

# Design of Adaptive Sliding Mode Controller for Robotic Manipulators Tracking Control

T. C. Kuo, Y. J. Huang, and B. W. Hong

**Abstract**—This paper proposes an adaptive sliding mode controller which combines adaptive control and sliding mode control to control a nonlinear robotic manipulator with uncertain parameters. We use an adaptive algorithm based on the concept of sliding mode control to alleviate the chattering phenomenon of control input. Adaptive laws are developed to obtain the gain of switching input and the boundary layer parameters. The stability and convergence of the robotic manipulator control system are guaranteed by applying the Lyapunov theorem. Simulation results demonstrate that the chattering of control input can be alleviated effectively. The proposed controller scheme can assure robustness against a large class of uncertainties and achieve good trajectory tracking performance.

**Keywords**—Robotic manipulators, sliding mode control, adaptive law, Lyapunov theorem, robustness.

## I. INTRODUCTION

A large number of robotic manipulators have been designed over the last half century[1]. There has been tremendous progress in the development of controllers for robotic systems, such as sliding mode control, fuzzy control, PD output feedback control, neural network, finite-time control, and so on [2-8]. It is well known that robotic manipulators have to encounter nonlinearities and various uncertainties in their dynamic models, such as friction, disturbance, and load changing, and it is very difficult to reach excellent performance when the control algorithm is simply based on the inaccurate plant model. Thus, designing a robotic manipulator controller is a serious challenge for engineers.

In the last few decades, the sliding mode control strategy has received much attention because this method provides a systematic approach to retaining asymptotic stability and robust performance. The sliding mode control is a robust technique to control nonlinear systems operating under uncertainty conditions and it can reduce the sensitivities to the variations of uncertain parameters and to external disturbances [9-12]. The sliding mode control is based on the design of a high-speed switching control law that drives the system's trajectory onto a user-chosen hyperplane in the state space, also known as sliding

surface. The main feature of sliding mode control are the following: (1) fast response and good transient performance; (2) robustness against a large class of perturbations or model uncertainties; and (3) the possibility of stabilizing some complex nonlinear systems which are difficult to stabilize by continuous state feedback laws. But, the discontinuous nature of the control law of sliding mode control creates chattering which may excite un-modeled high-frequency dynamics. The boundary-layer method, which attempts to eliminate the chattering phenomena, would require a trade-off between performance and chattering.

This paper proposes the solution to the problem of designing an adaptive controller for a nonlinear robotic manipulator with uncertain parameters. The controller comprises of sliding mode and adaptive components for uncertainty compensation. The adaptive algorithm is continuously refined based on sliding mode in order to improve the chattering phenomenon. It will be proven that the tracking error can converge to zero. Simulation results will be given to verify the effectiveness of the proposed scheme for high-performance trajectory tracking.

## II. DESCRIPTION OF THE ROBOTIC MANIPULATOR

Consider the dynamics of a general two-link robot manipulator with external disturbances to be described by the following Lagrange form [13]:

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) + T_d = \tau \quad (1)$$

where  $q \in R^2$  is the joint position of robotic manipulator,  $\dot{q} \in R^2$  is the joint velocity vector of robotic manipulator,  $\ddot{q} \in R^2$  is the joint acceleration vector of robotic manipulator,  $M(q) \in R^{2 \times 2}$  is the inertia matrix,  $C(q, \dot{q}) \in R^{2 \times 2}$  is the Coriolis and centrifugal torques,  $G(q) \in R^2$  is the gravity vector,  $T_d \in R^2$  is the external disturbance, and  $\tau \in R^2$  is the control vector representing the torque exerting on joints. Figure 1 shows the two-link robotic manipulator, where  $q_1$  is angle displacement of first joint,  $q_2$  is angle displacement of second joint,  $\dot{q}_1$  is angle displacement velocity of first joint,  $\dot{q}_2$  is angle displacement velocity of second joint,  $m_1$  is the mass of first joint,  $m_2$  is the mass of second joint,  $I_1$  is rotary inertia of first joint,  $I_2$  is rotary inertia of second joint,  $l_1$  is length of first joint,  $l_2$  is length of second joint,  $l_{c1}$  is distance of the starting point of first joint to the centroid, and  $l_{c2}$  is the distance

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of the starting point of second joint to the centroid.

