

Rayleigh Diagnostic Values

Nick Featherstone

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1 Overview of Diagnostic Outputs in Rayleigh

The purpose of this document is to describe Rayleigh’s internal menu system used for specifying diagnostic outputs. Rayleigh’s design includes an onboard diagnostics package that allows a user to output a variety of system quantities as the run evolves. These include system state variables, such as velocity and entropy, as well as derived quantities, such as the vector components of the Lorentz force and the kinetic energy density. Each diagnostic quantity is requested by adding its associated menu number to the *main_input* file. Radial velocity, for instance, has menu code 1, θ -component of velocity has menu code 2, etc.

A few points to keep in mind are

- **This document is intended to describe the diagnostics output menu only.** A complete description of Rayleigh’s diagnostic package is provided in Rayleigh/doc/Diagnostic_Plotting.pdf. A more in-depth description of the anelastic and Boussinesq modes available in Rayleigh is provided in Rayleigh/doc/user_guide.pdf.
- A number of *output methods* may be used to output any system diagnostic. No diagnostic is linked to a particular *output method*. The same diagnostic might be output in volume-averaged, azimuthally-averaged, and fully 3-D form, for instance.
- You may notice a good deal of redundancy in the available outputs. For instance, the azimuthal velocity, v_ϕ , and its zonal average, $\overline{v_\phi}$, are both available as outputs. Were the user to output both of these in an azimuthally-averaged format, the result would be the same. 3-D output, however, would not yield the same result. This redundancy has been added to help with post-processing calculations in which it can be useful to have all data products in a similar format.
- Given the degree of redundancy found in the list below, you may be surprised to notice that several values are not available for output at all. Some of these are best added as custom-user diagnostics and may be included in a future release. Many, however, may be obtained by considering either the sum, or difference, of those outputs already available.

2 Definitions and Conventions

2.1 Vector and Tensor Notation

All vector quantities are represented in bold italics. Components of a vector are indicated in non-bold italics, along with a subscript indicating the direction associated with that component. Unit vectors are written in lower-case, bold math font and are indicated by the use of a *hat* character. For example, a vector quantity \mathbf{a} would be represented as

$$\mathbf{a} = a_r \hat{\mathbf{r}} + a_\theta \hat{\boldsymbol{\theta}} + a_\phi \hat{\boldsymbol{\phi}}. \quad (1)$$

The symbols $(\hat{\mathbf{r}}, \hat{\boldsymbol{\theta}}, \hat{\boldsymbol{\phi}})$ indicate the unit vectors in the (r, θ, ϕ) directions, and (a_r, a_θ, a_ϕ) indicate the components of \mathbf{a} along those directions respectively.

Vectors may be written in lower case, as with the velocity field \mathbf{v} , or in upper case as with the magnetic field \mathbf{B} . Tensors are indicated by bold, upper-case, script font, as with the viscous stress tensor $\boldsymbol{\mathcal{D}}$. Tensor components are indicated in non-bold, and with directional subscripts (i.e., $\mathcal{D}_{r\theta}$).

2.2 Reference-State Values

The *hat* notation is also used to indicate reference-state quantities. These quantities are scalar, and they are not written in bold font. They vary only in radius and have no θ -dependence or ϕ -dependence. The reference-state density is indicated by $\hat{\rho}$ and the reference-state temperature by \hat{T} , for instance.

2.3 Averaged and Fluctuating Values

Most of the output variables have been decomposed into a zonally-averaged value, and a fluctuation about that average. The average is indicated by an overbar, such that

$$\bar{a} \equiv \frac{1}{2\pi} \int_0^{2\pi} a(r, \theta, \phi) d\phi. \quad (2)$$

Fluctuations about that average are indicated by a *prime* superscript, such that

$$a' \equiv a(r, \theta, \phi) - \bar{a}(r, \theta) \quad (3)$$

Finally, some quantities are averaged over the full sphere. These are indicated by a double-zero subscript (i.e. $\ell = 0, m = 0$), such that

$$a_{00} \equiv \frac{1}{4\pi} \int_0^{2\pi} \int_0^\pi a(r, \theta, \phi) r \sin \theta d\theta d\phi. \quad (4)$$

3 The Equation Sets Solved by Rayleigh

Rayleigh solves the Boussinesq or anelastic MHD equations in spherical geometry. Both the equations that Rayleigh solves and its diagnostics can be formulated either dimensionally or nondimensionally. A nondimensional Boussinesq formulation, as well as dimensional and nondimensional anelastic formulations (based on a polytropic reference state) are provided as part of Rayleigh. The user may employ alternative formulations via the custom Reference-state interface. To do so, they must specify the functions f_i and the constants c_i in equations 5–11 at input time (*in development*).

The general form of the momentum equation solved by Rayleigh is given by

$$f_1(r) \left[\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} + c_1 \hat{\mathbf{z}} \times \mathbf{v} \right] = c_2 f_2(r) \Theta \hat{\mathbf{r}} - c_3 f_1(r) \nabla \left(\frac{P}{f_1(r)} \right) + c_4 (\nabla \times \mathbf{B}) \times \mathbf{B} + c_5 \nabla \cdot \mathcal{D}, \quad (5)$$

where the stress tensor \mathcal{D} is given by

$$\mathcal{D}_{ij} = 2f_1(r) f_3(r) \left[e_{ij} - \frac{1}{3} \nabla \cdot \mathbf{v} \right]. \quad (6)$$

The velocity field is denoted by \mathbf{v} , the thermal anomaly by Θ , the pressure by P , and the magnetic field by \mathbf{B} . All four of these quantities are 3-dimensional functions of position, in contrast to the 1-dimensional coefficient functions f_i . The velocity and magnetic fields are subject to the constraints

$$\nabla \cdot (f_1(r) \mathbf{v}) = 0 \quad (7)$$

and

$$\nabla \cdot \mathbf{B} = 0 \quad (8)$$

respectively. The evolution of Θ is described

$$f_1(r) f_4(r) \left[\frac{\partial \Theta}{\partial t} + \mathbf{v} \cdot \nabla \Theta \right] = c_6 \nabla \cdot [f_1(r) f_4(r) f_5(r) \nabla \Theta] + f_6(r) + c_8 \Phi(r, \theta, \phi) + c_9 f_7(r) [\nabla \times \mathbf{B}]^2, \quad (9)$$

where the viscous heating Φ is given by

$$\Phi(r, \theta, \phi) = 2f_1(r) f_3(r) \left[e_{ij} e_{ij} - \frac{1}{3} (\nabla \cdot \mathbf{v})^2 \right]. \quad (10)$$

Finally, the evolution of \mathbf{B} is described by the induction equation

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B} - c_7 f_7(r) \nabla \times \mathbf{B}). \quad (11)$$

Equations 5–11 could have been formulated in other ways. For instance, we could combine f_1 and f_4 into a single function in Equation 10. The form of the equations presented here has been chosen to reflect that actually used in the code, which was originally written dimensionally. We now describe the dimensional anelastic and nondimensional Boussinesq formulations used in the code.

3.1 Dimensional Anelastic Formulation of the MHD Equations

When run in dimensional, anelastic mode (cgs units; **reference_type=2**), the following values are assigned to the functions f_i and the constants c_i :

$$\begin{array}{ll}
f_1(r) \rightarrow \hat{\rho}(r) & c_1 \rightarrow 2\Omega_0 \\
f_2(r) \rightarrow \frac{\hat{\rho}(r)}{c_P} g(r) & c_2 \rightarrow 1 \\
f_3(r) \rightarrow \nu(r) & c_3 \rightarrow 1 \\
f_4(r) \rightarrow \hat{T}(r) & c_4 \rightarrow \frac{1}{4\pi} \\
f_5(r) \rightarrow \kappa(r) & c_5 \rightarrow 1 \\
f_6(r) \rightarrow Q(r) & c_6 \rightarrow 1 \\
f_7(r) \rightarrow \eta(r) & c_7 \rightarrow 1. \\
c_8 \rightarrow 1 & c_9 \rightarrow \frac{1}{4\pi}.
\end{array}$$

Here, $\hat{\rho}$ and \hat{T} are the reference-state density and temperature respectively. g is the gravitational acceleration, c_P is the specific heat at constant pressure, and Ω_0 is the frame rotation rate. The viscous, thermal, and magnetic diffusivities are given by ν , κ , and η respectively. Finally, $Q(r)$ is an internal heating function; it might represent radiative heating or heating due to nuclear fusion, for instance. Note that in the anelastic formulation, the thermal variable Θ is interpreted as entropy s , rather than temperature T . When these substitutions are made, Equations 5–11 transform as follows.

$$\begin{array}{ll}
\hat{\rho}(r) \left[\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} + 2\Omega_0 \hat{\mathbf{z}} \times \mathbf{v} \right] = \frac{\hat{\rho}(r)}{c_P} g(r) \Theta \hat{\mathbf{r}} + \hat{\rho}(r) \nabla \left(\frac{P}{\hat{\rho}(r)} \right) + \frac{1}{4\pi} (\nabla \times \mathbf{B}) \times \mathbf{B} + \nabla \cdot \mathcal{D} & \text{Momentum} \\
\hat{\rho}(r) \hat{T}(r) \left[\frac{\partial \Theta}{\partial t} + \mathbf{v} \cdot \nabla \Theta \right] = \nabla \cdot \left[\hat{\rho}(r) \hat{T}(r) \kappa(r) \nabla \Theta \right] + Q(r) + \Phi(r, \theta, \phi) + \frac{\eta(r)}{4\pi} [\nabla \times \mathbf{B}]^2 & \text{Thermal Energy} \\
\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B} - \eta(r) \nabla \times \mathbf{B}) & \text{Induction} \\
\mathcal{D}_{ij} = 2\hat{\rho}(r) \nu(r) \left[e_{ij} - \frac{1}{3} \nabla \cdot \mathbf{v} \right] & \text{Viscous Stress Tensor} \\
\Phi(r, \theta, \phi) = 2\hat{\rho}(r) \nu(r) \left[e_{ij} e_{ij} - \frac{1}{3} (\nabla \cdot \mathbf{v})^2 \right] & \text{Viscous Heating} \\
\nabla \cdot (\hat{\rho}(r) \mathbf{v}) = 0 & \text{Solenoidal Mass Flux} \\
\nabla \cdot \mathbf{B} = 0 & \text{Solenoidal Magnetic Field}
\end{array}$$

3.2 Nondimensional Boussinesq Formulation of the MHD Equations

Rayleigh can also be run using a nondimensional, Boussinesq formulation of the MHD equations (**reference_type=1**). The nondimensionalization employed is as follows:

$$\begin{array}{ll}
\text{Length} \rightarrow L & (\text{Shell Depth}) \\
\text{Time} \rightarrow \frac{L^2}{\nu} & (\text{Viscous Timescale}) \\
\text{Temperature} \rightarrow \Delta T & (\text{Temperature Contrast Across Shell}) \\
\text{MagneticField} \rightarrow \sqrt{\rho \mu \eta \Omega_0}, &
\end{array}$$

where Ω_0 is the rotation rate of the frame, ρ is the (constant) density of the fluid, μ is the magnetic permeability, η is the magnetic diffusivity, and ν is the kinematic viscosity. After nondimensionalizing, the following nondimensional

numbers appear in our equations:

$$\begin{aligned}
Pr &= \frac{\nu}{\kappa} && \text{Prandtl Number} \\
Pr &= \frac{\nu}{\eta} && \text{Magnetic Prandtl Number} \\
E &= \frac{\nu}{\Omega_0 L^2} && \text{Ekman Number} \\
Ra &= \frac{\alpha g_0 \Delta T L^3}{\nu \kappa} && \text{Rayleigh Number,}
\end{aligned}$$

where α is coefficient of thermal expansion, g_0 is the gravitational acceleration, and κ is the thermal diffusivity. Adopting this nondimensionalization is equivalent to assigning values to f_i and the constants c_i :

$$\begin{aligned}
f_1(r) &\rightarrow 1 && c_1 \rightarrow \frac{2}{E} \\
f_2(r) &\rightarrow \left(\frac{r}{r_o}\right)^n && c_2 \rightarrow \frac{Ra}{E Pr} \\
f_3(r) &\rightarrow 1 && c_3 \rightarrow \frac{1}{E} \\
f_4(r) &\rightarrow 1 && c_4 \rightarrow \frac{1}{E Pm} \\
f_5(r) &\rightarrow 1 && c_5 \rightarrow 0 \\
f_6(r) &\rightarrow 0 && c_6 \rightarrow \frac{1}{Pr} \\
f_7(r) &\rightarrow 1 && c_7 \rightarrow \frac{1}{Pm}. \\
c_8 &\rightarrow 0 && c_9 \rightarrow 0.
\end{aligned}$$

Note that our choice of $f_2(r)$ allows gravity to vary with radius based on the value of the exponent n , which has a default value of 0 in the code. Note also that our definition of Ra assumes fixed-temperature boundary conditions. We might choose specify fixed-flux boundary conditions and/or an internal heating through a suitable choice $f_6(r)$, in which case the meaning of Ra in our equation set changes, with Ra denoting a flux Rayleigh number instead. In addition, ohmic and viscous heating, which do not appear in the Boussinesq formulation, are turned off when this nondimensionalization is specified at runtime. When these substitutions are made, Equations 5–11 transform as follows.

