SEAS Benchmark Problem BP2

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The problem set-up for this second benchmark is identical to that of the first benchmark problem (BP1) except for some changes in model parameters (D_{c} and t_{f}) and suggested cell sizes. All other parameters are unchanged. Please also note some changes in the sections related to data output: (1) locations for time series output are different from BP1 and (2) depth intervals for slip profile output depend on the cell size (as opposed to 500 m in BP1).

1 2D Problem Setup

The medium is assumed to be a homogeneous, isotropic, linear elastic half-space defined by

$$(x, y, z) \in (-\infty,\ \infty) \times (-\infty,\ \infty) \times (0,\ \infty),$$

with a free surface at $z = 0$ and z as positive downward. A vertical, strike-slip fault is embedded at $x = 0$, see Figure 1. We assume antiplane shear motion, letting $u = u(x, z, t)$ denote the displacement in the y-direction. Motion is governed by the equilibrium equation

$$0 = \frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{yz}}{\partial z}, \tag{1}$$

in the domain $(x, z) \in (-\infty, \infty) \times (0, \infty)$, and Hooke's law relates stresses to strains by

$$\sigma_{xy} = \mu \frac{\partial u}{\partial x}; \quad \sigma_{yz} = \mu \frac{\partial u}{\partial z} \tag{2}$$

for shear modulus μ.

2 Boundary and Interface Conditions

We supplement equations (1)–(2) with one boundary condition and two interface conditions. A free surface lies at $z = 0$, where all components of the traction vector equal 0. Mathematically, this condition is given by

$$\sigma_{yz}(x, 0, t) = 0. \tag{3}$$

At $x = 0$, the fault defines the interface. Superscripts "+" and "−" refer to the side of the fault with x positive, and x negative, respectively. We define slip by $\delta(z, t) = u(0^+, z, t) - u(0^-, z, t)$, i.e. the jump in displacement across the fault, with right-lateral motion yielding