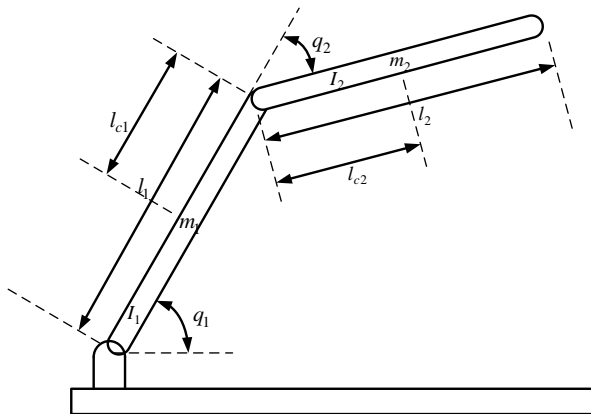


Fig. 1 Two-link robotic manipulator

The functions  $M(\cdot)$ ,  $C(\cdot)$ , and  $G(\cdot)$  are assumed to be continuous and defined on an appropriate open subset of the  $(q, \dot{q})$  phase space and assumed to be unknown. For simplicity, we assume that the above dynamics have the following properties [2].

(1) Property 1: The inertia matrix  $M(q)$  is symmetric and positive definite. It is assumed to be

$$m_1 I_2 \leq M(q) \leq m_2 I_2, \forall q \in R^2 \quad (2)$$

where  $m_1$  and  $m_2$  are positive constants, and  $I_2 \in R^{2 \times 2}$  is the identity matrix.

(2) Property 2: The matrix of Coriolis and centrifugal forces  $C(q, \dot{q})$  is bounded,

$$\|C(q, \dot{q})\| \leq \xi_c \|\dot{q}\|, \forall q, \dot{q} \in R^2 \quad (3)$$

where  $\xi_c$  is positive.

(3) Property 3:  $\dot{M}(q) - 2C(q, \dot{q})$  is a skew symmetric matrix and satisfies

$$x^T [\dot{M}(q) - 2C(q, \dot{q})] x = 0, \forall x \in R^2 \quad (4)$$

where  $x$  is a nonzero vector.

### III. SLIDING MODE CONTROLLER DESIGN

Sliding mode control design approach consists of two phases: (1) selection of a sliding surface so as to achieve the desired system behavior, when the control system reaches the sliding surface; and (2) selection of a control law such that the existence of sliding mode can be guaranteed.

In order to easier derivate the control law, let  $\dot{x} = \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix}$ ,

$$f = \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ -M^{-1}(q)(C(q, \dot{q})\dot{q} + G(q)) \end{bmatrix}, \quad b = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ M^{-1}(q) \end{bmatrix} \quad \text{and}$$

$$w = \begin{bmatrix} 0 \\ 0 \\ -M^{-1}(q)T_d \end{bmatrix}. \quad \text{Therefore, the dynamics of a general}$$

two-link robot manipulator (1) can be rewritten as

$$\dot{x} = f(x) + b(x)\tau + w \quad (5)$$

For tracking control purpose, the error state is defined as

$$e = x - x_d \quad (6)$$

where  $e = [e_1 \ e_2 \ \dot{e}_1 \ \dot{e}_2]^T$ ,  $x_d = [x_{d1} \ x_{d2} \ x_{d3} \ x_{d4}]^T = [q_{d1} \ q_{d2} \ \dot{q}_{d1} \ \dot{q}_{d2}]^T$ ,  $q_{di}$ ,  $i=1,2$ , represents the desired tracking of the first joint and second joint.

