Multiplicative input noise infusion

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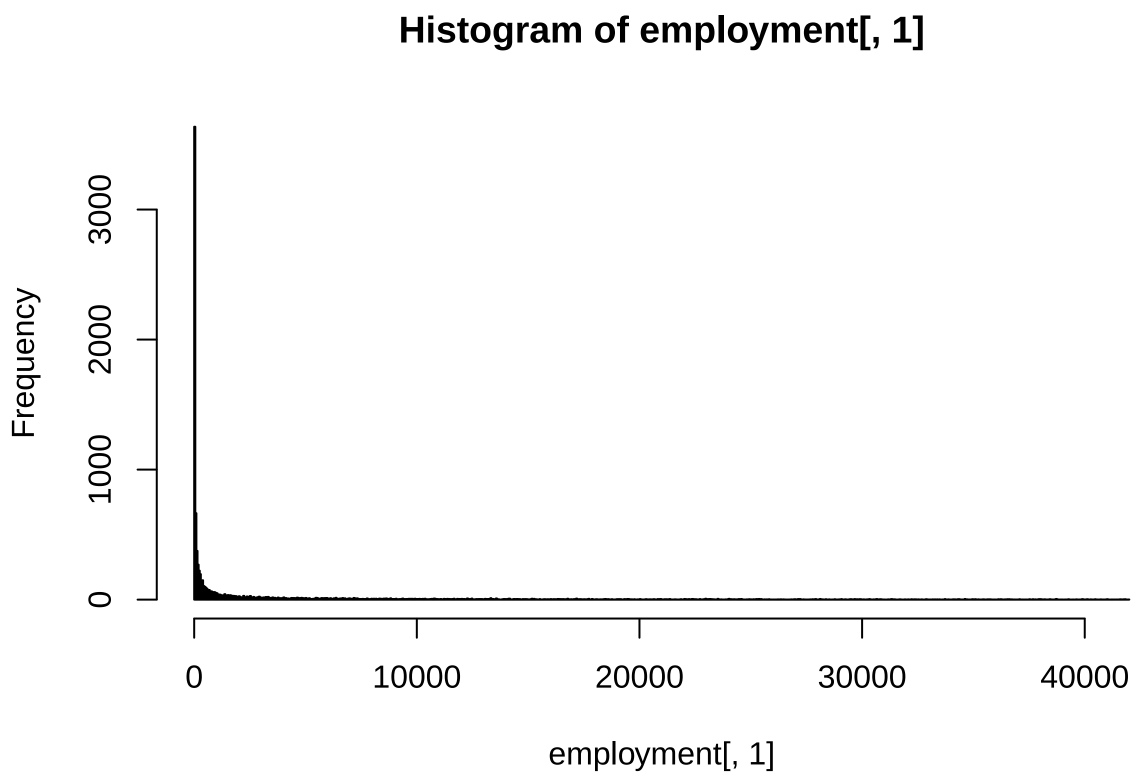
First proposed by Evans, Zayatz and Slanta (1998), multiplicative input noise infusion (henceforth simply "noise infusion") is used as a disclosure-avoidance measure. See also our implementation in the Quarterly Workforce Indicators (published in 2009, but first implemented in 2003).

Let's generate some random data:

```
employment <- round(as.data.frame(exp(runif(size,log(1),log(42000)))))
names(employment) <- c("Count")
```

This fake employment distribution looks like this (actually, real employment is different):



or for a closeup:

but most importantly, it has a **mean of 3879**, a **median of 215**, and **Q25 of 15**.

