

Special Event/Book Review

A Review of *Critical Mathematics Education* by Ole Skovsmose

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Ole Skovsmose (2023) opens this, his *magnum opus*, with the image of walking around a city. He provides a moving (sic) story of his early visit to São Paulo thirty years ago, and the colourful varieties of street life he saw and experienced. He fell in love with the city (and with his wife Miriam) and chose to settle and make his life there. He likens *critical mathematics education* to a city which can be traversed by many routes from different starting points and directions. The power of this simile is that it is organic and growing, not a systematic and rationally planned production. Ole Skovsmose not only applies this model to critical mathematics education but also to mathematics itself. Mathematics is defined by its practices, not some rational reconstruction or map.

Napoleon wanted to impose rationality on his capital, so the streets of the centre of Paris were pulled down and a new rational model was designed by Haussmann, with the main thoroughfares emanating radially from the Arc de Triomphe. But like the creepers overgrowing an abandoned temple in the jungle, organic growth expanded and spread over the capital, infilling the central streets and spreading the roots of the conurbation way out to the banlieues.

Likewise, the plans and maps of mathematics that tried to impose the rational order of logic (logicism), formal systems (formalism), intuitive reconstruction (intuitionism), or even the celestial city (Platonism) all failed to capture the buzz and colour of the city mathematical. Instead, Ole Skovsmose subscribes to naturalism; mathematics is what mathematical practice does. The meaning, philosophy and structure of mathematics are all given by this practice, nothing more, nothing less. This idea of meaning as practice, meaning as use takes us to Wittgenstein, who uses the city as an image of the language that his later philosophy investigates and clarifies.

Our language can be seen as an ancient city: a maze of little streets and squares, of old and new houses, and of houses with additions from various periods; and this surrounded by a multitude of new boroughs with straight regular streets and uniform houses. (Wittgenstein, 1953, para. 18)

Wittgenstein was influenced by St. Augustine but in his maturity he rejected the idea of the perfect City of God, and substituted instead the city as an image of humankind (Stewart, 1970). Mathematics like language is embodied in social practices, from the academy and computer App

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programmers to the shoppers and street mathematics on the thoroughfares and favelas of São Paulo.

Now the question is, what is the first path, the first street, the first route that Ole takes as he walks through the city of the philosophy of critical mathematics education? What is his outlook, his guiding concept or perspective? The answer is what any journey needs to begin with, namely hope. Hope is not something that is much talked about in mathematics education research. But hope is something every student of mathematics needs. Neither is hope something much treated in philosophy. As Ole tells us, it is denigrated by Descartes and Nietzsche but valued by Kant, Kierkegaard and Bloch. Although Ernst Bloch is the least well known of these philosophers, he published his monumental, three volume treatise *The Principle of Hope* in the 1950s. To Bloch, hope is formed through political visions and aspirations, it concerns socio-political utopias, which do not yet exist, but are dreams of optimistically possible futures. Hope is a city where every dwelling is dignified, where all can live and travel in comfort and joy, and where turning any corner brings a new vista of beauty and peace. Stepping back into our world, and closer to critical education Paulo Freire (2014) also offers a *Pedagogy of Hope*, where he claims that hope is an ontological need.

Thus, the positive principle or grounding value of critical mathematics education is that of hope. Hope concerns not only future socio-political utopias—that is an improved, fairer, and better society—but also personal dreams of improved opportunity and a better life. Hope can be expressed through Ole Skovsmose's concept of a student's *foreground*: meaning the possibilities available for that student, depending on their socio-cultural location. This concept of foreground provides a scaffolding of hope, taking it from what might be a nebulous aspiration to a concrete range of possibilities of growth and achievement. It aims to open paths for the downtrodden, deprived, or discriminated against that leads them into arenas of personal and political agency and empowerment.

As this corollary to the positive principle of hope implies, the negative principle of critical mathematics education is the recognition of obstacles to social improvement and personal betterment. This awareness includes a commitment to challenging these impediments and overcoming them. Critical mathematics education opposes social injustice. It is therefore necessary to identify and challenge causes and cases of social injustice, whether it be the exploitation of workers, structural racism, the erosion of democracy, or contributing to climate change. The commitment to social and environmental justice means that these obstacles must be attacked and overcome.

However, this negative aspect of critique is turned into a positive through the concept of *landscapes of investigation*. These provide learning contexts different from those of the school mathematics tradition. Instead of repetitive exercises, landscapes of investigation are real-world social and environmental problem areas. These could be the exploitation of workers, patterns of structural racism, erosions of democracy, and climate change. Here, students and teachers work cooperatively to identify and challenge cases of social injustice. These are not 'hot' or politicised issues imposed on the students, but jointly-identified problem areas analysed together. Anyone who doubts that school-aged children can identify and analyse issues of social or environmental injustice has only to recall the cases of Greta Thunberg and Malala Yousafzai who have inspired millions of young people through their activism.