Implementing the sliding mode control scheme to control a robotic manipulator generally involves two steps. An appropriate sliding surface must be selected first, capable of ensuring the stability of the equivalent dynamics in the sliding mode such that the error dynamics can converge to zero. A sliding mode control must then be determined to ensure not only the reaching of the sliding surface in finite time, but also that the state trajectory can remain on the sliding mode thereafter even when undergoing the system uncertainties. As mentioned earlier, a proper sliding surface must be designed to ensure the system stability in the sliding mode. The next step involves designing an adaptive sliding mode control scheme to drive the extended error system trajectories onto the sliding surface.

Firstly, the sliding surface is defined as

$$s(t) = \Lambda e \quad (7)$$

where  $\Lambda = \begin{bmatrix} \lambda_1 & 0 & 1 & 0 \\ 0 & \lambda_2 & 0 & 1 \end{bmatrix}$ ,  $\lambda_i > 0$ .

Consider the situation of parameters perturbation in the system, denoted as  $f = f_0 + \Delta f$ ,  $b = b_0 + \Delta b$ . Let the subscript '<sub>0</sub>' means the system nominal value, and symbol ' $\Delta$ ' means the system uncertain value, i.e., and it is supposed and satisfied as follows [14].

Assumption 1:  $\|\Delta f(x)\| \leq \zeta(x)$ ,  $\|\Delta b(x)\| \leq \varepsilon(x)$  and  $\|w\| \leq \xi$ , where  $\zeta(x)$ ,  $\varepsilon(x)$  and  $\xi$  are the upper limit of parameter perturbations and miscellaneous information.

Assumption 2:  $\det(\Lambda b_0) \neq 0$ .

Assumption 3:  $\|(\Lambda b_0(x))^{-1}(\Lambda \Delta b(x))\| < \delta < 1$ , where  $\delta$  is a positive real number.

Next, determinate the control law by satisfying the sliding condition  $s^T \dot{s} < 0$ . The control input is

$$\tau = \tau_0 + \tau_s \quad (8)$$

where  $\tau_0$  is the nominal control input and  $\tau_s$  is the switching control input.  $\tau_0$  takes care of the nominal part in (5), and  $\tau_s$  deals with the plant parameter variations and the external disturbances.

$$\tau_0 = -(\Lambda b_0(x))^{-1}(\Lambda f_0(x) - \Lambda \dot{x}_d) \quad (9)$$

$$\tau_s = -(\Lambda b_0(x))^{-1} \beta \frac{s}{\|s\|} \quad (10)$$

$$\beta > \frac{\|\Lambda\|(\|\zeta(x) + \xi\| + \delta\|\Lambda(\dot{x}_d - f_0(x))\|)}{1 - \delta} \quad (11)$$

Taking the derivative of  $s(t)$  with respect to time yields

$$\dot{s}(t) = \Lambda(\Delta f(x) + w) + (\Lambda b_0(x))^{-1}(\Lambda \Delta b(x))\Lambda(\dot{x}_d - f_0(x)) - [I_2 + (\Lambda b_0(x))^{-1}(\Lambda \Delta b(x))] \beta \frac{s}{\|s\|} \quad (12)$$

where  $I_2$  is the  $2 \times 2$  identity matrix. Multiplying both side of (14) by  $s^T$  obtains

$$\begin{aligned} s^T \dot{s} &= s^T (\Lambda \Delta f(x)) - s^T (I_2 + (\Lambda b_0(x))^{-1}(\Lambda \Delta b(x))) \beta \frac{s}{\|s\|} \\ &\quad - s^T ((\Lambda b_0(x))^{-1}(\Lambda \Delta b(x))\Lambda(\dot{x}_d - f_0(x)) + \Lambda w) \\ &\leq \|s\|(\|\Lambda\|(\|\Delta f(x)\| + \|w\|)) \\ &\quad + \|s\|(\|(\Lambda b_0(x))^{-1}(\Lambda \Delta b(x))\|\|\Lambda(\dot{x}_d - f_0(x))\|) \\ &\quad - s^T (I_2 + (\Lambda b_0(x))^{-1}(\Lambda \Delta b(x))) \beta \frac{s}{\|s\|} \end{aligned} \quad (13)$$

