Block-Diagonal Coding for Distributed Computing With Straggling Servers

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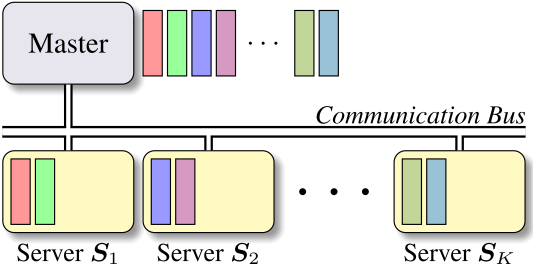
CHALMERS

Motivation



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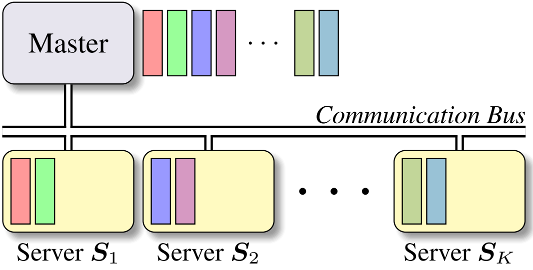

Problem addressed

- Given an $m \times n$ matrix $\boldsymbol{A}$ and N vectors $\boldsymbol{x}_1, \ldots, \boldsymbol{x}_N$, we want to compute $\boldsymbol{y}_1 = \boldsymbol{A}\boldsymbol{x}_1, \ldots, \boldsymbol{y}_N = \boldsymbol{A}\boldsymbol{x}_N$ using K servers.

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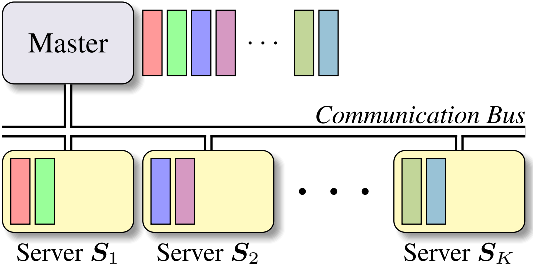

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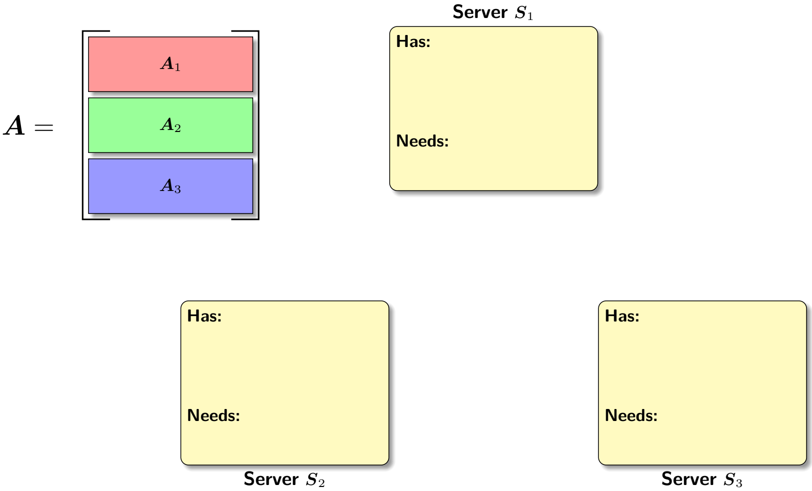
Performance metrics

- Communication load: Average number of messages sent over the network
- Computational delay: Average overall runtime of the computation

Bandwidth Scarcity

(Coded MapReduce, Li *et al.*, 2015)

$$\boldsymbol{y}_1 = \boldsymbol{A}\boldsymbol{x}_1, \boldsymbol{y}_2 = \boldsymbol{A}\boldsymbol{x}_2, \boldsymbol{y}_3 = \boldsymbol{A}\boldsymbol{x}_3$$



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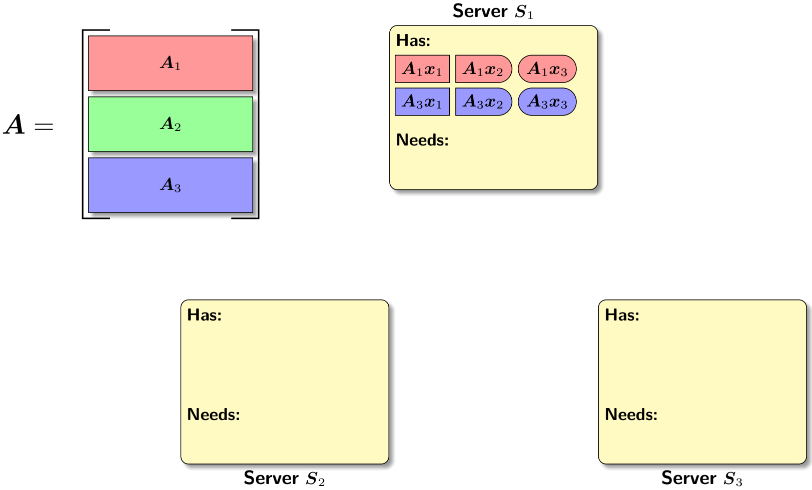

