NODUS 8.x: Methodological note

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February 2021

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Contents

Chapter 1

Introduction

1.1 What is Nodus?

Nodus is a GIS-based software specially designed for multi-modal freight transport network modeling.

1.2 Content of this document

Nodus 8.x is not provided with a complete reference or user guide. It is however important that the user has a good understanding of the methodological framework the software is based on. This is particularly true for the "Virtual Networks" and the different cost functions they need.

This document is largely inspired by the reference guide written for Nodus 4.0 (1999). Even if the later versions of the software are very different from version 4.0, the methodological framework remains essentially unchanged.

> Note, however, that Nodus 7.x maked it possible to define lines and services on Virtual Networks. This evolution is not discussed in this document. This functionality is (temporarily) disabled in Nodus 8.0.

Chapter 2

Modelling principles

2.1 Introduction

The aim of the development of NODUS is to offer a new solution to the problems posed by the flows of goods on a multi-modal transportation network. In that respect, this chapter, which is a short theoretic repetition of the concepts on which the development of transportation networks is based, will be essentially limited to the most important modal-choice and assignment models, so that these can be compared to the models that have been applied to NODUS. However, since it is necessary to have matrixes of origins-destinations (O-D) in order to apply the modal choice and the assignment, some generation and distribution methods will also be dealt with in this chapter, as they can proof useful during the practical applications[1] of NODUS.

But before passing on to the different points mentioned above, some general considerations on cost functions and on the behaviour of the different actors in a transportation system in relation to the demand for transportation, are certainly relevant here, since the (clear) definition of these functions and the understanding of the role of the different actors are the key to the succes of a model.

2.1.1 General considerations on cost functions

The notion of "cost function" will often be used in the different chapters that constitute this methodological note. These functions, however, are used in contexts that may differ greatly, and, moreover, they do not always refer to the same realities. Furthermore, their (non) linearity will often be discussed. In order to adequately clarify these concepts, the different types of cost used will be briefly dealt with.

[1] Although these matrixes are often treated as external data.

First of all there are the "cost functions linked to the links of a network". These are the costs [2] borne by a user who makes use of that segment of the network during a trip. The total cost of transportation will often be considered to be proportional to the distance covered and/or the quantity transported. Obviously, whenever there are fixed costs, the average cost of transportation will not be linear. (See figure 2.1).

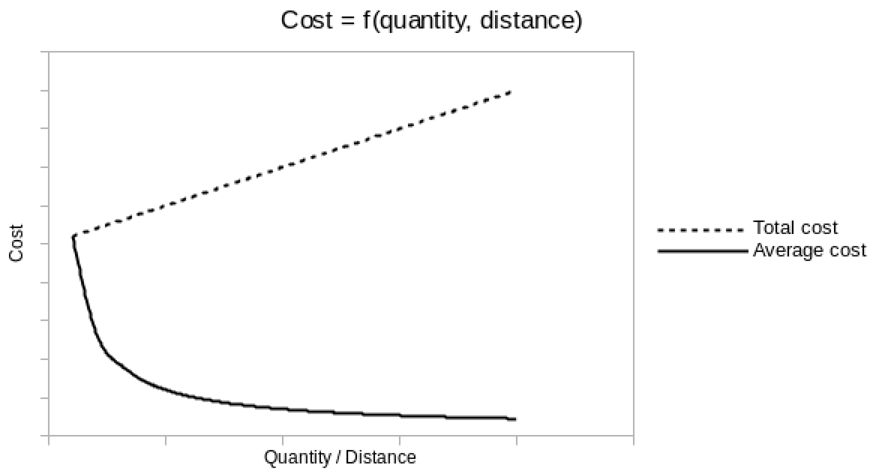

Figure 2.1: Relation between quantity/distance and cost

However, the total costs on the links are not always proportional to the quantity transported. The costs of handling the goods, for instance, are not proportional to the quantity of goods that are to be loaded and unloaded.

From the moment when congestion will have to be taken into account, the flow has to be introduced (total quantity transported on a certain link) as a variable of the cost function. Thus the total cost of transportation expands exponentially in proportion to that flow, as is illustrated in figure 2.2.