$$\begin{aligned}
\left[\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} + \frac{2}{E} \hat{\mathbf{z}} \times \mathbf{v} \right] &= \frac{Ra}{Pr} \left(\frac{r}{r_o} \right)^n \Theta \hat{\mathbf{r}} - \frac{1}{E} \nabla P + \frac{1}{E Pm} (\nabla \times \mathbf{B}) \times \mathbf{B} + \nabla^2 \mathbf{v} && \text{Momentum} \\
\left[\frac{\partial \Theta}{\partial t} + \mathbf{v} \cdot \nabla \Theta \right] &= \frac{1}{Pr} \nabla^2 \Theta && \text{Thermal Energy} \\
\frac{\partial \mathbf{B}}{\partial t} &= \nabla \times (\mathbf{v} \times \mathbf{B}) + \frac{1}{Pm} \nabla^2 \mathbf{B} && \text{Induction} \\
\nabla \cdot \mathbf{v} &= 0 && \text{Solenoidal Velocity Field} \\
\nabla \cdot \mathbf{B} &= 0 && \text{Solenoidal Magnetic Field}
\end{aligned}$$

3.3 Nondimensional Anelastic MHD Equations

To run in nondimensional anelastic mode, you must set **reference_type=3** in the Reference_Namelist. The reference state is assumed to be polytropic with a $\frac{1}{r^2}$ profile for gravity. Transport coefficients ν , κ , η are assumed to be constant in radius. When this mode is active, the following nondimensionalization is used (following Heimpel et al., 2016, *Nat. Geo*, 9, 19):

| | |
|---|---|
| Length $\rightarrow L$ | (Shell Depth) |
| Time $\rightarrow \frac{1}{\Omega_0}$ | (Rotational Timescale) |
| Temperature $\rightarrow T_o \equiv \hat{T}(r_{\max})$ | (Reference – State Temperature at Upper Boundary) |
| Density $\rightarrow \rho_o \equiv \hat{\rho}(r_{\max})$ | (Reference – State Density at Upper Boundary) |
| Entropy $\rightarrow \Delta s$ | (Entropy Constrast Across Shell) |
| Magnetic Field $\rightarrow \sqrt{\bar{\rho}(r_{\max})\mu\eta\Omega_0}$. | |

When run in this mode, Rayleigh employs a polytropic background state, with an assumed $\frac{1}{r^2}$ variation in gravity. These choices result in the functions f_i and the constants c_i (tildes indicate nondimensional reference-state variables):

| | |
|---|---|
| $f_1(r) \rightarrow \tilde{\rho}(r)$ | $c_1 \rightarrow 2$ |
| $f_2(r) \rightarrow \rho(r) \frac{r_{\max}^2}{r^2}$ | $c_2 \rightarrow \text{Ra}^*$ |
| $f_3(r) \rightarrow 1$ | $c_3 \rightarrow 1$ |
| $f_4(r) \rightarrow \tilde{T}(r)$ | $c_4 \rightarrow \frac{E}{\text{Pm}}$ |
| $f_5(r) \rightarrow 1$ | $c_5 \rightarrow E$ |
| $f_6(r) \rightarrow Q(r)$ | $c_6 \rightarrow \frac{E}{\text{Pr}}$ |
| $f_7(r) \rightarrow 1$ | $c_7 \rightarrow \frac{E}{\text{Pm}}$ |
| $c_8 \rightarrow \frac{E \text{Di}}{\text{Ra}^*}$ | $c_9 \rightarrow \frac{E^2 \text{Di}}{\text{Pm}^2 \text{Ra}^*}$ |

Two new nondimensional numbers appear in our equations. Di, the dissipation number, is defined by

$$\text{Di} = \frac{g_o L}{c_P T_o}, \quad (12)$$

where g_o and T_o are the gravitational acceleration and temperature at the outer boundary respectively. Once more, the thermal anomaly Θ should be interpreted as entropy s . The symbol Ra^* is the modified Rayleigh number, given by

$$\text{Ra}^* = \frac{g_o}{c_P \Omega_0^2} \frac{\Delta s}{L} \quad (13)$$

We arrive at the following nondimensionalized equations:

| | |
|---|---------------------------|
| $\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} + 2\hat{\mathbf{z}} \times \mathbf{v} = \text{Ra}^* \frac{r_{\max}^2}{r^2} \Theta \hat{\mathbf{r}} + \nabla \left(\frac{P}{\tilde{\rho}(r)} \right) + \frac{E}{\text{Pm} \tilde{\rho}} (\nabla \times \mathbf{B}) \times \mathbf{B} + \frac{E}{\rho(r)} \nabla \cdot \mathcal{D}$ | Momentum |
| $\tilde{\rho}(r) \tilde{T}(r) \left[\frac{\partial \Theta}{\partial t} + \mathbf{v} \cdot \nabla \Theta \right] = \frac{E}{\text{Pr}} \nabla \cdot \left[\tilde{\rho}(r) \tilde{T}(r) \nabla \Theta \right] + Q(r) + \frac{E \text{Di}}{\text{Ra}^*} \Phi(r, \theta, \phi) + \frac{\text{Di} E^2}{\text{Pm}^2 \text{Ra}^*} [\nabla \times \mathbf{B}]^2$ | Thermal Energy |
| $\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \left(\mathbf{v} \times \mathbf{B} - \frac{E}{\text{Pm}} \nabla \times \mathbf{B} \right)$ | Induction |
| $\mathcal{D}_{ij} = 2\tilde{\rho}(r) \left[e_{ij} - \frac{1}{3} \nabla \cdot \mathbf{v} \right]$ | Viscous Stress Tensor |
| $\Phi(r, \theta, \phi) = 2\tilde{\rho}(r) \left[e_{ij} e_{ij} - \frac{1}{3} (\nabla \cdot \mathbf{v})^2 \right]$ | Viscous Heating |
| $\nabla \cdot (\tilde{\rho}(r) \mathbf{v}) = 0$ | Solenoidal Mass Flux |
| $\nabla \cdot \mathbf{B} = 0.$ | Solenoidal Magnetic Field |

4 Diagnostic Code Tables

The remainder of this document contains tables enumerating the menu codes necessary to specify diagnostic outputs in Rayleigh.

4.1 Velocity Field

Output quantities related to the velocity field, its gradients, and its associated mass flux $f_1 \boldsymbol{v}$ are defined here.

| Value | Code | Variable | Value | Code | Variable |
|---|------|--------------|---|------|--------------|
| v_r | 1 | v_r | $\frac{\partial v_\phi}{\partial \theta}$ | 21 | dv_phi_dt |
| v_θ | 2 | v_theta | $\frac{\partial v'_r}{\partial \theta}$ | 22 | dvp_r_dt |
| v_ϕ | 3 | v_phi | $\frac{\partial v'_\theta}{\partial \theta}$ | 23 | dvp_theta_dt |
| v'_r | 4 | vp_r | $\frac{\partial v'_\phi}{\partial \theta}$ | 24 | dvp_phi_dt |
| v'_θ | 5 | vp_theta | $\frac{\partial \overline{v_r}}{\partial \theta}$ | 25 | dvm_r_dt |
| v'_ϕ | 6 | vp_phi | $\frac{\partial \overline{v_\theta}}{\partial \theta}$ | 26 | dvm_theta_dt |
| $\overline{v_r}$ | 7 | vm_r | $\frac{\partial \overline{v_\phi}}{\partial \theta}$ | 27 | dvm_phi_dt |
| $\overline{v_\theta}$ | 8 | vm_theta | $\frac{\partial v_r}{\partial \phi}$ | 28 | dv_r_dp |
| $\overline{v_\phi}$ | 9 | vm_phi | $\frac{\partial v_\theta}{\partial \phi}$ | 29 | dv_theta_dp |
| $\frac{\partial v_r}{\partial r}$ | 10 | dv_r_dr | $\frac{\partial v_\phi}{\partial \phi}$ | 30 | dv_phi_dp |
| $\frac{\partial v_\theta}{\partial r}$ | 11 | dv_theta_dr | $\frac{\partial v'_r}{\partial \phi}$ | 31 | dvp_r_dp |
| $\frac{\partial v_\phi}{\partial r}$ | 12 | dv_phi_dr | $\frac{\partial v'_\theta}{\partial \phi}$ | 32 | dvp_theta_dp |
| $\frac{\partial v'_r}{\partial r}$ | 13 | dvp_r_dr | $\frac{\partial v'_\phi}{\partial \phi}$ | 33 | dvp_phi_dp |
| $\frac{\partial v'_\theta}{\partial r}$ | 14 | dvp_theta_dr | $\frac{\partial \overline{v_r}}{\partial \phi}$ | 34 | dvm_r_dp |
| $\frac{\partial v'_\phi}{\partial r}$ | 15 | dvp_phi_dr | $\frac{\partial \overline{v_\theta}}{\partial \phi}$ | 35 | dvm_theta_dp |
| $\frac{\partial \overline{v_r}}{\partial r}$ | 16 | dvm_r_dr | $\frac{\partial \overline{v_\phi}}{\partial \phi}$ | 36 | dvm_phi_dp |
| $\frac{\partial \overline{v_\theta}}{\partial r}$ | 17 | dvm_theta_dr | $\frac{1}{r} \frac{\partial v_r}{\partial \theta}$ | 37 | dv_r_dtr |
| $\frac{\partial \overline{v_\phi}}{\partial r}$ | 18 | dvm_phi_dr | $\frac{1}{r} \frac{\partial v_\theta}{\partial \theta}$ | 38 | dv_theta_dtr |
| $\frac{\partial v_r}{\partial \theta}$ | 19 | dv_r_dt | $\frac{1}{r} \frac{\partial v_\phi}{\partial \theta}$ | 39 | dv_phi_dtr |
| $\frac{\partial v_\theta}{\partial \theta}$ | 20 | dv_theta_dt | $\frac{1}{r} \frac{\partial v'_r}{\partial \theta}$ | 40 | dvp_r_dtr |

| Value | Code | Variable | Value | Code | Variable |
|--|------|----------------|--|------|---------------|
| $\frac{1}{r} \frac{\partial v'_r}{\partial \theta}$ | 41 | dvp_theta_dtr | $\frac{\partial^2 \overline{v_r}}{\partial r^2}$ | 61 | dvm_r_d2r |
| $\frac{1}{r} \frac{\partial v'_\theta}{\partial \theta}$ | 42 | dvp_phi_dtr | $\frac{\partial^2 \overline{v_\theta}}{\partial r^2}$ | 62 | dvm_theta_d2r |
| $\frac{1}{r} \frac{\partial \overline{v_r}}{\partial \theta}$ | 43 | dvm_r_dtr | $\frac{\partial^2 \overline{v_\phi}}{\partial r^2}$ | 63 | dvm_phi_d2r |
| $\frac{1}{r} \frac{\partial \overline{v_\theta}}{\partial \theta}$ | 44 | dvm_theta_dtr | $\frac{\partial^2 v_r}{\partial \theta^2}$ | 64 | dv_r_d2t |
| $\frac{1}{r} \frac{\partial \overline{v_\phi}}{\partial \theta}$ | 45 | dvm_phi_dtr | $\frac{\partial^2 v_\theta}{\partial \theta^2}$ | 65 | dv_theta_d2t |
| $\frac{1}{r \sin \theta} \frac{\partial v_r}{\partial \phi}$ | 46 | dv_r_dprs | $\frac{\partial^2 v_\phi}{\partial \theta^2}$ | 66 | dv_phi_d2t |
| $\frac{1}{r \sin \theta} \frac{\partial v_\theta}{\partial \phi}$ | 47 | dv_theta_dprs | $\frac{\partial^2 v'_r}{\partial \theta^2}$ | 67 | dvp_r_d2t |
| $\frac{1}{r \sin \theta} \frac{\partial v_\phi}{\partial \phi}$ | 48 | dv_phi_dprs | $\frac{\partial^2 v'_\theta}{\partial \theta^2}$ | 68 | dvp_theta_d2t |
| $\frac{1}{r \sin \theta} \frac{\partial v'_r}{\partial \phi}$ | 49 | dvp_r_dprs | $\frac{\partial^2 v'_\phi}{\partial \theta^2}$ | 69 | dvp_phi_d2t |
| $\frac{1}{r \sin \theta} \frac{\partial v'_\theta}{\partial \phi}$ | 50 | dvp_theta_dprs | $\frac{\partial^2 \overline{v_r}}{\partial \theta^2}$ | 70 | dvm_r_d2t |
| $\frac{1}{r \sin \theta} \frac{\partial v'_\phi}{\partial \phi}$ | 51 | dvp_phi_dprs | $\frac{\partial^2 \overline{v_\theta}}{\partial \theta^2}$ | 71 | dvm_theta_d2t |
| $\frac{1}{r \sin \theta} \frac{\partial \overline{v_r}}{\partial \phi}$ | 52 | dvm_r_dprs | $\frac{\partial^2 \overline{v_\phi}}{\partial \theta^2}$ | 72 | dvm_phi_d2t |
| $\frac{1}{r \sin \theta} \frac{\partial \overline{v_\theta}}{\partial \phi}$ | 53 | dvm_theta_dprs | $\frac{\partial^2 v_r}{\partial \phi^2}$ | 73 | dv_r_d2p |
| $\frac{1}{r \sin \theta} \frac{\partial \overline{v_\phi}}{\partial \phi}$ | 54 | dvm_phi_dprs | $\frac{\partial^2 v_\theta}{\partial \phi^2}$ | 74 | dv_theta_d2p |
| $\frac{\partial^2 v_r}{\partial r^2}$ | 55 | dv_r_d2r | $\frac{\partial^2 v_\phi}{\partial \phi^2}$ | 75 | dv_phi_d2p |
| $\frac{\partial^2 v_\theta}{\partial r^2}$ | 56 | dv_theta_d2r | $\frac{\partial^2 v'_r}{\partial \phi^2}$ | 76 | dvp_r_d2p |
| $\frac{\partial^2 v_\phi}{\partial r^2}$ | 57 | dv_phi_d2r | $\frac{\partial^2 v'_\theta}{\partial \phi^2}$ | 77 | dvp_theta_d2p |
| $\frac{\partial^2 v'_r}{\partial r^2}$ | 58 | dvp_r_d2r | $\frac{\partial^2 v'_\phi}{\partial \phi^2}$ | 78 | dvp_phi_d2p |
| $\frac{\partial^2 v'_\theta}{\partial r^2}$ | 59 | dvp_theta_d2r | $\frac{\partial^2 \overline{v_r}}{\partial \phi^2}$ | 79 | dvm_r_d2p |
| $\frac{\partial^2 v'_\phi}{\partial r^2}$ | 60 | dvp_phi_d2r | $\frac{\partial^2 \overline{v_\theta}}{\partial \phi^2}$ | 80 | dvm_theta_d2p |

