In[1]:=

```
(*Complement graph*)
 補集
Comp[v_, k_, lam_, mu_] := {v, v - k - 1, v - 2 k + mu - 2, v - 2 k + lam}

(*Regular two-graph conds*)
ETFcond[v0_, k0_, lam0_, mu0_, assumption0_, G0_] :=
 Module[{v = v0, k = k0, lam = lam0, mu = mu0, assumption = assumption0, G = G0},
 模組
  cond = Simplify[ v == 2 × (2 k - lam - mu), assumption];
          化簡
  StringForm["The ETF cond of `` is ``", G, cond]]
  字串形式

(*Some numbers*)
Bino[n_, m_, q_] := Product[(q^(n-i) - 1)/(q^(m-i) - 1), {i, 0, m - 1}]
                    乘積

(*Eiganvalues r and s*)
reig[v_, k_, lam_, mu_] := 1/2 ((lam - mu) + Sqrt[(lam - mu)^2 + 4 (k - mu)])
seig[v_, k_, lam_, mu_] := 1/2 ((lam - mu) - Sqrt[(lam - mu)^2 + 4 (k - mu)])

(*P and Q matrices*)
Pmat[v_, k_, lam_, mu_] := {{1, k, v - k - 1},
                            {1, reig[v, k, lam, mu], -reig[v, k, lam, mu] - 1},
                            {1, seig[v, k, lam, mu], -seig[v, k, lam, mu] - 1}}
Qmat[v_, k_, lam_, mu_] := v Inverse[Pmat[v, k, lam, mu]]
                             逆
```

In[8]:=
```
(*Lattice graph*)
G = "L2(q)";
v = q^2;
k = 2 (q - 1);
lam = q - 2;
mu = 2;
ass = q ≥ 2;

ETFcond[v, k, lam, mu, ass, G]
StringForm["The Q-matrix of `` when ``=`` is ``",
字串形式
 G, q, 4, MatrixForm[Qmat[v, k, lam, mu] /. q → 4]]
           矩陣格式
StringForm["The Q-matrix of `` when ``=`` is ``",
字串形式
 G, q, 4, MatrixForm[Qmat[v, k, lam, mu] /. q → 4]]
           矩陣格式
StringForm["The Q-matrix of `` when ``=`` is ``",
字串形式
 G, q, 2, MatrixForm[Qmat[v, k, lam, mu] /. q → 2]]
           矩陣格式
```

Out[14]= The ETF cond of $L_2(q)$ is $8 + q^2 == 6q$

Out[15]= The Q-matrix of $L_2(q)$ when q=4 is $\begin{pmatrix} 1 & 6 & 9 \\ 1 & 2 & -3 \\ 1 & -2 & 1 \end{pmatrix}$

Out[16]= The Q-matrix of $L_2(q)$ when q=4 is $\begin{pmatrix} 1 & 6 & 9 \\ 1 & 2 & -3 \\ 1 & -2 & 1 \end{pmatrix}$

Out[17]= The Q-matrix of $L_2(q)$ when q=2 is $\begin{pmatrix} 1 & 2 & 1 \\ 1 & 0 & -1 \\ 1 & -2 & 1 \end{pmatrix}$

In[18]:=
```
(*Triangle graph*)
  三角形
G = "T(n)";
v = n (n - 1) / 2;
k = 2 (n - 2);
lam = n - 2;
mu = 4;
ass = n ≥ 5;

ETFcond[v, k, lam, mu, ass, G]
StringForm["The Q-matrix of `` when ``=`` is ``",
字串形式
 G, n, 8, MatrixForm[Qmat[v, k, lam, mu] /. n → 8]]
          矩陣格式
StringForm["The Q-matrix of `` when ``=`` is ``",
字串形式
 G, n, 5, MatrixForm[Qmat[v, k, lam, mu] /. n → 5]]
          矩陣格式
```

Out[24]= The ETF cond of T(n) is $40 + n^2 == 13\,n$

Out[25]= The Q-matrix of T(n) when n=8 is $\begin{pmatrix} 1 & 7 & 20 \\ 1 & \frac{7}{3} & -\frac{10}{3} \\ 1 & -\frac{7}{3} & \frac{4}{3} \end{pmatrix}$

Out[26]= The Q-matrix of T(n) when n=5 is $\begin{pmatrix} 1 & 4 & 5 \\ 1 & \frac{2}{3} & -\frac{5}{3} \\ 1 & -\frac{8}{3} & \frac{5}{3} \end{pmatrix}$

In[27]:=
```
(*Sp2n(q),O2n+1q*)
G = "Sp2 n(q),O2 n+1q";
```

$$v = \frac{q^n - 1}{q - 1}\left(t\,q^{n-1} + 1\right)\ /.\ t \to q;$$

$$k = q\,\frac{q^{n-1} - 1}{q - 1}\left(t\,q^{n-2} + 1\right)\ /.\ t \to q;$$

$$lam = \left(q^2\,\frac{q^{n-2} - 1}{q - 1}\left(t\,q^{n-3} + 1\right) + q - 1\right)\ /.\ t \to q;$$

$$mu = \frac{k}{q}\ /.\ t \to q;$$

$$ass = q \geq 2\ \&\&\ n \geq 2;$$

```
ETFcond[v, k, lam, mu, ass, G]
```

StringForm["It is equivalent to ``.", Factor$\left[-1 + q^{2n} - 4\,q^{-2+2n}\,(-1 + q)\right]\,q^2 == 0$]