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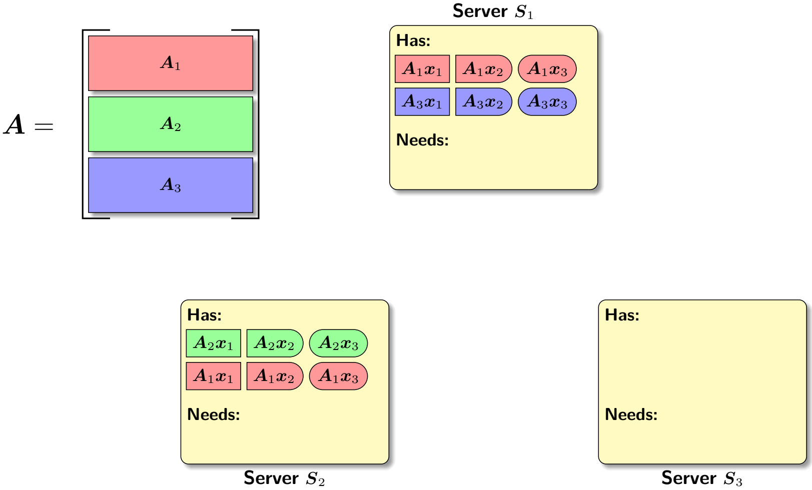

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$$\boldsymbol{A} = \begin{bmatrix} \boldsymbol{A}_1 \\ \boldsymbol{A}_2 \\ \boldsymbol{A}_3 \end{bmatrix}$$

Server S_1

Has:

$\boldsymbol{A}_1\boldsymbol{x}_1$ $\boldsymbol{A}_1\boldsymbol{x}_2$ $\boldsymbol{A}_1\boldsymbol{x}_3$

$\boldsymbol{A}_3\boldsymbol{x}_1$ $\boldsymbol{A}_3\boldsymbol{x}_2$ $\boldsymbol{A}_3\boldsymbol{x}_3$

Needs:

Has:

$\boldsymbol{A}_2\boldsymbol{x}_1$ $\boldsymbol{A}_2\boldsymbol{x}_2$ $\boldsymbol{A}_2\boldsymbol{x}_3$

$\boldsymbol{A}_1\boldsymbol{x}_1$ $\boldsymbol{A}_1\boldsymbol{x}_2$ $\boldsymbol{A}_1\boldsymbol{x}_3$

Needs:

Server S_2

Has:

$\boldsymbol{A}_3\boldsymbol{x}_1$ $\boldsymbol{A}_3\boldsymbol{x}_2$ $\boldsymbol{A}_3\boldsymbol{x}_3$

$\boldsymbol{A}_2\boldsymbol{x}_1$ $\boldsymbol{A}_2\boldsymbol{x}_2$ $\boldsymbol{A}_2\boldsymbol{x}_3$

Needs:

Server S_3

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(Coded MapReduce, Li *et al.*, 2015)

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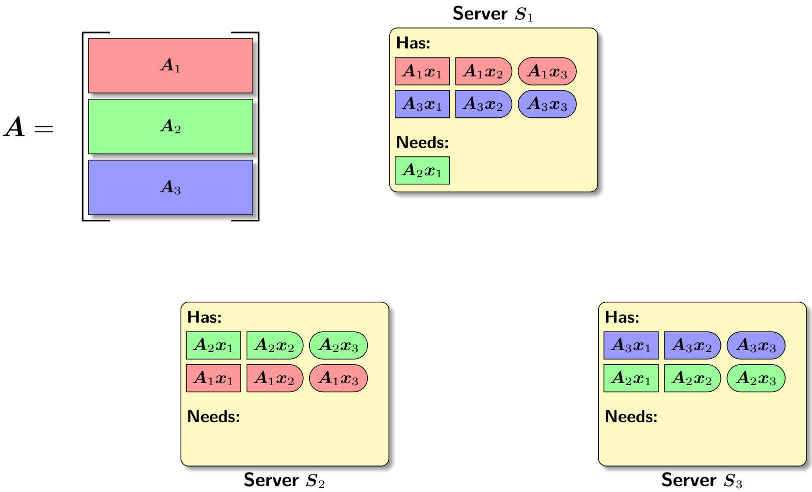

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Needs:
$\boldsymbol{A}_2\boldsymbol{x}_1$

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Needs:
$\boldsymbol{A}_3\boldsymbol{x}_2$