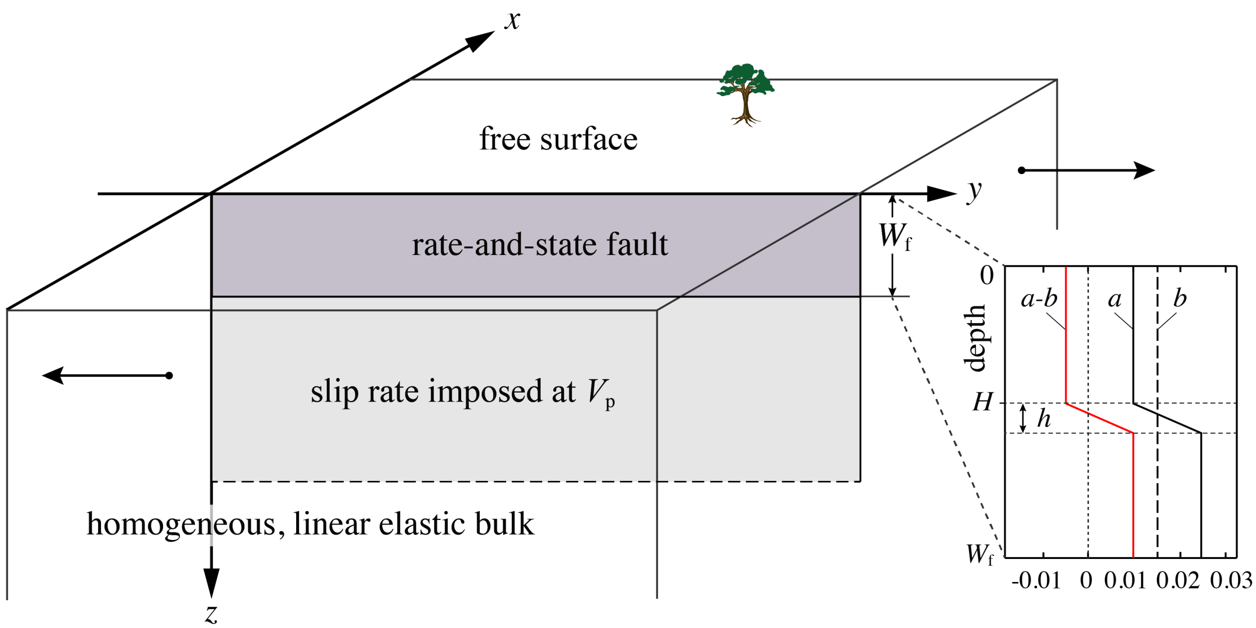

Figure 1: This second benchmark considers a planar fault embedded in a homogeneous, linear elastic half-space with a free surface. The fault is governed by rate-and-state friction down to the depth W_{f} and creeps at an imposed constant rate V_{p} down to the infinite depth. The simulations will include the nucleation, propagation, and arrest of earthquakes, and aseismic slip in the post- and inter-seismic periods.

positive values of δ. We require that components of the traction vector be equal and opposite, which reduces in antiplane shear to the condition

$$\sigma_{xy}(0^+, z, t) = \sigma_{xy}(0^-, z, t), \tag{4}$$

and denote the common value by τ^{qs} (shear stress due to quasi-static deformation). Note that positive values of τ^{qs} denotes stress that tends to cause right-lateral motion.

The second interface condition is depth dependent. Down to a depth of W_{f}, we impose rate-and-state friction, namely, that shear stress on the fault be equal to fault strength F, namely

$$\tau = F(V, \theta), \tag{5}$$

where $\tau = \tau^0 + \tau^{\mathrm{qs}} - \eta V$ is the sum of the prestress, the shear stress due to quasi-static deformation, and the radiation damping approximation to inertia, where $\eta = \mu/2c_{\mathrm{s}}$ is half the shear-wave impedance for shear wave speed $c_{\mathrm{s}} = \sqrt{\mu/\rho}$. The fault strength $F = \sigma_{\mathrm{n}} f(V, \theta)$, where $V = \frac{\partial u}{\partial t}(0^+, z, t) - \frac{\partial u}{\partial t}(0^-, z, t)$ is the slip rate, and θ is the state variable. σ_{n} is the effective normal stress on the fault. For this benchmark problem we assume σ_{n} is constant, given in Table 1. θ evolves according to the aging law

$$\frac{d\theta}{dt} = 1 - \frac{V\theta}{D_{\mathrm{c}}}, \tag{6}$$

where D_{c} is the critical slip distance. The friction coefficient f is given by a regularized formulation

$$f(V, \theta) = a \sinh^{-1}\left[\frac{V}{2V_0} \exp\left(\frac{f_0 + b\ln(V_0\theta/D_{\mathrm{c}})}{a}\right)\right] \tag{7}$$

for reference friction coefficient f_0, reference slip rate V_0, and rate-and-state parameters a and b. For this benchmark, b is constant as b_0 and a varies with depth (insert in Figure 1) as follows:

$$a(z) = \begin{cases} a_0, & 0 \le z < H \\ a_0 + (a_{\max} - a_0)(z - H)/h, & H \le z < H + h \\ a_{\max}, & H + h \le z < W_{\mathrm{f}} \end{cases} \tag{8}$$

Below depth W_{f}, the fault creeps at an imposed constant rate, given by the interface condition

$$V(z,t) = V_{\mathrm{p}}, \quad z \ge W_{\mathrm{f}}, \tag{9}$$

where V_{p} is the plate rate.

3 Initial Conditions, Simulation Time and Suggested Cell Size

Initial conditions on slip and the state variable are required. We consider that slip is initially zero everywhere in the domain, i.e.

$$\delta(z, 0) = 0. \tag{10}$$

The initial state on the fault is chosen so that the model can start with uniform slip rate and pre-stress at constant values V_{init} and τ^0, respectively. The pre-stress τ^0 corresponds to the steady-state stress with slip rate V_{init} at the depth of W_{f}, namely

$$\tau^0 = \sigma_{\mathrm{n}} a_{\max} \sinh^{-1}\left[\frac{V_{\mathrm{init}}}{2V_0}\exp\left(\frac{f_0 + b_0 \ln(V_0/V_{\mathrm{init}})}{a_{\max}}\right)\right] + \eta V_{\mathrm{init}}\,. \tag{11}$$

