```
In[1]:= (*some numbers*)

Bino[n_, m_, q_] := Product[(q^(n-i) - 1)/(q^(m-i) - 1), {i, 0, m - 1}]
                    乘積

(*descendant cond for one variable families and two variables families*)
Desc[v0_, k0_, lam0_, mu0_, var0_, G0_] :=
 Module[{v = v0, k = k0, lam = lam0, mu = mu0, var = var0, G = G0},
 模組
  cond = Simplify[k == 2 mu];
         化簡
  StringForm["The descendant cond of `` is ``", G, cond]]
  字串形式

(*Find the dimension*)
  找到
Dim[v_, k_, lam_, mu_, var_] :=
 Solve[Simplify[(8 d^2 k + 4 d^2 k^2 - 8 d k (v + 1) - 4 d^2 k (v + 1) - 4 d k^2 (v + 1) -
 求解   化簡
       (v + 1)^2 + d^2 (v + 1)^2 + 4 d k (v + 1)^2 + (v + 1)^3 - d (v + 1)^3 == 0) /. var], d]

(*Return informations of ETF*)
  返回
MN[v0_, k0_, lam0_, mu0_, var0_] :=
 Module[{v = v0, k = k0, lam = lam0, mu = mu0, var = var0},
 模組
  dim = Dim[v, k, lam, mu, var];
  Print["The (M,N) are "];
  列表          數值化
  M = Simplify[(v + 1) /. var];
      化簡
  Do[{Print[{M, N[[1]][[2]]}]}, {N, dim}]]
  D⋯  列表     數值化           數值化

(*Eiganvalues r and s*)
reig[v_, k_, lam_, mu_] := 1/2 ((lam - mu) + Sqrt[(lam - mu)^2 + 4 (k - mu)])

seig[v_, k_, lam_, mu_] := 1/2 ((lam - mu) - Sqrt[(lam - mu)^2 + 4 (k - mu)])

(*P and Q matrices*)
Pmat[v_, k_, lam_, mu_] := ( 1  k                      v - k - 1
                             1  reig[v, k, lam, mu]   -reig[v, k, lam, mu] - 1
                             1  seig[v, k, lam, mu]   -seig[v, k, lam, mu] - 1 )

Qmat[v_, k_, lam_, mu_] := v Inverse[Pmat[v, k, lam, mu]]
                             逆
```

In[9]:=
```
(*Lattice graph*)
G = "L₂(q)";
v = q^2;
k = 2 (q - 1);
lam = q - 2;
mu = 2;
ass = q ≥ 2;

Desc[v, k, lam, mu, q, G]
MN[v, k, lam, mu, q → 3]
```

Out[15]=
```
The descendant cond of L₂(q) is q == 3
The (M,N) are
{10, 5}
```

In[17]:=
```
(*Triangle graph*)
  三角形
G = "T(n)";
v = n (n - 1) / 2;
k = 2 (n - 2);
lam = n - 2;
mu = 4;
ass = n ≥ 5;

Desc[v, k, lam, mu, n, G]
MN[v, k, lam, mu, n → 6]
```

Out[23]=
```
The descendant cond of T(n) is n == 6
The (M,N) are
{16, 6}
{16, 10}
```

In[25]:=
```
(*Sp2n(q),O2n+1q*)
G = "Sp2 n(q),O2 n+1q";
```
$$v = \frac{q^n - 1}{q - 1} \left(t\, q^{n-1} + 1\right) \text{ /. } t \to q;$$
$$k = q\, \frac{q^{n-1} - 1}{q - 1} \left(t\, q^{n-2} + 1\right) \text{ /. } t \to q;$$
$$lam = \left(q^2\, \frac{q^{n-2} - 1}{q - 1} \left(t\, q^{n-3} + 1\right) + q - 1\right) \text{ /. } t \to q;$$
$$mu = \frac{k}{q} \text{ /. } t \to q;$$
$$ass = q \geq 2\; \&\&\; n \geq 2;$$

```
StringForm["The descendant cond of `` is ``", G, Desc[v, k, lam, mu, ass, G]]
```
字串形式
```
MN[v, k, lam, mu, q → 2]
```

Out[31]= The descendant cond of $Sp_{2n}(q)$,$O_{2n+1}q$ is

The descendant cond of $Sp_{2n}(q)$,$O_{2n+1}q$ is $\frac{(-2+q)\left(q^2 - q^{2n}\right)}{(-1+q)\, q} == 0$

The (M,N) are

$\left\{4^n,\ 2^{-1+n} \times (-1 + 2^n)\right\}$

$\left\{4^n,\ 2^{-1+n} \times (1 + 2^n)\right\}$

In[33]:=
```
(*O2n+(q)*)
G = "O2 n+(q)";
```
$$v = \frac{q^n - 1}{q - 1} \left(t\, q^{n-1} + 1\right) \text{ /. } t \to 1;$$
$$k = q\, \frac{q^{n-1} - 1}{q - 1} \left(t\, q^{n-2} + 1\right) \text{ /. } t \to 1;$$
$$lam = \left(q^2\, \frac{q^{n-2} - 1}{q - 1} \left(t\, q^{n-3} + 1\right) + q - 1\right) \text{ /. } t \to 1;$$
$$mu = \frac{k}{q} \text{ /. } t \to 1;$$
$$ass = q \geq 2\; \&\&\; n \geq 2;$$

```
StringForm["The descendant cond of `` is ``", G, Desc[v, k, lam, mu, ass, G]]
```
字串形式
```
MN[v, k, lam, mu, q → 2]
```

Out[39]= The descendant cond of $O_{2n}^{+}(q)$ is

The descendant cond of $O_{2n}^{+}(q)$ is $\frac{(-2+q)\left(-q + q^n\right)\left(q^2 + q^n\right)}{(-1+q)\, q} == 0$

The (M,N) are

$\left\{2^{-1+n}\times(1+2^n),\ \frac{1}{3}\times\left(-1+2^{2n}\right)\right\}$

$\left\{2^{-1+n}\times(1+2^n),\ \frac{1}{6}\times\left(2+3\times2^n+2^{2n}\right)\right\}$