Applying Assumptions 1 to 3 yields

$$\begin{aligned} s^T \dot{s} &\leq \|s\|(\|\Lambda\|(\|\zeta(x) + \xi\| + \delta\|\Lambda(\dot{x}_d - f_0(x))\|) \\ &\quad - (\|I_2\| + \|(\Lambda b_0(x))^{-1}(\Lambda \Delta b(x))\|)\beta) \\ &\quad - \left( \frac{\gamma + \|\Lambda\|(\|\zeta(x) + \xi\| + \delta\|\Lambda(\dot{x}_d - f_0(x))\|)}{(1 - \delta)} \right) \|s\| \\ &< -\gamma \|s\| \\ &\leq 0 \end{aligned} \quad (14)$$

The result of (14) implies that the system trajectories will asymptotically converge to sliding surface from any non-zero initial error, and guarantees the robust stability of the closed-loop system.

#### IV. DESIGN OF ADAPTIVE SLIDING MODE CONTROLLER

It is well known that the parameter variations of the system,

such as mass and inertia, are difficult to measure. And the exact value of the external disturbance is also difficult to measure in advance for practical applications. However, the application of control law given in Eqs. (9) to (11) is limited due to the chattering and the unknown bounds of the uncertainties. Also, it was generally selected conservatively based on the bounds of uncertainties. In this study, the goal is to replace the term  $\beta$  to alleviate chattering. The adaptive sliding mode control law is proposed as follows:

$$\tau = \tau_0 + \tau_{as} \quad (15)$$

$$\tau_0 = -(\Lambda b_0(x))^{-1}(\Lambda f_0(x)) \quad (16)$$

$$\tau_{as} = -(\Lambda b_0(x))^{-1} \hat{\beta} \phi(\hat{\alpha}, s) \quad (17)$$

$$\phi(\hat{\alpha}, s) = \begin{bmatrix} \phi_1(\hat{\alpha}_1, s_1) \\ \phi_2(\hat{\alpha}_2, s_2) \end{bmatrix}, \phi_i(\hat{\alpha}_i, s_i) = \frac{1 - \exp(-\hat{\alpha}_i s_i)}{1 + \exp(-\hat{\alpha}_i s_i)}, i = 1, 2. \quad (18)$$

The adaptive laws are

$$\dot{\hat{\beta}} = \eta_1 \phi^T(\hat{\alpha}, s) (\Lambda b_0(x)) \text{sgn}\left(\frac{\partial x}{\partial \tau}\right)^T e \quad (19)$$

$$\dot{\hat{\alpha}}_1 = \eta_2 (\Lambda b_0(x))_{11} e_1 \text{sgn}\left(\frac{\partial x_1}{\partial \tau_1}\right) s_1 \quad (20)$$

$$\dot{\hat{\alpha}}_2 = \eta_3 (\Lambda b_0(x))_{22} e_2 \text{sgn}\left(\frac{\partial x_2}{\partial \tau_2}\right) s_2 \quad (21)$$

where  $\eta_i$  are positive constants,  $i = 1, 2, 3$ .