Server S_2

Has:
$\boldsymbol{A}_3\boldsymbol{x}_1$ $\boldsymbol{A}_3\boldsymbol{x}_2$ $\boldsymbol{A}_3\boldsymbol{x}_3$
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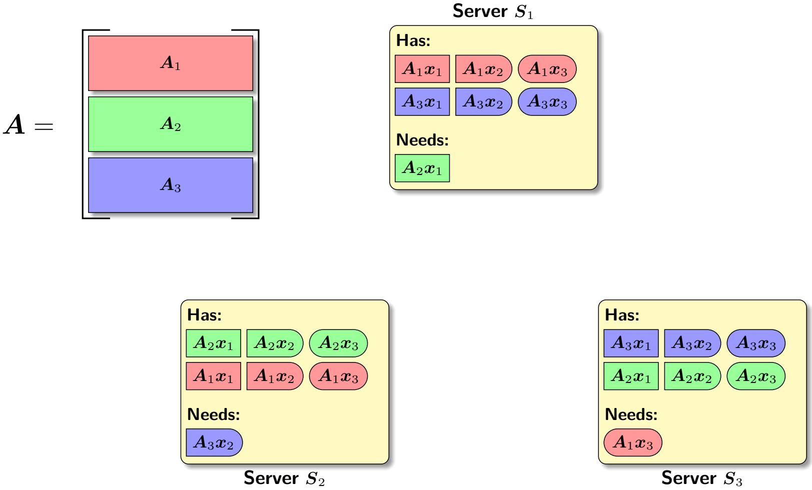

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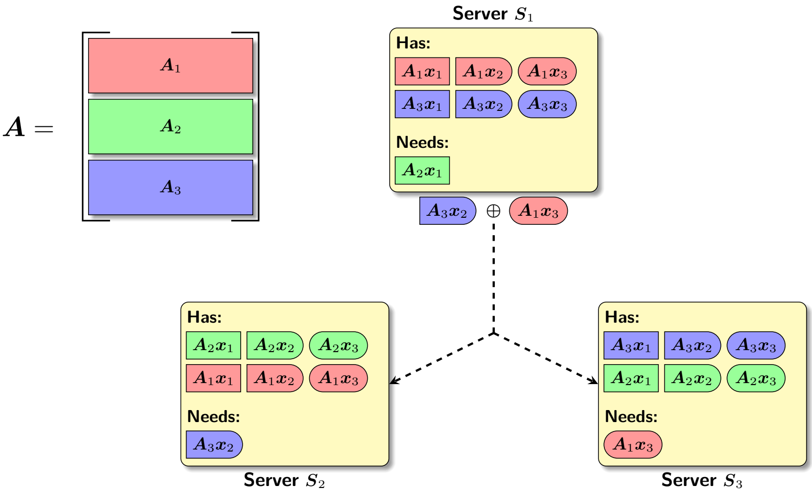

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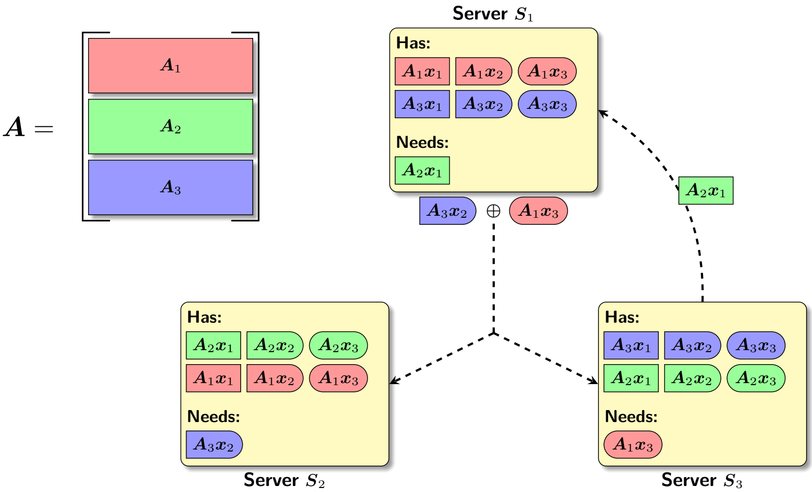

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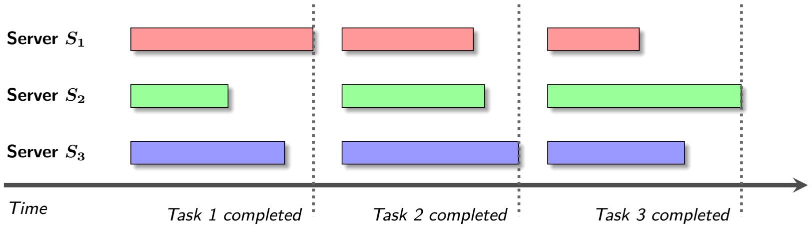

