THE EVOLUTIONARY UNIVERSE : CONDENSATION OF UNIVERSE STRUCTURES DUE TO SELF-GRAVITATION

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Abstract

We develop a virial analysis of self-gravitating spheroids and predict the effects of Universe expansion on the stability of these systems. We find that as the pressure and temperature of the Universe decreases in time, the critical mass that is unstable to collapse decreases. The largest self-gravitating body is the Universe and we show that the critical mass at the time of element formation corresponds with the present estimate of the mass of the Universe. Subsequently, as the temperature falls further, lower mass systems will collapse and form the next dominant form of self-gravitating bodies, i.e. the large-scale structures and clusters of the observable galaxies. Subsequently, as these clusters cool with Universe expansion, the next largest mass systems will condense out, namely the galaxies. Recent observations have shown that the Universe has seen new groups of galaxies being formed throughout its history. It appears to be populated by dominant species of galaxies at different times. Our analysis shows that the size and age of these galaxies is comprehended through our critical mass argument.

Introduction

Increasingly over the past few years, observations show that the Universe has changed its predominant galaxy population throughout its history. The earliest galaxies, born soon after the time of recombination, appear to be hundreds of times larger in mass than those of the present population, c.f. Tyson (1992). However, as recently as 2 billion years ago, the Universe was dominated by a population of young galaxies, called blue dwarfs, that were one-hundredth the size of the average present spiral, Cowie etal.(1992). These changing populations have led astronomers to speculate that the expansion of the Universe could be the cause of such evolution. Protogalactic gas spheroids are embedded in a background environment whose pressure and temperature decreases with expansion of the Universe. We shall show that such decreasing temperatures and pressures trigger collapse of these spheroids.

After the recombination of particles into hydrogen atoms, the era of matter domination began in the Universe. It has been shown, Davies (1991), that accumulation of the gas into clumps occurred soon after recombination due to non-linear interactions of the self-organising fluid motions. Within these clumps, it is expected that rotating self-gravitating isothermal spheroids were formed. Universe expansion and the corresponding external pressure and temperature changes will affect the stability of these spheroids.

We develop a virial approach in order to investigate the spheroid stability showing that, for a particular external pressure and temperature, instability occurs when the angular momentum per unit mass is too high. Also, when angular momentum is negligible, a critical mass exists above which no stable isothermal sphere can resist collapse at that particular temperature. These conditions delineate a stable region in the mass-temperature environment that, for a fixed angular momentum, allows stable spheroids to form. As temperature decreases with the expansion of the Universe, the largest mass spheroids are the first to become unstable to collapse, with smaller masses doing so as time progresses and temperature falls.

Virial Theorem

We use a virial method in order to elucidate the various energy components and their influences on stability of these spheroids. Even if we made changes in the basic assumptions, such as the isothermal conditions and solid-body rotation, the virial results would still be useful as they indicate the general properties of such spheroids. The virial theorem is derived from the basic law of momentum conservation, Chandrasekar (1961), by taking moments of the equation:

(1) $\rho[\delta\mathbf{u}/\delta t + (\mathbf{u}.\boldsymbol{\Delta})\mathbf{u}] = -\rho\Delta\Psi + \Delta P$

where ρ is density, $\mathbf{u}$ is vector velocity, $\boldsymbol{\Delta}$ is divergence vector operator, Δ is the gradient operator, Ψ is gravitational potential and P is the pressure.

For a spheroid under external pressure, P_e, the virial theorem so derived is:

(2) $1/2.d^2I/dt^2 = 2E_k + 3(\gamma-1)U + E_g - 3P_eV$

where V is spheroid volume, I is half the sum of the principle moments of inertia, E_k is kinetic energy, U is thermal energy where γ is the ratio of specific heats, and E_g is the gravitational energy. Each of these terms will be derived for a spheroid under solid-body rotation and with constant temperature and external pressure.

