

# Serviceability of Fabric-Formed Concrete Structures

Yadgar Tayfur, Antony Darby, Tim Ibell, Mark Evernden, John Orr

**Abstract**—Fabric form-work is a technique to cast concrete structures with a great advantage of saving concrete material of up to 40%. This technique is particularly associated with the optimized concrete structures that usually have smaller cross-section dimensions than equivalent prismatic members. However, this can make the structural system produced from these members prone to smaller serviceability safety margins. Therefore, it is very important to understand the serviceability issue of non-prismatic concrete structures. In this paper, an analytical computer-based model to optimize concrete beams and to predict load-deflection behaviour of both prismatic and non-prismatic concrete beams is presented. The model was developed based on the method of sectional analysis and integration of curvatures. Results from the analytical model were compared to load-deflection behaviour of a number of beams with different geometric and material properties from other researchers. The results of the comparison show that the analytical program can accurately predict the load-deflection response of concrete beams with medium reinforcement ratios. However, it over-estimates deflection values for lightly reinforced specimens. Finally, the analytical program acceptably predicted load-deflection behaviour of on-prismatic concrete beams.

**Keywords**—Concrete beams, deflections, fabric formwork, optimisation, serviceability.

## I. INTRODUCTION

SINCE concrete was invented, it has been cast in rigid forms. These forms were usually rectangular and were manufactured by using wood or steel panels. This is due to easier construction of these forms from rigid form-work panels. The structural members produced from this rigid form-work system, are rectangular prismatic solids and are likely to use more material than that required by the applied stress distribution. Thus, a member with more self-weight and more embodied carbon than an optimized member is produced.

New researches showed that nearly 40% of concrete used in the new office buildings has little contribution to structural functionality but more adds extra self-weight to the building [1]. Apart from this, one of the biggest sources of CO<sub>2</sub> emission in the world is the cement industry [2]. Therefore, structural optimization should be considered in order to obtain a member with variable cross-section which reflects the requirements of loading at each section.

Increasing interest in using optimized structures which results in consumption of less material and reduced embodied carbon may have been limited in the past due to complexity and being expensive to design and construct. Because of the lack of computers, analysis of these more sophisticated

shapes demanded a lot of time. However, with the introduction of computers, complicated calculations to design and analyze elements with variable geometric properties became easier to handle. Simultaneously, with the use of computers in engineering practice, strong and cost-effective polyolefin textiles, which made complicated non-prismatic and efficient structural members to be easily cast, became widely available. Having the computer and developed textile technology allows design and construction of reinforced concrete structures to be reconsidered [1], [3].

The shape of optimized members usually reflects the loading envelope that the member optimized for, therefore, optimized members can have an exceptional shape which makes them difficult to be cast using conventional prismatic molds and a more flexible form-work such as fabric formwork should be considered.

Fabric form-work is a technique of concrete construction in which rectangular fabric membranes are used as the main facing material instead of timber or steel forms in order to cast sophisticated shapes easily. Due to high flexibility, the material deflects under static pressure exerted by the wet concrete. The final shape shows curvature and very good surface texture that are not usually a property of concrete structures [1], [3].



Fig. 1 Fabric-formed concrete

Structural optimization offers a technique to minimize the material use required to resist a given loading. The prismatic and uniform shape of structural members we are familiar with today is more decided by commercial availability of the traditional form-work, not by optimal structural efficiency. For each member in a structural system, reduction in self-weight can take place; this decreases dead load on other members and this reduction accumulates through the structure for all members causing even more reduction in material used. However, in the structural design process, each member is expected to satisfy a number of different limit states or design requirements. There are minimum performance limits that are associated with strength and serviceability and any of these limits may govern the design of a certain element in the structural system. In order to have serviceability limit states fulfilled, a concrete structure must be serviceable and its

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aesthetic and structural function should not be impaired by disproportionate deflection or cracks [4]. This paper provides a method by which concrete beams are analyzed and optimized for strength and serviceability.

## II. METHODOLOGY

In order to develop a model that can efficiently optimize and predict load-deflection response of fabric-formed concrete members under service loads, a method of analysis with broad applicability should be chosen.

