Kinematic Sets

Ryan J. Buchanan

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Abstract

We construct a kinematic set out of some data associated to a path, which is defined here to be the target of an inclusion functor $i : \gamma \hookrightarrow \mathcal{T}op$, with γ a regular morphism.

One does not need to make any mention of applications for the concept of a "path," as they are so abundant as to be a part of the everyday nomenclature of even non-mathematicians, and indeed, even the children of non-mathematicians. So instead, we are free to focus on the technical details of paths.

1 Paths

Let $\mathscr{C}$ be a non-empty category. Let there be a single morphism $\gamma : A \to B$. We call γ a *path* if, for some injection $\mathscr{C} \hookrightarrow \mathcal{T}op$, we have

$$im(\gamma) = c$$

where c is a curve of dimension less than or equal to the dimension of the smallest subcategory $\mathtt{C}$ of $\mathcal{T}op$ with the following properties:

1. $\mathtt{C}$ has a Δ-complex structure.
2. $span(im(A) \sqcup im(B)) = max(span(\mathtt{C}))$

The second quality is equivalent to saying that A and B are antipodal. An equivalent formulation is to say that $d(A, B) \geq d(C, D)$ for all (C, D) in the infinitesimal extension of $\mathtt{C}$.

Definition 1.1. *An infinitesimal extension, ε of a Δ-complex $\tilde{\Delta} \in \mathcal{T}op$ is the smallest open set covering initial and terminal flags of a globally representative triangulation $Glo(\tilde{\Delta})$.*

Definition 1.2. *A representative triangulation is a triangulation*

$$\zeta \to \eta \to \theta \in \Delta$$

such that the $\mu(\Delta \setminus Glo(\tilde{\Delta}) \ll \mu(Glo(\tilde{\Delta}))$ where $\mu(\bullet)$ is the outer measure of $\bullet$, and such that each vertex of the triangulation's image lies within $\partial \mathtt{C}$ (i.e. $R \cap \partial \mathtt{C} = R$

Write $\mathtt{ob}(\mathscr{C})$ for the class $[c] \leftarrow \{c \mid c \in \mathscr{C}\}$. When γ is path, we have the morphism

$$\gamma : \mathtt{ob}(\mathscr{C}) \to \mathtt{ob}(\mathscr{C})$$

giving us

$$\gamma = ker(End(\mathtt{ob}(\mathscr{C}))$$

When $A \cap B \neq \emptyset$, then $|\gamma|$ is a Jordan curve, which is homeomorphic to the Tate circle.

2 Daseinization

Let $\mathtt{Amb}(\bullet)$ be the ambient space of $\bullet$, and define $\mathtt{Amb}(*) = \infty$ when $* = dom(\mathcal{T})$ for our chosen subcategory of $\mathcal{T}op$.

Let $\delta \simeq \gamma$, i.e., $\delta = (\omega \circ ... \circ f)(\gamma)$. Pick your favorite Grothendieck universe, $\mathcal{V}$. Since the map $g_1 \to \emptyset$ is epic for every $g_1 \in \mathcal{V}$ we can form the diagram

$$\begin{tikzcd}
g_1 \arrow[r, dashed, "\wp"] \arrow[d, two heads] \arrow[dr, phantom, "D_\delta \Uparrow"] & g_2 \\
\emptyset \arrow[r, "\neg"'] & \neg\emptyset \arrow[u, hook]
\end{tikzcd}$$

whose 2-cell we shall call the "dasseinisation" of δ.

Proof. Since for every $x \neq \emptyset$, we have $i : x \hookrightarrow X = \neg\emptyset$ for every inclusion functor $i \in \mathtt{Mor}(X)$. This means that $\neg\emptyset$ is free, and we can choose a term to rewrite it with. Select $\mathtt{amb}(\emptyset)$ and put

$$Re : \neg\emptyset \to \infty$$

Let there be a second universe $\mathcal{V}'$, and a flag

$$g_1 \subset \mathcal{V} \subset g_2 \subset \mathcal{V}'$$

consisting of inclusions $i = \bigcup i_0 ... i_n : \mathtt{ob}(inf(\mathcal{V})) \to \mathtt{ob}(sup(\mathcal{V}))$. We have

$$\#i_k = \#E(\mathcal{V})$$

for $0 < k \leq i$, where E is an edge in $graph(\mathcal{V})$. Since our diagram is a commutative square, this reduces to

$$i = \sqcup i_0 ... i_2$$

Next, we need to rewrite the term ∞ by

$$Re : \infty \to \mathbb{1}_\infty$$

where $\mathbb{1}_\infty$ is a model category. Then, we have the embedding

$$i_3 : \mathbb{1}_\infty \to \mathtt{ob}(sup(\mathcal{V}))$$

which simplifies to a lift

$$\wp : g_1 \nearrow g_2$$

We get a 2-cell: $D_\delta : \neg \to \wp$, which is the desired dasseinisation. $\square$