In[41]:=
```
(*O2n+2^-(q)*)
G = "O2 n+2^-(q)";
v = (q^n - 1)/(q - 1) (t q^(n-1) + 1) /. t → q^2;
k = q (q^(n-1) - 1)/(q - 1) (t q^(n-2) + 1) /. t → q^2;
lam = (q^2 (q^(n-2) - 1)/(q - 1) (t q^(n-3) + 1) + q - 1) /. t → q^2;
mu = k/q /. t → q^2;
ass = q ≥ 2 && n ≥ 2;

StringForm["The descendant cond of `` is ``", G, Desc[v, k, lam, mu, ass, G]]
字串形式
MN[v, k, lam, mu, q → 2]
```

Out[47]= The descendant cond of $O_{2n+2}^{-}(q)$ is

The descendant cond of $O_{2n+2}^{-}(q)$ is $\frac{(-2+q)\left(1+q^n\right)\left(-q+q^n\right)}{(-1+q)\,q} == 0$

The (M,N) are

$\left\{2^{n}\times\left(-1+2^{1+n}\right),\ \frac{1}{3}\times\left(1-3\times2^n+2^{1+2n}\right)\right\}$

$\left\{2^{n}\times\left(-1+2^{1+n}\right),\ \frac{1}{3}\times\left(-1+2^{2+2n}\right)\right\}$

In[49]:=
```
(*U_{2n}(√q)*)
G = "U_{2 n}(√q)";
```
$$v = \frac{q^n - 1}{q - 1}\left(t\, q^{n-1} + 1\right) \text{ /. } t \to q^{1/2};$$
$$k = q\, \frac{q^{n-1} - 1}{q - 1}\left(t\, q^{n-2} + 1\right) \text{ /. } t \to q^{1/2};$$
$$lam = \left(q^2\, \frac{q^{n-2} - 1}{q - 1}\left(t\, q^{n-3} + 1\right) + q - 1\right) \text{ /. } t \to q^{1/2};$$
$$mu = \frac{k}{q} \text{ /. } t \to q^{1/2};$$
```
ass = q ≥ 2 && n ≥ 2;

StringForm["The descendant cond of `` is ``", G, Desc[v, k, lam, mu, ass, G]]
```
[字串形式

```
MN[v, k, lam, mu, q → 2]
```

Out[55]= The descendant cond of $U_{2n}(\sqrt{q})$ is

The descendant cond of $U_{2n}(\sqrt{q})$ is $\frac{(-2 + q)\left(q - q^n\right)\left(q^{3/2} + q^n\right)}{(-1 + q)\sqrt{q}} == 0$

The (M,N) are

$$\left\{1 + \left(1 + 2^{-\frac{1}{2}+n}\right) \times (-1 + 2^n),\ \frac{2^n \times \left(1 + \sqrt{2} + 3 \times 2^n + 2^{\frac{3}{2}+n}\right) - \sqrt{8 + 27 \times 2^{2n} + 2^{4n} + 5 \times 2^{\frac{5}{2}+n} + 2^{3+n} + 3 \times 2^{\frac{3}{2}+2n} - 5 \times 2^{1+3n} - 2^{\frac{3}{2}+3n}}}{2 \times \left(4 + 3\sqrt{2}\right)}\right\}$$

$$\left\{1 + \left(1 + 2^{-\frac{1}{2}+n}\right) \times (-1 + 2^n),\ \frac{2^n \times \left(1 + \sqrt{2} + 3 \times 2^n + 2^{\frac{3}{2}+n}\right) + \sqrt{8 + 27 \times 2^{2n} + 2^{4n} + 5 \times 2^{\frac{5}{2}+n} + 2^{3+n} + 3 \times 2^{\frac{3}{2}+2n} - 5 \times 2^{1+3n} - 2^{\frac{3}{2}+3n}}}{2 \times \left(4 + 3\sqrt{2}\right)}\right\}$$

In[57]:=
```
(*U_{2n+1}(√q)*)
G = "U_{2 n+1}(√q)";
v = (q^n - 1)/(q - 1) (t q^(n-1) + 1) /. t → q^(3/2);
k = q (q^(n-1) - 1)/(q - 1) (t q^(n-2) + 1) /. t → q^(3/2);
lam = (q^2 (q^(n-2) - 1)/(q - 1) (t q^(n-3) + 1) + q - 1) /. t → q^(3/2);
mu = k/q /. t → q^(3/2);
ass = q ≥ 2 && n ≥ 2;

StringForm["The descendant cond of `` is ``", G, Desc[v, k, lam, mu, ass, G]]
字串形式
MN[v, k, lam, mu, q → 2]
```

Out[63]= The descendant cond of $U_{2n+1}(\sqrt{q})$ is

The descendant cond of $U_{2n+1}(\sqrt{q})$ is $\frac{(-2+q)\left(q-q^{n}\right)\left(\sqrt{q}+q^{n}\right)}{(-1+q)\sqrt{q}} == 0$

The (M,N) are

$$\left\{1+(-1+2^{n})\times\left(1+2^{\frac{1}{2}+n}\right),\ \frac{2^{n}\times\left(-1-\sqrt{2}+3\times 2^{\frac{1}{2}+n}+2^{2+n}\right)-\sqrt{4+27\times 2^{2n}-5\times 2^{2+n}-2^{\frac{5}{2}+n}+3\times 2^{\frac{3}{2}+2n}+5\times 2^{\frac{3}{2}+3n}+2^{2+3n}+2^{1+4n}}}{2\times\left(3+2\sqrt{2}\right)}\right\}$$