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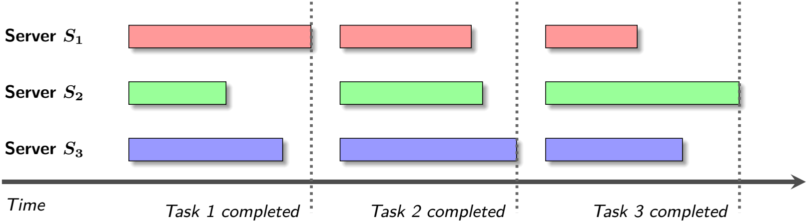
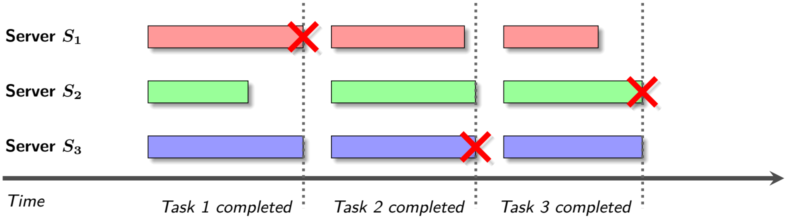




The Straggler Problem

$$y = Ax$$

$$A = \begin{bmatrix} A_1 \\ A_2 \end{bmatrix}$$

The Straggler Problem

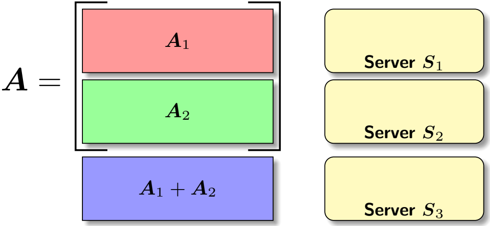
$$\boldsymbol{y} = \boldsymbol{A}\boldsymbol{x}$$

$$\boldsymbol{A} = \begin{bmatrix} \boldsymbol{A}_1 \\ \boldsymbol{A}_2 \end{bmatrix}$$

$$\boldsymbol{A}_1 + \boldsymbol{A}_2$$

The Straggler Problem

$$\boldsymbol{y} = \boldsymbol{A}\boldsymbol{x}$$



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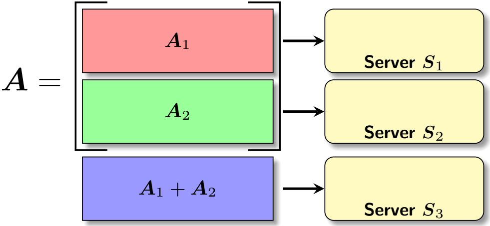

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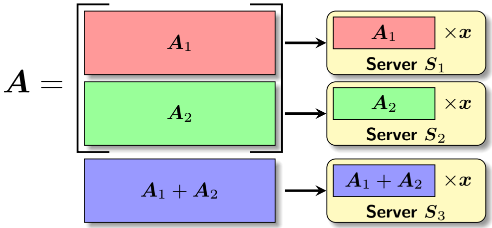

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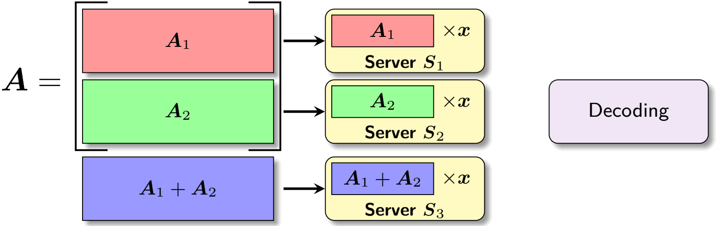

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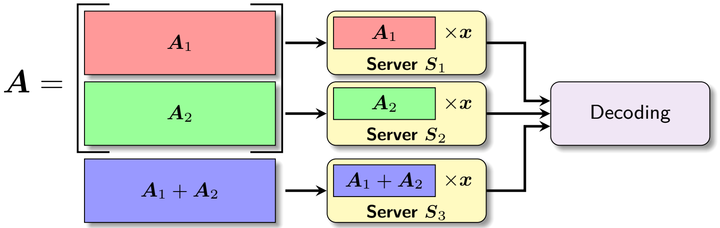

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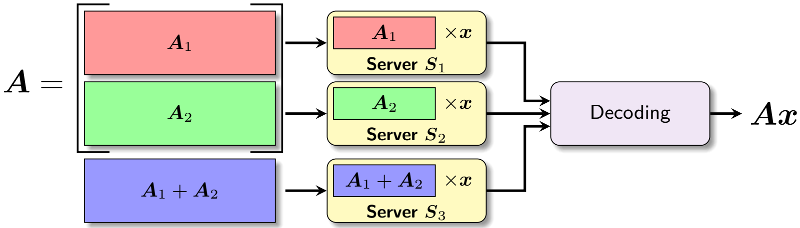