Daseinization was referred to in [1] as a standardized procedure for mapping propositions about the physical world to "clopen subobjects." A contact form like the one introduced in [2] should satisfy this property by, e.g. constructing a one link on a space whose naive Legendrian is zero.

3 Paths with time derivatives

In a bare 1-category, a morphism is directly distinguished from a path. This is why we must pass to topological categories (pick your favorite) in order to achieve the required daseinization. A further step we can then take is to write

$$\int \frac{d^k x}{d^k t} = \{\gamma\}$$

so that we obtain a kinematic set.

Definition 3.1. *A kinematic set is a set $\mathscr{K} = \{\{\gamma\}, D_\delta, sup(\mathcal{V}), \mathscr{Q}\}$ consisting of an integral curve, a Daseinization, a Grothendieck universe, and a quasi-category $\mathscr{Q}$ such that*

$$sup(\mathcal{V}) \overset{\sim}{\hookrightarrow} \mathscr{Q}$$

is an isofibration.

In specific, every Δ-complex structure in our universe extends to a full-blown simplicial complex in the quasi-category, and we get a functor

$$Gal_{\{\gamma\}} : (Gal_\gamma : \gamma \overset{\sim}{\hookrightarrow} \{\gamma\}) \longmapsto \delta \subseteq \mathscr{Q}$$

so that the morphism γ extends to an n-simplex in the codomain.

For the simplest dummy cases, we should begin by worrying about the case where $k = 1$. In such a case we are primarily interested in the Ricci flow of the kinematic set.

3.1 Ricci Graphs

Let $\mathfrak{G}$ be a graph. Identify the antipodal points with extremal vertices; i.e., if v_0, v_n are two vertices on a graph, they are extremal if the number of edges contained within the shortest path $v_0 \to v_n$ is such that

$$\#E(v_0 \to v_n) > \#else$$

for all other paths. We will call the number of edges in this path the "anchoring" constant.

Set the anchoring constant to q for step 0. We then introduce a "Poisson sprinkling" (much used in the theory of causal sets) to the graph. For vertices v_j with $0 < j < n$, attach a new edge and vertex to v_j. Then, embed the new graph $\mathfrak{G}'$ into $\tilde{\mathscr{Q}}$, which is the smooth representation of the pre-ordained quasi-category. We have

$$\mathscr{Q} \times \mathscr{Q} \mapsto \tilde{\mathscr{Q}} \cong Rep(\mathfrak{Cone}(\mathscr{Q}))$$

so that the representation of the mapping cone of a quasi-category admits a smooth embedding into its ambient space.

Construct some ring $\mathtt{r}$ so that there is a correspondence

$$cor_{\mathtt{r},\mathbb{E}} = \mathtt{r}\ cor\ \mathbb{E}_t$$

where $\mathbb{E}$ is the edge set of the graph at step t, such that the map

$$\mathbb{E}_t \to \mathbb{E}_{t+1} \cong Ric_{g_{i+1}}(cor_{\mathtt{r},\mathbb{E}})$$

induces an isofibration

$$\mathfrak{G} \overset{\sim}{\hookrightarrow} ih(\mathfrak{G}')$$

on the injective hull of the new graph.

Let $cor_{\mathtt{r},\mathbb{E}}$ be symbolized succinctly by an ordinary cardinal $c \in \mathtt{r}$ which is representative of the correspondence. I.e., to each edge in our graph, there is an object (possibly a numerical value) representing a proposition (possibly about a physical object), and that representative lives within our ring. Then, we have a map

$$Ric_{g_{i+1}}(c)$$

which witnesses the transformation $Aug : \{\gamma\}_1 \to \{\gamma\}_2$.