Thermal energy of this spheroid is dependent on the assumed equilibrium equation of state. We assume isothermal conditions throughout this equilibrium system:

(3a) $P = KT.\rho$

where T is the temperature and K the appropriate constant for the gas present. The thermal energy is thus:

(3b) $U = KTM/(\gamma-1)$

and the term in the virial equation becomes:

(4) $3(\gamma-1)U = 3KTM$

where the total mass of the spheroid is M.

Gravitational potential energy of the spheroid is expressed as:

(5a) $E_g = -1/2\ \Sigma_V\ \rho\Psi.dV$

McCrea (1957) has argued that, on account of its physical dimensions, this term is of the form:

(5b) $E_g = -A.GM^2/Z$

where G is gravitational constant, Z is a typical external radius of the spheroid and A is a positive dimensionless number approximately equal to one, dependent on the density distribution, and which varies slowly with spheroid size.

Kinetic energy of the spheroid under solid-body rotation Ω is:

(6) $E_k = 1/2.I_C.\Omega^2$

where the moment of inertia I_C of the isothermal spheroid about the central axis is given by the approximate relation:

(7) $I_C = 8\pi/3.\ \Sigma_Z\ q^4.dq$

The isothermal sphere through most of its radius q obeys the density versus radius relation:

(8) $\rho = \rho_e.Z^2/q^2$

where ρ_e is the external density, so that the mass of the sphere is approximately:

(9) $M = 4\pi\rho_e Z^3$

Thus the total kinetic energy is given by:

(10) $E_k = 2/9\ M.\Omega^2.Z^2$

On substitution of these terms into the virial relation (2), we get:

(11) $1/2.d^2I/dt^2 = 4/9\ M.\Omega^2.Z^2 + 3KTM - A.GM^2/Z - 3P_eV$

Using $V = 4/3.\pi.Z^3$, gives, finally, our original expression of the virial relation:

(12) $1/2.d^2I/dt^2 = 4/9\ M.\Omega^2.Z^2 + 3KTM - A.GM^2/Z - 4\pi.Z^3P_e$

Stability Relations

The virial relation applies to a body of gas released from rest in a given configuration and under given external pressure. If both dI/dt and d^2I/dt^2 are zero at this initial time, then these are necessary conditions for equilibrium. Otherwise, if dI/dt is greater than or less than zero, the gas body as a whole begins to expand or contract, respectively. However, the virial relation will not be valid during these expansions or contractions as mass motions have not been included therein.

The first term in this virial equation is kinetic energy, which is controlled by angular velocity, and is always positive. The second term corresponds to thermal energy and also is positive, with temperature decreases causing contraction. The third term represents the negative gravitational potential energy and the fourth, also negative, represents the effect of the external pressurized environment.

Assuming conservation of angular momentum, then, in addition to mass M, an invariant of the system is:

$$J = \Omega^2.Z^4$$

For an equilibrium system, we have, from (12):

(13) $4\pi P_e = 4/9\ M.J/Z^5 + 3KTM/Z^3 - A.GM^2/Z^4$

An example of this relation in the P, T, Z domain is shown in Figure 1. for constant M and J. As P and T decrease, it is seen that the solution bifurcates
with two solutions for radius Z becoming viable. These singular values are given by the condition of:

$\delta(d^2I/dt^2)/\delta Z = 0$; which is equivalent to $\delta P_e/\delta Z = 0$.

This yields:

(14) $Z(1,2) = 2/9.AGM/KT \pm [4/81.(AGM/KT)^2 - 20/27.J/KT]^{1/2}$

These two solutions are equal when:

(15) $J = 3(AGM)^2/5KT$

When J is less than this critical value, there are two real solutions for radius Z. When J is greater than this critical value, the solutions are complex and the spheroid will be unstable as the expansionary angular momentum and thermal terms dominate the gravitational contracting term.