To be consistent with slip rate and pre-stress everywhere, the initial state is variable with depth and not necessarily at the steady state:

$$\theta(z, 0) = \frac{D_{\mathrm{c}}}{V_0}\exp\left\{\frac{a}{b}\ln\left[\frac{2V_0}{V_{\mathrm{init}}}\sinh\left(\frac{\tau^0 - \eta V_{\mathrm{init}}}{a\sigma_{\mathrm{n}}}\right)\right] - \frac{f_0}{b}\right\} \tag{12}$$

Equations (1)–(2), along with boundary condition (3), interface conditions (4)–(5) and (9), and initial conditions (10) and (12) are solved over the time period $0 \le t \le t_{\mathrm{f}}$, where t_{f} is a specified final simulation time. Table 1 specifies t_{f} = 1,200 years. Note that if this final time is not sufficiently long enough to exit the spin-up period, we suggest increasing t_{f} until at least 4 large and small events have occurred after the spin-up cycle. All necessary parameter values for this benchmark problem are given in Table 1.

With an eye towards future benchmarks in 3D (when computational efficiency demands a large cell size), we want to explore how resolution of critical physical scales affects model outcome. At a rupture speed of 0^+, the quasi-static process zone, Λ_0, is expressed as:

$$\Lambda_0 = C\frac{\mu D_{\mathrm{c}}}{b\sigma_{\mathrm{n}}}, \tag{13}$$

where C is a constant on the order of 1. Another important length scale, the nucleation zone size, h^*, is expressed as:

$$h^* = \frac{2}{\pi}\frac{\mu b D_{\mathrm{c}}}{(b-a)^2\sigma_{\mathrm{n}}}. \tag{14}$$

With the provided model values, the process zone Λ_0 and h^* are nearly uniform within the VW region, with a size of ≈ 170 m and ≈ 1 km, respectively.

We suggest trying seven cell sizes, $\Delta z = 25$ m, 50 m, 100 m, 200 m, 300 m, 400 m and 800 m, for the simulations. These sizes correspond to three cases resolving Λ_0 with approximately 6, 3 and 1.7 grid points and four cases that do not resolve Λ_0. These cases resolve h^* with approximately 40, 20, 10, 5, 3, 2 and 1 grid points.

For methods that use multiple degrees of freedom along cell edges/faces, please take Δz = edge length / number of unique degrees of freedom. For instance, for a high-order finite element method, if h is the edge length and N the polynomial order then $\Delta z = h/N$.

We expect that some larger cell sizes may lead to numerical instability, unrealistically small time steps, or other issues that make simulations unfeasible. Please submit results only from successful simulations and report to us about when and how the simulations fail.

Table 1: Parameter values used in the benchmark problem

Parameter	Definition	Value, Units
ρ	density	2670 kg/m^3
c_s	shear wave speed	3.464 km/s
σ_n	effective normal stress on fault	50 MPa
a_0	rate-and-state parameter	0.010
a_{max}	rate-and-state parameter	0.025
b_0	rate-and-state parameter	0.015
D_c	critical slip distance	0.004 m
V_p	plate rate	10^{-9} m/s
V_{init}	initial slip rate	10^{-9} m/s
V_0	reference slip rate	10^{-6} m/s
f_0	reference friction coefficient	0.6
H	depth extent of uniform VW region	15 km
h	width of VW-VS transition zone	3 km
W_f	width of rate-and-state fault	40 km
Δz	suggested cell sizes	25 m, 50 m, 100 m, 200 m, 300 m, 400 m, 800 m
t_f	final simulation time	$1,200$ years

4 On-fault Time Series Output

Files are uploaded to the SCEC code validation web server at this address:

```
http://scecdata.usc.edu/cvws/cgi-bin/seas.cgi
```