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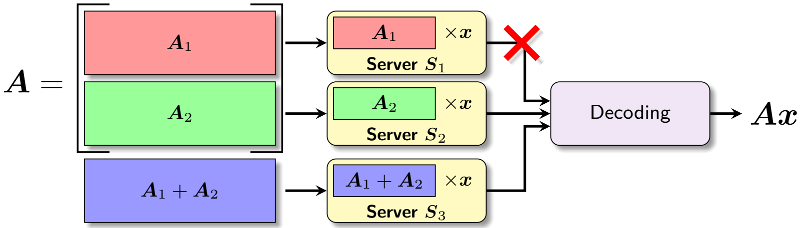

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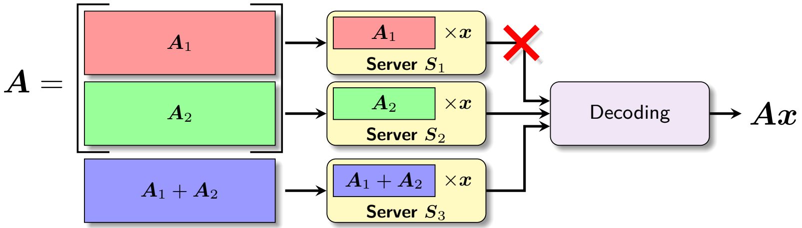

In general

- Introduce redundancy by encoding the input matrix A.

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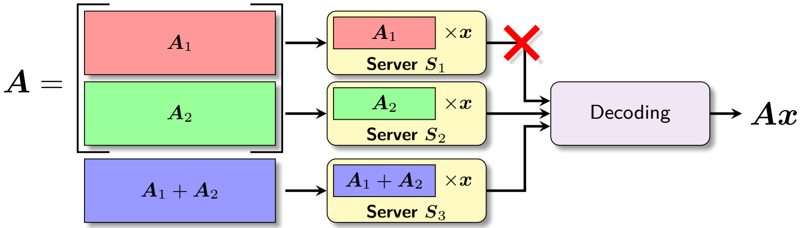

In general

- Introduce redundancy by encoding the input matrix A.
- Each server is given more work. However, this may still lower the computational delay!

The Unified Coded Framework

(A Unified Coding Framework for Distributed Computing with Straggling Servers, Li *et al.*, 2016)

Coded approach by Li *et al.*

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- Encode the columns of $\boldsymbol{A} \in \mathbb{F}^{m \times n}$ using an (r, m) MDS code by multiplying $\boldsymbol{A}$ with an $r \times n$ encoding matrix $\boldsymbol{\Psi}_{\text{MDS}}$, i.e., $\boldsymbol{C} = \boldsymbol{\Psi}_{\text{MDS}} \boldsymbol{A}$.

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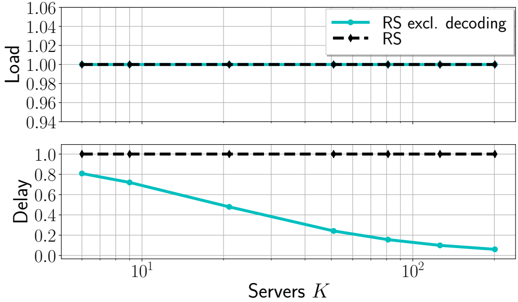

- 2000 rows assigned to each server, $n = 10000$ columns, $N = 2K/3$ vectors, and code rate $m/r = 2/3$.

In this talk

Block-Diagonal Coding

- We propose a block-diagonal encoding scheme with lower decoding complexity.

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Idea

- Partition $\boldsymbol{A}$ into T disjoint submatrices and apply smaller MDS codes to each submatrix,

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- Overall computational delay is lower than that of the scheme by Li *et al.*.
- Larger T may reduce computational delay further at the expense of higher communication load.

Block-Diagonal Coding

$$\Psi_{\mathsf{BDC}} = \underbrace{\begin{bmatrix} \psi_1 & & \\ & \psi_2 & \\ & & \psi_3 \end{bmatrix}}_{r \times m}$$

- Block-diagonal encoding with $T = 3$ partitions.

Block-Diagonal Coding

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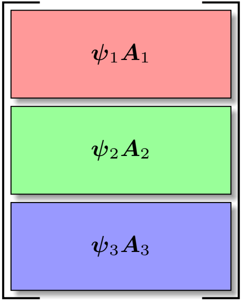
- Need any m/T out of r/T rows from each partition to decode.