字串形式 因式分解

Out[33]= The ETF cond of $Sp_{2n}(q), O_{2n+1}q$ is $\frac{-1 + q^{2n}}{-1 + q} == 4\,q^{-2+2n}$

Out[34]= It is equivalent to $\left(-q - 2\,q^n + q^{1+n}\right)\left(q - 2\,q^n + q^{1+n}\right) == 0$.

In[35]:=
```
(*O_{2n}^+ (q) *)
G = "O_{2 n}^+ (q)";
```

$$v = \frac{q^n - 1}{q - 1} \left(t\, q^{n-1} + 1\right) \text{ /. } t \to 1;$$

$$k = q\, \frac{q^{n-1} - 1}{q - 1} \left(t\, q^{n-2} + 1\right) \text{ /. } t \to 1;$$

$$lam = \left(q^2\, \frac{q^{n-2} - 1}{q - 1} \left(t\, q^{n-3} + 1\right) + q - 1\right) \text{ /. } t \to 1;$$

$$mu = \frac{k}{q} \text{ /. } t \to 1;$$

$$ass = q \ge 2 \;\&\&\; n \ge 2;$$

```
ETFcond[v, k, lam, mu, ass, G]
StringForm["It is equivalent to ``.",
```
字串形式

$$\text{Factor}\left[\left(1 + q^{-1+n}\right) \times \left(-1 + q^n\right) - 2\, q^{-3+n} \left(-q + q^2 + 2\, q^n\right) \; (-1 + q)\right] q^3 == 0\right]$$
因式分解

```
StringForm["The Q-matrix of `` when ``=`` is ``", G, {q, n},
```
字串形式

```
 {3, 2}, MatrixForm[Qmat[v, k, lam, mu] /. {q → 3, n → 2}]]
```
矩陣格式

Out[41]= The ETF cond of $O_{2\,n}{}^{+}(q)$ is $\frac{\left(1+q^{-1+n}\right)\left(-1+q^{n}\right)}{-1+q} == 2\, q^{-3+n}\left(-q+q^{2}+2\, q^{n}\right)$

Out[42]= It is equivalent to $\left(q - 2\, q^{n} + q^{1+n}\right)\left(-q^{2} - 2\, q^{n} + q^{1+n}\right) == 0.$

Out[43]= The Q-matrix of $O_{2\,n}{}^{+}(q)$ when $\{q, n\} = \{3, 2\}$ is $\begin{pmatrix} 1 & 6 & 9 \\ 1 & 2 & -3 \\ 1 & -2 & 1 \end{pmatrix}$

In[44]:=

```
(*O_{2n+2}^-(q)*)
G = "O_{2 n+2}^-(q)";
v = (q^n - 1)/(q - 1) (t q^(n-1) + 1) /. t → q^2;
k = q (q^(n-1) - 1)/(q - 1) (t q^(n-2) + 1) /. t → q^2;
lam = (q^2 (q^(n-2) - 1)/(q - 1) (t q^(n-3) + 1) + q - 1) /. t → q^2;
mu = k/q /. t → q^2;
ass = q ≥ 2 && n ≥ 2;

ETFcond[v, k, lam, mu, ass, G]
StringForm["It is equivalent to ``.",
字串形式
 Factor[(-1 + q^n) × (1 + q^(1+n)) - q^(-1+n) (2 - 2 q + 4 q^n) × (-1 + q)] q == 0]
 因式分解
```

Out[50]= The ETF cond of $O_{2\,n+2}^{-}(q)$ is $\frac{(-1+q^n)\,(1+q^{1+n})}{-1+q} == q^{-1+n}\,(2-2\,q+4\,q^n)$

Out[51]= It is equivalent to $(-1-2\,q^n+q^{1+n})\,(q-2\,q^n+q^{1+n}) == 0$.

In[52]:=

```
(*U_{2n}(√q)*)
G = "U_{2 n}(√q)";
v = (q^n - 1)/(q - 1) (t q^(n-1) + 1) /. t → q^(1/2);
k = q (q^(n-1) - 1)/(q - 1) (t q^(n-2) + 1) /. t → q^(1/2);
lam = (q^2 (q^(n-2) - 1)/(q - 1) (t q^(n-3) + 1) + q - 1) /. t → q^(1/2);
mu = k/q /. t → q^(1/2);
ass = q ≥ 2 && n ≥ 2;

ETFcond[v, k, lam, mu, ass, G]
```

Out[58]= The ETF cond of $U_{2\,n}(\sqrt{q})$ is $(q^{3/2}+2\,q^n-q^{1+n})\,(q-2\,q^n+q^{1+n}) == 0$