We then have

$$Aug(X) = Ric_{g_i}(X) \to Ric_{g_{i+1}}(X) = Ric_{g_{i\to i+1}}(X)$$

which can be written

$$\int \frac{dc}{dt}$$

and when normalized to 1 this gives us

$$\int \frac{dc}{dt} = e \sim 1 \in \mathbb{1}$$

so the augmentation is equivalent to a measure of the state transitions of a Ricci graph across a single step of time.

This step of time can be as large or small as we want, and we can arbitrarily manipulate its size by manipulating the size of our ε, and finding a sufficient G_δ to compensate.

4 Embeddings

Let $|\gamma| : \gamma \hookrightarrow \{\gamma\}$ be a presentation of a path $x \to y$ for two distinct (not necessarily antipodal) points on an n-sphere.

Definition 4.1. *A flat embedding*

$$|\gamma|^\flat : \gamma \hookrightarrow Aug(\gamma)$$

is an immersion of every disjoint point $* \in \gamma$ into every edge $e \in graph(\gamma)$ in the graph of a path.

Notice that, whence we have an extension of $|\gamma|$, there is a flat embedding of the extension, $|\gamma|^+$ into the extension of $im(\gamma)$.

$$\begin{tikzcd}
|\gamma| \arrow[rr, "\cup"] \arrow[d, "\flat"] \arrow[drr, "\sharp" description] & & |\gamma|^+ \arrow[d, dashed, "\flat"] \\
graph(\gamma) \arrow[rr, "\cup"] & & graph(\gamma \cup \iota)
\end{tikzcd}$$

The lift:

$$|\gamma| \nearrow graph(\gamma \cup \iota)$$

is actually a sharp morphism.

Proposition 4.1. *For every $\mathfrak{f} : x \nearrow y \in \mathscr{K}$, there is a map $sup(\mathcal{V}) \to \mathcal{V}$ given by*

$$D_\delta \otimes \mathfrak{f} \to (\mathscr{Q} \times \mathscr{Q})$$

Proof. For every quasi-category $\mathscr{Q}$, we are free to choose a normal element $q \in \mathscr{Q}$ with zero periodicity such that $q < \omega$ for the largest regular cardinal $\omega \in \mathcal{V}$.

Thus, for every $q' \in \mathscr{Q}$, there is a diagram

$$\begin{tikzcd}
q \arrow[r, hook, "i_1"'] \arrow[d, two heads, "i_2^{-1}"] & \mathscr{Q} \arrow[d, hook, "i_2"'] \\
q' \arrow[r, hook, "i_1'"] & \mathcal{V}
\end{tikzcd}$$

such that $i_1' \circ i_2^{-1} = i_2 \circ i_1$. We then gain a lift, $\mathfrak{f} : q \nearrow \mathcal{V}$, and there is an isofibration

$$sup([q, q']) \overset{\cong}{\hookrightarrow} sup(\mathcal{V})$$

By reversal, we have:

$$\begin{tikzcd}
q & \mathscr{Q} \times \mathscr{Q} \arrow[l, two heads, "\perp i^{-1}"'] \\
q' \times \mathfrak{g} \arrow[u, hook, "i_2''"] & \mathcal{V} \arrow[l, two heads, "\perp i'^{-1}"'] \arrow[u, hook, "i_2'"']
\end{tikzcd}$$

in $graph(q, \mathcal{V})^{op}$. We have an equivalence

$$i_2' \simeq D_\delta \otimes \mathfrak{f}$$

thus proving the proposition. $\square$

Theorem 4.1. *Every sharp embedding into the category of graphs*

$$|\gamma| \overset{\sharp}{\hookrightarrow} \mathfrak{Graphs}$$

forces an inclusion

$$i : \iota \hookrightarrow \mathfrak{Graphs}$$

such that $\iota \circ \gamma = \gamma^+$.

Proof. Take the retract

$$\flat^{-1} : graph(\gamma \cup \iota) \longrightarrow |\gamma|^+$$

Since $\flat$ is fully faithful, and exact, it suffices to show that it is invertible, so we have $(E(graph(\gamma \cup \iota)))^{-1} = \mathtt{mor}(\gamma^+)$ and $(V(graph(\gamma \cup \iota))^{-1} = \mathtt{ob}(\gamma^+)$. $\square$

5 References

[1] A. Döring, *Spectral presheaves as quantum state spaces*, (2015)
[2] R.J. Buchanan, *Constructing Legendrian links from chiral Reeb chords*, (2024)
[3] M. Futaki, K. Ueda, *Coamoeba and equivariant homological mirror symmetry for the projective space* (Date unknown)