over all frequencies.

Let us introduce the dimensionless parameters $x = \omega / \omega_C$, $\alpha_e = \Omega_e / \omega_C$ and $\alpha_M = \xi\omega_M / \omega_C$. Setting $n_- \equiv n$ for convenience, (7) becomes $n^2 = \varepsilon\mu$, where the effective permittivity and the effective permeability are given by

$$\varepsilon = \varepsilon_1 - \varepsilon_2 = 1 - \frac{\alpha_e^2}{x(x-1)}, \qquad \text{and} \qquad \mu = \mu_1 - \mu_2 = 1 - \frac{\alpha_M}{x-1}. \tag{8}$$

Inserting (8) into (7), we obtain the refractive index to be

$$n(x) = \pm\sqrt{\left[1 - \frac{\alpha_e^2}{x(x-1)}\right]\left[1 - \frac{\alpha_M}{x-1}\right]} \tag{9}$$

where the "+" sign is taken when ε and μ are simultaneously positive and the medium behaves as RHM; whereas the "-" sign is taken when ε and μ are simultaneously negative and the medium behaves as NRIM.

The condition $n^2 > 0$ may be attained when either for $\varepsilon, \mu > 0$ or $\varepsilon, \mu < 0$, simultaneously. For the former case the medium behaves as right-hand medium (RHM), referring to the relative orientation of the wave vector (**k**), electric field (**E**) and magnetic field (**H**) of the incident EMW which forms a right-handed triplets; whereas in the later case the medium behaves as left-handed medium (LHM), since the three vectors **k**, **E**, and **H** forms a left-handed triplets. Commonly LHM are also referred as negative refractive index medium (NRIM) or backward waves.

The main focus of the paper is the propagation of the extraordinary wave in the MPFG medium where the medium behave as RHM as well as LHM in different frequency domains. In the next sections, we consider the propagation of the extraordinary wave in the MPFG, in more detail.

2.4. Results and Discussions

For cold plasma, typical values of the plasma and the cyclotron frequencies are of the order of $\Omega_e \sim 2\pi$ (10 GHz) and $\omega_c \sim 2\pi$ (1GHz). The parameter $\alpha_M = (16\pi^2/3)\, a^3 N_g \chi_0$ strongly depends on the grain's radius a. For spherical grains of radius $a \approx 4 \times 10^{-4}$ cm, density number $Ng \approx 2.5 \times 10^5$ cm^{-3} and magnetic susceptibility $\chi_0 \approx 5 \times 10^3$, we get $\alpha_M \approx$ 0:85. For the sake of numerical evaluations we choose, $\omega_c = 2\pi$ (8 GHz), $\alpha_e = 2\pi$ (16 GHz) and the corresponding values of α_e = 2.00 and α_M = 0.85.

Figure 1 is the plot of the effective permittivity ε and the effective permeability μ of the MPFG versus the dimensionless frequency x in the range $0 < x < 3.5$. It is seen that for $0 < x < 1$ and $x > 2.56$, the permittivity is positive. This is consistent with the known result for the electron-ion plasma in an external field in the absence of the ferrite grains, where $\alpha_M = 0$. In the interval $1 < x < 2.56$ the permittivity is negative. In this case, the refractive index becomes purely imaginary and the MPFG is nontransparent to EMWs. The interval where ε is negative can be controlled by varying the density number of the electrons (i.e., the temperature) in the plasma.