In[59]:=
```
(*U_{2n+1}(√q)*)
G = "U_{2 n+1}(√q)";
```
$$v = \frac{q^n - 1}{q - 1}\left(t\,q^{n-1} + 1\right) \text{ /. } t \to q^{3/2};$$

$$k = q\,\frac{q^{n-1} - 1}{q - 1}\left(t\,q^{n-2} + 1\right) \text{ /. } t \to q^{3/2};$$

$$lam = \left(q^2\,\frac{q^{n-2} - 1}{q - 1}\left(t\,q^{n-3} + 1\right) + q - 1\right) \text{ /. } t \to q^{3/2};$$

$$mu = \frac{k}{q} \text{ /. } t \to q^{3/2};$$

```
ass = q ≥ 2 && n ≥ 2;

ETFcond[v, k, lam, mu, ass, G]
StringForm["It is equivalent to ``.",
```
字串形式

$$\text{Simplify}\left[\left(-1 + q^n\right) \times \left(1 + q^{\frac{1}{2}+n}\right) - 2\,q^{-\frac{3}{2}+n}\left(\sqrt{q} - q + 2\,q^n\right)(-1 + q) == 0,\ ass\right]\right]$$

化簡

Out[65]= The ETF cond of $U_{2\,n+1}(\sqrt{q})$ is $\frac{\left(-1 + q^n\right)\left(1 + q^{\frac{1}{2}+n}\right)}{-1 + q} == 2\,q^{-\frac{3}{2}+n}\left(\sqrt{q} - q + 2\,q^n\right)$

Out[66]= It is equivalent to $\left(-\sqrt{q} - 2\,q^n + q^{1+n}\right)\left(q - 2\,q^n + q^{1+n}\right) == 0$.

In[67]:=
```
(*Thm2.2.19*)
G = "graphs in Thm 2.2.19";
v = (1 + t) × (1 + q t);
k = t (q + 1);
lam = t - 1;
mu = k / t;
ass = q ≥ 2 && t > 0;

ETFcond[v, k, lam, mu, ass, G]
StringForm["The Q-matrix of `` when ``=`` is ``", G,
```
字串形式
```
 t, 1, Simplify[MatrixForm[Qmat[v, k, lam, mu] /. t → 1], ass]]
```
化簡 矩陣格式

Out[73]= The ETF cond of graphs in Thm 2.2.19 is $(-1 + q\,(-2 + t))\,(-1 + t) == 0$

Out[74]= The Q-matrix of graphs in Thm 2.2.19 when t=1 is $\begin{pmatrix} 1 & 2\,q & 1 \\ 1 & 0 & -1 \\ 1 & -2 & 1 \end{pmatrix}$

In[75]:=

In[76]:=
```
(*Thm2.2.20*)
G = "graph in Thm 2.2.20";
v = (q^4 + 1) (q^3 + 1) (q^2 + 1) (q + 1);
k = q Bino[5, 2, q];
lam = q - 1 + q^2 (q + 1) (q^2 + q + 1);
mu = Bino[4, 2, q];
ass = q ≥ 2;

ETFcond[v, k, lam, mu, ass, G]
StringForm["because `` is impossible.", Simplify[v - 2 × (2 k - lam - mu)] == 0]
```
字串形式 化簡

Out[82]= The ETF cond of graph in Thm 2.2.20 is False

Out[83]= because $1 + q + 3 q^2 - 4 q^5 - 2 q^6 - 2 q^7 + q^8 + q^9 + q^{10} == 0$ is impossible.

In[84]:=
```
(*NU_n(q): n even*)
G = "NU_n(q) for even n";
v = q^(n-1) (q^n - eps)/(q + 1) /. eps → 1;
k = (q^(n-1) + eps) (q^(n-2) - eps) /. eps → 1;
lam = (q^(2 n-5) (q + 1) - eps q^(n-2) (q - 1) - 2) /. eps → 1;
mu = q^(n-3) (q + 1) (q^(n-2) - eps) /. eps → 1;
ass = n ≥ 3 && q ≥ 2;

ETFcond[v, k, lam, mu, ass, G]
```

Out[90]= The ETF cond of $NU_n(q)$ for even n is $(-2 - 2 q + q^2) (q^2 + 2 q^3 - 2 q^n - 2 q^{1+n} + q^{2+n}) == 0$

In[91]:=
```
(*NU_n(q): n odd*)
G = "NU_n(q) for odd n";
v = q^(n-1) (q^n - eps)/(q + 1) /. eps → -1;
k = (q^(n-1) + eps) (q^(n-2) - eps) /. eps → -1;
lam = (q^(2 n-5) (q + 1) - eps q^(n-2) (q - 1) - 2) /. eps → -1;
mu = q^(n-3) (q + 1) (q^(n-2) - eps) /. eps → -1;
ass = n ≥ 3 && q ≥ 2;

ETFcond[v, k, lam, mu, ass, G]
```