The computational procedure presented in this paper carries out two main tasks; prediction of behaviour and structural optimization. The method of integration of curvatures, based on sectional analysis, is believed to be appropriate for this task because every section of the member is dealt with separately, allowing for considering variations in section dimensions along the member. For this reason, this method was adopted in this study to develop a numerical computer based model to optimize and predict load-deflection behaviour of fabric-formed concrete members. The computational procedure can be briefly outlined as below:

- 1- Finding the applied bending moments and shear stresses from the given loadings.
- 2- Optimization for strength is carried out to produce the profile of the beam which satisfies the ultimate limit states. This is done by dividing the member by a number of sections and finding the section geometry and layout of the reinforcement which resists the shear and bending moment for each section.
- 3- A form-finding procedure is applied to predict the shape of every section and eventually the full shape for a fabric-formed beam. At this point, a fabric-formed beam, optimized for strength is obtained.
- 4- The serviceability behaviour of the member is then assessed.
- 5- If the member fails to meet serviceability limit state conditions, optimization for serviceability is carried out to produce a fabric-formed beam fully optimized for both strength and serviceability.

### A. Prediction of Behaviour

#### 1. Sectional Analysis

The proposed model is based on finding moment-curvature behaviour of sections of a beam (Sectional model). This can be done by adopting a numerical procedure to calculate the sum of compression and tension forces on a section and solving the equilibrium conditions [5]. The overall depth of the section is divided into a number of reinforcement and concrete strips, as shown in Fig. 2. The force in each strip is derived from the strain in the strip and stress-strain relationship. The sectional equilibrium and compatibility conditions are then imposed. The contribution of tensile concrete is taken into account assuming a perfect bond between the reinforcement and concrete.

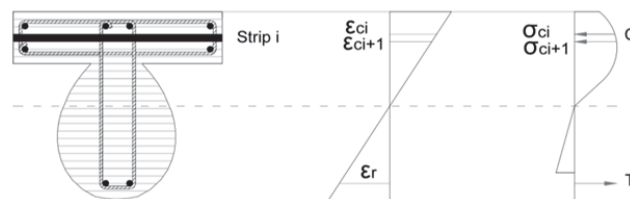


Fig. 2 Layered section approach

#### 2. Material Stress-Strain Models

A model to represent the stress-strain behaviour of concrete under uniaxial compressive stress is used in the analytical procedure. The model includes ascending (strain-hardening) and descending (strain-softening) branches. The expression suggested by Eurocode 2 [6], shows the relationship between absolute values of compressive stress ( $\sigma_c$ ) and shortening strain ( $\epsilon_c$ ), as shown in Fig. 3:

$$\sigma_c = \frac{k\eta - \eta^2}{1 + (k-2)\eta} f_{cm} \quad (1)$$

where

$$\eta = \frac{\epsilon_c}{\epsilon_{c1}} \quad (2)$$

$\epsilon_{c1}$  is the strain at peak

$$k = 1.05 E_{cm} \times \frac{|\epsilon_{c1}|}{f_{cm}} \quad (3)$$

This expression is valid for  $0 < |\epsilon_c| < |\epsilon_{cu1}|$ , where  $\epsilon_{cu1}$  is the nominal ultimate strain. Reinforcing steel is modelled as an elastic-plastic material which has a linear strain hardening branch after yield ( $\sigma_y$ ) for both tension and compression cases as shown in Fig. 4. ES1 and ES2 represent tangent modulus of the steel before and after yielding respectively.