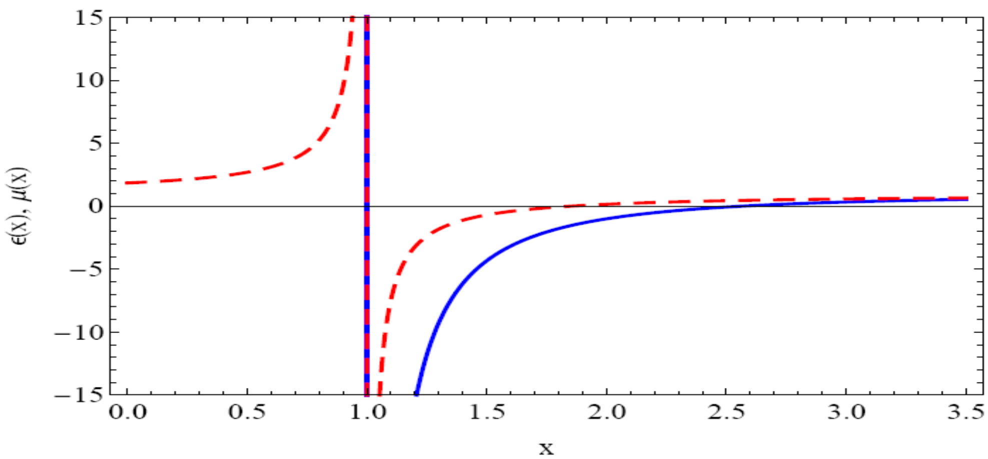

Figure 1: The effective permittivity (solid line) and permeability (dashed line) of MPFG versus the dimensionless frequency (x) for $\alpha_e = 2.00$ and $\alpha_M = 0.85$. (Color on line)

Moreover, Fig. 1 shows that in the frequency interval $0 < x < 1$ and $x > 1.85$, the permeability is positive whereas in the interval $1 < x < 1.85$ the permeability is negative. For RHM, the value of μ is positive and in most cases it is equal to one. But the permeability of the ferrite grains that are incorporated in the plasma is found to be negative in certain frequency range as shown in Fig. 1. This frequency domain completely lies the region where $\varepsilon < 0$. It is the presence of this overlapping frequency domain where ε and μ are simultaneously negative which makes MPFG an interesting, potentially viable alternative NRIM that can be employed to demonstrate the peculiar characteristics predicted in such media.

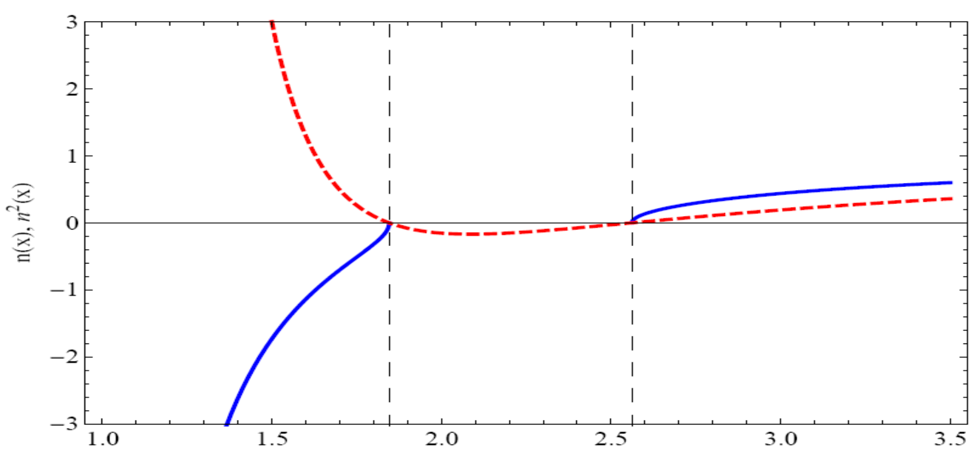

Figure 2: The refractive index (solid line) and its square (dashed line) versus the dimensionless frequency (x) of the MPFG for $\alpha_e = 2.00$ and $\alpha_M = 0.85$. (Color on line)