Out[97]= The ETF cond of $NU_n(q)$ for odd n is $(-2 - 2 q + q^2) (q^2 + 2 q^3 + 2 q^n + 2 q^{1+n} - q^{2+n}) == 0$

In[98]:=
```
(*NO2m+(2)*)
G = "NO2m+(2)";
v = (2^(2m-1) - eps 2^(m-1)) /. eps → 1;
k = (2^(2m-2) - 1) /. eps → 1;
lam = (2^(2m-3) - 2) /. eps → 1;
mu = (2^(2m-3) + eps 2^(m-2)) /. eps → 1;
ass = m ≥ 2;

ETFcond[v, k, lam, mu, ass, G]
StringForm["The Q-matrix of `` is ``",
 G, Simplify[MatrixForm[Qmat[v, k, lam, mu]], ass]]
```

字串形式 化簡 矩陣格式

Out[104]= The ETF cond of $NO_{2m}^{+}(2)$ is True

Out[105]= The Q-matrix of $NO_{2m}^{+}(2)$ is $\begin{pmatrix} 1 & \frac{1}{3}\left(-4+4^{m}\right) & \frac{1}{6}\left(-2+2^{m}\right)\left(-1+2^{m}\right) \\ 1 & \frac{1}{3}\left(-4+2^{m}\right) & \frac{1}{3}\left(1-2^{m}\right) \\ 1 & \frac{1}{3}\left(-2-2^{m}\right) & \frac{1}{3}\left(-1+2^{m}\right) \end{pmatrix}$

In[106]:=
```
(*NO2m-(2)*)
G = "NO2m-(2)";
v = (2^(2m-1) - eps 2^(m-1)) /. eps → -1;
k = (2^(2m-2) - 1) /. eps → -1;
lam = (2^(2m-3) - 2) /. eps → -1;
mu = (2^(2m-3) + eps 2^(m-2)) /. eps → -1;
ass = m ≥ 2;

ETFcond[v, k, lam, mu, ass, G]
StringForm["The Q-matrix of `` is ``",
 G, Simplify[MatrixForm[Qmat[v, k, lam, mu]], ass]]
```

字串形式 化簡 矩陣格式

Out[112]= The ETF cond of $NO_{2m}^{-}(2)$ is True

Out[113]= The Q-matrix of $NO_{2m}^{-}(2)$ is $\begin{pmatrix} 1 & \frac{1}{6}\left(1+2^{m}\right)\left(2+2^{m}\right) & \frac{1}{3}\left(-4+4^{m}\right) \\ 1 & \frac{1}{3}\left(1+2^{m}\right) & \frac{1}{3}\left(-4-2^{m}\right) \\ 1 & \frac{1}{3}\left(-1-2^{m}\right) & \frac{1}{3}\left(-2+2^{m}\right) \end{pmatrix}$

In[114]:=
```
(*NO2m+(3)*)
G = "NO2m+(3)";
v = 1/2 × 3^(m-1) × (3^m - eps) /. eps → 1;
k = 1/2 × 3^(m-1) × (3^(m-1) - eps) /. eps → 1;
lam = 1/2 × 3^(m-2) × (3^(m-1) + eps) /. eps → 1;
mu = 1/2 × 3^(m-1) × (3^(m-2) - eps) /. eps → 1;
ass = m ≥ 2;

ETFcond[v, k, lam, mu, ass, G]
StringForm["because `` is impossible", Simplify[v - 2 × (2 k - lam - mu) == 0]]
```
字串形式 化簡

Out[120]= The ETF cond of $NO_{2m}^{+}(3)$ is False

Out[121]= because $3^m (15 + 3^m) == 0$ is impossible

In[122]:=
```
(*NO2m+(3)*)
v = 1/2 × 3^(m-1) × (3^m - eps) /. eps → -1;
k = 1/2 × 3^(m-1) × (3^(m-1) - eps) /. eps → -1;
lam = 1/2 × 3^(m-2) × (3^(m-1) + eps) /. eps → -1;
mu = 1/2 × 3^(m-1) × (3^(m-2) - eps) /. eps → -1;
ass = m ≥ 2;

ETFcond[v, k, lam, mu, ass, G]
StringForm["because `` is impossible", Simplify[v - 2 × (2 k - lam - mu) == 0]]
```
字串形式 化簡

Out[127]= The ETF cond of $NO_{2m}^{+}(3)$ is $3^m == 15$

Out[128]= because $9^m == 5\,3^{1+m}$ is impossible

```
In[129]:= (*NO2m+1+(q)*)
G = "NO2 m+1+(q)";
v = 1/2 q^m (q^m + eps) /. eps → 1;
k = (q^(m-1) + eps) (q^m - eps) /. eps → 1;
lam = (2 (q^(2 m-2) - 1) + eps q^(m-1) (q - 1)) /. eps → 1;
mu = 2 q^(m-1) (q^(m-1) + eps) /. eps → 1;
ass = q ≥ 2 && m ≥ 1;

ETFcond[v, k, lam, mu, ass, G]
StringForm["The Q-matrix of `` for ``=`` is ``", G,
 q, 4, Simplify[MatrixForm[Qmat[v, k, lam, mu] /. q → 4], ass]]
StringForm["The Q-matrix of `` for ``=`` is ``", G, {q, m}, {7, 1},
 Simplify[MatrixForm[Qmat[v, k, lam, mu] /. {q → 7, m → 1}], ass]]
```