Block-Diagonal Coding



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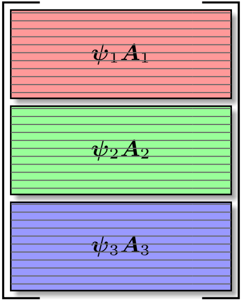

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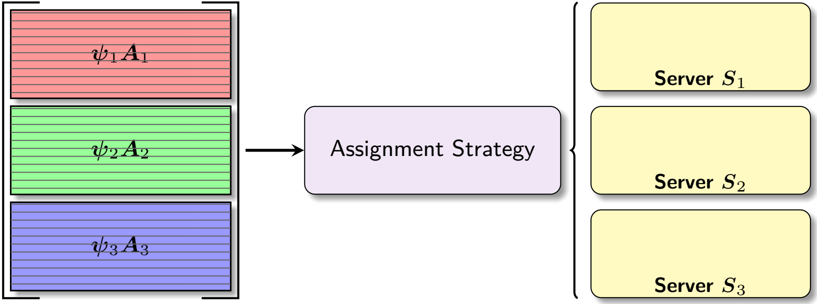

- Block-diagonal encoding with $T = 3$ partitions.
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- Need to assign coded rows to servers very carefully in some instances (such as when the number of servers is small).

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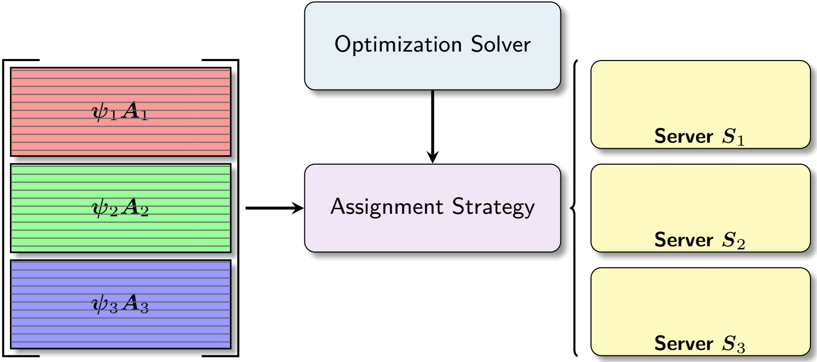

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- This assignment can be formulated as an optimization problem.

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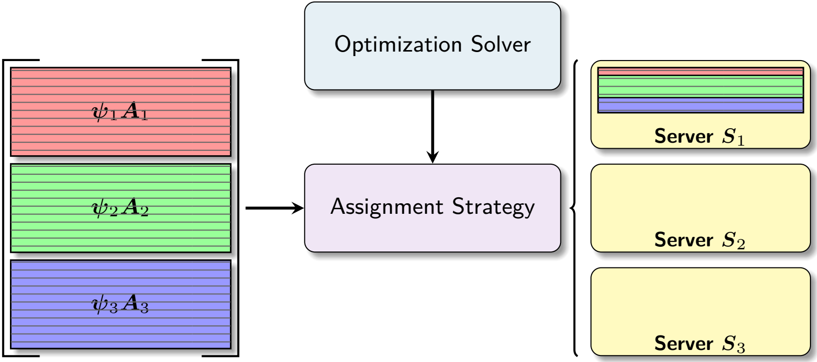

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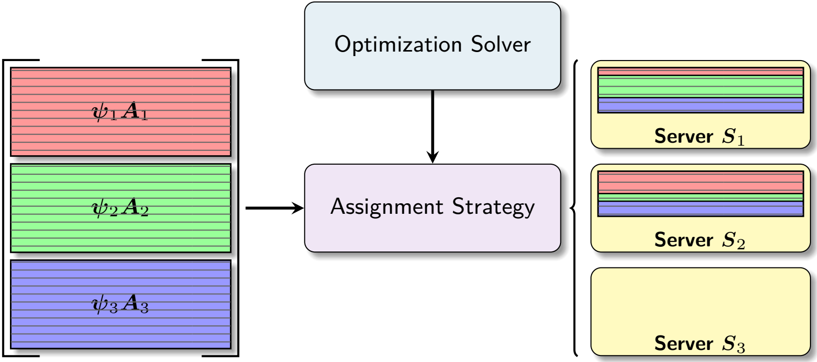

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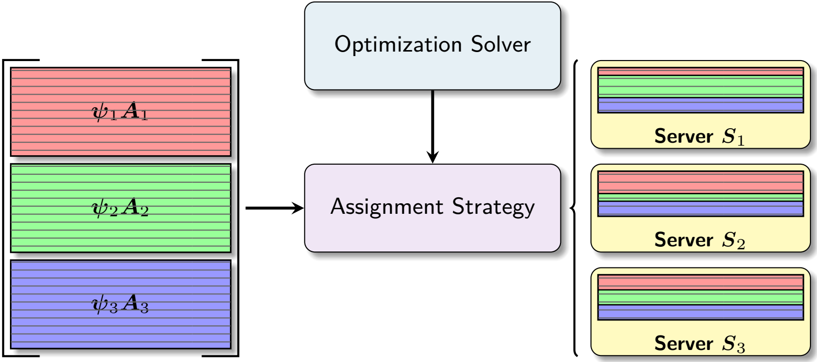

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Assignment Matrix

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- $K = 4$ servers, $T = 6$ partitions, $m = 6$ source matrix rows, and $r = 12$ coded rows, i.e., code rate $1/2$.