$$\left\{1+(-1+2^{n})\times\left(1+2^{\frac{1}{2}+n}\right),\ \frac{2^{n}\times\left(-1-\sqrt{2}+3\times 2^{\frac{1}{2}+n}+2^{2+n}\right)+\sqrt{4+27\times 2^{2n}-5\times 2^{2+n}-2^{\frac{5}{2}+n}+3\times 2^{\frac{3}{2}+2n}+5\times 2^{\frac{3}{2}+3n}+2^{2+3n}+2^{1+4n}}}{2\times\left(3+2\sqrt{2}\right)}\right\}$$

```
In[65]:= (*Thm2.2.19*)
G = "graphs in Thm 2.2.19";
v = (1 + t) × (1 + q t);
k = t (q + 1);
lam = t - 1;
mu = k / t;
ass = q ≥ 2 && t > 0;

StringForm["The descendant cond of `` is ``", G, Desc[v, k, lam, mu, ass, G]]
字串形式
StringForm["(q,d) is listed below, d is the dimension"]
字串形式
Solve[Simplify[
求解  化簡
  (8 d^2 k + 4 d^2 k^2 - 8 d k (v + 1) - 4 d^2 k (v + 1) - 4 d k^2 (v + 1) - (v + 1)^2 + d^2 (v + 1)^2 +
     4 d k (v + 1)^2 + (v + 1)^3 - d (v + 1)^3 == 0) /. t → 2], d, Integers]
                                                                  整數域

Out[71]= The descendant cond of graphs in Thm 2.2.19 is
  The descendant cond of graphs in Thm 2.2.19 is (1 + q) (-2 + t) == 0

Out[72]= (q,d) is listed below, d is the dimension

Out[73]= {{d → -90 if q == -14}, {d → -55 if q == -8}, {d → -44 if q == -6},
  {d → -39 if q == -5}, {d → -35 if q == -4 || q == -3}, {d → -3 if q == -1},
  {d → 1 if q == -1 || q == 0}, {d → 3 if q == 0}, {d → 5 if q == 1 || q == 1},
  {d → 6 if q == 2}, {d → 7 if q == 4}, {d → 8 if q == 10}, {d → 10 if q == -14 || q == 2},
  {d → 11 if q == -8}, {d → 12 if q == -6}, {d → 13 if q == -5},
  {d → 15 if q == -4}, {d → 21 if q == -3 || q == 4}, {d → 56 if q == 10}}

In[74]:= {v, k, lam, mu} /. t → 2
Out[74]= {3 × (1 + 2 q), 2 × (1 + q), 1, 1 + q}

In[75]:= {V, KK, LAM, MU} = {3 × (1 + 2 q), 2 × (1 + q), 1, 1 + q}

Out[75]= {3 × (1 + 2 q), 2 × (1 + q), 1, 1 + q}
```

In[76]:=
```
(*Thm2.2.20*)
G = "graph in Thm 2.2.20";
v = (q^4 + 1) (q^3 + 1) (q^2 + 1) (q + 1);
k = q Bino[5, 2, q];
lam = q - 1 + q^2 (q + 1) (q^2 + q + 1);
mu = Bino[4, 2, q];
ass = q ≥ 2;

Desc[v, k, lam, mu, q, G]
Solve[q^5 (2 + q + q^2) == 2 + q + 3 q^2, q, Integers]
```
求解 整數域

Out[82]= The descendant cond of graph in Thm 2.2.20 is $q^5 (2+q+q^2) == 2+q+3q^2$

Out[83]= { }

In[84]:=
```
(*NU_n(q): n even*)
G = "NU_n(q) for even n";
v = q^(n-1) (q^n - eps)/(q + 1) /. eps → 1;
k = (q^(n-1) + eps) (q^(n-2) - eps) /. eps → 1;
lam = (q^(2 n-5) (q + 1) - eps q^(n-2) (q - 1) - 2) /. eps → 1;
mu = q^(n-3) (q + 1) (q^(n-2) - eps) /. eps → 1;
ass = n ≥ 3 && q ≥ 2;

Desc[v, k, lam, mu, ass, G]
StringForm["It is equivalent to ``",
 Factor[(-1 + q^(-2+n)) × (1 + q^(-1+n)) - 2 q^(-3+n) (1 + q) × (-1 + q^(-2+n))] q^5 / (-q^2 + q^n) == 0]
```
字串形式 因式分解

Out[90]= The descendant cond of $NU_n(q)$ for even n is $(-1+q^{-2+n})(1+q^{-1+n}) == 2q^{-3+n}(1+q)(-1+q^{-2+n})$

Out[91]= It is equivalent to $q^3 - 2q^n - 2q^{1+n} + q^{2+n} == 0$

In[92]:=
```
(*NU_n(q): n odd*)
G = "NU_n(q) for odd n";
v = q^(n-1) (q^n - eps)/(q + 1) /. eps → -1;
k = (q^(n-1) + eps) (q^(n-2) - eps) /. eps → -1;
lam = (q^(2 n-5) (q + 1) - eps q^(n-2) (q - 1) - 2) /. eps → -1;
mu = q^(n-3) (q + 1) (q^(n-2) - eps) /. eps → -1;
ass = n ≥ 3 && q ≥ 2;

Desc[v, k, lam, mu, ass, G]
StringForm["It is equivalent to ``",
字串形式
 Factor[(1 + q^(-2+n)) × (-1 + q^(-1+n)) - 2 q^(-5+n) (1 + q) (q^2 + q^n)] q^5 / (q^2 + q^n) == 0]
 因式分解
MN[v, k, lam, mu, {q → 3, n → 3}]
```

Out[98]= The descendant cond of $NU_n(q)$ for odd n is $(1 + q^{-2+n}) (-1 + q^{-1+n}) == 2 q^{-5+n} (1 + q) (q^2 + q^n)$

Out[99]= It is equivalent to $-q^3 - 2 q^n - 2 q^{1+n} + q^{2+n} == 0$

The (M,N) are

{64, 28}

{64, 36}

In[101]:=
```
(*NO_2m^+(2)*)
G = "NO_2m^+(2)";
v = (2^(2 m-1) - eps 2^(m-1)) /. eps → 1;
k = (2^(2 m-2) - 1) /. eps → 1;
lam = (2^(2 m-3) - 2) /. eps → 1;
mu = (2^(2 m-3) + eps 2^(m-2)) /. eps → 1;
ass = m ≥ 2;

Desc[v, k, lam, mu, ass, G]
```

Out[107]= The descendant cond of $NO_{2m}^{+}(2)$ is $2 + 2^m == 0$

In[108]:=
```
(*NO_2m^-(2)*)
G = "NO_2m^-(2)";
v = (2^(2 m-1) - eps 2^(m-1)) /. eps → -1;
k = (2^(2 m-2) - 1) /. eps → -1;
lam = (2^(2 m-3) - 2) /. eps → -1;
mu = (2^(2 m-3) + eps 2^(m-2)) /. eps → -1;
ass = m ≥ 2;

Desc[v, k, lam, mu, ass, G]
```

