Just 131 Angels can Dance on the Head of a Pin!

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A new nonlinear Schrödinger equation is derived, which describes a cluster of bosons interacting via the strongest force. Its numerical solution points that the cluster is stable only if the number of particles is exactly 131. The relevant kinetic and potential energies of the particles are also calculated and interrelated via the virial theorem. The developed concept is applied to electrostatic forces as well.

How many angels can dance on the head of a pin had been a central problem for Medieval philosophy. If God is all-powerful, he should be able to bid an infinite number of angels anywhere. On the other hand, no infinite collection could appear to a human being. This paradox sheds light on human inability to process complexity, multiplicity and collapsed orders [1], which seems important nowadays to artificial intelligence as well. In physics one might be interested analogously in how many particles can occupy a given point in space. The prompt answer of classical physics is 1 as far as any classical particle possesses its own size. Quantum mechanics, however, dramatically changed the notion of particles. The orthodox interpretation, which dominates contemporary physics, principally exposes the crucial role of the observer. Generally, quantum particles are divided into two classes: fermions and bosons. Two fermions are restricted to occupying a single state unless they have opposite spin, i.e. the antient puzzle is naturally resolved to 2 angels, if they are fermions. On the other hand, bosons can undergo the Bose-Einstein condensation and it seems realistic to have many of them present at a single point. In any case, however, a finite number is expected, e.g. precisely 16 angels according to some wizard books [2]. The scope of the present paper is to prove rigorously the exact quantum number of 131 bosons.

Let us consider a cluster of N identical bosons, each with mass m. To keep as many particles as possible together, it requires the most powerful self-attraction, which is the fundamental strong force. The corresponding Yukawa potential φ obeys the dimensionless screened Poison equation [3]

$$\nabla^2\varphi = \varphi - \psi^2 \tag{1}$$

where ψ^2 is the local density of the self-interacting bosons. The screening length $\hbar/\mu c$ equals to the de Broglie wavelength of the neutral pi meson with mass μ, which is transmitting the strong interaction. Looking at the structure of Eq. (1) one can heuristically recognize that the wavefunction ψ coincides with the interaction potential φ. The ansatz $\psi = \varphi$ recalls to the de Broglie pilot wave interpretation of the wavefunction as a potential field guiding the quantum particles [4]. Expressing interaction via wavefunction is typical for quantum field theory [5], where Eq. (1) represents the relativistic Schrödinger equation for a neutral pi meson with φ being its wavefunction. Therefore, it can be rewritten as a dimensionless Schrödinger equation for the bosons:

$$\nabla^2\psi + \varphi\psi = \psi \tag{2}$$

The corresponding binding energy per particle equals to $\varepsilon = -(\mu c)^2/2m$, which is obviously due to momentum conservation. Using the typical value of $\mu = 140$ MeV/c^2 yields $\varepsilon = -10$ MeV for a particle with mass $m = 7\mu$, which matches to a neutron, for instance. The order of this energy correlates well with the nuclear binding energies of heavy atoms. As far as nucleons are fermions, a possible explanation of the coincidence is the interacting boson model, where dimer nuclear bosons form like Cooper's pairs in super conductivity [6, 7].

Combining Eqs. (1) and (2) together, the boson cluster wavefunction $\psi = \varphi$ will obey the following nonlinear Schrödinger equation, which is not present in the scientific literature,

$$\nabla^2\psi + \psi^2 = \psi \tag{3}$$

It is accomplished by the trivial boundary conditions $\nabla\psi(0) = 0$ at center and $\psi(\infty) = 0$ far from the cluster. Unfortunately, finding the analytical solution of Eq. (3) is mathematically not possible in 3D. However, its numerical 3D radial solution, as well as the 1D analytical one, are plotted in Fig. 1. Because $\varepsilon < 0$ the obtained peaks correspond to deep potential wells. Now it is possible to answer the main question of the Medieval thinkers, since the number of bosons in the cluster can be calculated from the wavefunction normalization $\int \psi^2 dV = N$. The obtained angel number 131 is a symbol of new beginnings and opportunities in numerology. It is close to the highest magic number 126 in nuclear physics and perhaps neutron stars are also micro-structured in such stable clusters. The simultaneous knowledge of the wavefunction ψ and potential φ allows calculation of the kinetic $\varepsilon_k = -\varepsilon$ and potential $\varepsilon_p = 2\varepsilon$ energies, respectively. Thus, the virial theorem $2\varepsilon_k = -\varepsilon_p$ confirms the reciprocal decay of the Yukawa interaction potential in 3D. To elucidate the obtained results one can analytically solve Eq. (3) in 1D space and the solution $\psi = 3/2\cosh^2(x/2)$ is also plotted in Fig.