| Value | Code | Variable | Value | Code | Variable |
|---|------|----------------|--|------|----------------|
| $\frac{\partial^2 \overline{v_\phi}}{\partial \phi^2}$ | 81 | dvm_phi_d2p | $\frac{\partial^2 v_\theta}{\partial \theta \partial \phi}$ | 101 | dv_theta_d2tp |
| $\frac{\partial^2 v_r}{\partial r \partial \theta}$ | 82 | dv_r_d2rt | $\frac{\partial^2 v_\phi}{\partial \theta \partial \phi}$ | 102 | dv_phi_d2tp |
| $\frac{\partial^2 v_\theta}{\partial r \partial \theta}$ | 83 | dv_theta_d2rt | $\frac{\partial^2 v'_r}{\partial \theta \partial \phi}$ | 103 | dvp_r_d2tp |
| $\frac{\partial^2 v_\phi}{\partial r \partial \theta}$ | 84 | dv_phi_d2rt | $\frac{\partial^2 v'_\theta}{\partial \theta \partial \phi}$ | 104 | dvp_theta_d2tp |
| $\frac{\partial^2 v'_r}{\partial r \partial \theta}$ | 85 | dvp_r_d2rt | $\frac{\partial^2 v'_\phi}{\partial \theta \partial \phi}$ | 105 | dvp_phi_d2tp |
| $\frac{\partial^2 v'_\theta}{\partial r \partial \theta}$ | 86 | dvp_theta_d2rt | $\frac{\partial^2 \overline{v_r}}{\partial \theta \partial \phi}$ | 106 | dvm_r_d2tp |
| $\frac{\partial^2 v'_\phi}{\partial r \partial \theta}$ | 87 | dvp_phi_d2rt | $\frac{\partial^2 \overline{v_\theta}}{\partial \theta \partial \phi}$ | 107 | dvm_theta_d2tp |
| $\frac{\partial^2 \overline{v_r}}{\partial r \partial \theta}$ | 88 | dvm_r_d2rt | $\frac{\partial^2 \overline{v_\phi}}{\partial \theta \partial \phi}$ | 108 | dvm_phi_d2tp |
| $\frac{\partial^2 \overline{v_\theta}}{\partial r \partial \theta}$ | 89 | dvm_theta_d2rt | | | |
| $\frac{\partial^2 \overline{v_\phi}}{\partial r \partial \theta}$ | 90 | dvm_phi_d2rt | | | |
| $\frac{\partial^2 v_r}{\partial r \partial \phi}$ | 91 | dv_r_d2rp | | | |
| $\frac{\partial^2 v_\theta}{\partial r \partial \phi}$ | 92 | dv_theta_d2rp | | | |
| $\frac{\partial^2 v_\phi}{\partial r \partial \phi}$ | 93 | dv_phi_d2rp | | | |
| $\frac{\partial^2 v'_r}{\partial r \partial \phi}$ | 94 | dvp_r_d2rp | | | |
| $\frac{\partial^2 v'_\theta}{\partial r \partial \phi}$ | 95 | dvp_theta_d2rp | | | |
| $\frac{\partial^2 v'_\phi}{\partial r \partial \phi}$ | 96 | dvp_phi_d2rp | | | |
| $\frac{\partial^2 \overline{v_r}}{\partial r \partial \phi}$ | 97 | dvm_r_d2rp | | | |
| $\frac{\partial^2 \overline{v_\theta}}{\partial r \partial \phi}$ | 98 | dvm_theta_d2rp | | | |
| $\frac{\partial^2 \overline{v_\phi}}{\partial r \partial \phi}$ | 99 | dvm_phi_d2rp | | | |
| $\frac{\partial^2 v_r}{\partial \theta \partial \phi}$ | 100 | dv_r_d2tp | | | |

| Value | Code | Variable | Value | Code | Variable |
|-----------------|------|-------------|---------------------------|------|-------------|
| $f_1 v_r$ | 201 | rhov_r | $f_1 v'_\phi$ | 206 | rhovp_phi |
| $f_1 v_\theta$ | 202 | rhov_theta | $f_1 \overline{v_r}$ | 207 | rhovm_r |
| $f_1 v_\phi$ | 203 | rhov_phi | $f_1 \overline{v_\theta}$ | 208 | rhovm_theta |
| $f_1 v'_r$ | 204 | rhovp_r | $f_1 \overline{v_\phi}$ | 209 | rhovm_phi |
| $f_1 v'_\theta$ | 205 | rhovp_theta | | | |

| Value | Code | Variable | Value | Code | Variable |
|-----------------------------------|------|--------------|---|------|----------------|
| ω_r | 301 | vort_r | $\overline{\omega} \cdot \overline{\omega}$ | 312 | enstrophy_mm |
| ω_θ | 302 | vort_theta | $\omega' \cdot \omega'$ | 313 | enstrophy_pp |
| ω_ϕ | 303 | vort_phi | ω_r^2 | 314 | vort_r_sq |
| ω'_r | 304 | vortp_r | ω_θ^2 | 315 | vort_theta_sq |
| ω'_θ | 305 | vortp_theta | ω_ϕ^2 | 316 | vort_phi_sq |
| ω'_ϕ | 306 | vortp_phi | $\omega_r'^2$ | 317 | vortp_r_sq |
| $\overline{\omega_r}$ | 307 | vortm_r | $\omega_\theta'^2$ | 318 | vortp_theta_sq |
| $\overline{\omega_\theta}$ | 308 | vortm_theta | $\omega_\phi'^2$ | 319 | vortp_phi_sq |
| $\overline{\omega_\phi}$ | 309 | vortm_phi | $\overline{\omega_r}^2$ | 320 | vortm_r_sq |
| $\omega \cdot \omega$ | 310 | enstrophy | $\overline{\omega_\theta}^2$ | 321 | vortm_theta_sq |
| $\omega' \cdot \overline{\omega}$ | 311 | enstrophy_pm | $\overline{\omega_\phi}^2$ | 322 | vortm_phi_sq |

4.2 Vorticity

Codes associated with the vorticity field ω are defined here. The vorticity field ω is given by

$$\omega = \nabla \times v. \quad (14)$$

| Value | Code | Variable | Value | Code | Variable |
|--|------|------------------|-------------------------------|------|-------------|
| $\frac{1}{2}f_1 \boldsymbol{v}^2$ | 401 | kinetic_energy | \boldsymbol{v}^2 | 413 | vsq |
| $\frac{1}{2}f_1 v_r^2$ | 402 | radial_ke | v_r^2 | 414 | radial_vsq |
| $\frac{1}{2}f_1 v_\theta^2$ | 403 | theta_ke | v_θ^2 | 415 | theta_vsq |
| $\frac{1}{2}f_1 v_\phi^2$ | 404 | phi_ke | v_ϕ^2 | 416 | phi_vsq |
| $\frac{1}{2}f_1 \overline{\boldsymbol{v}}^2$ | 405 | mkinetic_energy | $\overline{\boldsymbol{v}}^2$ | 417 | mvsq |
| $\frac{1}{2}f_1 \overline{v_r}^2$ | 406 | radial_mke | $\overline{v_r}^2$ | 418 | radial_mvsq |
| $\frac{1}{2}f_1 \overline{v_\theta}^2$ | 407 | theta_mke | $\overline{v_\theta}^2$ | 419 | theta_mvsq |
| $\frac{1}{2}f_1 \overline{v_\phi}^2$ | 408 | phi_mke | $\overline{v_\phi}^2$ | 420 | phi_mvsq |
| $\frac{1}{2}f_1 \boldsymbol{v}'^2$ | 409 | pkkinetic_energy | \boldsymbol{v}'^2 | 421 | pvsq |
| $\frac{1}{2}f_1 v_r'^2$ | 410 | radial_pke | $v_r'^2$ | 422 | radial_pvsq |
| $\frac{1}{2}f_1 v_\theta'^2$ | 411 | theta_pke | $v_\theta'^2$ | 423 | theta_pvsq |
| $\frac{1}{2}f_1 v_\phi'^2$ | 412 | phi_pke | $v_\phi'^2$ | 424 | phi_pvsq |

4.3 Kinetic Energy

Codes associated with the generalized kinetic energy density, $\frac{1}{2}f_1(r)\boldsymbol{v}^2$, are defined here.

4.4 Thermal Variables

Codes associated with the thermal variables Θ and P , and their gradients, are defined here.

| Value | Code | Variable | Value | Code | Variable |
|---|------|-------------------|---|------|-----------------|
| Θ | 501 | entropy | $\frac{\partial \Theta'}{\partial \phi}$ | 521 | entropy_p_dphi |
| P | 502 | pressure | $\frac{\partial P'}{\partial \phi}$ | 522 | pressure_p_dphi |
| Θ' | 503 | entropy_p | $\frac{\partial \bar{\Theta}}{\partial \phi}$ | 523 | entropy_m_dphi |
| P' | 504 | pressure_p | $\frac{\partial \bar{P}}{\partial \phi}$ | 524 | pressure_m_dphi |
| $\bar{\Theta}$ | 505 | entropy_m | $\frac{1}{r} \frac{\partial \Theta}{\partial \theta}$ | 525 | entropy_dtr |
| \bar{P} | 506 | pressure_m | $\frac{1}{r} \frac{\partial P}{\partial \theta}$ | 526 | pressure_dtr |
| $\frac{\partial \Theta}{\partial r}$ | 507 | entropy_dr | $\frac{1}{r} \frac{\partial \Theta'}{\partial \theta}$ | 527 | entropy_p_dtr |
| $\frac{\partial P}{\partial r}$ | 508 | pressure_dr | $\frac{1}{r} \frac{\partial P'}{\partial \theta}$ | 528 | pressure_p_dtr |
| $\frac{\partial \Theta'}{\partial r}$ | 509 | entropy_p_dr | $\frac{1}{r} \frac{\partial \bar{\Theta}}{\partial \theta}$ | 529 | entropy_m_dtr |
| $\frac{\partial P'}{\partial r}$ | 510 | pressure_p_dr | $\frac{1}{r} \frac{\partial \bar{P}}{\partial \theta}$ | 530 | pressure_m_dtr |
| $\frac{\partial \bar{\Theta}}{\partial r}$ | 511 | entropy_m_dr | $\frac{1}{r \sin \theta} \frac{\partial \Theta}{\partial \phi}$ | 531 | entropy_dprs |
| $\frac{\partial \bar{P}}{\partial r}$ | 512 | pressure_m_dr | $\frac{1}{r \sin \theta} \frac{\partial P}{\partial \phi}$ | 532 | pressure_dprs |
| $\frac{\partial \Theta}{\partial \theta}$ | 513 | entropy_dtheta | $\frac{1}{r \sin \theta} \frac{\partial \Theta'}{\partial \phi}$ | 533 | entropy_p_dprs |
| $\frac{\partial P}{\partial \theta}$ | 514 | pressure_dtheta | $\frac{1}{r \sin \theta} \frac{\partial P'}{\partial \phi}$ | 534 | pressure_p_dprs |
| $\frac{\partial \Theta'}{\partial \theta}$ | 515 | entropy_p_dtheta | $\frac{1}{r \sin \theta} \frac{\partial \bar{\Theta}}{\partial \phi}$ | 535 | entropy_m_dprs |
| $\frac{\partial P'}{\partial \theta}$ | 516 | pressure_p_dtheta | $\frac{1}{r \sin \theta} \frac{\partial \bar{P}}{\partial \phi}$ | 536 | pressure_m_dprs |
| $\frac{\partial \bar{\Theta}}{\partial \theta}$ | 517 | entropy_m_dtheta | $\frac{\partial^2 \Theta}{\partial r^2}$ | 537 | entropy_d2r |
| $\frac{\partial \bar{P}}{\partial \theta}$ | 518 | pressure_m_dtheta | $\frac{\partial^2 P}{\partial r^2}$ | 538 | pressure_d2r |
| $\frac{\partial \Theta}{\partial \phi}$ | 519 | entropy_dphi | $\frac{\partial^2 \Theta'}{\partial r^2}$ | 539 | entropy_p_d2r |
| $\frac{\partial P}{\partial \phi}$ | 520 | pressure_dphi | $\frac{\partial^2 P'}{\partial r^2}$ | 540 | pressure_p_d2r |

