Momentum Viewpoint of Quantum Bound State Part II

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We have suggested that exp(ipx) is a mathematical probability function such that: exp(i p1 dot r) exp(p2 dot r) = exp(i (p1+p2) dot r) and Integral exp(-ip1x)exp(ip2x) dx =0 unless p1 =p2.In other words, exp(ipx) is a special probability function which ensures that momentum is conserved, but it comes at the price of introducing a second variable x and introduce a wavelength hbar/p. Such a wavelength does not exist in Newtonian mechanics and we have suggested that this probabilistic approach is more general than Newtonian mechanics. It matches Newtonian mechanics when hbar/p reaches the size of dx which is considered to be almost zero.

Given the form exp(ipx) and the wavelength hbar/ p, one might ask if this causes problems for velocity which is constant, but one may simply use delta x which is larger than hbar/p. Special relativity, which is consistent with exp(ipx) through Action= -Et+px, suggests that a rest mass viewed from a frame moving at constant -v has both energy mocc/sqrt(1-vv/cc) and momentum p. In classical physics, V(x) (potential energy) indicates that at a given x one knows p and energy mocc/sqrt(1-vv/cc) approx= mocc + .5movv (nonrelativistic limit), but that given that p changes in x, V(x) accounts for the extra energy such that total energy is constant. It seems to be a tallying value as the energy is possibly spread out in fields.

Given the hbar/p length, one cannot talk about a point x. If one wishes to retain V(x), it is at best an average. We argued that exp(ipx) is needed for momentum conservation, but given that p is well defined, so too is mocc/sqrt(1-vv/cc). Neither contain x. We suggested that a bound state is a statistical collection of p values with a(p) weights such that a*(p)a(p) represents P(p) through: Integral W*(x)W(x)dx, where W(x)= Sum over p a(p)exp(ix). Given that one can no longer view V(x) as acting at a point or p acting at a point x and receiving an impulse hit, there are some things one may retain.

V(x) is ultimately associated with force which changes one p to another. Each p is associated with energy, not just pp/2m, but some energy linked to V(x). Given that one cannot think in terms of energy at x, one may note that V(x) is associated with delivering k momenta values to p, but must do so in a manner consistent with the probability scheme, i.e. exp(ikx)exp(ipx)=exp(i (k+p)x). This suggests V(x) = sum over k Vk exp(ikx). Thus, if V(x) delivers k momentum to any p value, it is linked with a Vk energy existing in a field (like in the electromagnetic case). Thus, V(x) can change different p-k momenta to p bringing in different Vk a(p-k) values. If Integral W*(x)W(x)dx=1, one may also suggest that Integral dx W* (hbarhbar/2m (-d/dx)d/dx + V(x)) W is the average energy E. E= E Integral W*W dx, so one may create an equation for each exp(ipx) or a differential equation for W.

This raises the question: Why is total energy used? Given V(x), one might just as well use: velocity average = v(x) = sqrt(2m(E-V(x)). At first glance, there may be no reason to treat energy as special. We argue that given the form exp(ipx) for momentum conservation, a bound state has a negative energy which indicates the tightness of binding and this should also be a conserved quantity, given by the probability exp(-iEt). One may argue that since one uses a differential operator -id/dx on exp(ipx) to bring in the conserved quantity p, to be consistent one should use the operator id/dt on exp(-iEt) to obtain the conserved quantity E.

Classical Mechanics and the Conserved Momentum Probability

Classical mechanics follows a particle in x(t) ,which matches experimental observations. The particle changes velocity in a tiny dx and so has an energy mocc/sqrt(1-vv/cc) from special relativity at x and if the overall total energy is E, there is leftover energy V(x). This potential energy may exist in a field whereas kinetic energy and mo seem to be tied directly to the particle. Thus, it is perhaps a little misleading to note that one has potential energy at x. This energy is possibly spread out in a field, but V(x) is used for tallying and -dV/dx does give the force.

We suggest that a key idea of classical physics is momentum conservation and desire a probability function which shows this conservation and also Pcons(p1)Pcons(p2) = Pcons(p1+p2). Unfortunately, one cannot create a function in only p. A second variable is needed and one requires orthogonality in this variable (x), which means there must be a length, hbar/hp. (hbar is a constant which yields proper units). It follows from considerations of a photon.

The problem now is that exp(ipx) ,which serves as momentum conserving probability , requires a region of space. For example, Integral exp(-ip1x) exp(ipx) dx requires hbar/ |p1-p| for establishing orthogonality. One can no longer have physics happening at an x point. Classical mechanics, however, does just this and matches observation. In such a case, hbar/p may be very small as to appear as a point, but in other cases may be large compared to system lengths.