字串形式 化簡 矩陣格式 字串形式 化簡 矩陣格式

Out[135]= The ETF cond of $NO_{2\,m+1}{}^{+}(q)$ is $(-4+q)\left(-3\,q-4\,q^m+q^{1+m}\right) == 0$

Out[136]= The Q-matrix of $NO_{2\,m+1}{}^{+}(q)$ for q=4 is $\begin{pmatrix} 1 & \frac{1}{6}\left(-1+4^m\right)\left(4+4^m\right) & \frac{1}{3}\left(-1+16^m\right) \\ 1 & \frac{1}{3}\left(-2+4^m\right) & \frac{1}{3}\left(-1-4^m\right) \\ 1 & \frac{1}{3}\left(-4-4^m\right) & \frac{1}{3}\left(1+4^m\right) \end{pmatrix}$

Out[137]= The Q-matrix of $NO_{2\,m+1}{}^{+}(q)$ for $\{q, m\}=\{7, 1\}$ is $\begin{pmatrix} 1 & 7 & 20 \\ 1 & \frac{7}{3} & -\frac{10}{3} \\ 1 & -\frac{7}{3} & \frac{4}{3} \end{pmatrix}$

In[138]:=
```
(*NO2m+1^-(q)*)
G = "NO2 m+1^-(q)";
v = 1/2 q^m (q^m + eps) /. eps → -1;
k = (q^(m-1) + eps) (q^m - eps) /. eps → -1;
lam = (2 (q^(2 m-2) - 1) + eps q^(m-1) (q - 1)) /. eps → -1;
mu = 2 q^(m-1) (q^(m-1) + eps) /. eps → -1;
ass = q ≥ 2 && m ≥ 1;

ETFcond[v, k, lam, mu, ass, G]
StringForm["The Q-matrix of `` for ``=`` is ``", G,
字串形式
 q, 4, Simplify[MatrixForm[Qmat[v, k, lam, mu] /. q → 4], ass]]
       化簡          矩陣格式
StringForm["The Q-matrix of `` for ``=`` is ``", G, {q, m}, {3, 2},
字串形式
 Simplify[MatrixForm[Qmat[v, k, lam, mu] /. {q → 3, m → 2}], ass]]
 化簡          矩陣格式
```

Out[144]= The ETF cond of $NO_{2\,m+1}^{-}(q)$ is $(-4+q)\left(3\,q-4\,q^{m}+q^{1+m}\right)=0$

Out[145]= The Q-matrix of $NO_{2\,m+1}^{-}(q)$ for q=4 is $\begin{pmatrix} 1 & \frac{1}{3}\left(-1+16^{m}\right) & \frac{1}{6}\left(-4+4^{m}\right)\left(1+4^{m}\right) \\ 1 & \frac{1}{3}\left(-1+4^{m}\right) & \frac{1}{3}\left(-2-4^{m}\right) \\ 1 & \frac{1}{3}\left(1-4^{m}\right) & \frac{1}{3}\left(-4+4^{m}\right) \end{pmatrix}$

Out[146]= The Q-matrix of $NO_{2\,m+1}^{-}(q)$ for $\{q, m\}=\{3, 2\}$ is $\begin{pmatrix} 1 & 20 & 15 \\ 1 & 2 & -3 \\ 1 & -4 & 3 \end{pmatrix}$

In[147]:=
```
(*[3.5.1]*)
G = "graphs in 3.5.1";
v = Bino[n, 2, q];
k = (q + 1) (Bino[n - 1, 1, q] - 1);
lam = Bino[n - 1, 1, q] + q^2 - 2;
mu = (q + 1)^2;
r = q^2 Bino[n - 3, 1, q] - 1;
s = -q - 1;
ass = q ≥ 2 && n ≥ 4;

ETFcond[v, k, lam, mu, ass, G]
StringForm["It is equivalent to ``",
字串形式
 Factor[q + 2 q^2 + 4 q^5 + 4 q^6 + q^n + q^(2 n) + 3 q^(1+n) - 2 q^2 (2 q + 3 q^2 + q^n + 2 q^(1+n))] == 0]
 因式分解
```

Out[155]= The ETF cond of graphs in 3.5.1 is
$q+2\,q^{2}+4\,q^{5}+4\,q^{6}+q^{n}+q^{2\,n}+3\,q^{1+n}=2\,q^{2}\left(2\,q+3\,q^{2}+q^{n}+2\,q^{1+n}\right)$

Out[156]= It is equivalent to $\left(q-2\,q^{3}+q^{n}\right)\left(1+2\,q-2\,q^{2}-2\,q^{3}+q^{n}\right)=0$