| Value | Code | Variable | Value | Code | Variable |
|--|------|-----------------|---|------|-----------------|
| $\frac{\partial^2 \Theta}{\partial r^2}$ | 541 | entropy_m_d2r | $\frac{\partial^2 \Theta}{\partial r \partial \phi}$ | 561 | entropy_d2rp |
| $\frac{\partial^2 \bar{P}}{\partial r^2}$ | 542 | pressure_m_d2r | $\frac{\partial^2 P}{\partial r \partial \phi}$ | 562 | pressure_d2rp |
| $\frac{\partial^2 \Theta}{\partial \theta^2}$ | 543 | entropy_d2t | $\frac{\partial^2 \Theta'}{\partial r \partial \phi}$ | 563 | entropy_p_d2rp |
| $\frac{\partial^2 P}{\partial \theta^2}$ | 544 | pressure_d2t | $\frac{\partial^2 P'}{\partial r \partial \phi}$ | 564 | pressure_p_d2rp |
| $\frac{\partial^2 \Theta'}{\partial \theta^2}$ | 545 | entropy_p_d2t | $\frac{\partial^2 \bar{\Theta}}{\partial r \partial \phi}$ | 565 | entropy_m_d2rp |
| $\frac{\partial^2 P'}{\partial \theta^2}$ | 546 | pressure_p_d2t | $\frac{\partial^2 \bar{P}}{\partial r \partial \phi}$ | 566 | pressure_m_d2rp |
| $\frac{\partial^2 \bar{\Theta}}{\partial \theta^2}$ | 547 | entropy_m_d2t | $\frac{\partial^2 \Theta}{\partial \theta \partial \phi}$ | 567 | entropy_d2tp |
| $\frac{\partial^2 \bar{P}}{\partial \theta^2}$ | 548 | pressure_m_d2t | $\frac{\partial^2 P}{\partial \theta \partial \phi}$ | 568 | pressure_d2tp |
| $\frac{\partial^2 \Theta}{\partial \phi^2}$ | 549 | entropy_d2p | $\frac{\partial^2 \Theta'}{\partial \theta \partial \phi}$ | 569 | entropy_p_d2tp |
| $\frac{\partial^2 P}{\partial \phi^2}$ | 550 | pressure_d2p | $\frac{\partial^2 P'}{\partial \theta \partial \phi}$ | 570 | pressure_p_d2tp |
| $\frac{\partial^2 \Theta'}{\partial \phi^2}$ | 551 | entropy_p_d2p | $\frac{\partial^2 \bar{\Theta}}{\partial \theta \partial \phi}$ | 571 | entropy_m_d2tp |
| $\frac{\partial^2 P'}{\partial \phi^2}$ | 552 | pressure_p_d2p | $\frac{\partial^2 \bar{P}}{\partial \theta \partial \phi}$ | 572 | pressure_m_d2tp |
| $\frac{\partial^2 \bar{\Theta}}{\partial \phi^2}$ | 553 | entropy_m_d2p | $\frac{\partial}{\partial r} \left(\frac{P}{\rho} \right)$ | 573 | rhopressure_dr |
| $\frac{\partial^2 \bar{P}}{\partial \phi^2}$ | 554 | pressure_m_d2p | $\frac{\partial}{\partial r} \left(\frac{P'}{\rho} \right)$ | 574 | rhopressurep_dr |
| $\frac{\partial^2 \Theta}{\partial r \partial \theta}$ | 555 | entropy_d2rt | $\frac{\partial}{\partial r} \left(\frac{\bar{P}}{\rho} \right)$ | 575 | rhopressurem_dr |
| $\frac{\partial^2 P}{\partial r \partial \theta}$ | 556 | pressure_d2rt | | | |
| $\frac{\partial^2 \Theta'}{\partial r \partial \theta}$ | 557 | entropy_p_d2rt | | | |
| $\frac{\partial^2 P'}{\partial r \partial \theta}$ | 558 | pressure_p_d2rt | | | |
| $\frac{\partial^2 \bar{\Theta}}{\partial r \partial \theta}$ | 559 | entropy_m_d2rt | | | |
| $\frac{\partial^2 \bar{P}}{\partial r \partial \theta}$ | 560 | pressure_m_d2rt | | | |

| Value | Code | Variable | Value | Code | Variable |
|--------------------------|------|---------------------|------------------------------|------|--------------------|
| $f_1 f_4 \Theta$ | 701 | thermal_energy_full | $(f_1 f_4 \Theta)^2$ | 707 | thermal_energy_sq |
| $f_1 f_4 \Theta$ | 702 | thermal_energy_p | $(f_1 f_4 \Theta)^2$ | 708 | thermal_energyp_sq |
| $f_1 f_4 \bar{\Theta}$ | 703 | thermal_energy_m | $(f_1 f_4 \bar{\Theta})^2$ | 709 | thermal_energym_sq |
| $c_P \hat{\rho} T$ | 704 | enthalpy_full | $(c_P \hat{\rho} T)^2$ | 710 | enthalpy_sq |
| $c_P \hat{\rho} T'$ | 705 | enthalpy_p | $(c_P \hat{\rho} T')^2$ | 711 | enthalpyp_sq |
| $c_P \hat{\rho} \bar{T}$ | 706 | enthalpy_m | $(c_P \hat{\rho} \bar{T})^2$ | 712 | enthalpym_sq |

4.5 thermal energy

Codes associated with the thermal energy density and the enthalpy are defined here.

4.6 Magnetic Field

Codes associated with the magnetic field \boldsymbol{B} and its gradients appear here.

| Value | Code | Variable | Value | Code | Variable |
|---|------|--------------|---|------|--------------|
| B_r | 801 | b_r | $\frac{\partial B_\phi}{\partial \theta}$ | 821 | db_phi_dt |
| B_θ | 802 | b_theta | $\frac{\partial B'_r}{\partial \theta}$ | 822 | dbp_r_dt |
| B_ϕ | 803 | b_phi | $\frac{\partial B'_\theta}{\partial \theta}$ | 823 | dbp_theta_dt |
| B'_r | 804 | bp_r | $\frac{\partial B'_\phi}{\partial \theta}$ | 824 | dbp_phi_dt |
| B'_θ | 805 | bp_theta | $\frac{\partial \overline{B_r}}{\partial \theta}$ | 825 | dbm_r_dt |
| B'_ϕ | 806 | bp_phi | $\frac{\partial \overline{B_\theta}}{\partial \theta}$ | 826 | dbm_theta_dt |
| $\overline{B_r}$ | 807 | bm_r | $\frac{\partial \overline{B_\phi}}{\partial \theta}$ | 827 | dbm_phi_dt |
| $\overline{B_\theta}$ | 808 | bm_theta | $\frac{\partial B_r}{\partial \phi}$ | 828 | db_r_dp |
| $\overline{B_\phi}$ | 809 | bm_phi | $\frac{\partial B_\theta}{\partial \phi}$ | 829 | db_theta_dp |
| $\frac{\partial B_r}{\partial r}$ | 810 | db_r_dr | $\frac{\partial B_\phi}{\partial \phi}$ | 830 | db_phi_dp |
| $\frac{\partial B_\theta}{\partial r}$ | 811 | db_theta_dr | $\frac{\partial B'_r}{\partial \phi}$ | 831 | dbp_r_dp |
| $\frac{\partial B_\phi}{\partial r}$ | 812 | db_phi_dr | $\frac{\partial B'_\theta}{\partial \phi}$ | 832 | dbp_theta_dp |
| $\frac{\partial B'_r}{\partial r}$ | 813 | dbp_r_dr | $\frac{\partial B'_\phi}{\partial \phi}$ | 833 | dbp_phi_dp |
| $\frac{\partial B'_\theta}{\partial r}$ | 814 | dbp_theta_dr | $\frac{\partial \overline{B_r}}{\partial \phi}$ | 834 | dbm_r_dp |
| $\frac{\partial B'_\phi}{\partial r}$ | 815 | dbp_phi_dr | $\frac{\partial \overline{B_\theta}}{\partial \phi}$ | 835 | dbm_theta_dp |
| $\frac{\partial \overline{B_r}}{\partial r}$ | 816 | dbm_r_dr | $\frac{\partial \overline{B_\phi}}{\partial \phi}$ | 836 | dbm_phi_dp |
| $\frac{\partial \overline{B_\theta}}{\partial r}$ | 817 | dbm_theta_dr | $\frac{1}{r} \frac{\partial B_r}{\partial \theta}$ | 837 | db_r_dtr |
| $\frac{\partial \overline{B_\phi}}{\partial r}$ | 818 | dbm_phi_dr | $\frac{1}{r} \frac{\partial B_\theta}{\partial \theta}$ | 838 | db_theta_dtr |
| $\frac{\partial B_r}{\partial \theta}$ | 819 | db_r_dt | $\frac{1}{r} \frac{\partial B_\phi}{\partial \theta}$ | 839 | db_phi_dtr |
| $\frac{\partial B_\theta}{\partial \theta}$ | 820 | db_theta_dt | $\frac{1}{r} \frac{\partial B'_r}{\partial \theta}$ | 840 | dbp_r_dtr |

| Value | Code | Variable | Value | Code | Variable |
|--|------|----------------|--|------|---------------|
| $\frac{1}{r} \frac{\partial B'_r}{\partial \theta}$ | 841 | dbp_theta_dtr | $\frac{\partial^2 \overline{B_r}}{\partial r^2}$ | 861 | dbm_r_d2r |
| $\frac{1}{r} \frac{\partial B'_r}{\partial \theta}$ | 842 | dbp_phi_dtr | $\frac{\partial^2 \overline{B_\theta}}{\partial r^2}$ | 862 | dbm_theta_d2r |
| $\frac{1}{r} \frac{\partial \overline{B_r}}{\partial \theta}$ | 843 | dbm_r_dtr | $\frac{\partial^2 \overline{B_\phi}}{\partial r^2}$ | 863 | dbm_phi_d2r |
| $\frac{1}{r} \frac{\partial \overline{B_\theta}}{\partial \theta}$ | 844 | dbm_theta_dtr | $\frac{\partial^2 B_r}{\partial \theta^2}$ | 864 | db_r_d2t |
| $\frac{1}{r} \frac{\partial \overline{B_\phi}}{\partial \theta}$ | 845 | dbm_phi_dtr | $\frac{\partial^2 B_\theta}{\partial \theta^2}$ | 865 | db_theta_d2t |
| $\frac{1}{r \sin \theta} \frac{\partial B_r}{\partial \phi}$ | 846 | db_r_dprs | $\frac{\partial^2 B_\phi}{\partial \theta^2}$ | 866 | db_phi_d2t |
| $\frac{1}{r \sin \theta} \frac{\partial B_\theta}{\partial \phi}$ | 847 | db_theta_dprs | $\frac{\partial^2 B'_r}{\partial \theta^2}$ | 867 | dbp_r_d2t |
| $\frac{1}{r \sin \theta} \frac{\partial B_\phi}{\partial \phi}$ | 848 | db_phi_dprs | $\frac{\partial^2 B'_\theta}{\partial \theta^2}$ | 868 | dbp_theta_d2t |
| $\frac{1}{r \sin \theta} \frac{\partial B'_r}{\partial \phi}$ | 849 | dbp_r_dprs | $\frac{\partial^2 B'_\phi}{\partial \theta^2}$ | 869 | dbp_phi_d2t |
| $\frac{1}{r \sin \theta} \frac{\partial B'_\theta}{\partial \phi}$ | 850 | dbp_theta_dprs | $\frac{\partial^2 \overline{B_r}}{\partial \theta^2}$ | 870 | dbm_r_d2t |
| $\frac{1}{r \sin \theta} \frac{\partial B'_\phi}{\partial \phi}$ | 851 | dbp_phi_dprs | $\frac{\partial^2 \overline{B_\theta}}{\partial \theta^2}$ | 871 | dbm_theta_d2t |
| $\frac{1}{r \sin \theta} \frac{\partial \overline{B_r}}{\partial \phi}$ | 852 | dbm_r_dprs | $\frac{\partial^2 \overline{B_\phi}}{\partial \theta^2}$ | 872 | dbm_phi_d2t |
| $\frac{1}{r \sin \theta} \frac{\partial \overline{B_\theta}}{\partial \phi}$ | 853 | dbm_theta_dprs | $\frac{\partial^2 B_r}{\partial \phi^2}$ | 873 | db_r_d2p |
| $\frac{1}{r \sin \theta} \frac{\partial \overline{B_\phi}}{\partial \phi}$ | 854 | dbm_phi_dprs | $\frac{\partial^2 B_\theta}{\partial \phi^2}$ | 874 | db_theta_d2p |
| $\frac{\partial^2 B_r}{\partial r^2}$ | 855 | db_r_d2r | $\frac{\partial^2 B_\phi}{\partial \phi^2}$ | 875 | db_phi_d2p |
| $\frac{\partial^2 B_\theta}{\partial r^2}$ | 856 | db_theta_d2r | $\frac{\partial^2 B'_r}{\partial \phi^2}$ | 876 | dbp_r_d2p |
| $\frac{\partial^2 B_\phi}{\partial r^2}$ | 857 | db_phi_d2r | $\frac{\partial^2 B'_\theta}{\partial \phi^2}$ | 877 | dbp_theta_d2p |
| $\frac{\partial^2 B'_r}{\partial r^2}$ | 858 | dbp_r_d2r | $\frac{\partial^2 B'_\phi}{\partial \phi^2}$ | 878 | dbp_phi_d2p |
| $\frac{\partial^2 B'_\theta}{\partial r^2}$ | 859 | dbp_theta_d2r | $\frac{\partial^2 \overline{B_r}}{\partial \phi^2}$ | 879 | dbm_r_d2p |
| $\frac{\partial^2 B'_\phi}{\partial r^2}$ | 860 | dbp_phi_d2r | $\frac{\partial^2 \overline{B_\theta}}{\partial \phi^2}$ | 880 | dbm_theta_d2p |