Ole Skovsmose offers several examples of landscapes of investigation, and these are explored in some depth. One is that of racism, which is analysed from the perspective of representation and the mathematical analysis of proportionalities. A second example is the climate emergency and investigating mathematical models of the climate. A third concerns water usage and its environmental implications. A fourth landscape is called the erosions of democracy. The

explanation of democracy given in the text involves exploring systems of voting, fairness among groups of people, social equity, and deliberation in the sense of discussion, reflection, and consideration. Each of these dimensions is illuminated through mathematical analysis. But this last one, democracy, is not just one of the landscapes of investigation. It is also included both as a process and as a classroom practice involving the students in decisions about the content, organization, and the directions of inquiry in their mathematical learning. This represents one way in which dialogical processes and relationships are central to this exposition of critical mathematics education.

One of the many innovations of the text is to bring the idea of *risk* into critical mathematics education. Mathematical modelling works by creating a set of equations or theory that creates a model of selected aspects of the material world; a ‘mathematics-based creation of a parallel world.’ The use of such a model requires boundary parameters and assumptions on which the model rests. Such a basis comes with assumptions about the risks expressed as probabilities of negative (and positive) outcomes. For example, the process of estimating the risk of nuclear accidents as 1 in 10,000 is analysed and questioned in the text. Such simple risks can be expressed in a single assumption. However, mathematical models incorporate a whole complex of interrelated risks. They are used as maps that format our understandings of the world. Beyond this, they also format the kinds of responses we can imagine for dealing with real-world problems and crises such as the climate emergency. There are major risks in limiting possible solutions for problems to those deemed imaginable within a necessarily restricted (and inaccurate) model. *Formatting* in this important sense is a concept introduced by Ole Skovsmose several decades ago and in its present, elaborated form is a key underpinning of the critical mathematics education critique of modelling and the impact of the growing mathematization of society.

As befits a book that aims to offer a philosophy of critical mathematics education, the key concepts considered are analysed philosophically. Thus, the teaching and learning of mathematics are not taken for granted as primitive notions. The analysis provided starts at a more fundamental level, looking at how, in a social setting, learning, action, intention, foreground, motive, mathematics, and lifeworld are all interrelated. Many traditional positions in the philosophy and psychology of education take learning for granted, as a social necessity both in and out of school. The reasons why perhaps the teaching and learning of mathematics are beneficial both to society and individuals are explored also. But in this text, this is not a point of view that is taken for granted. Here student intentions and motives are made the central focus. How does mathematics figure in their lifeworld and in their foreground, the imagined landscape of future possibilities and life trajectories? How do learners see—and *can they* see—mathematics as folded into their future actions and lives? What makes this a philosophical inquiry into *critical* mathematics education is a concern with how the teaching and learning of mathematics is failing important groups of learners. How and why are we creating low achievers in mathematics, reluctant learners, students not interested in addressing apparently relevant real-life problems? How are low achievement in mathematics and students’ motives for learning mathematics formatted by social, political, and economic factors?

The account in this book sees learning as action. Rather than viewing learning as something the learner is subject to, learning is something actively performed by the learner. This perspective fully accommodates student agency. Students might want to participate in the learning process, but they might also have reasons for withdrawing from it. The account goes on to relate action and intention, for intentionality is crucial for interpreting human actions. Any human action, including the activation of consciousness itself is directed towards something. This is an important point, one that is shared with Activity Theory. In this theory all activities have an objective, a goal.

By seeing learning as action, any learning activity is intentional and goal-directed. Here Skovsmose uses his concept of foreground, the possibilities and futures that are available for a particular student. The probabilities of fulfilling intentions and attaining goals, especially long-term ones, partly depend on the learner's foreground. Very different foregrounds are spread out in front of a student, depending on their social location, including economic, political, culture, religious, as well as discursive factors.

Foregrounds provide the environment for the formation of intentionalities. They influence what possibilities, risks, and dangers we might perceive or ignore. A foreground might be a horizon of hope, appearing rich with possibilities, but it might also appear devoid of them. In exploring motives, the question is raised as to how we might locate students' motives for learning mathematics? The proposal is to look into students' foregrounds, to explore how mathematics might appear in these foregrounds. For some, mathematics is seen as a necessary qualification for future career prospects, an avenue for exploring learner group interests, a set of capabilities for economic and democratic participation (both 'reading' and 'writing the world,' after Gutstein, 2006), and so on. Reading and writing in this sense are the critical understanding and action on the world that is so central to the mission of critical mathematics education.

There are many motives for learning mathematics including fascination with the subject. But there is also an inverse phenomenon that needs to be considered. This is mathematics refusal, or auto-exclusions from mathematics. Some students express a fear of mathematics and consequently abandon it.