The graph of the refractive index *n(x)* and the square of the refractive index $n^2(x)$ for the extraordinary wave versus the relative frequency x is shown in Fig. 2, for $\alpha_e = 2.00$ and $\alpha_M = 0.85$. It is seen that in the frequency domains $0 < x < 1.85$ and $x > 2.56$, $n^2(x)$ is positive and the MPFG is transparent to EMWs, whereas in the domain $1.85 < x < 2$, $n^2(x)$ is negative and the medium is nontransparent to EMWs. The graph of *n(x)* versus x shows that in the frequency interval $1 < x < 1.85$, the refractive index is negative and consequently the medium behaves as a LHM, in this frequency domain. Note that in the region $1 < x < 1.85$, ε and μ are simultaneously negative as shown in Fig. 1, so that the refractive index $n(x) = \pm (\varepsilon\mu)^{1/2}$ is negative but real. Note that the conventional magnetized plasma is nontransparent in this frequency domain. However, the addition of the ferrite grains to the plasma makes it possible for EMWs to propagate in the combined system - MPFG. In the other frequency domains $0 < x < 1$ and $x > 2.56$, the refractive index is positive and the medium behaves as RHM. However, for $1.85 < I < 2.56$, $\varepsilon > 0$; $\mu < 0$ and the corresponding refractive index $n(x) = \pm (\varepsilon\mu)^{1/2}$ is purely imaginary - the MPFG is nontransparent to EMWs.

The extent of the nontransparent region is a function of the parameters of the electric and magnetic subsystem. It may be varied (tuned) by appropriate choice of the values of the temperature of the MPFG (the density number of the plasma), the radius, or/and the density number of the grains. Below, we show the case where the nontransparent frequency domain observed in Fig. 2 is completely eliminated.

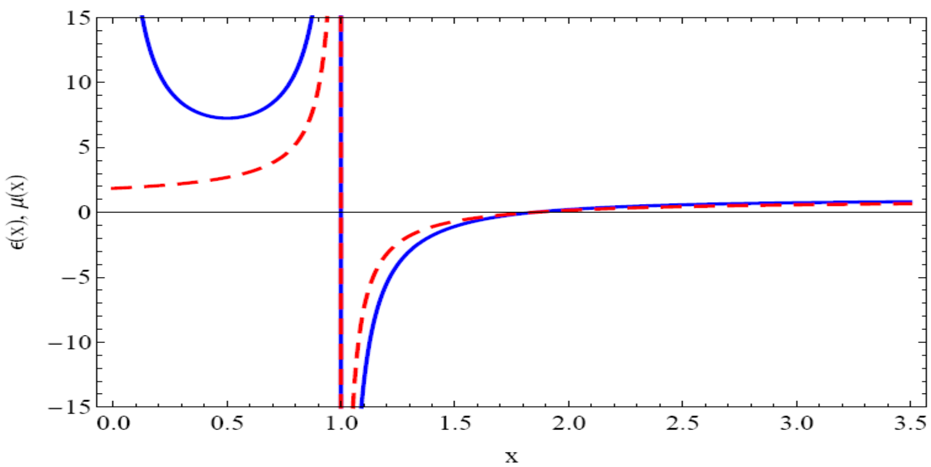

Figure 3: The effective permittivity (solid line) and permeability (dashed line) of MPFG versus the dimensionless frequency (x) for $\alpha_e = 1.25$ and $\alpha_M = 0.85$. (Color on line)

Suppose that the MPFG system is tuned in such a way that $\alpha_e = 1.25$, keeping $\alpha_M = 0.85$ as in the previous case. Figure 3 is the plot of the effective permittivity ε and the effective permeability μ of the MPFG versus the dimensionless frequency x, for the tuned parameters $\alpha_e = 1.25$ and $\alpha_M = 0.85$ in the range $0 < x < 3.5$. It is seen that for $0 < x < 1$ and $x > 1.85$, the permittivity is positive; whereas in the interval $1 < x < 1.85$ the permittivity is negative. Similarly, Fig. 3 shows that in the frequency interval $0 < x < 1$ and $x > 1.85$, the permeability is positive whereas in the interval $1 < x < 1.85$ the permeability is negative. It is observed that the frequency domain where ε is negative completely coincides (overlaps) with that where $\mu < 0$. This overlapping frequency domain where ε and μ are simultaneously negative enables the medium to be transparent to EMWs over the whole frequency domain. However, the mode of propagation is different in different frequency domains with the medium behaving as RHM as well as NRIM.