| Value | Code | Variable | Value | Code | Variable |
|---|------|----------------|--|------|----------------|
| $\frac{\partial^2 \overline{B_\phi}}{\partial \phi^2}$ | 881 | dbm_phi_d2p | $\frac{\partial^2 B_\theta}{\partial \theta \partial \phi}$ | 901 | db_theta_d2tp |
| $\frac{\partial^2 \overline{B_r}}{\partial r \partial \theta}$ | 882 | db_r_d2rt | $\frac{\partial^2 B_\phi}{\partial \theta \partial \phi}$ | 902 | db_phi_d2tp |
| $\frac{\partial^2 B_\theta}{\partial r \partial \theta}$ | 883 | db_theta_d2rt | $\frac{\partial^2 B'_r}{\partial \theta \partial \phi}$ | 903 | dbp_r_d2tp |
| $\frac{\partial^2 B_\phi}{\partial r \partial \theta}$ | 884 | db_phi_d2rt | $\frac{\partial^2 B'_\theta}{\partial \theta \partial \phi}$ | 904 | dbp_theta_d2tp |
| $\frac{\partial^2 B'_r}{\partial r \partial \theta}$ | 885 | dbp_r_d2rt | $\frac{\partial^2 B'_\phi}{\partial \theta \partial \phi}$ | 905 | dbp_phi_d2tp |
| $\frac{\partial^2 B'_\theta}{\partial r \partial \theta}$ | 886 | dbp_theta_d2rt | $\frac{\partial^2 \overline{B_r}}{\partial \theta \partial \phi}$ | 906 | dbm_r_d2tp |
| $\frac{\partial^2 B'_\phi}{\partial r \partial \theta}$ | 887 | dbp_phi_d2rt | $\frac{\partial^2 \overline{B_\theta}}{\partial \theta \partial \phi}$ | 907 | dbm_theta_d2tp |
| $\frac{\partial^2 \overline{B_r}}{\partial r \partial \theta}$ | 888 | dbm_r_d2rt | $\frac{\partial^2 \overline{B_\phi}}{\partial \theta \partial \phi}$ | 908 | dbm_phi_d2tp |
| $\frac{\partial^2 \overline{B_\theta}}{\partial r \partial \theta}$ | 889 | dbm_theta_d2rt | | | |
| $\frac{\partial^2 \overline{B_\phi}}{\partial r \partial \theta}$ | 890 | dbm_phi_d2rt | | | |
| $\frac{\partial^2 B_r}{\partial r \partial \phi}$ | 891 | db_r_d2rp | | | |
| $\frac{\partial^2 B_\theta}{\partial r \partial \phi}$ | 892 | db_theta_d2rp | | | |
| $\frac{\partial^2 B_\phi}{\partial r \partial \phi}$ | 893 | db_phi_d2rp | | | |
| $\frac{\partial^2 B'_r}{\partial r \partial \phi}$ | 894 | dbp_r_d2rp | | | |
| $\frac{\partial^2 B'_\theta}{\partial r \partial \phi}$ | 895 | dbp_theta_d2rp | | | |
| $\frac{\partial^2 B'_\phi}{\partial r \partial \phi}$ | 896 | dbp_phi_d2rp | | | |
| $\frac{\partial^2 \overline{B_r}}{\partial r \partial \phi}$ | 897 | dbm_r_d2rp | | | |
| $\frac{\partial^2 \overline{B_\theta}}{\partial r \partial \phi}$ | 898 | dbm_theta_d2rp | | | |
| $\frac{\partial^2 \overline{B_\phi}}{\partial r \partial \phi}$ | 899 | dbm_phi_d2rp | | | |
| $\frac{\partial^2 B_r}{\partial \theta \partial \phi}$ | 900 | db_r_d2tp | | | |

| Value | Code | Variable | Value | Code | Variable |
|-----------------------------------|------|----------|---|------|-------------|
| \mathcal{J}_r | 1001 | j_r | $\overline{\mathcal{J}} \cdot \overline{\mathcal{J}}$ | 1012 | jm_sq |
| \mathcal{J}'_r | 1002 | jp_r | $\overline{\mathcal{J}} \cdot \mathcal{J}'$ | 1013 | jpm_sq |
| $\overline{\mathcal{J}}_r$ | 1003 | jm_r | $(\mathcal{J}_r)^2$ | 1014 | j_r_sq |
| \mathcal{J}_θ | 1004 | j_theta | $(\mathcal{J}'_r)^2$ | 1015 | jp_r_sq |
| \mathcal{J}'_θ | 1005 | jp_theta | $(\overline{\mathcal{J}}_r)^2$ | 1016 | jm_r_sq |
| $\overline{\mathcal{J}}_\theta$ | 1006 | jm_theta | $(\mathcal{J}_\theta)^2$ | 1017 | j_theta_sq |
| \mathcal{J}_ϕ | 1007 | j_phi | $(\mathcal{J}'_\theta)^2$ | 1018 | jp_theta_sq |
| \mathcal{J}'_ϕ | 1008 | jp_phi | $(\overline{\mathcal{J}}_\theta)^2$ | 1019 | jm_theta_sq |
| $\overline{\mathcal{J}}_\phi$ | 1009 | jm_phi | $(\mathcal{J}_\phi)^2$ | 1020 | j_phi_sq |
| $\mathcal{J} \cdot \mathcal{J}$ | 1010 | j_sq | $(\mathcal{J}'_\phi)^2$ | 1021 | jp_phi_sq |
| $\mathcal{J}' \cdot \mathcal{J}'$ | 1011 | jp_sq | $(\overline{\mathcal{J}}_\phi)^2$ | 1022 | jm_phi_sq |

We use the shorthand \mathcal{J} to denote the curl of \mathbf{B} , namely

$$\mathcal{J} \equiv \nabla \times \mathbf{B}. \quad (15)$$

4.7 $\nabla \times \mathbf{B}$

| Value | Code | Variable | Value | Code | Variable |
|---|------|------------------|---------------------------------------|------|------------------|
| $\frac{1}{2}c_4\mathbf{B}^2$ | 1101 | magnetic_energy | $\frac{1}{2}c_4\overline{B_\theta}^2$ | 1107 | theta_mme |
| $\frac{1}{2}c_4B_r^2$ | 1102 | radial_me | $\frac{1}{2}c_4\overline{B_\phi}^2$ | 1108 | phi_mme |
| $\frac{1}{2}c_4B_\theta^2$ | 1103 | theta_me | $\frac{1}{2}c_4\mathbf{B}'^2$ | 1109 | pmagnetic_energy |
| $\frac{1}{2}c_4B_\phi^2$ | 1104 | phi_me | $\frac{1}{2}c_4B_r'^2$ | 1110 | radial_pme |
| $\frac{1}{2}c_4\overline{\mathbf{B}}^2$ | 1105 | mmagnetic_energy | $\frac{1}{2}c_4B_\theta'^2$ | 1111 | theta_pme |
| $\frac{1}{2}c_4\overline{B_r}^2$ | 1106 | radial_mme | $\frac{1}{2}c_4B_\phi'^2$ | 1112 | phi_pme |

4.8 Magnetic Energy Density

Output codes related to the generalized magnetic energy density, $\frac{1}{2}c_4\mathbf{B}^2$, are defined here.

| Value | Code | Variable | Value | Code | Variable |
|---|------|----------------------|--|------|-----------------------|
| $f_1 [\mathbf{v} \cdot \nabla \mathbf{v}]_r$ | 1201 | v_grad_v_r | $-c_1 f_1 [\hat{\mathbf{z}} \times \mathbf{v}]_\phi$ | 1221 | Coriolis_Force_phi |
| $f_1 [\mathbf{v} \cdot \nabla \mathbf{v}]_\theta$ | 1202 | v_grad_v_theta | $-c_1 f_1 [\hat{\mathbf{z}} \times \mathbf{v}']_r$ | 1222 | Coriolis_pForce_r |
| $f_1 [\mathbf{v} \cdot \nabla \mathbf{v}]_\phi$ | 1203 | v_grad_v_phi | $-c_1 f_1 [\hat{\mathbf{z}} \times \mathbf{v}']_\theta$ | 1223 | Coriolis_pForce_theta |
| $f_1 [\mathbf{v}' \cdot \nabla \bar{\mathbf{v}}]_r$ | 1204 | vp_grad_vm_r | $-c_1 f_1 [\hat{\mathbf{z}} \times \mathbf{v}']_\phi$ | 1224 | Coriolis_pForce_phi |
| $f_1 [\mathbf{v}' \cdot \nabla \bar{\mathbf{v}}]_\theta$ | 1205 | vp_grad_vm_theta | $-c_1 f_1 [\hat{\mathbf{z}} \times \bar{\mathbf{v}}]_r$ | 1225 | Coriolis_mForce_r |
| $f_1 [\mathbf{v}' \cdot \nabla \bar{\mathbf{v}}]_\phi$ | 1206 | vp_grad_vm_phi | $-c_1 f_1 [\hat{\mathbf{z}} \times \bar{\mathbf{v}}]_\theta$ | 1226 | Coriolis_mForce_theta |
| $f_1 [\bar{\mathbf{v}} \cdot \nabla \mathbf{v}']_r$ | 1207 | vm_grad_vp_r | $-c_1 f_1 [\hat{\mathbf{z}} \times \bar{\mathbf{v}}]_\phi$ | 1227 | Coriolis_mForce_phi |
| $f_1 [\bar{\mathbf{v}} \cdot \nabla \mathbf{v}']_\theta$ | 1208 | vm_grad_vp_theta | $c_5 [\nabla \cdot \mathcal{D}]_r$ | 1228 | viscous_Force_r |
| $f_1 [\bar{\mathbf{v}} \cdot \nabla \mathbf{v}']_\phi$ | 1209 | vm_grad_vp_phi | $c_5 [\nabla \cdot \mathcal{D}]_\theta$ | 1229 | viscous_Force_theta |
| $f_1 [\mathbf{v}' \cdot \nabla \mathbf{v}']_r$ | 1210 | vp_grad_vp_r | $c_5 [\nabla \cdot \mathcal{D}]_\phi$ | 1230 | viscous_Force_phi |
| $f_1 [\mathbf{v}' \cdot \nabla \mathbf{v}']_\theta$ | 1211 | vp_grad_vp_theta | $c_5 [\nabla \cdot \mathcal{D}']_r$ | 1231 | viscous_pForce_r |
| $f_1 [\mathbf{v}' \cdot \nabla \mathbf{v}']_\phi$ | 1212 | vp_grad_vp_phi | $c_5 [\nabla \cdot \mathcal{D}']_\theta$ | 1232 | viscous_pForce_theta |
| $f_1 [\bar{\mathbf{v}} \cdot \nabla \bar{\mathbf{v}}]_r$ | 1213 | vm_grad_vm_r | $c_5 [\nabla \cdot \mathcal{D}']_\phi$ | 1233 | viscous_pForce_phi |
| $f_1 [\bar{\mathbf{v}} \cdot \nabla \bar{\mathbf{v}}]_\theta$ | 1214 | vm_grad_vm_theta | $c_5 [\nabla \cdot \bar{\mathcal{D}}]_r$ | 1234 | viscous_mForce_r |
| $f_1 [\bar{\mathbf{v}} \cdot \nabla \bar{\mathbf{v}}]_\phi$ | 1215 | vm_grad_vm_phi | $c_5 [\nabla \cdot \bar{\mathcal{D}}]_\theta$ | 1235 | viscous_mForce_theta |
| $c_2 f_2 \Theta$ | 1216 | buoyancy_force | $c_5 [\nabla \cdot \bar{\mathcal{D}}]_\phi$ | 1236 | viscous_mForce_phi |
| $c_2 f_2 \Theta'$ | 1217 | buoyancy_pforce | $-c_3 f_1 \frac{\partial}{\partial r} \left(\frac{P}{f_1} \right)$ | 1237 | pressure_Force_r |
| $c_2 f_2 \bar{\Theta}$ | 1218 | buoyancy_mforce | $-c_3 \frac{1}{r} \frac{\partial P}{\partial \theta}$ | 1238 | pressure_Force_theta |
| $-c_1 f_1 [\hat{\mathbf{z}} \times \mathbf{v}]_r$ | 1219 | Coriolis_Force_r | $-c_3 \frac{1}{r \sin \theta} \frac{\partial P}{\partial \phi}$ | 1239 | pressure_Force_phi |
| $-c_1 f_1 [\hat{\mathbf{z}} \times \mathbf{v}]_\theta$ | 1220 | Coriolis_Force_theta | $-c_3 f_1 \frac{\partial}{\partial r} \left(\frac{P'}{f_1} \right)$ | 1240 | pressure_pForce_r |

4.9 momentum equation

All terms from the momentum equation, and their Reynolds decomposition, are defined here.