Critical Mass Spheroids

When the spheroid has little angular momentum, it will approach an isothermal self-gravitating sphere under external pressure and the virial relation will be, at equilibrium:

(16) $4\pi P_e = 3KTM/Z^3 - A.GM^2/Z^4$

which is identical to eqn (3.2) of McCrea (1957). He shows that such a sphere will collapse at a particular temperature and under a particular external pressure if the mass of the body exceeds a critical value. In order to determine this critical mass, Ebert's (1955) numerical solution of the isothermal Emden equation is utilised. It is shown, eqn (9.3) of McCrea (1957), that this critical mass is given by:

(17) $M_c = 8.91\ T^2(P/K)^{-1/2}$

where the mass is measured in solar masses. Relating pressure to temperature and number density of particles, where n is number of particles per cubic centimetre, gives:

(18) $M_c = 41.1\ T^{3/2}\ n^{-1/2}$

with mass M_c expressed in solar masses.

As the Universe expands, the external pressure acting on these spheroids will decrease. Also the prevailing temperature of these isothermal systems will decrease with the expansion. In order to understand the evolution of these spheroids as a function of time, the stability of these systems must be mapped. For the rotating spheroid, the critical angular momentum requirement can be expressed from equation (15) as:

(19) $\log M = 1/2(\log J + \text{Log } T + C_1)$

where C_1 is a constant. When M is less than this critical value, expansionary instability will occur; above this critical value, the systems will be stable for that particular J and T.

For the non-rotating spheroid, the critical mass from equation (18) is:

(20) $\log M_c = 3/4(\text{Log } T + C_2)$

where C_2 is a constant. When the system mass is less than this critical value, stability reigns and when the mass is greater, collapse instability occurs.

For a fixed value of J, the log M versus log T domain decomposes into four regions, one being unstable under both conditions and two being unstable for one of the conditions. The other region is stable under both conditions and occurs for a sufficiently high temperature.

Masses in the Universe

As the Universe expands and the temperature drops, the spheroids with the largest masses will become unstable first. In an expanding Universe, Weinberg (1972), the matter density varies as R^{-3} and the temperature as R^{-2}, where R is the radius of the Universe. The variation of temperature and density with Universe radius implies, from McCrea's formula, that M_c is proportional to $R^{-3/2}$. An approximate value of these critical masses can be obtained for different eras of the Universe.

Using estimates of present-day intergalactic values of T = 10K and an intergalactic number density range of about 10^{-10} particles per cc., equation (3) gives the present value of critical mass of about 10^{8} solar masses. We have shown that as the Universe expands, temperature and pressure decrease with ever-smaller spheroids becoming unstable and collapsing. Subsequently, we can estimate an approximate value of these critical masses for the different self-gravitating bodies of the Universe. At the time of decoupling, when the radius was approximately 10^{-4} that of the present Universe, the critical mass is thus predicted by equation (3) to be in the range of 10^{14} solar masses. This is of the order of the observed mass of galaxy super-clusters and large scale structures, which are the largest observed self-gravitating bodies in our Universe. This confirms our previous result, Davies (1991), that at this time clumping occurred with the self-organisation of matter into distinct higher-density regions separated by lower-density voids.

Observations by Tyson (1992) indicate that the earliest galaxies were formed inside the clusters within a few billion years after the time of recombination and were of the order 100 times the mass of the Milky Way. These values are consistent with those predicted above for the critical masses at that time and Universe radius, given the uncertainty in our knowledge of present day intergalactic density. Similarly, Cowie et al. (1992) observe that, about two billion years ago, small young galaxies are observed to dominate the Universe with masses about one-hundredth that of the Milky Way and thus again consistent with our model results.

At the time of formation of the elements, when the radius was about 10^{-9} of the present, the critical mass is thus in the range of about 10^{21} solar masses which is equivalent to a mass of 10^{54} gms. This compares favorably with the estimate of $1.6\text{x}10^{54}$ gms for the present baryon mass of the Universe using a Hubble radius of 10^{28} cm and a present baryon matter density of about $4\text{x}10^{-31}$ gm/cc. Thus we conclude that the particular value of the baryon mass of the Universe is a consequence of the thermodynamic conditions under which the atomic matter was formed and the stability requirements of this particular self-gravitating body, the baryon Universe.

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