Out[114]= The descendant cond of $NO_{2m}^{-}(2)$ is $2^m == 2$

In[115]:=
```
(*NO2m+(3)*)
G = "NO2m+(3)";
v = 1/2 × 3^(m-1) × (3^m - eps) /. eps → 1;
k = 1/2 × 3^(m-1) × (3^(m-1) - eps) /. eps → 1;
lam = 1/2 × 3^(m-2) × (3^(m-1) + eps) /. eps → 1;
mu = 1/2 × 3^(m-1) × (3^(m-2) - eps) /. eps → 1;
ass = m ≥ 2;

Desc[v, k, lam, mu, ass, G]
```

Out[121]= The descendant cond of $NO_{2m}^{+}(3)$ is $3^{2+m} + 9^m == 0$

In[122]:=
```
(*NO2m-(3)*)
G = "NO2m-(3)";
v = 1/2 × 3^(m-1) × (3^m - eps) /. eps → -1;
k = 1/2 × 3^(m-1) × (3^(m-1) - eps) /. eps → -1;
lam = 1/2 × 3^(m-2) × (3^(m-1) + eps) /. eps → -1;
mu = 1/2 × 3^(m-1) × (3^(m-2) - eps) /. eps → -1;
ass = m ≥ 2;

Desc[v, k, lam, mu, ass, G]
MN[v, k, lam, mu, m → 2]
```

Out[128]= The descendant cond of $NO_{2m}^{-}(3)$ is $9^m == 3^{2+m}$

The (M,N) are

{16, 6}

{16, 10}

In[130]:=
```
(*NO2m+1^+(q)*)
G = "NO2 m+1^+(q)";
v = 1/2 q^m (q^m + eps) /. eps → 1;
k = (q^(m-1) + eps) (q^m - eps) /. eps → 1;
lam = (2 (q^(2 m-2) - 1) + eps q^(m-1) (q - 1)) /. eps → 1;
mu = 2 q^(m-1) (q^(m-1) + eps) /. eps → 1;
ass = q ≥ 2 && m ≥ 1;

Desc[v, k, lam, mu, ass, G]
MN[v, k, lam, mu, {q → 5, m → 1}]
```

Out[136]= The descendant cond of $NO_{2\,m+1}{}^{+}(q)$ is $\frac{\left(q+q^{m}\right)\left(-q-4\,q^{m}+q^{1+m}\right)}{q} == 0$

The (M,N) are

{16, 6}

{16, 10}

In[138]:=
```
(*NO2m+1^-(q)*)
G = "NO2 m+1^-(q)";
v = 1/2 q^m (q^m + eps) /. eps → -1;
k = (q^(m-1) + eps) (q^m - eps) /. eps → -1;
lam = (2 (q^(2 m-2) - 1) + eps q^(m-1) (q - 1)) /. eps → -1;
mu = 2 q^(m-1) (q^(m-1) + eps) /. eps → -1;
ass = q ≥ 2 && m ≥ 1;

Desc[v, k, lam, mu, ass, G]
```

Out[144]= The descendant cond of $NO_{2\,m+1}{}^{-}(q)$ is $\frac{\left(-q+q^{m}\right)\left(q-4\,q^{m}+q^{1+m}\right)}{q} == 0$

In[145]:=
```
(*[3.5.1]*)
G = "graphs in 3.5.1";
v = Bino[n, 2, q];
k = (q + 1) (Bino[n - 1, 1, q] - 1);
lam = Bino[n - 1, 1, q] + q^2 - 2;
mu = (q + 1)^2;
r = q^2 Bino[n - 3, 1, q] - 1;
s = -q - 1;
ass = q ≥ 2 && n ≥ 4;

Desc[v, k, lam, mu, ass, G]
StringForm["It is equivalent to ``",
字串形式
 Factor[(1 + q) × (-1 + (-1 + q^(-1+n))/(-1 + q)) - 2 (1 + q)^2] (-1 + q) q / (1 + q) == 0]
 因式分解

MN[v, k, lam, mu, {q → 2, n → 4}]
```

Out[153]= The descendant cond of graphs in 3.5.1 is $(1+q)\left(-1+\frac{-1+q^{-1+n}}{-1+q}\right) == 2\,(1+q)^2$

Out[154]= It is equivalent to $2\,q - q^2 - 2\,q^3 + q^n == 0$

The (M,N) are

{36, 15}

{36, 21}

In[156]:=
```
(*E_{6,1}(q)*)
G = "E_{6,1}(q)";
v = ((q^12 - 1) (q^9 - 1))/((q^4 - 1) (q - 1));
k = q (q^3 + 1) Bino[8, 1, q];
lam = q^2 (q^2 + 1) Bino[5, 1, q] + q - 1;
mu = (q^3 + 1) Bino[4, 1, q];
ass = q ≥ 2 && m ≥ 1;

Desc[v, k, lam, mu, q, G]
Solve[q + q^4 + q^6 + q^7 + q^8 + q^10 == 2 + 2 q^2 + q^3 + q^5, q, Integers]
求解                                                                整數域
```

Out[162]= The descendant cond of $E_{6,1}(q)$ is $q + q^4 + q^6 + q^7 + q^8 + q^{10} == 2 + 2\,q^2 + q^3 + q^5$

Out[163]= $\{\{q \to -1\}, \{q \to 1\}\}$

In[164]:=
```
(*P(q),P*(q)*)
G = "P(q),P*(q)";
v = 4 t + 1;
k = 2 t;
lam = t - 1;
mu = t;
ass = t ≥ 1;

Desc[v, k, lam, mu, q, G]
StringForm["Dimension is ``",
字串形式
 Solve[Simplify[(8 d^2 k + 4 d^2 k^2 - 8 d k (v + 1) - 4 d^2 k (v + 1) - 4 d k^2 (v + 1) -
 求解  化簡
      (v + 1)^2 + d^2 (v + 1)^2 + 4 d k (v + 1)^2 + (v + 1)^3 - d (v + 1)^3 == 0)], d]]
StringForm["(M,N) is (``,``)", v + 1, 1 + 2 t]
字串形式              數值化
```

Out[170]= The descendant cond of P(q),P*(q) is True

Out[171]= Dimension is {{d → 1 + 2 t}}

Out[172]= (M,N) is (2 + 4 t,1 + 2 t)

In[173]:=
```
(*Van Lint-Schrijver graphs: t is odd*)
G = "Van Lint-Schrijver graph: t is odd";
v = q;
```
$$k = \frac{q-1}{e};$$
$$lam = \frac{q - 3 e + 1 + (e - 1) (e - 2) \sqrt{q}}{e^2};$$
$$mu = \frac{q - e + 1 - (e - 2) \sqrt{q}}{e^2};$$
```
ass = q ≥ 2 && e > 2;

Desc[v, k, lam, mu, q, G]
```