| Value | Code | Variable | Value | Code | Variable |
|---|------|-----------------------|---|------|-------------------|
| $-c_3 \frac{1}{r} \frac{\partial P'}{\partial \theta}$ | 1241 | pressure_pForce_theta | $c_4 [(\nabla \times \mathbf{B}') \times \mathbf{B}']_\theta$ | 1261 | jp_cross_bp_theta |
| $-c_3 \frac{1}{r \sin \theta} \frac{\partial P'}{\partial \phi}$ | 1242 | pressure_pForce_phi | $c_4 [(\nabla \times \mathbf{B}') \times \mathbf{B}']_\phi$ | 1262 | jp_cross_bp_phi |
| $-c_3 f_1 \frac{\partial}{\partial r} \left(\frac{\bar{P}}{f_1} \right)$ | 1243 | pressure_mForce_r | | | |
| $-c_3 \frac{1}{r} \frac{\partial \bar{P}}{\partial \theta}$ | 1244 | pressure_mForce_theta | | | |
| $-c_3 \frac{1}{r \sin \theta} \frac{\partial \bar{P}}{\partial \phi}$ | 1245 | pressure_mForce_phi | | | |
| $c_2 f_2 \Theta_{00}$ | 1246 | buoyancy_force_ell0 | | | |
| $-c_3 f_1 \frac{\partial}{\partial r} \left(\frac{P_{00}}{f_1} \right)$ | 1247 | pressure_force_ell0_r | | | |
| $c_4 [(\nabla \times \mathbf{B}) \times \mathbf{B}]_r$ | 1248 | j_cross_b_r | | | |
| $c_4 [(\nabla \times \mathbf{B}) \times \mathbf{B}]_\theta$ | 1249 | j_cross_b_theta | | | |
| $c_4 [(\nabla \times \mathbf{B}) \times \mathbf{B}]_\phi$ | 1250 | j_cross_b_phi | | | |
| $c_4 [(\nabla \times \mathbf{B}') \times \bar{\mathbf{B}}]_r$ | 1251 | jp_cross_bm_r | | | |
| $c_4 [(\nabla \times \mathbf{B}') \times \bar{\mathbf{B}}]_\theta$ | 1252 | jp_cross_bm_theta | | | |
| $c_4 [(\nabla \times \mathbf{B}') \times \bar{\mathbf{B}}]_\phi$ | 1253 | jp_cross_bm_phi | | | |
| $c_4 [(\nabla \times \bar{\mathbf{B}}) \times \mathbf{B}']_r$ | 1254 | jm_cross_bp_r | | | |
| $c_4 [(\nabla \times \bar{\mathbf{B}}) \times \mathbf{B}']_\theta$ | 1255 | jm_cross_bp_theta | | | |
| $c_4 [(\nabla \times \bar{\mathbf{B}}) \times \mathbf{B}']_\phi$ | 1256 | jm_cross_bp_phi | | | |
| $c_4 [(\nabla \times \bar{\mathbf{B}}) \times \bar{\mathbf{B}}]_r$ | 1257 | jm_cross_bm_r | | | |
| $c_4 [(\nabla \times \bar{\mathbf{B}}) \times \bar{\mathbf{B}}]_\theta$ | 1258 | jm_cross_bm_theta | | | |
| $c_4 [(\nabla \times \bar{\mathbf{B}}) \times \bar{\mathbf{B}}]_\phi$ | 1259 | jm_cross_bm_phi | | | |
| $c_4 [(\nabla \times \mathbf{B}') \times \mathbf{B}']_r$ | 1260 | jp_cross_bp_r | | | |

4.10 thermal energy equation

Terms from the thermal energy equation, and their Reynolds decomposition, are defined here.

| Value | Code | Variable | Value | Code | Variable |
|--|------|-----------------|--|------|---------------|
| $f_1 f_4 \mathbf{v} \cdot \nabla \Theta$ | 1401 | rhotv_grad_s | $-c_6 \nabla \cdot \mathbf{F}_{cond}$ | 1421 | s_diff |
| $f_1 f_4 \mathbf{v}' \cdot \nabla \Theta'$ | 1402 | rhotvp_grad_sp | $-c_6 \nabla \cdot \mathbf{F}'_{cond}$ | 1422 | sp_diff |
| $f_1 f_4 \mathbf{v}' \cdot \nabla \bar{\Theta}$ | 1403 | rhotvp_grad_sm | $-c_6 \nabla \cdot \bar{\mathbf{F}}_{cond}$ | 1423 | sm_diff |
| $f_1 f_4 \bar{\mathbf{v}} \cdot \nabla \bar{\Theta}$ | 1404 | rhotvm_grad_sm | $c_6 f_1 f_4 f_5 \left(\frac{\partial^2 \Theta}{\partial r^2} + \frac{\partial \Theta}{\partial r} \left[\frac{2}{r} + \frac{d}{dr} \ln \{f_1 f_4 f_5\} \right] \right)$ | 1424 | s_diff_r |
| $f_1 f_4 \bar{\mathbf{v}} \cdot \nabla \Theta'$ | 1405 | rhotvm_grad_sp | $c_6 f_1 f_4 f_5 \left(\frac{\partial^2 \Theta'}{\partial r^2} + \frac{\partial \Theta'}{\partial r} \left[\frac{2}{r} + \frac{d}{dr} \ln \{f_1 f_4 f_5\} \right] \right)$ | 1425 | sp_diff_r |
| $f_1 f_4 v_r \frac{\partial s}{\partial r}$ | 1406 | rhotvr_grad_s | $c_6 f_1 f_4 f_5 \left(\frac{\partial^2 \bar{\Theta}}{\partial r^2} + \frac{\partial \bar{\Theta}}{\partial r} \left[\frac{2}{r} + \frac{d}{dr} \ln \{f_1 f_4 f_5\} \right] \right)$ | 1426 | sm_diff_r |
| $f_1 f_4 v'_r \frac{\partial \Theta'}{\partial r}$ | 1407 | rhotvpr_grad_sp | $c_6 \frac{f_1 f_4 f_5}{r^2} \left(\frac{\partial^2 \Theta}{\partial \theta^2} + \cot \theta \frac{\partial s}{\partial \theta} \right)$ | 1427 | s_diff_theta |
| $f_1 f_4 v'_r \frac{\partial \bar{\Theta}}{\partial r}$ | 1408 | rhotvpr_grad_sm | $c_6 \frac{f_1 f_4 f_5}{r^2} \left(\frac{\partial^2 \Theta'}{\partial \theta^2} + \cot \theta \frac{\partial \Theta'}{\partial \theta} \right)$ | 1428 | sp_diff_theta |
| $f_1 f_4 \bar{v}_r \frac{\partial \bar{\Theta}}{\partial r}$ | 1409 | rhotvmr_grad_sm | $c_6 \frac{f_1 f_4 f_5}{r^2} \left(\frac{\partial^2 \bar{\Theta}}{\partial \theta^2} + \cot \theta \frac{\partial \bar{\Theta}}{\partial \theta} \right)$ | 1429 | sm_diff_theta |
| $f_1 f_4 \bar{v}_r \frac{\partial \Theta'}{\partial r}$ | 1410 | rhotvmr_grad_sp | $c_6 \frac{f_1 f_4 f_5}{r^2 \sin^2 \theta} \frac{\partial^2 \Theta}{\partial \phi^2}$ | 1430 | s_diff_phi |
| $f_1 f_4 \frac{v_\theta}{r} \frac{\partial \Theta}{\partial \theta}$ | 1411 | rhotvt_grad_s | $c_6 \frac{f_1 f_4 f_5}{r^2 \sin^2 \theta} \frac{\partial^2 \Theta'}{\partial \phi^2}$ | 1431 | sp_diff_phi |
| $f_1 f_4 \frac{v'_\theta}{r} \frac{\partial \Theta'}{\partial \theta}$ | 1412 | rhotvpt_grad_sp | $c_6 \frac{f_1 f_4 f_5}{r^2 \sin^2 \theta} \frac{\partial^2 \bar{\Theta}}{\partial \phi^2}$ | 1432 | sm_diff_phi |
| $f_1 f_4 \frac{v'_\theta}{r} \frac{\partial \bar{\Theta}}{\partial \theta}$ | 1413 | rhotvpt_grad_sm | $F_Q(r)$ | 1431 | vol_heat_flux |
| $f_1 f_4 \frac{\bar{v}_\theta}{r} \frac{\partial \bar{\Theta}}{\partial \theta}$ | 1414 | rhotvmt_grad_sm | $f_6(r)$ | 1432 | vol_heating |
| $f_1 f_4 \frac{\bar{v}_\theta}{r} \frac{\partial \Theta'}{\partial \theta}$ | 1415 | rhotvmt_grad_sp | $c_5 \Phi(r, \theta, \phi)$ | 1433 | visc_heating |
| $f_1 f_4 \frac{v_\phi}{r \sin \theta} \frac{\partial \Theta}{\partial \phi}$ | 1416 | rhotvp_grad_s | $f_7 c_4 (\mathcal{J}' \cdot \mathcal{J}')$ | 1434 | ohmic_heat |
| $f_1 f_4 \frac{v'_\phi}{r \sin \theta} \frac{\partial \Theta'}{\partial \phi}$ | 1417 | rhotvpp_grad_sp | $f_7 c_4 (\mathcal{J}' \cdot \mathcal{J}')$ | 1435 | ohmic_heat_pp |
| $f_1 f_4 \frac{v'_\phi}{r \sin \theta} \frac{\partial \bar{\Theta}}{\partial \phi}$ | 1418 | rhotvpp_grad_sm | $f_7 c_4 (\bar{\mathcal{J}} \cdot \bar{\mathcal{J}})$ | 1436 | ohmic_heat_pm |
| $f_1 f_4 \frac{\bar{v}_\phi}{r \sin \theta} \frac{\partial \bar{\Theta}}{\partial \phi}$ | 1419 | rhotvmp_grad_sm | $f_7 c_4 (\bar{\mathcal{J}} \cdot \mathcal{J}')$ | 1437 | ohmic_heat_mm |
| $f_1 f_4 \frac{\bar{v}_\phi}{r \sin \theta} \frac{\partial \Theta'}{\partial \phi}$ | 1420 | rhotvmp_grad_sp | $f_1 f_4 v_r \Theta$ | 1438 | rhot_vr_s |

| Value | Code | Variable | Value | Code | Variable |
|---|------|-------------------|---|------|-------------------|
| $f_1 f_4 v'_r \Theta'$ | 1439 | rhot_vrp_sp | $c_P f_1 v'_r \overline{T}$ | 1459 | enth_flux_rpm |
| $f_1 f_4 v'_r \overline{\Theta}$ | 1440 | rhot_vrp_sm | $c_P f_1 v'_\theta \overline{T}$ | 1460 | enth_flux_thetapm |
| $f_1 f_4 \overline{v_r} \Theta'$ | 1441 | rhot_vrm_sp | $c_P f_1 v'_\phi \overline{T}$ | 1461 | enth_flux_phipm |
| $f_1 f_4 \overline{v_r} \overline{\Theta}$ | 1442 | rhot_vrm_sm | $c_P f_1 \overline{v_r} T'$ | 1462 | enth_flux_rmp |
| $f_1 f_4 v_\theta \Theta$ | 1443 | rhot_vt_s | $c_P f_1 \overline{v_\theta} T'$ | 1463 | enth_flux_thetamp |
| $f_1 f_4 v'_\theta \Theta'$ | 1444 | rhot_vtp_sp | $c_P f_1 \overline{v_\phi} T'$ | 1464 | enth_flux_phimp |
| $f_1 f_4 v'_\theta \overline{\Theta}$ | 1445 | rhot_vtp_sm | $c_P f_1 \overline{v_r} \overline{T}$ | 1465 | enth_flux_rmm |
| $f_1 f_4 \overline{v_\theta} \Theta'$ | 1446 | rhot_vtm_sp | $c_P f_1 \overline{v_\theta} \overline{T}$ | 1466 | enth_flux_thetamm |
| $f_1 f_4 \overline{v_\theta} \overline{\Theta}$ | 1447 | rhot_vtm_sm | $c_P f_1 \overline{v_\phi} \overline{T}$ | 1467 | enth_flux_phimm |
| $f_1 f_4 v_\phi \Theta$ | 1448 | rhot_vp_s | $-c_6 f_1 f_4 f_5 \frac{\partial \Theta}{\partial r}$ | 1468 | cond_flux_r |
| $f_1 f_4 v'_\phi \Theta'$ | 1449 | rhot_vpp_sp | $-c_6 f_1 f_4 f_5 \frac{1}{r} \frac{\partial \Theta}{\partial \theta}$ | 1469 | cond_flux_theta |
| $f_1 f_4 v'_\phi \overline{\Theta}$ | 1450 | rhot_vpp_sm | $-c_6 f_1 f_4 f_5 \frac{1}{r \sin \theta} \frac{\partial \Theta}{\partial \phi}$ | 1470 | cond_flux_phi |
| $f_1 f_4 \overline{v_\phi} \Theta'$ | 1451 | rhot_vpm_sp | $-c_6 f_1 f_4 f_5 \frac{\partial \Theta'}{\partial r}$ | 1471 | cond_fluxp_r |
| $f_1 f_4 \overline{v_\phi} \overline{\Theta}$ | 1452 | rhot_vpm_sm | $-c_6 f_1 f_4 f_5 \frac{1}{r} \frac{\partial \Theta'}{\partial \theta}$ | 1472 | cond_fluxp_theta |
| $c_P f_1 v_r T$ | 1453 | enth_flux_r | $-c_6 f_1 f_4 f_5 \frac{1}{r \sin \theta} \frac{\partial \Theta'}{\partial \phi}$ | 1473 | cond_fluxp_phi |
| $c_P f_1 v_\theta T$ | 1454 | enth_flux_theta | $-c_6 f_1 f_4 f_5 \frac{\partial \overline{\Theta}}{\partial r}$ | 1474 | cond_fluxm_r |
| $c_P f_1 v_\phi T$ | 1455 | enth_flux_phi | $-c_6 f_1 f_4 f_5 \frac{1}{r} \frac{\partial \overline{\Theta}}{\partial \theta}$ | 1475 | cond_fluxm_theta |
| $c_P f_1 v'_r T'$ | 1456 | enth_flux_rpp | $-c_6 f_1 f_4 f_5 \frac{1}{r \sin \theta} \frac{\partial \overline{\Theta}}{\partial \phi}$ | 1476 | cond_fluxm_phi |
| $c_P f_1 v'_\theta T'$ | 1457 | enth_flux_thetapp | | | |
| $c_P f_1 v'_\phi T'$ | 1458 | enth_flux_phipp | | | |