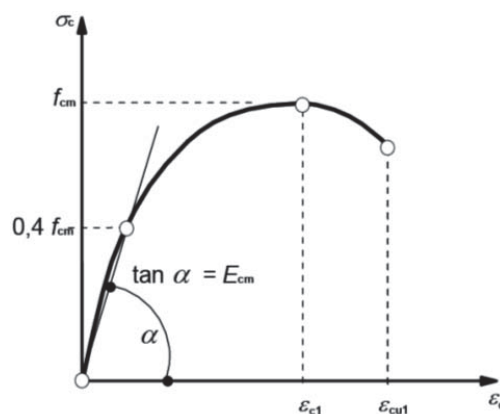


Fig. 3 Schematic representation of the stress-strain relation for structural analysis [6]

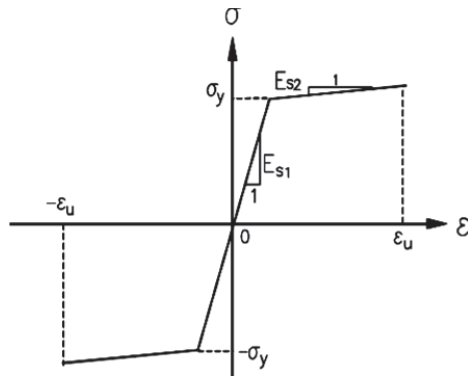


Fig. 4 Idealized stress-strain relationships for (a) steel [7]

### 3. Method of Integration of Curvatures

Conventional methods for evaluating short-term serviceability displacement of concrete structures can be divided by two categories: those methods which combine effective flexural rigidity ( $EI_{eff}$ ) of the concrete member and an elastic deflection equation; and those methods that find the deflected shape of the member by calculating integration of curvature in different points throughout the length of the member [8].

Methods that use effective flexural stiffness usually average the value of ( $EI_{eff}$ ) for the whole member for pre-cracking and post-cracking stages. This method can be good for prismatic beams with the same cross-section properties along the member not probably for beams with variable cross-section geometries that have variable flexural rigidities.

#### B. Form-Finding of Fabric-Formed Concrete Beams

The boundary conditions applied by the irregular shape of fabric and its supported edges, the prestressing of fabric and the nonlinear characteristics of fabric material make the shape of a fluid filled membrane unknown in advance. Therefore, a form-finding technique is necessary to predict the deformed shape of fabric under the hydrostatic load of wet concrete [9].

A number of different form-finding methods have been used in the past. Garbett [10] introduced a number of empirical equations based on test data to predict the shape of fabric under static pressure of wet concrete.

Dynamic relaxation which was used by Veenendaal [9], is the process of solving static problems by considering the equilibrium state of damped structural motion. This uses a pseudo-dynamic approach with the application of Newton's second law of motion.

The step-wise procedure presented by Foster [11] is used in the current work as it can easily be implemented in Excel spreadsheets or MATLAB program. Given the basic relationship between hydraulic height ( $z$ ), Curvature in the fabric ( $k$ ), density of wet concrete ( $\rho$ ) and gravitational acceleration ( $g$ ), the constant tension force in the fabric ( $T_o$ ) is given as:

$$T_o = \frac{\rho g z}{k} \quad (4)$$

A walking-out procedure is undertaken by starting from (0, 0) point on the lowest point of the hung fabric. A constant increment of  $\delta L_1$  is chosen as shown in Fig. 5.

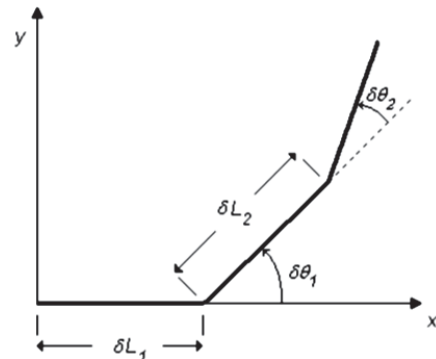


Fig. 5 Characterisation of the walking-out procedure [11]

The depth of  $n^{th}$  increment and the angle between this depth and the previous increment are given by (5) and (6) respectively [11]:

$$Z_n = z - y_n \quad (5)$$

$$\delta\theta = KZ_n \quad (6)$$

where ( $Z_n$ ) is the depth of  $n^{th}$  increment, ( $z$ ) is the total hydraulic depth, and ( $y_n$ ) is the coordinates of  $n^{th}$  step;  $\Theta_n$  is the angle of the  $n^{th}$  increment,  $K$  is a coefficient.