1. The corresponding number of bosons is 6, which represents the 'diameter' of the 3D cluster. The relevant kinetic $\varepsilon_k = -\varepsilon/5$ and potential $\varepsilon_p = 6\varepsilon/5$ energies are naturally lower. The corresponding virial theorem $2\varepsilon_k = -\varepsilon_p/3$ reflects the weaker potential decay in 1D as shown in Fig. 1:

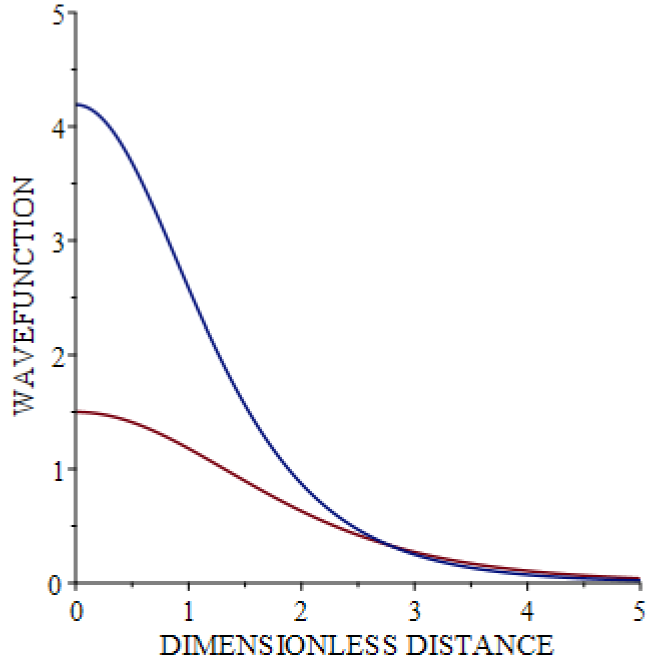

Fig. 1 The wavefunction ψ from Eq. (3): 3D solution (blue line) and 1D solution (red line)

It would be interesting whether the developed concept is applicable to electrostatic interactions as well. In this case the plasma contains particles with positive and negative charges, which can be considered as gender of the angels. The corresponding partial wavefunctions $\psi_\pm$ obey the dimensionless Schrödinger equation, in general,

$$\nabla^2\psi_\pm \pm \phi\psi_\pm = \psi_\pm \tag{4}$$

where ϕ is the dimensionless electrostatic potential of the cluster. The new characteristic length here is naturally proportional to the Bohr radius $a_0 = 4\pi\varepsilon_0\hbar^2/me^2$. Following the de Broglie pilot wave theory, the potential can be expressed as the difference $\phi = \psi_+ - \psi_-$. Thus, one arrives from Eq. (4) to the screened Poisson equation again, where the last two terms represent the local charge density,

$$\nabla^2\phi = \phi + \psi_-^2 - \psi_+^2 \tag{5}$$

This equation can be easily rewritten in the form $\psi_+ + \psi_- = 1 - \nabla^2\phi/\phi$, where the last term resembles the Bohm quantum potential. Therefore, the partial wavefunctions equal to

$$\psi_\pm = (1 - \nabla^2\phi/\phi \pm \phi)/2 \tag{6}$$

Substituting Eq. (6) in the Schrödinger equation (4) yields the Poison equation for the electric potential in the closed mean-field form:

$$\nabla^2\phi = \phi - \phi^3 + \phi\nabla^2(\nabla^2\phi/\phi) \tag{7}$$

The analysis of this equation shows that the corresponding clusters must be hollow, which is typical for clusters of self-interacting quantum particles [8]. If the quantum potential is nearly constant, the last quadrupolar term can be neglected and Eq. (7) reduces to the Gross-Pitaevskii equation [9, 10]

$$\nabla^2\phi + \phi^3 = \phi \tag{8}$$

The 1D analytical solution of the latter reads $\phi = \sqrt{2}/\cosh(x)$. Since in this approximation the partial wavefunctions are $\psi_\pm = \pm\phi/2$, the stable 1D cluster consists of 1 positive and 1 negative particles as a family. This is not surprising, however, because the electrostatic interaction is 137 times weaker than the strong force. The virial theorem follows again $2\varepsilon_k = -\varepsilon_p$.

In conclusion, the de Broglie interpretation of the wavefunction is revived, which leads naturally to nonlinear Schrödinger equations typical for quantum field theory. It is applied to a cluster of bosons, interacting via strong force, which appears to be stable only if the number of bosons is exactly 131 in 3D. The corresponding kinetic and potential energies of the particles are reasonable and interrelated via the virial theorem. The application of the developed concept to quantum electrostatics results in nonlinear Poison equation, which reduces approximately to the Gross-Pitaevskii equation. Furthermore, the partial wavefunctions of the oppositely charged particles are also derived. These results point to a new promising pilot wave reinterpretation.

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