| Value | Code | Variable | Value | Code | Variable |
|---|------|---------------------|--|------|-------------------------|
| $[\mathbf{B} \cdot \nabla \mathbf{v}]_r$ | 1601 | induct_shear_r | $[\overline{\mathbf{B}} \cdot \nabla \overline{\mathbf{v}}]_\theta$ | 1621 | induct_shear_vmbm_theta |
| $-(\nabla \cdot \mathbf{v}) B_r$ | 1602 | induct_comp_r | $-(\overline{\nabla} \cdot \overline{\mathbf{v}}) \overline{B_\theta}$ | 1622 | induct_comp_vmbm_theta |
| $-\mathbf{v} \cdot \nabla \mathbf{B}]_r$ | 1603 | induct_advec_r | $-\overline{\mathbf{v}} \cdot \nabla \overline{\mathbf{B}}]_\theta$ | 1623 | induct_advec_vmbm_theta |
| $[\nabla \times (\mathbf{v} \times \mathbf{B})]_r$ | 1604 | induct_r | $[\nabla \times (\overline{\mathbf{v}} \times \overline{\mathbf{B}})]_\theta$ | 1624 | induct_vmbm_theta |
| $-c_7 [\nabla \times (\mathbf{f}_7 \nabla \times \mathbf{B})]_r$ | 1605 | induct_diff_r | $-c_7 [\nabla \times (\mathbf{f}_7 \nabla \times \overline{\mathbf{B}})]_\theta$ | 1625 | induct_diff_bm_theta |
| $[\mathbf{B} \cdot \nabla \mathbf{v}]_\theta$ | 1606 | induct_shear_theta | $[\overline{\mathbf{B}} \cdot \nabla \overline{\mathbf{v}}]_\phi$ | 1626 | induct_shear_vmbm_phi |
| $-(\nabla \cdot \mathbf{v}) B_\theta$ | 1607 | induct_comp_theta | $-(\overline{\nabla} \cdot \overline{\mathbf{v}}) \overline{B_\phi}$ | 1627 | induct_comp_vmbm_phi |
| $-\mathbf{v} \cdot \nabla \mathbf{B}]_\theta$ | 1608 | induct_advec_theta | $-\overline{\mathbf{v}} \cdot \nabla \overline{\mathbf{B}}]_\phi$ | 1628 | induct_advec_vmbm_phi |
| $[\nabla \times (\mathbf{v} \times \mathbf{B})]_\theta$ | 1609 | induct_theta | $[\nabla \times (\overline{\mathbf{v}} \times \overline{\mathbf{B}})]_\phi$ | 1629 | induct_vmbm_phi |
| $-c_7 [\nabla \times (\mathbf{f}_7 \nabla \times \mathbf{B})]_\theta$ | 1610 | induct_diff_theta | $-c_7 [\nabla \times (\mathbf{f}_7 \nabla \times \overline{\mathbf{B}})]_\phi$ | 1630 | induct_diff_bm_phi |
| $[\mathbf{B} \cdot \nabla \mathbf{v}]_\phi$ | 1611 | induct_shear_phi | $[\mathbf{B}' \cdot \nabla \overline{\mathbf{v}}]_r$ | 1631 | induct_shear_vmbp_r |
| $-(\nabla \cdot \mathbf{v}) B_\phi$ | 1612 | induct_comp_phi | $-(\overline{\nabla} \cdot \overline{\mathbf{v}}) B'_r$ | 1632 | induct_comp_vmbp_r |
| $-\mathbf{v} \cdot \nabla \mathbf{B}]_\phi$ | 1613 | induct_advec_phi | $-\overline{\mathbf{v}} \cdot \nabla \mathbf{B}'_r$ | 1633 | induct_advec_vmbp_r |
| $[\nabla \times (\mathbf{v} \times \mathbf{B})]_\phi$ | 1614 | induct_phi | $[\nabla \times (\overline{\mathbf{v}} \times \mathbf{B}')]_r$ | 1634 | induct_vmbp_r |
| $-c_7 [\nabla \times (\mathbf{f}_7 \nabla \times \mathbf{B})]_\phi$ | 1615 | induct_diff_phi | $-c_7 [\nabla \times (\mathbf{f}_7 \nabla \times \mathbf{B}')]_r$ | 1635 | induct_diff_bp_r |
| $[\overline{\mathbf{B}} \cdot \nabla \overline{\mathbf{v}}]_r$ | 1616 | induct_shear_vmbm_r | $[\mathbf{B}' \cdot \nabla \overline{\mathbf{v}}]_\theta$ | 1636 | induct_shear_vmbp_theta |
| $-(\overline{\nabla} \cdot \overline{\mathbf{v}}) \overline{B_r}$ | 1617 | induct_comp_vmbm_r | $-(\overline{\nabla} \cdot \overline{\mathbf{v}}) B'_\theta$ | 1637 | induct_comp_vmbp_theta |
| $-\overline{\mathbf{v}} \cdot \nabla \overline{\mathbf{B}}]_r$ | 1618 | induct_advec_vmbm_r | $-\overline{\mathbf{v}} \cdot \nabla \mathbf{B}'_\theta$ | 1638 | induct_advec_vmbp_theta |
| $[\nabla \times (\overline{\mathbf{v}} \times \overline{\mathbf{B}})]_r$ | 1619 | induct_vmbm_r | $[\nabla \times (\overline{\mathbf{v}} \times \mathbf{B}')]_\theta$ | 1639 | induct_vmbp_theta |
| $-c_7 [\nabla \times (\mathbf{f}_7 \nabla \times \overline{\mathbf{B}})]_r$ | 1620 | induct_diff_bm_r | $-c_7 [\nabla \times (\mathbf{f}_7 \nabla \times \mathbf{B}')]_\theta$ | 1640 | induct_diff_bp_theta |

4.11 Induction Equation

Terms from the induction equation, and their Reynolds decomposition, are described here.

| Value | Code | Variable | Value | Code | Variable |
|--|------|-------------------------|--|------|-------------------------|
| $[\mathbf{B}' \cdot \nabla \bar{\mathbf{v}}]_\phi$ | 1641 | induct_shear_vmbp_phi | $[\nabla \times (\mathbf{v} \times \mathbf{B}')]_r$ | 1661 | induct_vpbp_r |
| $-(\nabla \cdot \bar{\mathbf{v}}) B'_\phi$ | 1642 | induct_comp_vmbp_phi | $[\mathbf{B}' \cdot \nabla \mathbf{v}']_\theta$ | 1662 | induct_shear_vpbp_theta |
| $-\bar{\mathbf{v}} \cdot \nabla \mathbf{B}'_\phi$ | 1643 | induct_advec_vmbp_phi | $-(\nabla \cdot \mathbf{v}') B'_\theta$ | 1663 | induct_comp_vpbp_theta |
| $[\nabla \times (\bar{\mathbf{v}} \times \mathbf{B}')]_\phi$ | 1644 | induct_vmbp_phi | $-\mathbf{v}' \cdot \nabla \mathbf{B}'_\theta$ | 1664 | induct_advec_vpbp_theta |
| $-c_7 [\nabla \times (\mathbf{f}_7 \nabla \times \mathbf{B}')]_\phi$ | 1645 | induct_diff_bp_phi | $[\nabla \times (\mathbf{v} \times \mathbf{B}')]_\theta$ | 1665 | induct_vpbp_theta |
| $[\bar{\mathbf{B}} \cdot \nabla \mathbf{v}']_r$ | 1646 | induct_shear_vpbm_r | $[\mathbf{B}' \cdot \nabla \mathbf{v}']_\phi$ | 1666 | induct_shear_vpbp_phi |
| $-(\nabla \cdot \mathbf{v}') \bar{B}_r$ | 1647 | induct_comp_vpbm_r | $-(\nabla \cdot \mathbf{v}') B'_\phi$ | 1667 | induct_comp_vpbp_phi |
| $-\mathbf{v}' \cdot \nabla \bar{\mathbf{B}}_r$ | 1648 | induct_advec_vpbm_r | $-\mathbf{v}' \cdot \nabla \mathbf{B}'_\phi$ | 1668 | induct_advec_vpbp_phi |
| $[\nabla \times (\mathbf{v}' \times \bar{\mathbf{B}})]_r$ | 1649 | induct_vpbm_r | $[\nabla \times (\mathbf{v} \times \mathbf{B}')]_\phi$ | 1669 | induct_vpbp_phi |
| $[\bar{\mathbf{B}} \cdot \nabla \mathbf{v}']_\theta$ | 1650 | induct_shear_vpbm_theta | | | |
| $-(\nabla \cdot \mathbf{v}') \bar{B}_\theta$ | 1651 | induct_comp_vpbm_theta | | | |
| $-\mathbf{v}' \cdot \nabla \bar{\mathbf{B}}_\theta$ | 1652 | induct_advec_vpbm_theta | | | |
| $[\nabla \times (\mathbf{v}' \times \bar{\mathbf{B}})]_\theta$ | 1653 | induct_vpbm_theta | | | |
| $[\bar{\mathbf{B}} \cdot \nabla \mathbf{v}']_\phi$ | 1654 | induct_shear_vpbm_phi | | | |
| $-(\nabla \cdot \mathbf{v}') \bar{B}_\phi$ | 1655 | induct_comp_vpbm_phi | | | |
| $-\mathbf{v}' \cdot \nabla \bar{\mathbf{B}}_\phi$ | 1656 | induct_advec_vpbm_phi | | | |
| $[\nabla \times (\mathbf{v}' \times \bar{\mathbf{B}})]_\phi$ | 1657 | induct_vpbm_phi | | | |
| $[\mathbf{B}' \cdot \nabla \mathbf{v}']_r$ | 1658 | induct_shear_vpbp_r | | | |
| $-(\nabla \cdot \mathbf{v}') B'_r$ | 1659 | induct_comp_vpbp_r | | | |
| $-\mathbf{v}' \cdot \nabla \mathbf{B}'_r$ | 1660 | induct_advec_vpbp_r | | | |

| Value | Code | Variable | Value | Code | Variable |
|--|------|-------------------|---|------|--------------------|
| $r \sin\theta f_1 [\mathbf{v}' \cdot \nabla \mathbf{v}']_\phi$ | 1801 | samom_advec_pp | $f_1 r \sin\theta \overline{v_\theta} \overline{v_\phi}$ | 1810 | famom_dr_theta |
| $r \sin\theta f_1 [\overline{\mathbf{v}} \cdot \nabla \overline{\mathbf{v}}]_\phi$ | 1802 | samom_advec_mm | $\frac{c_1}{2} f_1 r^2 \sin^2\theta \overline{v_r}$ | 1811 | famom_mean_r |
| $-c_1 f_1 r \sin\theta (\cos\theta v_\theta + \sin\theta v_r)$ | 1803 | samom_coriolis | $\frac{c_1}{2} f_1 r^2 \sin^2\theta \overline{v_\theta}$ | 1812 | famom_mean_theta |
| $r \sin\theta [\nabla \cdot \overline{\mathcal{D}}]_\phi$ | 1804 | samom_diffusion | $f_1 \nu \sin\theta \left(v_\phi - r \frac{\partial \overline{v_\phi}}{\partial r} \right)$ | 1813 | famom_diff_r |
| $r \sin\theta c_4 [(\nabla \times \overline{\mathbf{B}}) \times \overline{\mathbf{B}}]_\phi$ | 1805 | samom_lorentz_mm | $f_1 \nu \left(\cos\theta \overline{v_\phi} - \sin\theta \frac{\partial \overline{v_\phi}}{\partial \theta} \right)$ | 1814 | famom_diff_theta |
| $r \sin\theta c_4 [(\nabla \times \mathbf{B}') \times \mathbf{B}']_\phi$ | 1806 | samom_lorentz_pp | $-r \sin\theta c_4 B'_r B'_\phi$ | 1815 | famom_maxstr_r |
| $f_1 r \sin\theta v'_r v'_\phi$ | 1807 | famom_fluct_r | $-r \sin\theta c_4 B'_\theta B'_\phi$ | 1816 | famom_maxstr_theta |
| $f_1 r \sin\theta v'_\theta v'_\phi$ | 1808 | famom_fluct_theta | $-r \sin\theta c_4 \overline{B_r} \overline{B_\phi}$ | 1817 | famom_magtor_r |
| $f_1 r \sin\theta \overline{v_r} \overline{v_\phi}$ | 1809 | famom_dr_r | $-r \sin\theta c_4 \overline{B_\theta} \overline{B_\phi}$ | 1818 | famom_magtor_theta |

4.12 Angular Momentum

Terms from the angular momentum equation and their associated fluxes are defined here. Only those terms contributing to the axisymmetric mean are calculated. Terms of form $a' \overline{a}$, which do not contribute to the mean, are omitted.