Choosing a constant increment length, the expression of finding coordinates of points on the perimeter is given as:

$$(X_n, Y_n) = [\delta L \sum_{i=1}^n \cos \sum_{j=1}^{i-1} \delta\theta_j, \delta L \sum_{i=1}^n \sin \sum_{j=1}^{i-1} \delta\theta_j] \quad (7)$$

From the above procedure, the cross-section shape of a fabric filled with a liquid can be found iteratively at discrete points along the length of the member.

#### C. Optimisation

##### 1. Optimisation for Strength

Optimisation for strength for a simply supported beam is relatively easy and straightforward. The goal is to satisfy the requirements imposed by bending moment and shear forces due to the applied loading envelope at any section along the beam. From the sectional analysis mentioned in section 2.1, moment-curvature relationship of every section can be found, then an iterative procedure chooses a section that has a moment capacity greater than the applied moments. This procedure is repeated until the whole profile of the beam is created.

In order to satisfy shear force requirements at each section, the following equation, given by ACI 310M-11 [12], was used.

$$V_s = \frac{A_v f_{yt} d}{s} \quad (8)$$

where  $V_s$  is applied shear force,  $A_v$  is the area of shear reinforcement within spacing  $s$ ,  $f_{yt}$  is the yield strength of shear links, and  $d$  is effective depth. From this equation, the area of shear reinforcement for a certain section depth can be chosen.

## 2. Optimisation for Serviceability

The common serviceability limit states according to Eurocode 2 [6] are, deflection control, crack control and stress limitation. In this work, optimisation for deflections only is presented. When a member is optimized for strength only, it may fail to satisfy serviceability criterion, especially when the member is simply supported. The material, is only just sufficient to resist bending and shear stresses at any section. That means a reduction of material from anywhere in the member would make it fail in flexure or shear. Therefore, serviceability optimization can only be done by adding material to certain sections along the member. The question now is where to add material. Candidate sections for adding material should be chosen based on locations which reduce the maximum deflection value the most. The optimization procedure used in this work adds material to the section that has maximum curvature value at any step of loading. This addition continues until deflection limit state is met. This returns a member which is buildable and fully optimized for both strength and serviceability.

## III. RESULTS

### A. Load-Deflection Prediction

In order to validate the computational analysis adopted in this study, the load-deflection data of a number of concrete beams taken from the literature were compared to that predicted by the program developed to simulate load-deflection response of reinforced concrete beams. To assess the analysis program against a wide range of parameters, the members considered in this work had various section geometries, reinforcement types and ratios, and concrete compressive strengths.

The members were all simply supported, including prismatic and fabric-formed concrete beams. The beams were singly or doubly reinforced with steel rebars and had low to high reinforcement ratios.

Beams were cast with concrete of different strength classes from normal strength to high strength. Details of all beams are shown in Table I. Beams T1MA and J4 were presented by Kwak & Kim [7], beams B36-L1 and B18-2 were tested by Kalkan [13]. All specimens were under-reinforced.

Two fabric-formed concrete Single and Double T-beams tested by Orr [14] were also considered in this work. Beam 5-1-H-C was tested under seven-point load arranged symmetrically over the length of the beam with one-point load on overhang part on the left and right hand sides. Beam 6-4-1 was tested under five-point loading applied within the span between the supports at equal distances from each other

Starting with Beams T1MA and J4, the comparison of the actual and the predicted load-deflection responses are shown

in Figs. 6 and 7, respectively. The computer program provides satisfactory prediction for load-deflection behaviour.

Slight overestimation of flexural stiffness can be seen in Fig. 6 and more notable overestimation in Fig. 7. This might be due to bond slip between the steel and the surrounding concrete and the assumption of perfect bond in the analysis procedure. In the cracked region, the bond-slip impact can be dominant for under-reinforced concrete [7].