The total cost on the transportation system remains to be considered. Although there are several assignment methods on a network, they all have in common that they minimize an objective function. These objective functions can also take different forms. From a social viewpoint, it is important to minimize the total cost of transportation on the system. From the viewpoint of the user, however, it is important to minimize the average cost of the trip.

Generally speaking, when the cost functions are proportional to the quantities, and when congestion does not have to be taken into account, so that it is always possible to assign the transportations by choosing the solution (choice of a mode, a means or a route) with the lowest cost by applying an efficient shortest-path algorithm, there is, properly speaking, no objective function as

[2] To avoid complicating things at this stage, the term "cost" will be used for all financial charges. It may concern, for instance, a cost such as the wage of the driver when he uses his lorry.

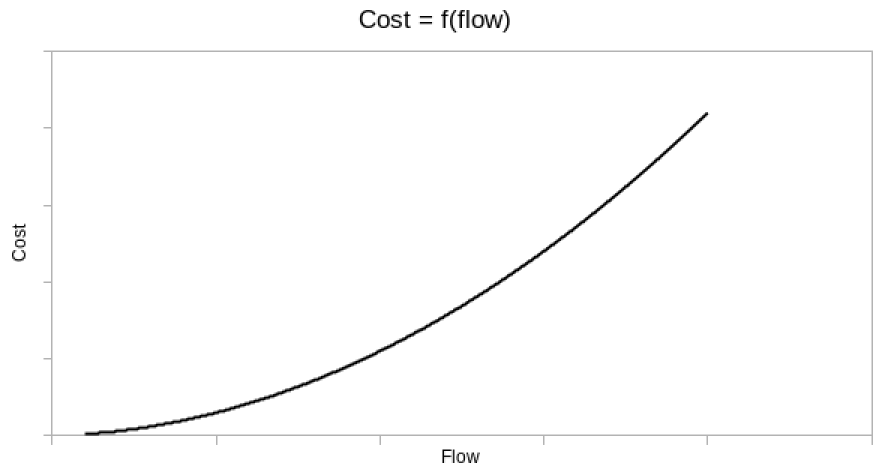

Figure 2.2: Relation between flow and cost

long as there is discontinuity between the different possible solutions. But if the costs of the different convex combinations of the solutions for the assignment of the quantities are distinguished, the obtained cost function will be a linear function since all the cost functions on the different links are proportional to the quantities transported on those particular links.

However, when non-linear (convex) cost functions are used on the links, which has to be the case for the models that take the level of congestion on a network into consideration, the solutions will generally not be of the "all or nothing"-type, but there will be a repartition of the quantities between the different available modes, means and routes. The set of those possible solutions constitutes a continuous convex function of the total cost resulting from the addition of the different convex functions.

Several concepts relating to costs have to be taken into consideration, and for this reason, all through this text, it will be clearly indicated what the costs used refer to.

2.1.2 The behaviour of the actors on a transportation system

Traditionally, the actors that are taken into consideration are the producers of goods (that play the role of shippers), the consumers and the carriers. The producers and the consumers are not situated at the same location. The analysis of the mechanisms that control the links between those different actors is the reason for which transportation economics exists and for which computer programs such as NODUS have been developed these recent years.

The shippers are economic actors that decide to assign and to distribute a certain flow of goods between an origin and a certain number of destinations. The carriers represent in reality a set of different entities that are implied in the decision of transportation. This set covers for instance the departments "shipments", "distribution" or "deliveries" of the firms. The decision of transportation may be very intricate, but 'the only motivation that leads to transport is an economic reason', as has been well explained by Roberts [3]. Thus the decision of the carrier depends upon the fluctuations of supply and demand, and of the prices on the market.

When the shipper decides to transport goods, he chooses a carrier to effect the transportation. In transportation economics, the carrier is always looked upon as an economic agent that produces a service (transportation) while trying to maximise his profit.

The relationship between the shippers and the carriers is compared to the relationship between a consumer and a producer of services. By his decision to confide a shipment to a carrier, the shipper creates a demand for the output produced by that carrier. The carrier, on the other hand, will ask a certain price for effecting the transportation task and will thus create a certain level of service.