Ramp distribution

The most common noise distribution used is a ramp distribution. So what is a ramp distribution?

$$p\left(\delta_j\right)=\begin{cases}\dfrac{1+b-\delta}{(b-a)^2} & , \quad \delta \in \ [1+a, 1+b] \\ \dfrac{\delta-(1-b)}{(b-a)^2} & , \quad \delta \in [1-b, 1-a] \\ 0 & , \quad \text{otherwise}\end{cases}$$

with a cumulative distribution of

$$F\left(\delta_j\right)=\begin{cases} 0, \ \delta<2-b \\ \left[(\delta-(1-b))^2\right] \Big/ \left[2\,(b-a)^2\right], \ \delta \in [1-b, 1-a] \\ 0.5, \ \delta \in \ (1-a, 1+a) \\ 0.5+\left[(b-a)^2-(1+b-\delta)^2\right] \Big/ \left[2\,(b-a)^2\right], \ \delta \in \ [1+a, 1+b] \\ 1, \ \delta>1+b \end{cases}$$

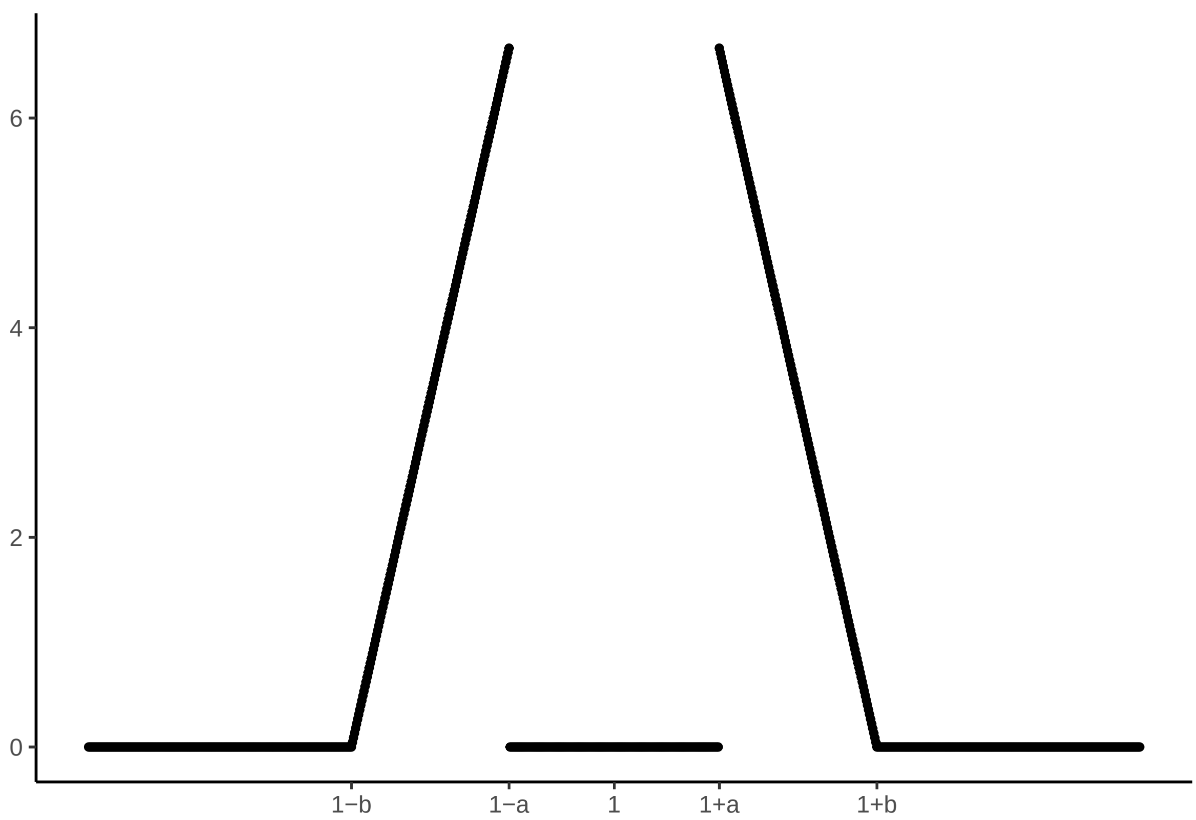
```
dramp <- function(x,a,b) {
  if ( b< a) {
    c <- a
    a <- b
    b <- c
  }
  part1 <- which(x < 1- b )
  part2 <- intersect(which (x >= 1-b),which(x <= 1-a))
```