If we draw on the phenomenologists' idea of a *lifeworld* as representing our immediate daily life experiences, how can we locate mathematics in our lifeworlds? Ole Skovsmose observes that answering this apparently straightforward question has turned out to be quite complicated due to two interconnected processes: mathematisation and demathematisation. The *mathematisation* of the world means that much of our experience of the world, including mobile phone and computer Apps, shopping and travelling by public transport, and so on, depends on the complex mathematical algorithms and software that structures and performs such functions for us. The *demathematisation* is the process whereby this mathematisation is rendered invisible and does not appear to be part of daily life practices. These two interconnected processes lead to a paradox. Modern life in virtually all of its organised aspects depends deeply on the widespread and near universal mathematisation of social functions based on a myriad of interconnected algorithms. However, the human interface of this underpinning software is increasingly hidden, so that mathematics appears less frequently in our lifeworld and experience. An outcome is the belief that 'mathematics is everywhere' is being replaced by it appearing to be 'nowhere.' Most students do not see mathematics as something important in their lives, and for students in marginalised positions, mathematics only appears in an opaque format in their foregrounds. This is due to both to processes of demathematisation, but also because their daily life problems are so urgent that their foregrounds can be ruined.

Could learning mathematics change structural features of foregrounds and lifeworlds? Changing social structures is a political act. Can learning change social structures? Can learning mathematics form or reform foregrounds and lifeworlds? What is being offered in this book is an explicit example of turning the learning of mathematics into a political act. Through making this observation, the conceptual circle that Ole Skovsmose set out to explore has been completed. Learning mathematics can change features of students' lifeworlds, and also the lifeworlds of broader communities. In this sense it is a political action. Learning mathematics is not just a classroom activity. It is a way of being, and a way of being political.

As this sampling of themes shows, this book introduces and develops many new concepts and analyses. It draws on many of the most significant philosophers of the past centuries, philosophers that do not usually appear in mathematics education research. But the point of this is to draw together the most powerful ideas to weld together in constructing a philosophy of critical mathematics education. This is, first of all, a deep theory of mathematics education, taking very seriously the contexts, crises, and conversations involved. It foregrounds the roles of power, positioning and pedagogy in critiquing the teaching and learning of mathematics and in rolling out a more just education.

But secondly it also provides a new philosophy of mathematics. It fills a giant hole in contemporary philosophy of mathematics. It offers a philosophy of applied mathematics, something that traditional philosophers have overlooked or turned away from. This neglect is in part this due to the ideology of *purism* that values pure over applied mathematics, and that of *absolutism*, which locates the objects and truths of mathematics in some superhuman other world. This leads to a philosophical attitude that turns away from practical philosophies and philosophies of practice.

Focussing on a philosophy of applied as opposed to pure mathematics is not a reduction but an expansion of ambition. A philosophy of applied mathematics not only treats the ontological and epistemological issues that a traditional philosophy of mathematics does. Nor does it expand its goals by merely including the themes of a philosophy of mathematical practice. Ole Skovsmose's philosophy is much more ambitious, for it includes the nature, possibilities, and roles of the applications of mathematics throughout citizens' lifeworlds, throughout society and across the world we live in.

The critical philosophy of mathematics offered here is so important because of the power of mathematics in learners' and all citizens' lives and throughout society. This is revealed through the analyses of the power of mathematics to format peoples' lives and society that are presented. Naturally, as befits a philosophy of applied mathematics, the ethics of mathematics is explored. This is something else traditional philosophies of mathematics have also shied away from. But ethics is an inescapable dimension of a philosophy of applied mathematics. Ole Skovsmose fully grasps the nettle that mathematics in society and education is performative; constitutive of how we see social functioning and crises. Mathematics is an engine, not a camera to use MacKenzie's (2008) metaphor. In this case how can we erase ethics from our discussions?

The book ends as it begins, with an enlightening personal tale. This includes the history of Ole Skovsmose's ideas and their development, seen in the context of others' more partial philosophies of critical mathematics education. It also recounts his experiences away from the comforts of Europe, in Brazil and South Africa. Here, he saw injustices and inequalities beyond anything we know in Western Europe, both in terms of criticality and mathematics education. Here he saw the hopes of the deprived etched in powerful relief on the faces of the young. But this is not a book of liberation theology. It is a philosophical manual of critical mathematics education as it is needed everywhere, among the rich and poor, the developed and the developing. It offers the critical means by which democracy can be safeguarded everywhere, through a well-armed critical citizenry, with all the power that a mathematical education can confer. It constitutes a philosophy for the technological crisis of the Anthropocene. Only if we take a philosophy like this seriously will we and our children and their planet have a chance of survival.

I have been privileged to share some of Ole Skovsmose's journey with him. I admired his first English language publications on critical mathematics education, and we have been in touch for almost forty years. I was fortunate to share the stage with him at ICME in Quebec, 1992, and also his important trip to South Africa in 1993. Ole and his students helped me by working through

the drafts of my book on social constructivism as a philosophy of mathematics (Ernest, 1998). His ideas have sparked mine and I flatter myself that I have had some small influence on his thinking. I have been honoured with his support for the *Philosophy of Mathematics Education Journal*. I have watched the whole nexus of his vital ideas and contributions flower and come together in this, his *magnum opus*. His enterprise is bigger than any one of us and I am delighted to be in the position to introduce, via review, this very important, special and definitive philosophy of critical mathematics education.

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