You need to upload on-fault ($x = 0$) time series files, which give slip δ, base 10 log of the slip rate V, base 10 log of the state variable (i.e. $\log_{10}(\theta)$), and shear stress τ, for each on-fault station at representative time steps. We define the simulation periods as either aseismic (when $\max(V) < 10^{-3}$ m/s, where $\max(V)$ is the maximum slip rate over the entire fault) or seismic (when $\max(V) \geq 10^{-3}$ m/s). When outputting modeling results, use larger time intervals (e.g., ∼0.1 yr) during aseismic periods and smaller time intervals (e.g., ∼0.1 s) during seismic periods. More variable time steps are OK. Please keep the total number of time steps in the data file on the order of 10^4–10^5.

Time series data is supplied as ASCII files, one file for each station. There are 12 stations in total, as follows:

`fltst_dp000`: $z = 0.0$ km (at the free surface)
`fltst_dp024`: $z = 2.4$ km
`fltst_dp048`: $z = 4.8$ km
`fltst_dp072`: $z = 7.2$ km
`fltst_dp096`: $z = 9.6$ km
`fltst_dp120`: $z = 12.0$ km
`fltst_dp144`: $z = 14.4$ km
`fltst_dp168`: $z = 16.8$ km
`fltst_dp192`: $z = 19.2$ km
`fltst_dp240`: $z = 24.0$ km
`fltst_dp288`: $z = 28.8$ km
`fltst_dp360`: $z = 36.0$ km

Each time series has 5 data fields, as follows.

Field Name	Description, Units and Sign Convention
`t`	Time (s)
`slip`	Out-of-plane slip (m). Positive for right-lateral motion.
`slip_rate`	$\log_{10}$ of the out-of-plane slip-rate ($\log_{10}$ m/s). Positive for right-lateral motion.
`shear_stress`	Shear stress (MPa). Positive for shear stress that tends to cause right-lateral motion.
`state`	$\log_{10}$ of state variable ($\log_{10}$ s).

The on-fault time series file consists of three sections, as follows:

File Section	Description
File Header	A series of lines, each beginning with a # symbol, that give the following information: • Benchmark problem (No.1) • Code name • Code version (optional) • Modeler • Date • Node spacing or element size • Station location • Minimum time step (optional) • Maximum time step (optional) • Number of time steps in file (optional) • Anything else you think is relevant (optional) • Descriptions of data columns (5 lines)
Field List	A single line, which lists the names of the 5 data fields, in column order, separated by spaces. It should be: `t slip slip_rate shear_stress state` (all on one line). The server examines this line to check that your file contains the correct data fields.
Time History	A series of lines. Each line contains 5 numbers, which give the data values for a single time step. The lines must appear in order of increasing time. **Make sure to use double-precision when saving all fields**. **C/C++ users:** We recommend using 21.13E or 21.13e floating-point format for the time field and 14.6E or 14.6e format for all other data fields. **Fortran users:** We recommend using E22.14 or 1PE22.13 floating-point format for the time field and E15.7 or 1PE15.6 format for other data fields. The server accepts most common numeric formats. If the server cannot understand your file, you will see an error message when you attempt to upload the file.

Here is an example of an on-fault time-series file, with invented data.

```
# This is the file header:
# problem=SEAS Benchmark No.2
# code=MYcode
# version=1.0
# modeler=A.Modeler
# date=2018/10/01
# element_size=35 m
# location= on fault, 7.2km depth
# minimum_time_step=0.1
# maximum_time_step=3.157e6
# num_time_steps=2400
# Column #1 = Time (s)
# Column #2 = Slip (m)
# Column #3 = Slip rate (log10 m/s)
# Column #4 = Shear stress (MPa)
# Column #5 = State (log10 s)
#
```

```
# The line below lists the names of the data fields
t slip slip_rate shear_stress state
# Here is the time-series data.
0.000000E+00 0.000000E+00 -9.000000E+00 3.000000E+01 7.000000E+00 ...
5.000000E-02 0.000000E+00 -9.000000E+00 3.000000E+01 7.000000E+00 ...
1.000000E-01 0.000000E+00 -9.000000E+00 3.000000E+01 7.000000E+00 ...
1.500000E-01 0.000000E+00 -9.000000E+00 3.000000E+01 7.000000E+00 ...
# ...  and so on.
```