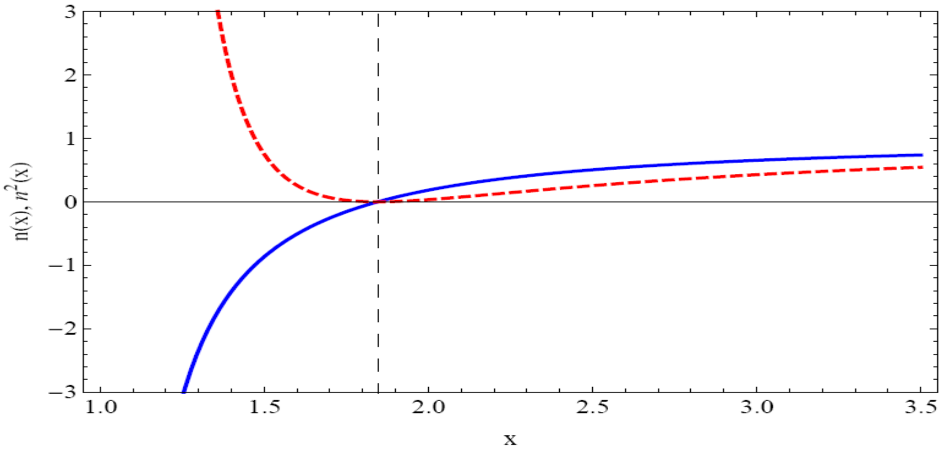

Figure 4: The refractive index and its square versus the dimensionless frequency (x) of the MPFG for α_e = 1.25 and α_M = 0.85.

The corresponding graph of the refractive index n(x) and the square of the refractive index $n^2(x)$ of the extraordinary wave versus the relative frequency for α_e = 1.25 and α_M = 0.85 is shown in Fig. 4. The graph (solid line) shows that $n^2(x)$ is positive over all frequencies. It means that the MPFG is transparent to EMWs over the entire frequency range. Also, the plot of n versus x depicts that in the frequency domain $1 < x < 1.85$, where the effective permittivity and permeability are simultaneously negative, the refractive index is negative and consequently the medium behaves as LHM. In the other hand, for the frequency domain $x > 1.85$, both ε and μ are simultaneously positive and hence the refractive index is positive so that the medium behaves as RHM. Unlike the previous cases, i.e., Fig. 2, here the nontransparent frequency domain is completely eliminated enabling the MPFG to be transparent to EMWs over the entire frequency.

3. CONCLUSIONS

In this work we showed that MPFG behaves as LHM in certain frequency range. In the MPFG, the plasma provides negative permittivity whereas the ferrite grains provide negative permeability in the range $\omega_c < \omega < 1.85\omega_c$. In particular, for $\alpha_M = 0.85$ and $\alpha_e = 2$, $\varepsilon < 0$ in the frequency range $\omega_c < \omega < 2.56\omega_c$ and $\mu < 0$ in the range $\omega_c < \omega < 1.85\omega_c$. It means that ε and μ are simultaneously negative in the range $\omega_c < \omega < 1.85\omega_c$ giving rise to negative refractive index in the given region. In the range $1.85\omega_c < \omega < 2.56\omega_c$, $\varepsilon < 0$, $\mu > 0$ and n becomes purely imaginary so that the medium is nontransparent to EMWs. For large frequencies ($\omega > 2.56$), $n(\omega) > 0$, and the MPFG behaves as RHM.

The nontransparent region is mainly a function of the parameters of the grain subsystem. By appropriately selecting (tuning) the parameter α_e, while keeping α_M constant the nontransparent domain can be completely eliminated. With α_M = 1.25 and α_e = 0.85, the MPFG is made transparent to EMWs over the entire frequency ranges. Thus, we conclude that in the MPFG negative refractive index can be attained in a certain frequency range and as a result the MPFG behaves as left-handed medium in the SHF (i.e., in the microwave) region.

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