In[157]:=
```
(*E_{6,1}(q)*)
G = "E_{6,1}(q)";
v = ((q^12 - 1) (q^9 - 1))/((q^4 - 1) (q - 1));
k = q (q^3 + 1) Bino[8, 1, q];
lam = q^2 (q^2 + 1) Bino[5, 1, q] + q - 1;
mu = (q^3 + 1) Bino[4, 1, q];
ass = q ≥ 2 && m ≥ 1;

ETFcond[v, k, lam, mu, ass, G]
StringForm["because `` is impossible", Simplify[v - 2 × (2 k - lam - mu)] == 0]
```
字串形式 化簡

Out[163]= The ETF cond of $E_{6,1}(q)$ is False

Out[164]= because $1 + q + q^2 + 3 q^3 - 4 q^7 - 3 q^8 - 2 q^9 - 2 q^{10} - 2 q^{11} + 2 q^{12} + q^{13} + q^{14} + q^{15} + q^{16} == 0$ is impossible

In[165]:=
```
(*P(q),P*(q)*)
G = "P(q),P*(q)";
v = 4 t + 1;
k = 2 t;
lam = t - 1;
mu = t;
ass = t ≥ 1;

ETFcond[v, k, lam, mu, ass, G]
```

Out[171]= The ETF cond of $P(q)$,$P^*(q)$ is False

In[172]:=
```
(*Van Lint-Schrijver graphs: t is odd*)
G = "Van Lint-Schrijver graph: t is odd";
v = q;
k = (q - 1)/e;
lam = (q - 3 e + 1 + (e - 1) (e - 2) Sqrt[q])/e^2;
mu = (q - e + 1 - (e - 2) Sqrt[q])/e^2;
ass = q ≥ 2 && e ≥ 2;
ETFcond[v, k, lam, mu, ass, G]
StringForm["The possible (b=Sqrt[q],e) are ``",
 Solve[Simplify[(4 + 2 × (4 + e^2) Sqrt[q] + (4 + e^2) q == 4 e (1 + Sqrt[q])^2) /. q → b^2,
   b > 0 && e ≥ 2], {b, e}, Integers]]
StringForm["The parameters are `` and Q-matrix is ``",
 {v, k, lam, mu} /. q → 4 /. e → 3, MatrixForm[Qmat[v, k, lam, mu] /. q → 4 /. e → 3]]
```

字串形式 求解 化簡 整數域 字串形式 矩陣格式

Out[178]= The ETF cond of Van Lint-Schrijver graph: t is odd is $4+2\left(4+e^2\right)\sqrt{q}+\left(4+e^2\right)q==4e\left(1+\sqrt{q}\right)^2$

Out[179]= The possible $(b=\sqrt{q},e)$ are {{b → -4, e → 3}, {b → -3, e → 4}, {b → -2, e → 1}, {b → 0, e → 1}, {b → 1, e → 4}, {b → 2, e → 3}, {b → -1, e → 0}}

Out[180]= The parameters are {4, 1, 0, 0} and Q-matrix is $\begin{pmatrix} 1 & 1 & 2 \\ 1 & 1 & -2 \\ 1 & -1 & 0 \end{pmatrix}$

In[181]:=
```
(*Van Lint-Schrijver graphs: t is even*)
G = "Van Lint-Schrijver graph: t is even";
v = q;
```
$k = \frac{q-1}{e};$

$lam = \frac{q - 3e + 1 - (e-1)(e-2)\sqrt{q}}{e^2};$

$mu = \frac{q - e + 1 + (e-2)\sqrt{q}}{e^2};$

$ass = q \geq 2 \,\&\&\, e \geq 2;$

```
ETFcond[v, k, lam, mu, ass, G]
```
StringForm["The possible (b=$\sqrt{q}$,e) are ``",
字串形式

Solve[Simplify[$\left(4 - 2\times(4+e^2)\sqrt{q} + (4+e^2)\,q == 4e\left(-1+\sqrt{q}\right)^2\right)$ /. q → b^2, b > 0],
求解 化簡

{b, e}, Integers]]
整數域

StringForm["The parameters are `` and Q-matrix is ``",
字串形式

{v, k, lam, mu} /. q → 16 /. e → 3, MatrixForm[Qmat[v, k, lam, mu] /. q → 16 /. e → 3]]
矩陣格式

Out[187]= The ETF cond of Van Lint-Schrijver graph: t is even is $4 - 2(4+e^2)\sqrt{q} + (4+e^2)\,q == 4e\left(-1+\sqrt{q}\right)^2$

Out[188]= The possible (b=$\sqrt{q}$,e) are {{b → -2, e → 3}, {b → -1, e → 4}, {b → 0, e → 1}, {b → 2, e → 1}, {b → 3, e → 4}, {b → 4, e → 3}, {b → 1, e → 0}}

Out[189]= The parameters are {16, 5, 0, 2} and Q-matrix is $\begin{pmatrix} 1 & 10 & 5 \\ 1 & 2 & -3 \\ 1 & -2 & 1 \end{pmatrix}$