The following discussion establishes that if the control input  $\tau$  is appropriately designed as (15)-(18) with adaption laws (19)-(21), then the trajectory of the error dynamics converges to the sliding surface. Now consider the following Lyapunov function candidate

$$V = \frac{1}{2} e^T e = \frac{1}{2} (e_1^2 + e_2^2 + \dot{e}_1^2 + \dot{e}_2^2) \quad (22)$$

The derivative of Lyapunov function is

$$\begin{aligned} \frac{dV}{dt} &= \frac{\partial V}{\partial e} \frac{\partial e}{\partial x} \frac{\partial x}{\partial \tau} \frac{\partial \tau}{\partial \hat{\beta}} \frac{\partial \hat{\beta}}{\partial t} \\ &= e \frac{\partial x}{\partial \tau} \frac{\partial}{\partial \hat{\beta}} \left( -(\Lambda b_0(x))^{-1} (\Lambda f_0(x) + \hat{\beta} \phi(\hat{\alpha}, s)) \right) \frac{\partial \hat{\beta}}{\partial t} \\ &= e \frac{\partial x}{\partial \tau} \left( -(\Lambda b_0(x))^{-1} \phi(\hat{\alpha}, s) \right) \dot{\hat{\beta}} \end{aligned} \quad (23)$$

Substituting (19) into (23) yields

$$\begin{aligned}\frac{dV}{dt} &= -\eta_1 \left( e^T e \right) \left[ \left( \frac{\partial x}{\partial \tau} \right)^T \left( \frac{\partial x}{\partial \tau} \right) \right] \left[ \left( \phi(\hat{\alpha}, s) \right)^T \left( \phi(\hat{\alpha}, s) \right) \right] \\ &= -\eta_1 \left( e^T e \right) \left\| \frac{\partial x}{\partial \tau} \right\|^2 \left\| \phi(\hat{\alpha}, s) \right\|^2 \\ &\leq 0\end{aligned}\quad (24)$$

By direct computation [15],

$$\begin{aligned}\frac{dV}{dt} &= \frac{\partial V}{\partial e_1} \frac{\partial e_1}{\partial x_1} \frac{\partial x_1}{\partial \tau_1} \frac{\partial \tau_1}{\partial \hat{\alpha}_1} \frac{\partial \hat{\alpha}_1}{\partial t} \\ &= -e_1 \frac{\partial x_1}{\partial \tau_1} \hat{\beta}(\Lambda b_0(x))_{11}^{-1} \frac{2s_1 \exp(-\hat{\alpha}_1 s_1)}{(1 + \exp(-\hat{\alpha}_1 s_1))^2} \dot{\hat{\alpha}}_1\end{aligned}\quad (25)$$

Substituting (20) into (25) yields

$$\frac{dV}{dt} = -\eta_2 e_1^2 \hat{\beta}_1^2 \frac{2 \exp(-\hat{\alpha}_1 s_1)}{(1 + \exp(-\hat{\alpha}_1 s_1))^2} \leq 0 \quad (26)$$

With the same procedure, the derivative of Lyapunov function is found to be

$$\begin{aligned}\frac{dV}{dt} &= \frac{\partial V}{\partial e_2} \frac{\partial e_2}{\partial x_2} \frac{\partial x_2}{\partial \tau_2} \frac{\partial \tau_2}{\partial \hat{\alpha}_2} \frac{\partial \hat{\alpha}_2}{\partial t} \\ &= -e_2 \frac{\partial x_2}{\partial \tau_2} \hat{\beta}(\Lambda b_0(x))_{22}^{-1} \frac{2s_2 \exp(-\hat{\alpha}_2 s_2)}{(1 + \exp(-\hat{\alpha}_2 s_2))^2} \dot{\hat{\alpha}}_2\end{aligned}\quad (27)$$

Substituting (21) into (27) yields

$$\frac{dV}{dt} = -\eta_2 e_2^2 \hat{\beta}_2^2 \frac{2 \exp(-\hat{\alpha}_2 s_2)}{(1 + \exp(-\hat{\alpha}_2 s_2))^2} \leq 0 \quad (28)$$

Since  $\frac{dV}{dt} < 0$  while  $V > 0$ , as defined,  $V$  reaches zero in finite time, and  $e = 0$ . Therefore, the elimination of tracking error can be guaranteed.