It can be seen in Figs. 8 and 9, that the model provides a good correlation with the actual load-deflection curves for specimens B36-L1 and B18-2 that had larger dimensions, high steel ratios and comparatively high concrete strengths. Most importantly for the current research, Figs. 10 and 11 show that a good prediction of behaviour can be obtained for non-prismatic fabric-formed concrete beams. This suggests that the analysis model is suitable for predicting deflections for both prismatic and non-prismatic beams and therefore can be used to help optimize beam shape, for both serviceability and ultimate limit states.

### B. Optimization

The verified model used in this project to predict load-deflection behaviour of concrete beams can be utilized for flexural members of any shape. Therefore, the model was used to design concrete members optimized for strength and serviceability. When the member is optimized considering the ultimate limit state only, it has been seen that, about 40% of saving in material is possible, compared to an equivalent prismatic beam. The resulting variation in beam depth is shown in Fig. 12.

When the member is optimized for serviceability, material added to locations of maximum curvature of the member which was already optimized for strength. This results in an increase in depth of the beam towards the ends (as shown in Fig. 12) with no increase at the center of the beam. This decreases the amount of material that can be saved to about 25%, depending on some parameters such as reinforcement type and ratio, material properties, the size of the element and loading type. Adding material to the sections that have maximum curvature value flattens the curvature curve of the member and a member with a uniform curvature distribution under service loads is obtained. This has added the advantage that a larger region of the beam is liable to crack with smaller crack widths at serviceability rather than large local cracks.

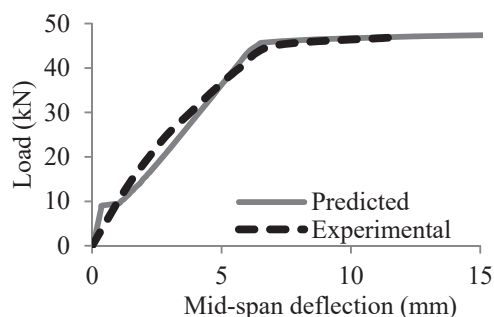


Fig. 6 Comparison of predicted and experimental load-deflection behaviour of Beam T1MA



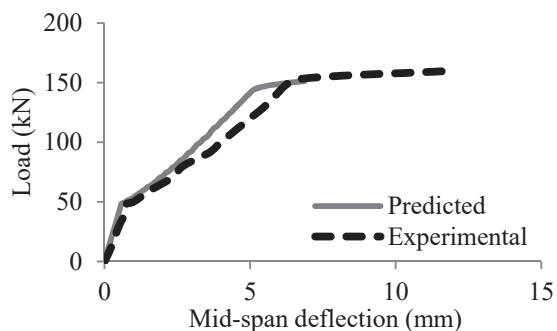


Fig. 7 Comparison of predicted and experimental load-deflection behaviour of Beam J4