5 Slip Evolution Output

The slip evolution output file (with a name `devol.dat`) is a single ASCII file that records the spatial distribution of slip on a subset of fault nodes at representative time steps during the aseismic and seismic phases of the simulation. Data can be saved using representative time intervals of approximately 5 yr and 1 s for results in aseismic and seismic phases, respectively, or with variable time steps. Either way, data will be interpolated to plot slip every 5 yr during the aseismic phase, and every 1 s during the coseismic phase. The data should include a subset of nodes (e.g., with a spacing of ∼100 m, or Δz if $\Delta z > 100$ m) down to a depth of 40 km (the width of rate-and-state frictional fault, W_{f}). The file also contains the time series of maximum slip rates (taken over the entire fault), so that we can precisely differentiate aseismic and seismic phases.

NOTE: Please submit this data via email to Brittany Erickson (bae@uoregon.edu). Please ensure that the file is ∼10s of MBs or less.

The data file has 4 data fields, as follows:

Field Name	Description, Units and Sign Convention
`z`	Depth (m) at increments of approximately $\max(100\text{ m}, \Delta z)$, down to 40 km. Positive for all depths.
`t`	Time (s). Nonuniform time steps.
`max_slip_rate`	The $\log_{10}$ of maximum out-of-plane slip-rate (taken over the entire fault) ($\log_{10}$ m/s). Positive for right-lateral motion.
`slip`	Out-of-plane slip (m). Positive for right-lateral motion.

The data output consists of three sections, as follows:

File Section	Description
File Header	A series of lines, each beginning with a # symbol, that give the following information: • Benchmark problem (No.2) • Modeler • Date • Code • Code version (if desired) • Node spacing or element size • Descriptions of data fields (3 lines) • Anything else you think is relevant (e.g. computational domain size)
Field List	Two lines. The first line lists z. The second lists the names of the 3 other data fields on one line, separated by spaces. It should be: `z` `t max_slip_rate slip` (last three fields on one line).
Slip History	A series of lines that form a 2-dimensional array of rows and columns. The first row/line lists the numbers 0, 0 (to maintain a consistent array size), followed by the spatial nodes with increasing depth as you go across the row. Starting from the second row/line, each row/line contains time, maximum slip rate, and slip of all nodes at the time. These lines appear in order of increasing time (from top to bottom) and slip is recorded with increasing depth (from left to right). **Make sure to use double-precision when saving all fields**. **C/C++ users:** We recommend using 21.13E or 21.13e floating-point format for the time field and 14.6E or 14.6e format for all other data fields. **Fortran users:** We recommend using E22.14 or 1PE22.13 floating-point format for the time field and E15.7 or 1PE15.6 format for other data fields.

Note that z should appear in the first row, preceded by two zero numbers, for nodes with a spacing of approximately 100 m down to a depth of 40 km (width of rate-and-state frictional fault). Time and maximum slip rate should appear as two single columns that start on the second row, with time increasing as you go down. Slip history (the remaining block) is represented by a two-dimensional array with time increasing as you go down the rows/lines, and z increasing as you go across the columns (approximately 401 columns). The entire output array should be of size ($N_t + 1, \sim 403$), where N_t is the total number of time steps. This means that you output slip at selected nodes at one time step and move on to the next time step. (To keep the file on the order of 10s of MB, N_t should be on the order of 10,000).