## V. SIMULATION RESULTS

In this section, we show the design process of the proposed adaptive sliding mode control algorithm on a two-link manipulator. The equation of motion for this robot system is defined as in [16]. The desired trajectories are given by

$$q_d = \begin{bmatrix} q_{d1} \\ q_{d2} \end{bmatrix} = \begin{bmatrix} 1.6 - 1.6 \exp(-8t) - 12.8t \exp(-8t) \\ 1.6 - 1.6 \exp(-8t) - 12.8t \exp(-8t) \end{bmatrix} \quad (29)$$

The external disturbance is

$$T_d = \begin{bmatrix} 3.2 + 2 \cos(0.02t) \\ 3.5 + 1.7 \sin(0.02t) \end{bmatrix} \quad (30)$$

The proposed sliding mode controller, (9)-(11), is applied. The gain matrix of control input and the gain matrix of sliding function are

$$\beta = \begin{bmatrix} 30 & 0 \\ 0 & 30 \end{bmatrix}, \Lambda = \begin{bmatrix} 10 & 0 & 1 & 0 \\ 0 & 10 & 0 & 1 \end{bmatrix} \quad (30)$$

Figures 2 to 5 show the simulation results of the sliding mode control. The output tracking results of joint 1 and joint 2 are as shown in Fig. 2, the solid line is the joint position and the dashed line is the desired position. Fig. 3 demonstrates the control inputs performance of joint 1 and joint 2. It shows that the control inputs have high frequency chattering phenomenon. This result of chattering control input will cause machinery damage in practical application. Figure 4 illustrates sliding surfaces time response of joint 1 and joint 2. Fig. 5 shows the tracking error of joint 1 and joint 2.