This idea of wavelength causes problems for the notion of a bound particle. Certainly, one cannot have one p value given an average V(x). What about the notion of velocity? Even though there is hbar/p, delta x/dleta t =constant may still hold if delta x > hbar/p (and delta t> hbar/E, where E= mocc/sqrt(1-vv/cc)). We argued in part II that the potential must create other p values and that in this statistical approach one has weight a(p) exp(ipx) such that:

Integral dx W*(x)W(x)=1 where W(x)= Sum over p a(p)exp(ipx) ((1))

Using Average Energy

In Part II, we suggested using operators such as <W* hbarhbar/2m (-d/dx d/dx) W> to find average nonrelativistic kinetic energy. An immediate question which may arise is: Why should one calculate average energy? Given V(x), which is an average, one has the notion of energy, but also:

v(x) = average velocity = sqrt(2m (E-V(x)) ((2))

Given that p=mov(x) (nonrelativistic), why doesn’t one calculate average p, i..e Integral dx W* |(-id/dx) W| ? This would lead to a different result. For example, for the ground state of an oscillator, an energy approach yields W= Cexp(-bxx) so:

-1/2m d/dx d/dx W= -1/2m C (2b-4bbxx) W ((3a))

while : ..5m vave vave = .5mo 4bbxx W ((3b))

((3a)) and ((3b)) are not the same.

We suggest using an energy centric approach because for a bound system, total energy is conserved quantity. Overall, there should be an E associated with the bound state and one may use the probability exp(-iEt) to show it is conserved. This suggests an id/dt operator (partial) like the (-id/dx) used for exp(ipx).

Bound State Using Energy Ideas

We consider energy average values as in Part II based on the argument in the previous section. Given that one describes statistically the bound system by a set of weighted p values, i.e. a(p)exp(ipx), these automatically have an movement-rest mass energy of mocc/sqrt(1-vv/cc) form special relativity which is consistent with exp(ipx) through the action= -Et+px (for a free particle, both relativistically and nonrelativisitcally). Originally V(x) represented the excess field energy spread throughout space, but linked to x, the point at which p exists classically. Now with exp(ipx), there is no exp(ipx) , but there may still be changing p values due to V(x) which is an average result.

Given that one uses a probability scheme exp(ipx) and that p is changed by V(x) and exp(ip1x)exp(ip2x) =exp(i (p1+p2)x), it seems that V(x) must be written in this formalism, i.e.

V(x) = Sum over k Vk exp(ikx) ((4))

This suggests that V(x) can deliver a momentum k, but automatically brings Vk of energy along with it. Thus, one may have:

Integral dx W* (-1/2m d/dx d/dx) W + Integral W* V(x)W dx = average Energy ((5))

Given that: average energy = average energy Integral W*Wdx, because the integral is 1, one may write ((5)) as a differential equation:

-1/2m d/dx d/dx W + V(x) W = En W ((6))

With wavelengths, there are no sharp classical turning points anymore, but W*W should tend to zero as one moves to +/- infinite as a bound particle should be localized.

As a result, one uses exp(ipx)s to indicate momentum conservation and does not worry about exp(-i mocc/sqt(1-vv/cc)) t) because one is only interested in an overall energy, not elastic scattering. Overall, however, En is conserved and one is interested in exp(-iEnt).

As a result, the quantum approach is a little nebulous in space, and represents averaging using p values which have received a hit from the potential, i.e. one averages in time.

Conclusion

In conclusion, we argue that one may introduce a momentum conserving probability, exp(ipx), such that Integral dx exp(-ip1x)exp(ipx) = 0 if p1 not = p2, but this comes at the cost of introducing the variable x and defining a length hbar/p. In the above integral hbar / |p1-p| is needed to establish orthogonality. Thus, the idea of interactions and changes at points from Newtonian mechanics is gone, unless hbar/p is very tiny, i.e. of the order of dx.

We argue, however, that one may still have a bound, i.e.localized state, and still consider V(x), the classical potential, as an average. It is still the case that one has a set of p values with different weights, but no x information, i.e. Integral W*(x)W(x)dx=1, with W(x)=Sum over p a(p)exp(ipx). Given V(x), one may consider averages of energy, but one may also consider an average momentum through v(x) average = sqrt(E-V(x)). The average energy approach would yield a vrms(x). We suggest that one should use the energy approach, because even though one does not consider energy conservation for exp(ipx), the average E for a bound state should be a conserved quantity. We thus suggest that it should be linked to a probability exp(-iEt) and the operator ihbar d/dt partial W(x,t).

Given that V(x) must change p values from one to another, one should describe this using probabilities . This seems to force one to write: V(x)=Sum over k Vkexp(ikx), so that any delivery of k momentum is linked with Vk of potential energy existing spread out in fields.

In such a case: Integral dx W* (-1/2m d/dx d/dx) W + Integral dx W* V(x)W should yield an average energy which is a number. This may be multiplied by Integral dx W*W and one may pull out Integral dx W* to obtain a differential equation.