Out[179]= The descendant cond of Van Lint-Schrijver graph: t is odd is $\frac{(-2+e)\left(1+\sqrt{q}\right)}{e} == 0$

In[180]:=
```
(*Van Lint-Schrijver graphs: t is even*)
G = "Van Lint-Schrijver graph: t is even";
v = q;
```
$k = \frac{q-1}{e};$

$lam = \frac{q - 3e + 1 - (e-1)(e-2)\sqrt{q}}{e^2};$

$mu = \frac{q - e + 1 + (e-2)\sqrt{q}}{e^2};$

```
ass = q ≥ 2 && e > 2;

Desc[v, k, lam, mu, q, G]
```

Out[186]= The descendant cond of Van Lint-Schrijver graph: t is even is $\frac{(-2+e)\left(-1+\sqrt{q}\right)}{e} == 0$

In[187]:=
(*$H_q(2,m)$*)
G = "$H_q(2,m)$";
$v = q^{2m};$
$k = (q+1)\left(q^m - 1\right);$
$lam = q^m + (q-2)(q+1);$
$mu = q(q+1);$
ass = m ≥ 2 && q ≥ 2;

Desc[v, k, lam, mu, ass, G]

Out[193]= The descendant cond of $H_q(2,m)$ is $(1+q)\left(-1-2q+q^m\right) == 0$

In[194]:=
(*$VO_{2m}^{+}(q)$*)
G = "$VO_{2m}^{+}(q)$";
$v = q^{2m};$
$k = \left(q^m - eps\right)\left(q^{m-1} + eps\right)$ /. eps → 1;
$lam = \left(q\left(q^{m-1} - eps\right)\left(q^{m-2} + eps\right) + q - 2\right)$ /. eps → 1;
$mu = q^{m-1}\left(q^{m-1} + eps\right)$ /. eps → 1;
ass = q ≥ 2 && m ≥ 1;

Desc[v, k, lam, mu, ass, G]
MN[v, k, lam, mu, {q → 3, m → 1}]

Out[200]= The descendant cond of $VO_{2m}^{+}(q)$ is $\frac{\left(q+q^m\right)\left(-q-2q^m+q^{1+m}\right)}{q} == 0$

The (M,N) are

{10, 5}

In[202]:=
```
(*VO2m⁻(q)*)
G = "VO2m⁻(q)";
v = q^(2 m);
k = (q^m - eps) (q^(m-1) + eps) /. eps → -1;
lam = (q (q^(m-1) - eps) (q^(m-2) + eps) + q - 2) /. eps → -1;
mu = q^(m-1) (q^(m-1) + eps) /. eps → -1;
ass = q ≥ 2 && m ≥ 1;

Desc[v, k, lam, mu, ass, G]
StringForm["The `` has parameters `` when ``",
 G, Simplify[{v, k, lam, mu} /. m → 1], m == 1]
```
字串形式 化簡

Out[208]= The descendant cond of $VO_{2m}^{-}(q)$ is $\frac{(-q+q^m)(q-2q^m+q^{1+m})}{q} == 0$

Out[209]= The $VO_{2m}^{-}(q)$ has parameters $\{q^2, 0, -q, 0\}$ when $m == 1$

In[210]:=
```
(*[3.4.2]*)
G = "graphs in 3.4.2";
v = q^10;
k = (q^2 + 1) (q^5 - 1);
lam = q^5 + q^4 - q^2 - 2;
mu = q^2 (q^2 + 1);
ass = q ≥ 2;
Desc[v, k, lam, mu, q, G]
Solve[q^5 + q^7 == 1 + 3 q^2 + 2 q^4, q, Integers]
```
求解 整數域

Out[216]= The descendant cond of graphs in 3.4.2 is $q^5 + q^7 == 1 + 3q^2 + 2q^4$

Out[217]= { }

In[218]:=
```
(*VD5,5(q)*)
G = "VD5,5(q)";
v = q^16;
k = (q^8 - 1) (q^3 + 1);
lam = q^8 + q^6 - q^3 - 2;
mu = q^3 (q^3 + 1);
ass = q ≥ 2;
Desc[v, k, lam, mu, q, G]
Solve[q^8 + q^11 == 1 + 3 q^3 + 2 q^6, q, Integers]
```
求解 整數域

Out[224]= The descendant cond of $VD_{5,5}(q)$ is $q^8 + q^{11} == 1 + 3q^3 + 2q^6$