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- Store each coded row at 2 servers to enable coded multicasting.

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$$\begin{array}{c} (S_1, S_2) \\ (S_1, S_3) \\ (S_1, S_4) \\ (S_2, S_3) \\ (S_2, S_4) \\ (S_3, S_4) \end{array} \begin{pmatrix} \quad \\ \quad \\ \quad \\ \quad \\ \quad \\ \quad \end{pmatrix}$$

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$$
\begin{array}{c c}
 & \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\
\begin{array}{c} (S_1, S_2) \\ (S_1, S_3) \\ (S_1, S_4) \\ (S_2, S_3) \\ (S_2, S_4) \\ (S_3, S_4) \end{array} & \left(\begin{array}{cccccc} & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \end{array} \right)
\end{array}
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\begin{pmatrix}
2 & & & & & \\
 & 2 & & & & \\
 & & 2 & & & \\
 & & & 2 & & \\
 & & & & 2 & \\
 & & & & & 2
\end{pmatrix}
\end{array}
$$

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2 & & & & & \\
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\end{array}$$

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\begin{pmatrix}
1 & 1 & & & & \\
 & & 1 & 1 & & \\
 & & & & 1 & 1 \\
1 & 1 & & & & \\
 & & 1 & 1 & & \\
 & & & & 1 & 1
\end{pmatrix}
\end{array}
$$

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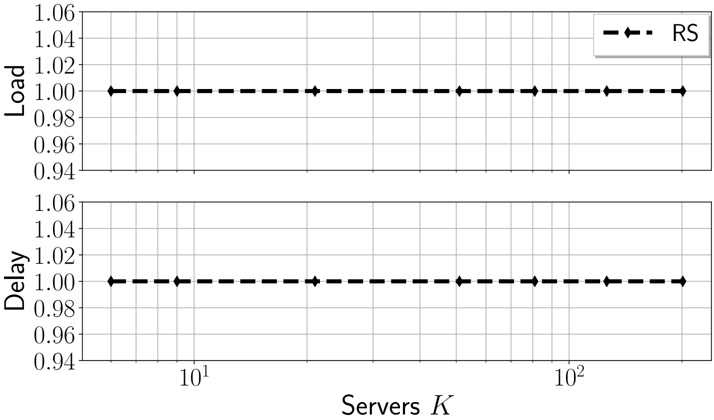
\begin{pmatrix}
1 & 1 & & & & \\
 & & 1 & 1 & & \\
 & & & & 1 & 1 \\
1 & 1 & & & & \\
 & & 1 & 1 & & \\
 & & & & 1 & 1
\end{pmatrix}
\end{array}
$$

Numerical results



- 2000 rows assigned to each server, $n = 10000$ columns, $m/T = 10$ rows per partition, $N = 2K/3$ vectors, and code rate $m/r = 2/3$.

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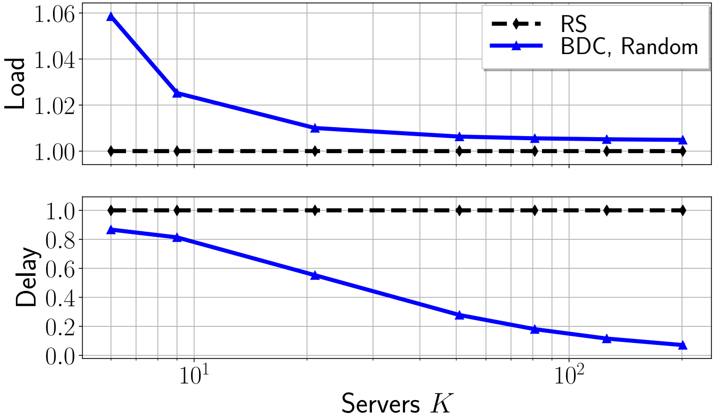

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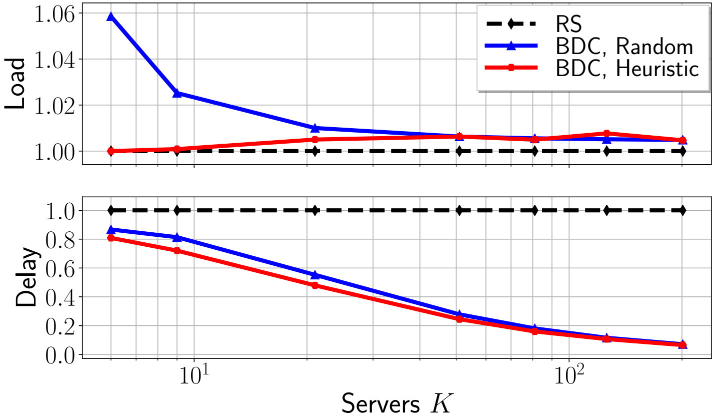