If the role played by the State is added to this, the existing relationships between the different actors can be represented as shown in figure 2.3:

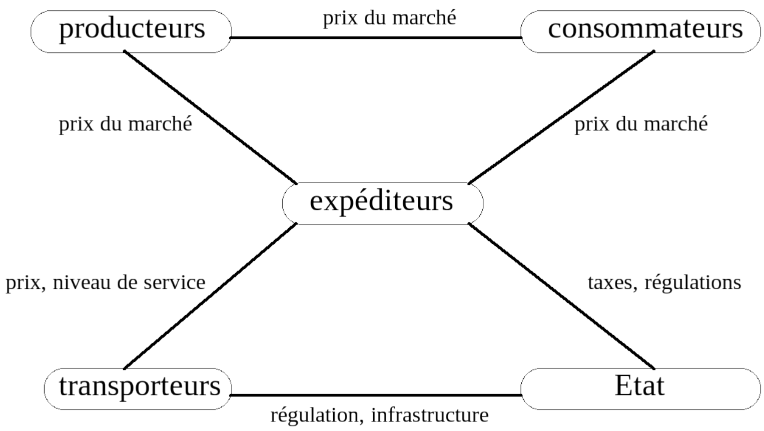

Figure 2.3: The actors in a transport system

Nevertheless, it remains clear that all these interactions can only exist if there is a demand for transportation. That demand depends upon certain explanatory factors among which :

- The price of the service of transportation : similar to the demand for whatever product, the demand for transportation is in principle a decreasing function of the price of transportation.

[3]Roberts, P.O., 1976, "Forecasting Freight Flows Using a Disaggregated Freight Demand Model", CTS Report 76-1, Center for Transportation Studies, MIT.

- The price of the other modes of transportation : whereas certain categories of goods are (almost) exclusively transported by one mode of transportation, certain other categories qualify for several modes. A price variation of one mode of transportation can as a result influence the demand for another mode.

- The price of the complementary goods : transportation can be considered as a part of a production process. A classical example is the electric energy production of a power plant that has its coals or fuel supplied by barges or by trucks. It is clear that the demand for transportation is a direct function of the level of energy production.

- The duration : transportation is a particular product since the user is "implied" in the production of the service of transport operation by the time it takes for the goods to be moved. The duration of the transportation plays a highly important role for the demand for transportation and for the modal choice. The demand for transportation is inversely related to the amount of time needed for the transport cycle. The time-elasticity is therefore negative. The demand for transportation is also influenced by the speed, but speed is rather an element determining the duration of the transportation than a supplementary explanatory factor.

- The freedom of choice : the shippers are highly sensible to this factor, which partly explains the succes of road transportation.

- The capacity : the correspondence between the characteristics of the goods that are to be transported (volume, weight) and those of the mode of transportation often determine the modal choice.

- The security and the observance of the time schedules: the transportations of goods are influenced by the inherent risks (although the perception of that risk is often subjective) and by the time constraints that are imposed .

2.2 Generation and distribution techniques

This section focusses on some basic techniques concerning the generation and the distribution[4]. As mentioned above, NODUS does not contain any generation or distribution modules : as a consequence, the O-D matrixes always have to be provided, and the few paragraphs that will follow have to be considered as a short theoretic explanation, that may be useful when the O-D matrixes are not available or when they are incomplete or out-of-date. The generation offers the possibility to estimate the total flows coming from and going to the different parts of the network, whereas the distribution offers the possibility to create an O-D matrix based on those total flows.

[4]For those interested in a more detailed description of these techniques we refer to a book by J. de D. Ortuzar and L.G. Willumsen, "Modelling Transport", Wiley, 2011.

Once again, these pages are certainly not meant as an in-depth analysis of this subject; the aim here is merely to help the reader who is not familiar with the basic techniques of the traditional modelling.

2.2.1 Generation

The generation phase tries to determine the global needs as to import and export of each zone of a studied geographical area. At this stage, the links that may exist between an origin and a particular destination are not yet taken into consideration. The movements that come and go from a certain node are estimated by classifying the flows according to categories (aim, period, type of goods, ...) and by determining the factors that influence those flows. In the case of freight transportation, the following criteria are usually taken into consideration :

- the type of goods,
- the population of the zone,
- the level of income,
- the number of employees in the companies,
- the number of transactions,
- the sphere of influence of the companies,
- etc.