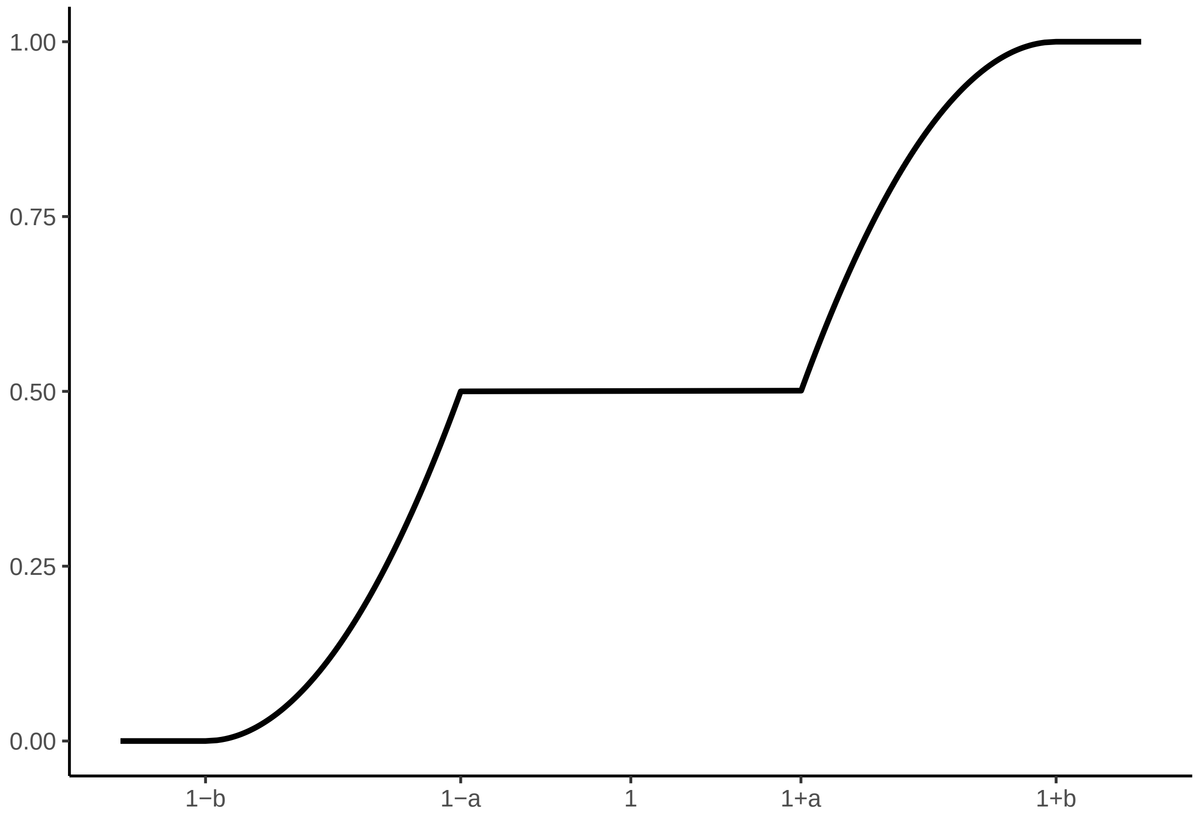
```
  part3 <- intersect(which (x > 1-a), which(x< 1+a))
  part4 <- intersect(which (x >= 1+a),which(x <= 1+b))
  part5 <- which(x > 1+ b )

  y <- x
  y[part1] <- 0
  y[part2] <- (x[part2] - (1-b))/ ( b - a )^2
  y[part3] <- 0
  y[part4] <- ( 1 + b -x[part4])/ ( b - a )^2
  y[part5] <- 0
  return(y)
}

invcramp <- function(y,a,b) {
  part1 <- intersect(which(y>0.5),which(y<=1))
  part2 <- intersect(which(y<=0.5),which(y>=0))
  part3 <- intersect(which(y<0),which(y>1))
  x <- y
  x[part1] <-  1+b - ((1-2*(y[part1]-0.5))*(b-a)^2)^0.5
  x[part2] <-  1-b + ((  2*(y[part2]    ))*(b-a)^2)^0.5
  x[part3] <-  0
  return(x)
  }
```

where $a = c/100$ and $b = d/100$ are constants chosen such that the true value is distorted by a minimum of c percent and a maximum of d percent. This produces a random noise factor centered around 1 with distortion of at least c and at most d percent.