In[190]:=
```
(*Hq(2,m)*)
G = "Hq(2,m)";
v = q^(2 m);
k = (q + 1) (q^m - 1);
lam = q^m + (q - 2) (q + 1);
mu = q (q + 1);
ass = m ≥ 2 && q ≥ 2;

ETFcond[v, k, lam, mu, ass, G]
StringForm["It is equivalent to ``", Factor[4 q + 4 q^2 + q^(2 m) - 2 q^m (1 + 2 q)] == 0]
StringForm["The Q-matrix of `` for ``=`` is ``", G, {q, m}, {2, 2},
 Simplify[MatrixForm[Qmat[v, k, lam, mu] /. {q → 2, m → 2}], ass]]
```

Out[196]= The ETF cond of $H_q(2,m)$ is $4 q + 4 q^2 + q^{2 m} == 2 q^m (1 + 2 q)$

Out[197]= It is equivalent to $(-2 - 2 q + q^m) (-2 q + q^m) == 0$

Out[198]= The Q-matrix of $H_q(2,m)$ for $\{q, m\}=\{2, 2\}$ is $\begin{pmatrix} 1 & 9 & 6 \\ 1 & 1 & -2 \\ 1 & -3 & 2 \end{pmatrix}$

In[199]:=
```
(*VO2m+(q)*)
G = "VO2m+(q)";
v = q^(2 m);
k = (q^m - eps) (q^(m-1) + eps) /. eps → 1;
lam = (q (q^(m-1) - eps) (q^(m-2) + eps) + q - 2) /. eps → 1;
mu = q^(m-1) (q^(m-1) + eps) /. eps → 1;
ass = q ≥ 2 && m ≥ 1;

ETFcond[v, k, lam, mu, ass, G]
StringForm["The Q-matrix of `` for ``=`` is ``", G,
 q, 2, Simplify[MatrixForm[Qmat[v, k, lam, mu] /. q → 2], ass]]
StringForm["The Q-matrix of `` for ``=`` is ``", G, {q, m}, {4, 1},
 Simplify[MatrixForm[Qmat[v, k, lam, mu] /. {q → 4, m → 1}], ass]]
```

Out[205]= The ETF cond of $VO_{2m}^{+}(q)$ is $(-2 + q) (-2 q - 2 q^m + q^{1+m}) == 0$

Out[206]= The Q-matrix of $VO_{2m}^{+}(q)$ for q=2 is $\begin{pmatrix} 1 & \frac{1}{2} (-1 + 2^m) (2 + 2^m) & 2^{-1+m} (-1 + 2^m) \\ 1 & -1 + 2^{-1+m} & -2^{-1+m} \\ 1 & -1 - 2^{-1+m} & 2^{-1+m} \end{pmatrix}$

Out[207]= The Q-matrix of $VO_{2m}^{+}(q)$ for $\{q, m\}=\{4, 1\}$ is $\begin{pmatrix} 1 & 6 & 9 \\ 1 & 2 & -3 \\ 1 & -2 & 1 \end{pmatrix}$

In[208]:=
```
(*VO2m^-(q)*)
G = "VO2m^-(q)";
v = q^(2m);
k = (q^m - eps) (q^(m-1) + eps) /. eps → -1;
lam = (q (q^(m-1) - eps) (q^(m-2) + eps) + q - 2) /. eps → -1;
mu = q^(m-1) (q^(m-1) + eps) /. eps → -1;
ass = q ≥ 2 && m ≥ 1;

ETFcond[v, k, lam, mu, ass, G]
StringForm["The Q-matrix of `` for ``=`` is ``", G,
字串形式
 q, 2, Simplify[MatrixForm[Qmat[v, k, lam, mu] /. q → 2], ass]]
        化簡          矩陣格式
```

Out[214]= The ETF cond of $VO_{2m}^{-}(q)$ is $(-2+q)\left(2q-2q^m+q^{1+m}\right)=0$

Out[215]= The Q-matrix of $VO_{2m}^{-}(q)$ for q=2 is $\begin{pmatrix} 1 & 2^{-1+m}\left(1+2^m\right) & \frac{1}{2}\left(-2-2^m+4^m\right) \\ 1 & 2^{-1+m} & -1-2^{-1+m} \\ 1 & -2^{-1+m} & -1+2^{-1+m} \end{pmatrix}$

In[216]:=
```
(*[3.4.2]*)
G = "graphs in 3.4.2";
v = q^10;
k = (q^2 + 1) (q^5 - 1);
lam = q^5 + q^4 - q^2 - 2;
mu = q^2 (q^2 + 1);
ass = q ≥ 2;

ETFcond[v, k, lam, mu, ass, G]
StringForm["because `` is impossible.", Simplify[v - 2 × (2 k - lam - mu)] == 0]
字串形式                                  化簡
```

Out[222]= The ETF cond of graphs in 3.4.2 is False

Out[223]= because $q^2\left(4+4q^2-2q^3-4q^5+q^8\right)=0$ is impossible.