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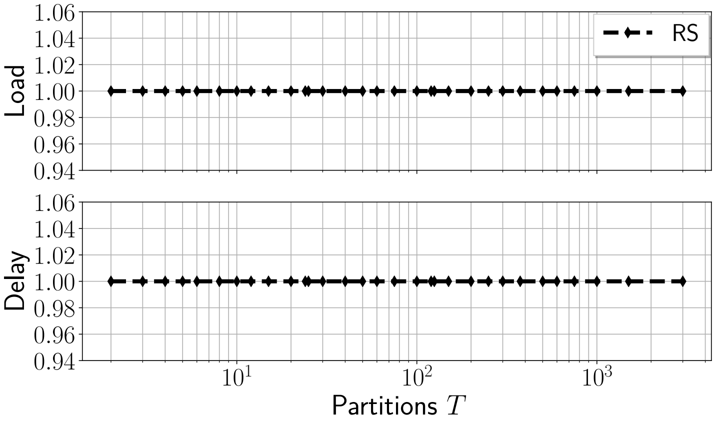
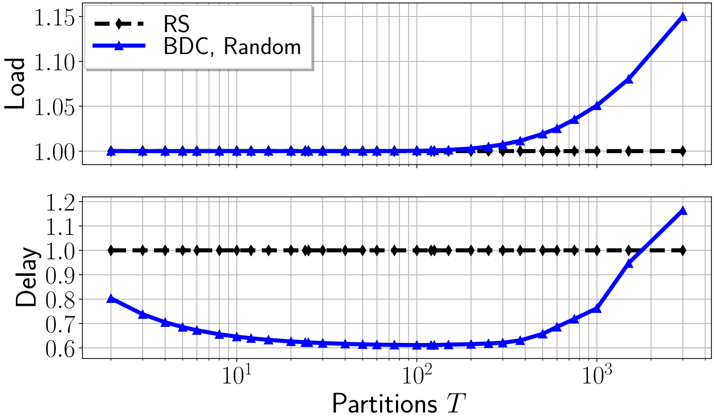


- $m = 6000$ source matrix rows, $n = 6000$ columns, $N = 6$ vectors, $K = 9$ servers, and code rate $m/r = 2/3$.

Numerical results



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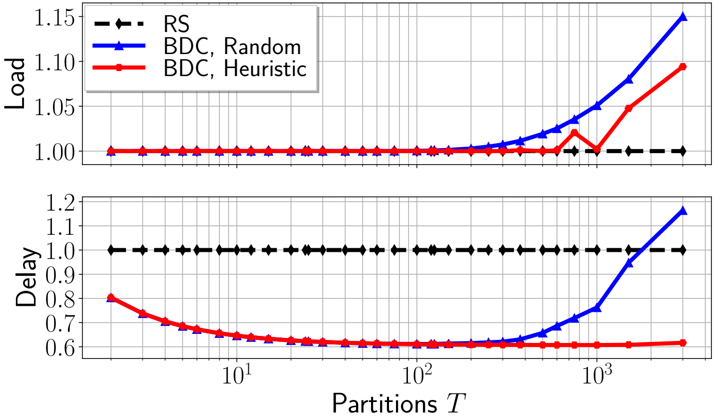

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Optimal Assignment

Theorem

- Up to a given number of partitions T, the
  - communication load, and the
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- of the BDC scheme are equal to those of the scheme by Li *et al.*.

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  - communication load, and the
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The overall computational delay of our scheme is much lower than that of the scheme by Li *et al.* due to its lower decoding complexity.

Conclusion

Take-home message...

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- The delay of the scheme by Li *et al.* is dominated by the decoding delay. Our scheme addresses this issue with little to no impact on the load.
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  - $> 90\%$ reduced delay over the scheme by Li *et al.* for a matrix with 134000 rows and 10000 columns with $< 1\%$ increased load.

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- Encoding delay, LT codes under inactivation decoding...
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- Slides and code on Github: github.com/severinson/coded-computing-tools

References

[1] K. Lee et al. "Speeding up distributed machine learning using codes". In: *Proc. IEEE Int. Symp. Inf. Theory*. Barcelona, Spain, July 2016, pp. 1143–1147. DOI: 10.1109/ISIT.2016.7541478.

[2] S. Li, M. A. Maddah-Ali, and A. S. Avestimehr. "Coded MapReduce". In: *Proc. Annual Allerton Conf. Commun., Control, and Computing*. Monticello, IL, Sept. 2015, pp. 964–971. DOI: 10.1109/ALLERTON.2015.7447112.

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