Out[225]= $\{\{q \to -1\}\}$

In[226]:=

```
In[227]:= Sporadic = {{15, 6, 1, 3}, {35, 16, 6, 8}, {49, 24, 11, 12}, {55, 18, 9, 4},
    {64, 18, 2, 6}, {64, 27, 10, 12}, {66, 20, 10, 4}, {77, 16, 0, 4},
    {81, 16, 7, 2}, {81, 32, 13, 12}, {81, 40, 19, 20}, {100, 22, 0, 6},
    {100, 36, 14, 12}, {120, 56, 28, 24}, {126, 25, 8, 4}, {169, 72, 31, 30},
    {176, 70, 18, 34}, {243, 22, 1, 2}, {243, 110, 37, 60}, {253, 42, 21, 4},
    {253, 112, 36, 60}, {256, 45, 16, 6}, {256, 75, 26, 20}, {256, 102, 38, 42},
    {256, 120, 56, 56}, {275, 112, 30, 56}, {276, 44, 22, 4}, {289, 96, 35, 30},
    {351, 126, 45, 45}, {361, 144, 59, 56}, {416, 100, 36, 20}, {529, 264, 131, 132},
    {625, 144, 43, 30}, {625, 240, 95, 90}, {729, 104, 31, 12}, {729, 224, 61, 72},
    {841, 168, 47, 30}, {961, 240, 71, 56}, {961, 360, 139, 132},
    {1288, 495, 206, 180}, {1681, 480, 149, 132}, {1782, 416, 100, 96},
    {2016, 975, 462, 480}, {2048, 276, 44, 36}, {2048, 759, 310, 264},
    {2209, 1104, 551, 552}, {2300, 891, 378, 324}, {2401, 240, 59, 20},
    {2401, 480, 119, 90}, {2401, 720, 229, 210}, {2401, 960, 389, 380},
    {3510, 693, 180, 126}, {4060, 1755, 730, 780}, {4096, 315, 74, 20},
    {4096, 1575, 614, 600}, {5041, 840, 179, 132}, {6241, 1560, 419, 380},
    {6561, 1440, 351, 306}, {7921, 2640, 899, 870}, {14 080, 3159, 918, 648},
    {15 625, 744, 143, 30}, {15 625, 7560, 3655, 3660}, {31 671, 3510, 693, 351},
    {130 816, 32 319, 7742, 8064}, {137 632, 28 431, 6030, 5832},
    {306 936, 31 671, 3510, 3240}, {531 441, 65 520, 8559, 8010}};

In[228]:= MNsp[v0_, k0_, lam0_, mu0_] := Module[{v = v0, k = k0, lam = lam0, mu = mu0},
                                         模組
  dim =
   Solve[Simplify[(8 d^2 k + 4 d^2 k^2 - 8 d k (v + 1) - 4 d^2 k (v + 1) - 4 d k^2 (v + 1) - (v + 1)^2 +
   求解   化簡
        d^2 (v + 1)^2 + 4 d k (v + 1)^2 + (v + 1)^3 - d (v + 1)^3 == 0)], d];
  Print["The  (M,N)  are "];
  列表              數值化
  M = Simplify[(v + 1)];
      化簡
  Do[{Print[{M, N[[1]][[2]]}]}, {N, dim}]]
  D…  列表        數值化            數值化

In[229]:= Do[MNsp[srg[[1]], srg[[2]], srg[[3]], srg[[4]]], {srg, Sporadic}]
          Do迴圈
```

The (M,N) are

{16, 6}

{16, 10}

The (M,N) are

{36, 15}

{36, 21}

The (M,N) are

{50, 25}

The (M,N) are

$\left\{56, \frac{7}{17} \times \left(68 - 9\sqrt{34}\right)\right\}$

$\left\{56, \frac{7}{17} \times \left(68 + 9\sqrt{34}\right)\right\}$

The (M,N) are

$\left\{65, \frac{13}{394} \times \left(985 - 27\sqrt{985}\right)\right\}$

$\left\{65, \frac{13}{394} \times \left(985 + 27\sqrt{985}\right)\right\}$

The (M,N) are

$\left\{65, \frac{65}{674} \times \left(337 - 9\sqrt{337}\right)\right\}$

$\left\{65, \frac{65}{674} \times \left(337 + 9\sqrt{337}\right)\right\}$

The (M,N) are

$\left\{67, \frac{67 \times \left(889 - 25\sqrt{889}\right)}{1778}\right\}$

$\left\{67, \frac{67 \times \left(889 + 25\sqrt{889}\right)}{1778}\right\}$

The (M,N) are

$\left\{78, \frac{13}{17} \times \left(51 - 2\sqrt{561}\right)\right\}$

$\left\{78, \frac{13}{17} \times \left(51 + 2\sqrt{561}\right)\right\}$

The (M,N) are

$\left\{82, \frac{41}{73} \times \left(73 - 8\sqrt{73}\right)\right\}$

$\left\{82, \frac{41}{73} \times \left(73 + 8\sqrt{73}\right)\right\}$

The (M,N) are

$\left\{82, \frac{41}{145} \times \left(145 - 8\sqrt{145}\right)\right\}$

$\left\{82, \frac{41}{145} \times \left(145 + 8\sqrt{145}\right)\right\}$

The (M,N) are

$\{82, 41\}$

The (M,N) are

$\left\{101, \frac{101}{274} \times \left(137 - 11\sqrt{137}\right)\right\}$

$\left\{101, \frac{101}{274} \times \left(137 + 11\sqrt{137}\right)\right\}$

The (M,N) are

$\left\{101, \frac{101 \times \left(1129 - 27\sqrt{1129}\right)}{2258}\right\}$

$$\left\{101,\ \frac{101\times\left(1129+27\sqrt{1129}\right)}{2258}\right\}$$

The (M,N) are

$$\left\{121,\ \frac{968}{23}\right\}$$

$$\left\{121,\ \frac{1815}{23}\right\}$$

The (M,N) are

$$\left\{127,\ \frac{127\times\left(681-25\sqrt{681}\right)}{1362}\right\}$$

$$\left\{127,\ \frac{127\times\left(681+25\sqrt{681}\right)}{1362}\right\}$$

The (M,N) are

$$\left\{170,\ \frac{85}{313}\times\left(313-12\sqrt{313}\right)\right\}$$

$$\left\{170,\ \frac{85}{313}\times\left(313+12\sqrt{313}\right)\right\}$$

The (M,N) are

$$\left\{177,\ \frac{59\times\left(1929-35\sqrt{1929}\right)}{1286}\right\}$$

$$\left\{177,\ \frac{59\times\left(1929+35\sqrt{1929}\right)}{1286}\right\}$$

The (M,N) are

$$\left\{244,\ \frac{61}{31}\times\left(62-11\sqrt{31}\right)\right\}$$

$$\left\{244,\ \frac{61}{31}\times\left(62+11\sqrt{31}\right)\right\}$$

The (M,N) are

$$\left\{244,\ \frac{61}{91}\times\left(182-11\sqrt{91}\right)\right\}$$

$$\left\{244,\ \frac{61}{91}\times\left(182+11\sqrt{91}\right)\right\}$$

The (M,N) are

$$\left\{254,\ \frac{127\times\left(7309-84\sqrt{7309}\right)}{7309}\right\}$$

$$\left\{254,\ \frac{127\times\left(7309+84\sqrt{7309}\right)}{7309}\right\}$$

The (M,N) are

$$\left\{254,\ \frac{127}{449}\times\left(449-14\sqrt{449}\right)\right\}$$

$\left\{254, \frac{127}{449} \times \left(449 + 14 \sqrt{449}\right)\right\}$

The (M,N) are

$\left\{257, \frac{257 \times \left(28249 - 165 \sqrt{28249}\right)}{56498}\right\}$

$\left\{257, \frac{257 \times \left(28249 + 165 \sqrt{28249}\right)}{56498}\right\}$

The (M,N) are

$\left\{257, \frac{257 \times \left(12049 - 105 \sqrt{12049}\right)}{24098}\right\}$