These data are subsequently essentially used for :

- regression analysis,
- crossed classification techniques,
- prevision techniques,
- stability tests of the obtained results.

2.2.2 Distribution

Logically, the following step is the phase in which the total flows are distributed between the different centres that have been discerned in the generation phase. In other words, an O-D matrix now has to be estimated.

In the literature on the subject several solutions are proposed among which classical ones such as the growth rate methods and the synthetic methods.

Growth rate methods.

These methods are based on the prior existence of a first O-D matrix (t). An expected growth rate is then used to estimate a new matrix.

The basic method is used if only a general growth rate τ is available. For each pair (i,j), $T_{ij} = \tau t_{ij}$ is applied.

If a specific growth rate is available for each origin and each destination τ_i or τ_j, the method is called the growth rate method with a single constraint.

In that case, $T_{ij} = \tau_i t_{ij}$ for an "origin"-growth rate and $T_{ij} = \tau_j t_{ij}$ for a "destination"-growth rate.

An example (see table 2.1) based on forecasted information on the flows on the origins (targets):

i\j	1	2	3	4	$\sum_i$	Targets O_i
1	5	50	100	200	355	400
2	50	5	100	300	455	460
3	50	100	5	100	255	400
4	100	200	250	20	570	702
$\sum_i$	205	355	455	620	1635	1962

Table 2.1: Prevision at origins and destinations

The problem can easily be solved by multiplying each line by the ratio $O_i/\sum_j t_{ij}$, that represents a growth rate (see schedule 2.2).

i\j	1	2	3	4	$\sum_i$	Targets O_i
1	5.6	56.3	112.7	225.4	400	400
2	50.5	5.1	101.1	303.3	460	460
3	78.4	156.9	7.8	156.9	400	400
4	123.2	246.3	307.9	24.6	702	702
$\sum_i$	257.7	464.3	529.5	701.2	1962	1962

Table 2.2: Application of the growth rate

The growth rate method with double constraint is applied when estimations on the futur number of movements at the origins and destinations are available. In that case, for each node, there is an attraction τ_i and generation Γ_j growth rate. The use of an average factor $F_{ij} = 0.5(\tau_i + \Gamma_j)$ is merely an approximate compromise. To solve the problem, iterative processes are used, among which the most known is that of Furness[5] , who introduces the notion of "balancing

[5]Furness K. P., 1965, "Time Function Iteration", Traffic Engineering and Control, 7(7), 458-60

factors" A_i and B_j.

$$T_{ij} = t_{ij}\tau_i\Gamma_j A_i B_j$$

or

$$T_{ij} = t_{ij}a_i b_j \; with \; a_i = \tau_i A_i \; et \; b_j = \Gamma_j B_j$$

The iterative process is the following one:

- Initialize all the b_j to 0 and solve for a_i (satisfy the generation constraint).
- Calculate the b_j by using the a_i previously obtained (satisfy the attraction constraint).
- Recalculate the a_i with the obtained b_j. Repeat the previous steps until stable parameters are obtained.

The methods based on the growth rate are easy since they are based on the pre-existence of observed O-D matrixes and estimated growth rates. Their weakness consists of course in the use of pre-existing O-D matrixes. For this reason, only short-term previsions can be realized on networks that remain very stable.

The synthetic methods. Several methods have been elaborated in order to realize previsions on networks characterized, for instance, by wide flow variations. The most known among these methods is the gravity model, which is generally formulated as follows :

$$T_{ij} = \alpha O_i D_j f(c_{ij})$$

where O_i et D_j represent the total emissions and attractions, α is a relative weight and $f(c_{ij})$ is a generalized transportation cost function known as "deterrence function". [6].

The gravity model is based on the principle that the probability to make a trip between two nodes that are close to each is higher than the probability of making a trip between two nodes that are far away from each other. The concept of distance has to be interpreted in the broadest way. Whereas physical distance can be used as a weighting factor, two nodes can also be looked upon as close

[6]For instance :

- exponential $f(c_{ij}) = exp(-\beta c_{ij})$
- power $f(c_{ij}) = c_{ij}^n$
- combined $f(c_{ij}) = c_{ij}^n exp(-\beta c_{ij})$

to each other because they are linked by the same type of economic activity or by complementary economic activities. This explains why there is more traffic between two industrial regions than between two agricultural regions.