In[224]:=
```
(*VD5,5(q)*)
G = "VD5,5(q)";
v = q^16;
k = (q^8 - 1) (q^3 + 1);
lam = q^8 + q^6 - q^3 - 2;
mu = q^3 (q^3 + 1);
ass = q ≥ 2;

ETFcond[v, k, lam, mu, ass, G]
StringForm["because `` is impossible.", Simplify[v - 2 × (2 k - lam - mu)] == 0]
字串形式                                  化簡
```

Out[230]= The ETF cond of $VD_{5,5}(q)$ is False

Out[231]= because $q^3\left(4+4q^3-2q^5-4q^8+q^{13}\right)=0$ is impossible.

In[232]:=
```
Sporadic = {{15, 6, 1, 3}, {35, 16, 6, 8}, {49, 24, 11, 12}, {55, 18, 9, 4},
   {64, 18, 2, 6}, {64, 27, 10, 12}, {66, 20, 10, 4}, {77, 16, 0, 4},
   {81, 16, 7, 2}, {81, 32, 13, 12}, {81, 40, 19, 20}, {100, 22, 0, 6},
   {100, 36, 14, 12}, {120, 56, 28, 24}, {126, 25, 8, 4}, {169, 72, 31, 30},
   {176, 70, 18, 34}, {243, 22, 1, 2}, {243, 110, 37, 60}, {253, 42, 21, 4},
   {253, 112, 36, 60}, {256, 45, 16, 6}, {256, 75, 26, 20}, {256, 102, 38, 42},
   {256, 120, 56, 56}, {275, 112, 30, 56}, {276, 44, 22, 4}, {289, 96, 35, 30},
   {351, 126, 45, 45}, {361, 144, 59, 56}, {416, 100, 36, 20}, {529, 264, 131, 132},
   {625, 144, 43, 30}, {625, 240, 95, 90}, {729, 104, 31, 12}, {729, 224, 61, 72},
   {841, 168, 47, 30}, {961, 240, 71, 56}, {961, 360, 139, 132},
   {1288, 495, 206, 180}, {1681, 480, 149, 132}, {1782, 416, 100, 96},
   {2016, 975, 462, 480}, {2048, 276, 44, 36}, {2048, 759, 310, 264},
   {2209, 1104, 551, 552}, {2300, 891, 378, 324}, {2401, 240, 59, 20},
   {2401, 480, 119, 90}, {2401, 720, 229, 210}, {2401, 960, 389, 380},
   {3510, 693, 180, 126}, {4060, 1755, 730, 780}, {4096, 315, 74, 20},
   {4096, 1575, 614, 600}, {5041, 840, 179, 132}, {6241, 1560, 419, 380},
   {6561, 1440, 351, 306}, {7921, 2640, 899, 870}, {14080, 3159, 918, 648},
   {15625, 744, 143, 30}, {15625, 7560, 3655, 3660}, {31671, 3510, 693, 351},
   {130816, 32319, 7742, 8064}, {137632, 28431, 6030, 5832},
   {306936, 31671, 3510, 3240}, {531441, 65520, 8559, 8010}};
```

In[233]:=
```
ETFsp[v0_, k0_, lam0_, mu0_] := Module[{v = v0, k = k0, lam = lam0, mu = mu0},
                                └模組
  Q = Qmat[v, k, lam, mu];
  If[Q[[2, 2]] + Q[[3, 2]] == 0 || Q[[3, 2]] + Q[[3, 3]] == 0,
 └如果
   Print[StringForm["The SRG is `` and its Q matrix is ``",
   └列表   └字串形式
     {v, k, lam, mu}, MatrixForm[Q]]], False]
                      └矩陣格式          └假
 ]
Do[ETFsp[srg[[1]], srg[[2]], srg[[3]], srg[[4]]], {srg, Sporadic}]
└Do迴圈
```

The SRG is {64, 27, 10, 12} and its Q matrix is $\begin{pmatrix} 1 & 36 & 27 \\ 1 & 4 & -5 \\ 1 & -4 & 3 \end{pmatrix}$

The SRG is {120, 56, 28, 24} and its Q matrix is $\begin{pmatrix} 1 & 35 & 84 \\ 1 & 5 & -6 \\ 1 & -5 & 4 \end{pmatrix}$

The SRG is {176, 70, 18, 34} and its Q matrix is $\begin{pmatrix} 1 & 154 & 21 \\ 1 & \frac{22}{5} & -\frac{27}{5} \\ 1 & -\frac{22}{5} & \frac{17}{5} \end{pmatrix}$

The SRG is {256, 120, 56, 56} and its Q matrix is $\begin{pmatrix} 1 & 120 & 135 \\ 1 & 8 & -9 \\ 1 & -8 & 7 \end{pmatrix}$

The SRG is {2016, 975, 462, 480} and its Q matrix is $\begin{pmatrix} 1 & 1365 & 650 \\ 1 & 21 & -22 \\ 1 & -21 & 20 \end{pmatrix}$

In[235]:=
```
Export["ETFs_mathematica_output.pdf", EvaluationNotebook[]]
```
匯出 計算過的筆記本

Out[◦]= ETFs_mathematica_output.pdf