$\left\{257, \frac{257 \times \left(12049 + 105 \sqrt{12049}\right)}{24098}\right\}$

The (M,N) are

$\left\{257, \frac{257 \times \left(725 - 51 \sqrt{145}\right)}{1450}\right\}$

$\left\{257, \frac{257 \times \left(725 + 51 \sqrt{145}\right)}{1450}\right\}$

The (M,N) are

$\left\{257, \frac{257 \times \left(1249 - 15 \sqrt{1249}\right)}{2498}\right\}$

$\left\{257, \frac{257 \times \left(1249 + 15 \sqrt{1249}\right)}{2498}\right\}$

The (M,N) are

$\{276, 23\}$

$\{276, 253\}$

The (M,N) are

$\left\{277, \frac{277 \times \left(36073 - 187 \sqrt{36073}\right)}{72146}\right\}$

$\left\{277, \frac{277 \times \left(36073 + 187 \sqrt{36073}\right)}{72146}\right\}$

The (M,N) are

$\left\{290, \frac{145 \times \left(2593 - 48 \sqrt{2593}\right)}{2593}\right\}$

$\left\{290, \frac{145 \times \left(2593 + 48 \sqrt{2593}\right)}{2593}\right\}$

The (M,N) are

$\left\{352, \frac{22}{43} \times \left(344 - 49 \sqrt{43}\right)\right\}$

$\left\{352, \frac{22}{43} \times \left(344 + 49\sqrt{43}\right)\right\}$

The (M,N) are

$\left\{362, \frac{181 \times \left(1657 - 36\sqrt{1657}\right)}{1657}\right\}$

$\left\{362, \frac{181 \times \left(1657 + 36\sqrt{1657}\right)}{1657}\right\}$

The (M,N) are

$\left\{417, \frac{139 \times \left(15\,963 - 215\sqrt{5321}\right)}{10\,642}\right\}$

$\left\{417, \frac{139 \times \left(15\,963 + 215\sqrt{5321}\right)}{10\,642}\right\}$

The (M,N) are

$\{530, 265\}$

The (M,N) are

$\left\{626, \frac{313 \times \left(28\,849 - 168\sqrt{28\,849}\right)}{28\,849}\right\}$

$\left\{626, \frac{313 \times \left(28\,849 + 168\sqrt{28\,849}\right)}{28\,849}\right\}$

The (M,N) are

$\left\{626, \frac{313 \times \left(5809 - 72\sqrt{5809}\right)}{5809}\right\}$

$\left\{626, \frac{313 \times \left(5809 + 72\sqrt{5809}\right)}{5809}\right\}$

The (M,N) are

$\left\{730, \frac{266\,085}{68\,329 + 260\sqrt{68\,329}}\right\}$

$\left\{730, \frac{365 \times \left(68\,329 + 260\sqrt{68\,329}\right)}{68\,329}\right\}$

The (M,N) are

$\left\{730, \frac{365 \times \left(20\,329 - 140\sqrt{20\,329}\right)}{20\,329}\right\}$

$\left\{730, \frac{365 \times \left(20\,329 + 140\sqrt{20\,329}\right)}{20\,329}\right\}$

The (M,N) are

$\left\{842, \frac{421 \times \left(64\,345 - 252\sqrt{64\,345}\right)}{64\,345}\right\}$

$\left\{842, \frac{421 \times \left(64\,345 + 252\sqrt{64\,345}\right)}{64\,345}\right\}$

The (M,N) are

$$\left\{962, \frac{481 \times \left(58561 - 240\sqrt{58561}\right)}{58561}\right\}$$

$$\left\{962, \frac{481 \times \left(58561 + 240\sqrt{58561}\right)}{58561}\right\}$$

The (M,N) are

$$\left\{962, \frac{481 \times \left(15361 - 120\sqrt{15361}\right)}{15361}\right\}$$

$$\left\{962, \frac{481 \times \left(15361 + 120\sqrt{15361}\right)}{15361}\right\}$$

The (M,N) are

$$\left\{1289, \frac{1289 \times \left(93361 - 297\sqrt{93361}\right)}{186722}\right\}$$

$$\left\{1289, \frac{1289 \times \left(93361 + 297\sqrt{93361}\right)}{186722}\right\}$$

The (M,N) are

$$\left\{1682, \frac{841 \times \left(131281 - 360\sqrt{131281}\right)}{131281}\right\}$$

$$\left\{1682, \frac{841 \times \left(131281 + 360\sqrt{131281}\right)}{131281}\right\}$$

The (M,N) are

$$\left\{1783, \frac{6354612}{907729 + 949\sqrt{907729}}\right\}$$

$$\left\{1783, \frac{1783 \times \left(907729 + 949\sqrt{907729}\right)}{1815458}\right\}$$

The (M,N) are

$$\left\{2017, \frac{2017 \times \left(12289 - 65\sqrt{12289}\right)}{24578}\right\}$$

$$\left\{2017, \frac{2017 \times \left(12289 + 65\sqrt{12289}\right)}{24578}\right\}$$

The (M,N) are

$$\left\{2049, \frac{8392704}{2243217 + 1495\sqrt{2243217}}\right\}$$

$$\left\{2049, \frac{683 \times \left(2243217 + 1495\sqrt{2243217}\right)}{1495478}\right\}$$

The (M,N) are

$$\left\{2049, \frac{683 \times \left(288033 - 529\sqrt{288033}\right)}{192022}\right\}$$

$\left\{2049, \frac{683 \times \left(288033 + 529 \sqrt{288033}\right)}{192022}\right\}$

The (M,N) are

{2210, 1105}

The (M,N) are

$\left\{2301, \frac{767 \times \left(92163 - 517 \sqrt{30721}\right)}{61442}\right\}$

$\left\{2301, \frac{767 \times \left(92163 + 517 \sqrt{30721}\right)}{61442}\right\}$

The (M,N) are

$\left\{2402, \frac{2883601}{924001 + 960 \sqrt{924001}}\right\}$

$\left\{2402, \frac{1201 \times \left(924001 + 960 \sqrt{924001}\right)}{924001}\right\}$

The (M,N) are

$\left\{2402, \frac{2883601}{520801 + 720 \sqrt{520801}}\right\}$

$\left\{2402, \frac{1201 \times \left(520801 + 720 \sqrt{520801}\right)}{520801}\right\}$