| Value | Code | Variable | Value | Code | Variable |
|---|------|----------------|---|------|-------------------|
| $-c_3 f_1 \mathbf{v} \cdot \nabla \left(\frac{P}{f_1} \right)$ | 1901 | press_work | $c_4 \bar{\mathbf{v}} \cdot [(\nabla \times \bar{\mathbf{B}}) \times \bar{\mathbf{B}}]$ | 1922 | mag_work_mmm |
| $-c_3 f_1 \mathbf{v}' \cdot \nabla \left(\frac{P'}{f_1} \right)$ | 1902 | press_work_pp | $\frac{1}{2} f_1 v_r v^2$ | 1922 | ke_flux_radial |
| $-c_3 f_1 \bar{\mathbf{v}} \cdot \nabla \left(\frac{\bar{P}}{f_1} \right)$ | 1903 | press_work_mm | $\frac{1}{2} f_1 v_\theta v^2$ | 1923 | ke_flux_theta |
| $c_2 v_r f_2 \Theta$ | 1904 | buoy_work | $\frac{1}{2} f_1 v_\phi v^2$ | 1924 | ke_flux_phi |
| $c_2 v'_r f_2 \Theta'$ | 1905 | buoy_work_pp | $\frac{1}{2} f_1 \bar{v}_r \bar{v}^2$ | 1925 | mke_mflux_radial |
| $c_2 \bar{v}_r f_2 \bar{\Theta}$ | 1906 | buoy_work_mm | $\frac{1}{2} f_1 \bar{v}_\theta \bar{v}^2$ | 1926 | mke_mflux_theta |
| $c_5 \mathbf{v} \cdot [\nabla \cdot \mathcal{D}]$ | 1907 | visc_work | $\frac{1}{2} f_1 \bar{v}_\phi \bar{v}^2$ | 1927 | mke_mflux_phi |
| $c_5 \mathbf{v}' \cdot [\nabla \cdot \mathcal{D}']$ | 1908 | visc_work_pp | $\frac{1}{2} f_1 \bar{v}_r \bar{v}'^2$ | 1928 | pke_mflux_radial |
| $c_5 \bar{\mathbf{v}} \cdot [\nabla \cdot \bar{\mathcal{D}}]$ | 1909 | visc_work_mm | $\frac{1}{2} f_1 \bar{v}_\theta \bar{v}'^2$ | 1929 | pke_mflux_theta |
| $f_1 \mathbf{v} \cdot [\mathbf{v} \cdot \nabla \mathbf{v}]$ | 1910 | advec_work | $\frac{1}{2} f_1 \bar{v}_\phi \bar{v}'^2$ | 1930 | pke_mflux_phi |
| $f_1 \mathbf{v}' \cdot [\mathbf{v}' \cdot \nabla \mathbf{v}']$ | 1911 | advec_work_ppp | $\frac{1}{2} f_1 v'_r v'^2$ | 1931 | pke_pflux_radial |
| $f_1 \bar{\mathbf{v}} \cdot [\bar{\mathbf{v}} \cdot \nabla \bar{\mathbf{v}}]$ | 1912 | advec_work_mpp | $\frac{1}{2} f_1 v'_\theta v'^2$ | 1932 | pke_pflux_theta |
| $f_1 \mathbf{v}' \cdot [\bar{\mathbf{v}} \cdot \nabla \mathbf{v}']$ | 1913 | advec_work_pmp | $\frac{1}{2} f_1 v'_\phi v'^2$ | 1933 | pke_pflux_phi |
| $f_1 \mathbf{v}' \cdot [\mathbf{v}' \cdot \nabla \bar{\mathbf{v}}]$ | 1914 | advec_work_ppm | $c_5 [\mathbf{v} \cdot \mathcal{D}]_r$ | 1934 | visc_flux_r |
| $f_1 \bar{\mathbf{v}} \cdot [\bar{\mathbf{v}} \cdot \nabla \bar{\mathbf{v}}]$ | 1915 | advec_work_mmm | $c_5 [\mathbf{v} \cdot \mathcal{D}]_\theta$ | 1935 | visc_flux_theta |
| $c_4 \mathbf{v} \cdot [(\nabla \times \mathbf{B}) \times \mathbf{B}]$ | 1916 | mag_work | $c_5 [\mathbf{v} \cdot \mathcal{D}]_\phi$ | 1936 | visc_flux_phi |
| $c_4 \mathbf{v}' \cdot [(\nabla \times \mathbf{B}') \times \mathbf{B}']$ | 1918 | mag_work_ppp | $c_5 [\mathbf{v}' \cdot \mathcal{D}']_r$ | 1937 | visc_fluxpp_r |
| $c_4 \bar{\mathbf{v}} \cdot [(\nabla \times \mathbf{B}') \times \mathbf{B}']$ | 1919 | mag_work_mpp | $c_5 [\mathbf{v}' \cdot \mathcal{D}']_\theta$ | 1938 | visc_fluxpp_theta |
| $c_4 \mathbf{v}' \cdot [(\nabla \times \bar{\mathbf{B}}) \times \mathbf{B}']$ | 1920 | mag_work_pmp | $c_5 [\mathbf{v}' \cdot \mathcal{D}']_\phi$ | 1939 | visc_fluxpp_phi |
| $c_4 \mathbf{v}' \cdot [(\nabla \times \mathbf{B}') \times \bar{\mathbf{B}}]$ | 1921 | mag_work_ppm | $c_5 [\bar{\mathbf{v}} \cdot \bar{\mathcal{D}}]_r$ | 1940 | visc_fluxmm_r |

4.13 Kinetic Energy Equation

Terms appearing in the kinetic energy equation ($\mathbf{v} \cdot \frac{\partial \hat{\rho} \mathbf{v}}{\partial t}$) are described here.

| Value | Code | Variable | Value | Code | Variable |
|---|------|----------------------|-------|------|----------|
| $c_5 [\overline{\mathbf{v}} \cdot \overline{\mathcal{D}}]_\theta$ | 1941 | visc_fluxmm_theta | | | |
| $c_5 [\overline{\mathbf{v}} \cdot \overline{\mathcal{D}}]_\phi$ | 1942 | visc_fluxmm_phi | | | |
| $-c_3 v_r P$ | 1943 | press_flux_r | | | |
| $-c_3 v_\theta P$ | 1944 | press_flux_theta | | | |
| $-c_3 v_\phi P$ | 1945 | press_flux_phi | | | |
| $-c_3 v'_r P'$ | 1943 | press_fluxpp_r | | | |
| $-c_3 v'_\theta P'$ | 1944 | press_fluxpp_theta | | | |
| $-c_3 v'_\phi P'$ | 1945 | press_fluxpp_phi | | | |
| $-c_3 \overline{v_r} \overline{P}$ | 1943 | press_fluxmm_r | | | |
| $-c_3 \overline{v_\theta} \overline{P}$ | 1944 | press_fluxmm_theta | | | |
| $-c_3 \overline{v_\phi} \overline{P}$ | 1945 | press_fluxmm_phi | | | |
| -- | 1946 | production_shear_ke | | | |
| -- | 1947 | production_shear_pke | | | |
| -- | 1948 | production_shear_mke | | | |

4.14 Magnetic Energy Equation

Terms appearing in the Magnetic energy equation ($\mathbf{B} \cdot \frac{\partial \mathbf{B}}{\partial t}$) are described here.

| Value | Code | Variable | Value | Code | Variable |
|--|------|-------------------|---|------|-----------------|
| $[(\mathbf{v} \times \mathbf{B} - \eta \mathcal{J}) \times \mathbf{B}]_r$ | 2001 | ecrossb_r | $\mathbf{B}' \cdot [\nabla \times (\mathbf{v}' \times \overline{\mathbf{B}})]$ | 2018 | induct_work_ppm |
| $[(\mathbf{v} \times \mathbf{B} - \eta \mathcal{J}) \times \mathbf{B}]_\theta$ | 2002 | ecrossb_theta | $\mathbf{B}' \cdot [\nabla \times (\overline{\mathbf{v}} \times \mathbf{B}')]_r$ | 2019 | induct_work_pmp |
| $[(\mathbf{v} \times \mathbf{B} - \eta \mathcal{J}) \times \mathbf{B}]_\phi$ | 2003 | ecrossb_phi | $\overline{\mathbf{B}} \cdot [\nabla \times (\mathbf{v}' \times \mathbf{B}')]_\theta$ | 2020 | induct_work_mpp |
| $[(\mathbf{v}' \times \mathbf{B}' - \eta \mathcal{J}') \times \mathbf{B}']_r$ | 2004 | ecrossb_ppp_r | $\overline{\mathbf{B}} \cdot [\nabla \times (\overline{\mathbf{v}} \times \overline{\mathbf{B}})]_\phi$ | 2021 | induct_work_mmm |
| $[(\mathbf{v}' \times \mathbf{B}' - \eta \mathcal{J}') \times \mathbf{B}']_\theta$ | 2005 | ecrossb_ppp_theta | $\mathbf{B} \cdot [\mathbf{B} \cdot \nabla \mathbf{v}]$ | 2022 | ishear_work |
| $[(\mathbf{v}' \times \mathbf{B}' - \eta \mathcal{J}') \times \mathbf{B}']_\phi$ | 2006 | ecrossb_ppp_phi | $-\mathbf{B} \cdot [\mathbf{v} \cdot \nabla \mathbf{B}]$ | 2023 | iadvec_work |
| $[(\overline{\mathbf{v}} \times \overline{\mathbf{B}} - \eta \overline{\mathcal{J}}) \times \overline{\mathbf{B}}]_r$ | 2007 | ecrossb_mmm_r | $-\mathbf{B} \cdot (\nabla \cdot \mathbf{v}) \mathbf{B}$ | 2024 | icomp_work |
| $[(\overline{\mathbf{v}} \times \overline{\mathbf{B}} - \eta \overline{\mathcal{J}}) \times \overline{\mathbf{B}}]_\theta$ | 2008 | ecrossb_mmm_theta | $\mathbf{B}' \cdot [\overline{\mathbf{B}} \cdot \nabla \mathbf{v}']$ | 2025 | ishear_work_pmp |
| $[(\overline{\mathbf{v}} \times \overline{\mathbf{B}} - \eta \overline{\mathcal{J}}) \times \overline{\mathbf{B}}]_\phi$ | 2009 | ecrossb_mmm_phi | $-\mathbf{B}' \cdot [\overline{\mathbf{v}} \cdot \nabla \mathbf{B}']$ | 2026 | iadvec_work_pmp |
| $[(\mathbf{v}' \times \mathbf{B}') \times \overline{\mathbf{B}}]_r$ | 2010 | ecrossb_ppm_r | $-\mathbf{B}' \cdot (\nabla \cdot \overline{\mathbf{v}}) \mathbf{B}'$ | 2027 | icomp_work_pmp |
| $[(\mathbf{v}' \times \mathbf{B}') \times \overline{\mathbf{B}}]_\theta$ | 2011 | ecrossb_ppm_theta | $\mathbf{B}' \cdot [\mathbf{B}' \cdot \nabla \overline{\mathbf{v}}]$ | 2025 | ishear_work_ppm |
| $[(\mathbf{v}' \times \mathbf{B}') \times \overline{\mathbf{B}}]_\phi$ | 2012 | ecrossb_ppm_phi | $-\mathbf{B}' \cdot [\mathbf{v}' \cdot \nabla \overline{\mathbf{B}}]$ | 2026 | iadvec_work_ppm |
| $[(\mathbf{v}' \times \overline{\mathbf{B}}) \times \mathbf{B}']_r$ | 2013 | ecrossb_pmp_r | $-\mathbf{B}' \cdot (\nabla \cdot \mathbf{v}') \overline{\mathbf{B}}$ | 2027 | icomp_work_ppm |
| $[(\mathbf{v}' \times \overline{\mathbf{B}}) \times \mathbf{B}']_\theta$ | 2014 | ecrossb_pmp_theta | $\overline{\mathbf{B}} \cdot [\overline{\mathbf{B}} \cdot \nabla \overline{\mathbf{v}}]$ | 2028 | ishear_work_mmm |
| $[(\mathbf{v}' \times \overline{\mathbf{B}}) \times \mathbf{B}']_\phi$ | 2015 | ecrossb_pmp_phi | $-\overline{\mathbf{B}} \cdot [\overline{\mathbf{v}} \cdot \nabla \overline{\mathbf{B}}]$ | 2029 | iadvec_work_mmm |
| $[(\overline{\mathbf{v}} \times \mathbf{B}') \times \mathbf{B}']_r$ | 2013 | ecrossb_mpp_r | $-\overline{\mathbf{B}} \cdot (\nabla \cdot \overline{\mathbf{v}}) \overline{\mathbf{B}}$ | 2030 | icomp_work_mmm |
| $[(\overline{\mathbf{v}} \times \mathbf{B}') \times \mathbf{B}']_\theta$ | 2014 | ecrossb_mpp_theta | $\overline{\mathbf{B}} \cdot [\mathbf{B}' \cdot \nabla \mathbf{v}']$ | 2031 | ishear_work_mpp |
| $[(\overline{\mathbf{v}} \times \mathbf{B}') \times \mathbf{B}']_\phi$ | 2015 | ecrossb_mpp_phi | $-\overline{\mathbf{B}} \cdot [\mathbf{v}' \cdot \nabla \mathbf{B}']$ | 2032 | iadvec_work_mpp |
| $\mathbf{B} \cdot [\nabla \times (\mathbf{v} \times \mathbf{B})]$ | 2016 | induct_work | $-\overline{\mathbf{B}} \cdot (\nabla \cdot \mathbf{v}') \mathbf{B}'$ | 2033 | icomp_work_mpp |
| $\mathbf{B}' \cdot [\nabla \times (\mathbf{v}' \times \mathbf{B}')]_\theta$ | 2017 | induct_work_ppp | $\mathbf{B}' \cdot [\mathbf{B}' \cdot \nabla \mathbf{v}']$ | 2034 | ishear_work_ppp |

| Value | Code | Variable | Value | Code | Variable |
|--|------|-----------------|-------|------|----------|
| $-\mathbf{B}' \cdot [\mathbf{v}' \cdot \nabla \mathbf{B}']$ | 2035 | iadvec_work_ppp | | | |
| $-\mathbf{B}' \cdot (\nabla \cdot \mathbf{v}') \mathbf{B}'$ | 2036 | icomp_work_ppp | | | |
| $-c_7 \mathbf{B} \cdot [\nabla \times (\mathbf{f}_7 \nabla \times \mathbf{B})]$ | 2037 | idiff_work | | | |
| $-c_7 \mathbf{B}' \cdot [\nabla \times (\mathbf{f}_7 \nabla \times \mathbf{B}')]_\theta$ | 2038 | idiff_work_pp | | | |
| $-c_7 \overline{\mathbf{B}} \cdot [\nabla \times (\mathbf{f}_7 \nabla \times \overline{\mathbf{B}})]_\phi$ | 2039 | idiff_work_mmm | | | |