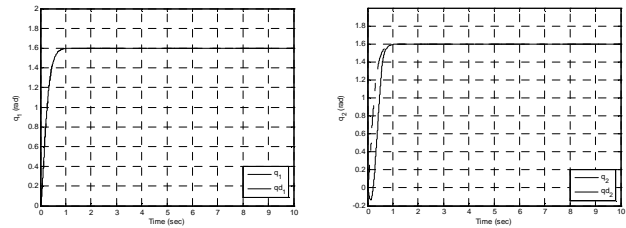


Fig. 2. Output tracking of sliding mode control

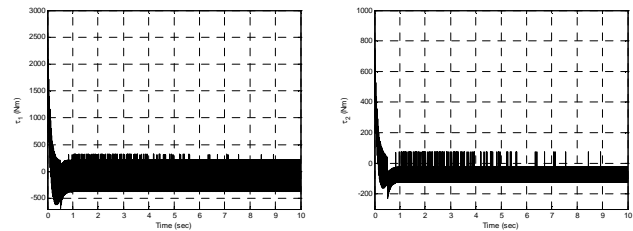


Fig. 3. Control input of sliding mode control

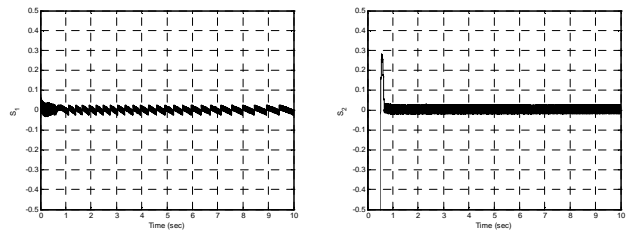


Fig. 4. Sliding surface of sliding mode control

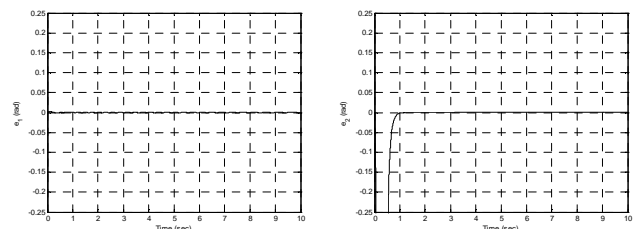


Fig. 5. Tracking error of sliding mode control

For comparison purpose, the performances of adaptive sliding mode control law (15)-(21) are demonstrated next. The initial value of adaptive law is  $\hat{\beta}(0) = 200$ ,  $\hat{\Gamma}(0) = 300$ ,  $\hat{\alpha}(0) = [20 \ 20]^T$ , and  $\alpha = 0.001$ . The learning rates are  $\eta_1 = 1$ ,  $\eta_2 = 1$ ,  $\eta_3 = 1$ . The simulation results of the adaptive

sliding mode control are shown in Figs. 6 to 9. From Fig. 6, we can see that joint 1 arrives steady state after  $t \geq 1$  s and joint 2 enters steady state after  $t \geq 1$ . Figure 7 shows the performance of control law without chattering phenomenon. We can see the performance of adaptive sliding mode control is better than the sliding mode control in the presence of the external disturbance. Figure 8 shows sliding surfaces time response. Figure 9 illustrates the position tracking errors with the proposed adaptive sliding mode controller.

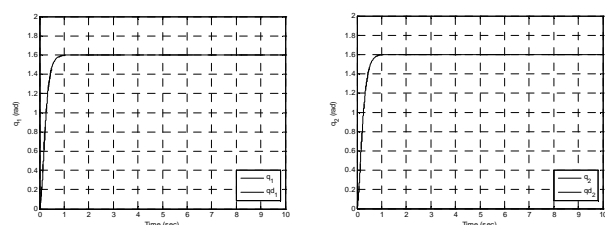


Fig. 6. Output tracking of adaptive sliding mode control

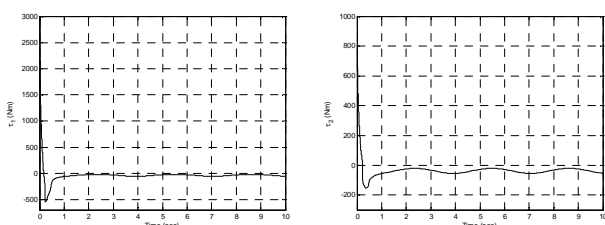


Fig. 7. Control input of adaptive sliding mode control

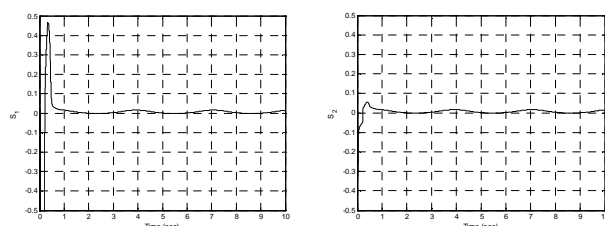


Fig. 8. Sliding surface of adaptive sliding mode control

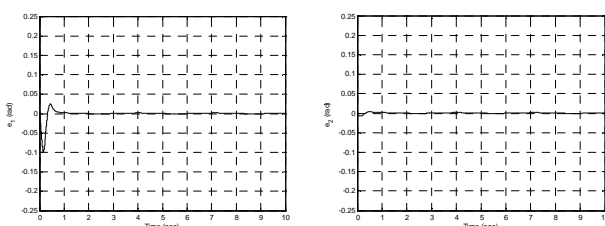


Fig. 9. Tracking error of adaptive sliding mode control

## VI. CONCLUSION

This paper proposes a novel control method for a robotic manipulator with external disturbance. Using the Lyapunov stability method, the proposed method guarantees robustness as well as tracking performance. A self-tuning law enables the automatic adaptation of the adjustable control gain parameter and the adjustable boundary layer width to the terminal values. The control input chattering is eliminated effectively. Smooth control activity and excellent tracking performance are shown. Simulations performed on a two-DOF robot demonstrate the

effectiveness of the proposed controller.

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