The (M,N) are

$\left\{2402, \frac{2883601}{232801 + 480 \sqrt{232801}}\right\}$

$\left\{2402, \frac{1201 \times \left(232801 + 480 \sqrt{232801}\right)}{232801}\right\}$

The (M,N) are

$\left\{2402, \frac{1201 \times \left(60001 - 240 \sqrt{60001}\right)}{60001}\right\}$

$\left\{2402, \frac{1201 \times \left(60001 + 240 \sqrt{60001}\right)}{60001}\right\}$

The (M,N) are

$\left\{3511, \frac{24647220}{4521169 + 2123 \sqrt{4521169}}\right\}$

$\left\{3511, \frac{3511 \times \left(4521169 + 2123 \sqrt{4521169}\right)}{9042338}\right\}$

The (M,N) are

$\left\{4061, \frac{4061 \times \left(317641 - 549 \sqrt{317641}\right)}{635282}\right\}$

$\left\{4061, \frac{4061 \times \left(317641 + 549 \sqrt{317641}\right)}{635282}\right\}$

The (M,N) are

$$\left\{4097, \frac{33\,562\,624}{12\,022\,609 + 3465\,\sqrt{12\,022\,609}}\right\}$$

$$\left\{4097, \frac{4097 \times \left(12\,022\,609 + 3465\,\sqrt{12\,022\,609}\right)}{24\,045\,218}\right\}$$

The (M,N) are

$$\left\{4097, \frac{4097 \times \left(909\,409 - 945\,\sqrt{909\,409}\right)}{1\,818\,818}\right\}$$

$$\left\{4097, \frac{4097 \times \left(909\,409 + 945\,\sqrt{909\,409}\right)}{1\,818\,818}\right\}$$

The (M,N) are

$$\left\{5042, \frac{12\,708\,361}{2\,827\,441 + 1680\,\sqrt{2\,827\,441}}\right\}$$

$$\left\{5042, \frac{2521 \times \left(2\,827\,441 + 1680\,\sqrt{2\,827\,441}\right)}{2\,827\,441}\right\}$$

The (M,N) are

$$\left\{6242, \frac{19\,478\,161}{2\,439\,841 + 1560\,\sqrt{2\,439\,841}}\right\}$$

$$\left\{6242, \frac{3121 \times \left(2\,439\,841 + 1560\,\sqrt{2\,439\,841}\right)}{2\,439\,841}\right\}$$

The (M,N) are

$$\left\{6562, \frac{21\,526\,641}{3\,392\,161 + 1840\,\sqrt{3\,392\,161}}\right\}$$

$$\left\{6562, \frac{3281 \times \left(3\,392\,161 + 1840\,\sqrt{3\,392\,161}\right)}{3\,392\,161}\right\}$$

The (M,N) are

$$\left\{7922, \frac{31\,375\,081}{1\,750\,321 + 1320\,\sqrt{1\,750\,321}}\right\}$$

$$\left\{7922, \frac{3961 \times \left(1\,750\,321 + 1320\,\sqrt{1\,750\,321}\right)}{1\,750\,321}\right\}$$

The (M,N) are

$$\left\{14\,081, \frac{396\,520\,960}{60\,289\,441 + 7761\,\sqrt{60\,289\,441}}\right\}$$

$$\left\{14\,081, \frac{14\,081 \times \left(60\,289\,441 + 7761\,\sqrt{60\,289\,441}\right)}{120\,578\,882}\right\}$$

The (M,N) are

$$\left\{15\,626, \frac{122\,078\,125}{49\,972\,249 + 7068\,\sqrt{49\,972\,249}}\right\}$$

$$\left\{15\,626,\ \frac{7813 \times \left(49\,972\,249 + 7068\ \sqrt{49\,972\,249}\right)}{49\,972\,249}\right\}$$

The (M,N) are

$$\left\{15\,626,\ \frac{7813 \times \left(79\,129 - 252\ \sqrt{79\,129}\right)}{79\,129}\right\}$$

$$\left\{15\,626,\ \frac{7813 \times \left(79\,129 + 252\ \sqrt{79\,129}\right)}{79\,129}\right\}$$

The (M,N) are

$$\left\{31\,672,\ \frac{7\,375\,617}{2\,234\,372 + 725\ \sqrt{9\,496\,081}}\right\}$$

$$\left\{31\,672,\ \frac{3959 \times \left(2\,234\,372 + 725\ \sqrt{9\,496\,081}\right)}{558\,593}\right\}$$

The (M,N) are

$$\left\{130\,817,\ \frac{4\,477\,331\,305\,922\,048}{572\,967\,810\,580\,481 + 66\,177\ \sqrt{74\,953\,930\,076\,706\,782\,977}}\right\}$$

$$\left\{130\,817,\ \frac{572\,967\,810\,580\,481 + 66\,177\ \sqrt{74\,953\,930\,076\,706\,782\,977}}{8\,759\,837\,186}\right\}$$

The (M,N) are

$$\left\{137\,633,\ \frac{5\,214\,282\,649\,944\,896}{897\,942\,725\,928\,737 + 80\,769\ \sqrt{123\,586\,551\,197\,749\,859\,521}}\right\}$$

$$\left\{137\,633,\ \frac{897\,942\,725\,928\,737 + 80\,769\ \sqrt{123\,586\,551\,197\,749\,859\,521}}{13\,048\,363\,778}\right\}$$

The (M,N) are

$$\left\{306\,937,\ \frac{3\,845\,306\,736}{7 \times \left(172\,999\,351 + 34\,799\ \sqrt{24\,714\,193}\right)}\right\}$$

$$\left\{306\,937,\ \frac{306\,937 \times \left(172\,999\,351 + 34\,799\ \sqrt{24\,714\,193}\right)}{345\,998\,702}\right\}$$

The (M,N) are

$$\left\{531\,442,\ \frac{141\,215\,033\,961}{40\,080\,571\,441 + 200\,200\ \sqrt{40\,080\,571\,441}}\right\}$$

$$\left\{531\,442,\ \frac{265\,721 \times \left(40\,080\,571\,441 + 200\,200\ \sqrt{40\,080\,571\,441}\right)}{40\,080\,571\,441}\right\}$$

In[230]:=
```
Export["descendant.pdf", EvaluationNotebook[]]
```
匯出 計算過的筆記本