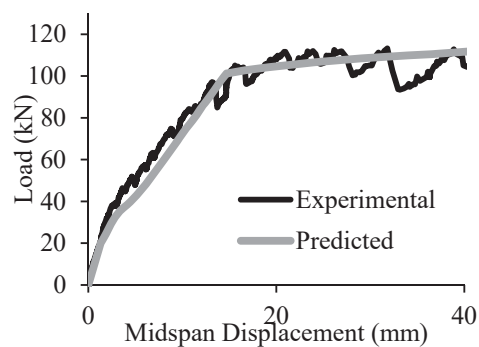


Fig. 9 Prediction of deflection of Beam B318-2

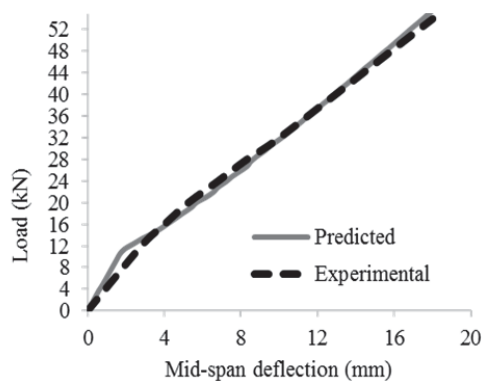


Fig. 8 Prediction of deflection of Beam B3

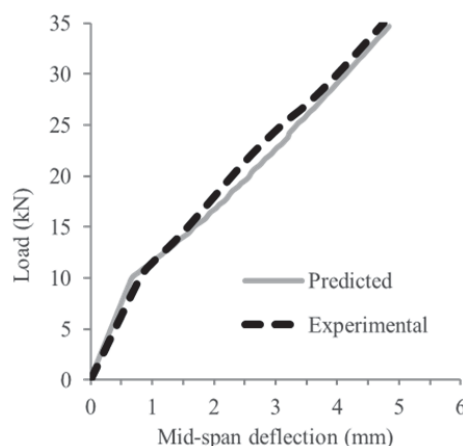


Fig. 10 Load-deflection plots of fabric-formed double T-beams

TABLE I  
DETAILS OF THE BEAMS CONSIDERED

Specimen	Type	b (mm)	h (mm)	d (mm)	L (mm)	$f_y$ (MPa)	$f_c'$ (MPa)	$\rho\%$	$\rho' \%$
T1MA [7]	Prismatic	152.4	304.8	272.3	2700	317.5	31.7	0.62	0
J4 [7]	Prismatic	203.2	508	457.2	3600	309.7	33.35	0.99	0
B36-L1 [13]	Prismatic	76	914	774.7	11890	426.6	54.47	2.9	0
B18-2 [13]	Prismatic	38	457	388.6	3660	358.5	78.05	3.4	0
5-1-H-C [14]	Fabric-formed	965	Varies	Varies	4000	575	40	Varies	Varies
6-4-1 [14]	Fabric-formed	965	Varies	Varies	4000	575	23.7	Varies	Varies

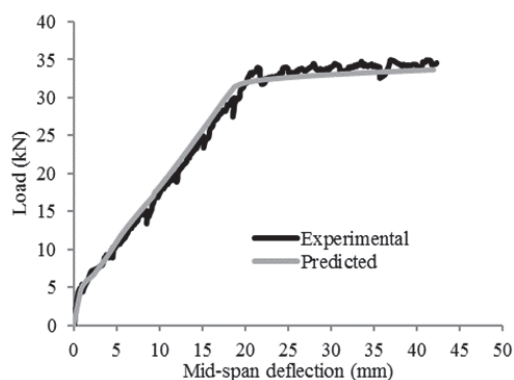


Fig. 11 Load-deflection plots of fabric-formed single T-beam

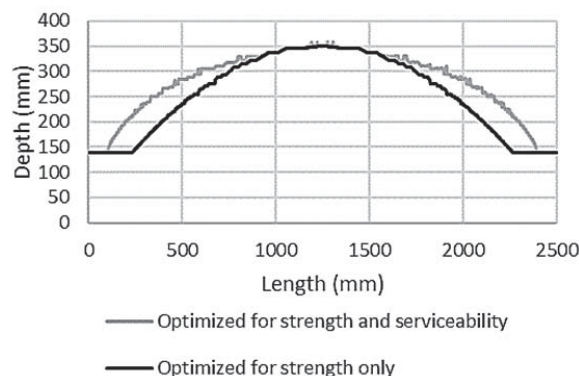


Fig. 12 Profile of simply supported beams optimized for different limit state criterion

#### IV. CONCLUSIONS

In this paper, a computer model was developed to numerically predict load-deflection behaviour of prismatic and fabric-formed concrete members optimize their shape for both serviceability and ultimate limit state performance, the main conclusions are briefly outlined below:

- 1- The methods of sectional analysis and integration of curvatures can provide a good indication of flexural behaviour for simply supported concrete beams.
- 2- The assumption of perfect bond between reinforcement and concrete can lead to some overestimation in predicting flexural rigidity. This effect becomes more obvious at later stages of crack propagation.
- 3- Optimizing for strength only can produce a beam with about 40% less material than an equivalent prismatic beam. However, it may more flexible and may fail to satisfy serviceability limit state criteria.
- 4- The technique used in this work, theoretically produces a fully optimized concrete beam that can safely function at serviceability and ultimate limit states. By adding material in optimizing for serviceability, material saving of about 25% is still achievable.

#### ACKNOWLEDGMENTS

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