The two-dimensional array should therefore be of the form: $\begin{bmatrix} 0 & 0 & z \\ T & \max(V) & \text{slip} \end{bmatrix}$

Here is an example of a slip-evolution file, with invented data.

```
# This is the file header:
# problem=SEAS Benchmark No.2
# modeler=A.Modeler
# date=2018/10/01
# code=MyCode
# code_version=3.7
```

```
# element_size=35 m
# Row #1 = Depth (m) with two zeros first
# Column #1 = Time (s)
# Column #2 = Max slip rate (log10 m/s)
# Columns #3-83 = Slip (m)
# Computational domain size:  depth 100 km, distance off fault 100 km
# The line below lists the names of the data fields
z
t max_slip_rate slip
# Here are the data
0.000000E+00  0.000000E+00 0.000000E+00 1.000000E+02 ...  4.000000E+04
0.000000E+00 -9.000000E+00 0.000000E+00 0.000000E+00 ...  0.000000E+00
3.140000E+05 -9.000000E+00 1.340000E-05 1.340000E-05 ...  3.140000E-05
1.227000E+07 -9.000000E+00 1.560000E-05 1.560000E-05 ...  1.220000E-02
4.690000E+07 -9.000000E+00 1.580000E-05 1.580000E-05 ...  4.680000E-02
...
9.467078E+10 -4.500000E-01 9.050000E+01 9.050000E+01 ...  9.461000E+01
```

6 Using the Web Server

The web server lets you upload your modeling results (section 4). Once uploaded, you and other modelers can view the data in various ways.

6.1 Logging in and Selecting a Problem

To log in, start your web browser and go to the home page at:

`http://scecdata.usc.edu/cvws/cgi-bin/seas.cgi`

Click on "Upload Files," and then log in using your user name and password. Remember that passwords are case-sensitive. You are then presented with a list of code validation problems. Find the problem you want, and click the "Select" button. You will then see a list of files for the problem.

6.2 Navigating the Site

You navigate through the site by clicking buttons on the web pages. Avoid using your browser's Back or Forward buttons. If you use the Back or Forward buttons, you may get error messages from your browser.

6.3 Uploading Files

To upload a file, do the following steps:

- Find the file you want to upload, and click "Select." The server displays a page for you to upload the file.
- Select the data file on your computer. The exact method for file selection varies depending on operating system and web browser.
- Click on "Click Once to Upload." The file you selected is uploaded to the server.

When you upload a file, the web server immediately checks for correct formatting. There are three possible results:

- If the file is correctly formatted, the server displays a page noting the successful upload.
- If the file contains errors, the server displays an error log. The error log lists the errors that were detected in the file, each identified as specifically as possible.
- If the file is correctly formatted, but is questionable in some way (for example, a missing time step), then the server displays a warning log, which describes the problem.

When uploading time series files, the website may issue a warning that the time series cannot be filtered. Modelers should ignore this warning. After uploading a file, the file list shows the date and time that you uploaded the file. Remember that any file you upload will be visible to anyone who has access to the web site.

Additional help is available by clicking the "Help" link in the upper right corner of the webpage. Modelers who want to upload multiple versions of the benchmark (for example, using different element sizes), can do so using the "Change Version" feature of the website, which is described in the help screens. Direct further questions to Michael Barall.

6.4 Graphing, Viewing, and Deleting Files

After uploading a file, additional functions become available. These functions let you graph, view, or delete the uploaded file.

Graphing: To graph a file, find the file you want and click "Graph." For a time-series file, the server displays graphs of all the data fields in the file. At the bottom of each graph page, there is a box you can use to adjust graphing preferences. Graphing a file is a good way to check that the server is interpreting your data as you intended.

Viewing: To view the text of a file, find the file you want and click "View."

Deleting: To delete a file from the server, find the file you want and click "Delete." The server displays a page asking you to confirm the deletion.

7 Benchmark Tips

Numerical boundary conditions (to truncate the half-space when defining the computational domain) will most likely change results at least quantitatively, or even qualitatively. We suggest extending these boundaries until you see results appear independent of the computational domain size. We ask participants to use the cell-sizes specified in Table 1. Each person can submit (at most) results from seven different spatial resolutions and two different computational domain sizes.

As a sanity check for the model results, the total simulation time of 1,200 years would consist of about 10 large and 11 small earthquakes that nucleate at a depth of $\sim$15 km and arrest at a depth of $\sim$18 km. The large events occur every $\sim$90 years (with a small event nucleating $\sim$25 years after the large), with surface slip of $\sim$3 m and max surface slip rate on the order of $\sim$1 m/s (for the large event).