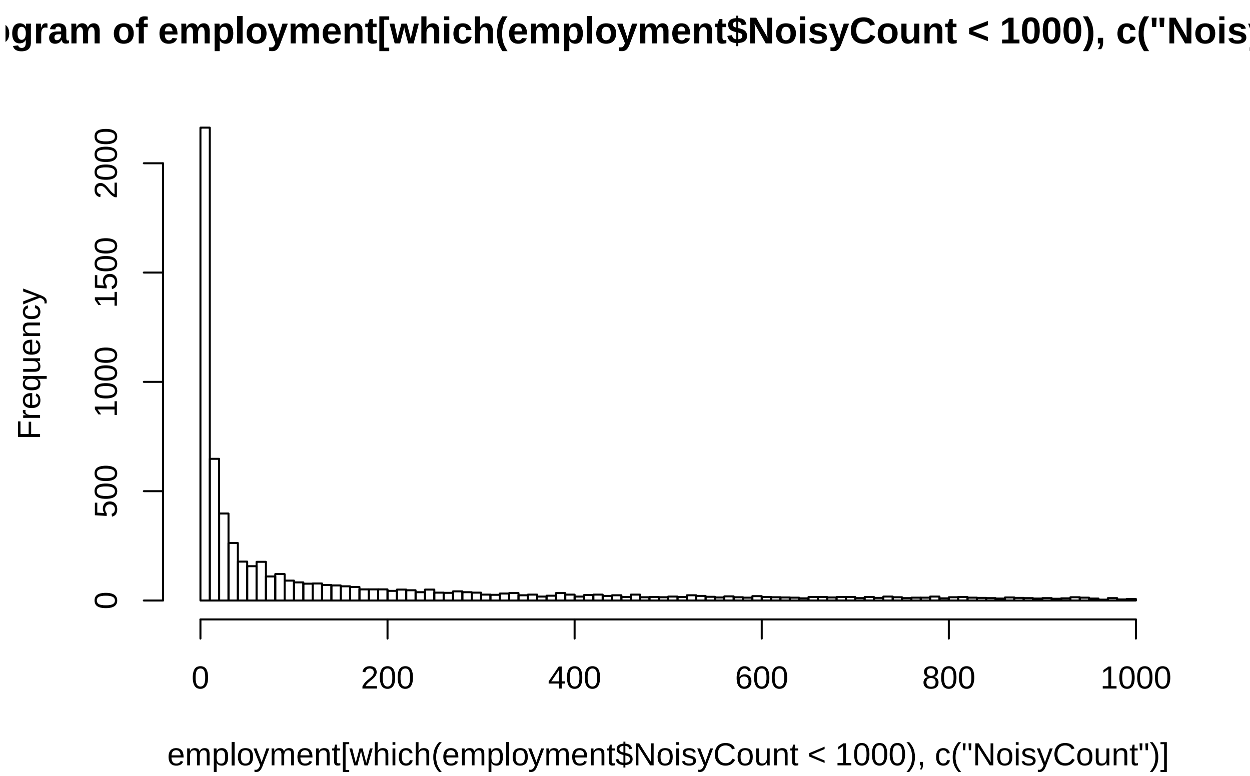
Distorting the data

Applying the multiplicative noise to the counts yields protected counts. Since the mean of the noise distribution is unity by design, the two distributions are likely to have similar means.

```
employment$uniform <- runif(nrow(employment))
employment$fuzzfactor <- invcramp(employment$uniform,0.1,0.25)
employment$NoisyCount <- round(employment$Count * employment$fuzzfactor,0)
```

If we compare the original data

against the protected data



we see very similar distributions. The user can verify that the univariate statistics are very similar: the raw data has a mean of 3878.9 against a mean of 3860.2 in the protected data.