When a constraint (a single or a double one) is introduced, the Furness method is used by replacing the factor α by the balancing factors A_i and B_j :

$$T_{ij} = A_i O_i B_j D_j f(c_{ij})$$

In the case of a single constraint model with a single obligation, A_i or B_j equals the unity. In the opposite case, the interdependence of the balancing factors requires an iterative process, as the one that has been explained above.

Example with exponential deterrence function [7] ($\beta = .10$) (see tables 2.3 to 2.5).

					Targets (flow)
i\j	1	2	3	4	O_i
1	3	11	18	22	400
2	12	3	13	19	460
3	15.5	13	5	7	400
4	15.5	13	5	7	400
D_j	260	400	500	802	1962

Table 2.3: Cost matrix (in minutes)

					Targets (flows)
i\j	1	2	3	4	O_i
1	0.74	0.33	0.17	0.11	400
2	0.30	0.74	0.27	0.15	460
3	0.14	0.27	0.61	0.50	400
4	0.09	0.17	0.45	0.61	702
D_j	260	400	500	802	1962

Table 2.4: Results of the deterrence function $exp(-\beta cost)$

2.3 Modal choice techniques

The concept of "virtual network", as applied in the NODUS-software, combines the modal choice and generation step of the traditional four stages model. Nevertheless it remains useful here to describe a few alternative techniques so that

[7] Several researchers have tried to propose estimations for the parameter β. See for instance Blauwens G., 1975, "Interpreting Coefficients in Gravity Model", werknota 32, UFSIA-SESO, Antwerp.

i\j	1	2	3	4	O_i	a_i
1	162	98.5	65.5	74	400	405.2
2	162	98.5	65.5	74	400	405.2
3	18.2	47.2	140.6	194	400	237.1
4	23.4	55.4	192.2	431	702	439.4
D_j	265.1	405.6	499.1	792.2		1962/1959.9
b_j	1.5	1.7	2.0	2.6		

Table 2.5: Gravity matrix

Level of aggregation	Models
Aggregate	Derived demand models
	Probabilistic models
Disaggregate	Probabilistic models
	Inventory theories

Table 2.6: Types of models according to the level of aggregation

it becomes clear what exactly is the contribution of the virtual network in terms of ease of development.

The analysis of the factors determining the modal choice has always drawn attention for the study of the competition between the modes of transportation. For that reason, a wide range of models has been developed. According to Wilson[8], the demand models can be classified in two categories : the aggregate models and the disaggregate models. These models differ in the degree the data used is aggregate (see table 2.6).

2.3.1 Aggregated models

In the aggregated models, the producers maximize their profits and transportation are to be included in their production process. Thus, the demand function of transportation is based on the estimated cost function of the firm.

The aggregated approach implies the use of temporal and/or crossed-time series to estimate the structural relationships that describe the behaviour of one or more actors of the transportation system. This type of approach generaly leads to the estimations of general cost or production functions applicable to a particular firm or to an entire industry. In general, this approach ignores the details of a transportation network : therefore it is not suitable to analyse the flows on a complex network.

[8]Wilson A. G., 1981, "A Disaggregate Model of the Demand for Intercity Freight Transportation", Econometrica, 49, 981-1006.

Derived demand models

The researches of Sloss [9], Oum [10] and of Friedlander and Spady [11] are based on this type of approach. The cost functions are specified as in the neo-classical production theory, in which the shipper is supposed to minimize the transportation cost taking into account different technological constraints.

The adopted approach is the translogarithmic function, that does not impose restrictions on the price and substitution elasticities. These functions belong to the family of 'flexible', and that can be used to obtain a second order approximation of the Taylor's serie.

Oum uses aggregate data in a chronological order relating to the flows of goods between certain Canadian cities, whereas Friedlander and Spady use instantaneous, more disaggregate data.

The derived demand models consider transportation as a necessary input in a production process. Therefore, the demand for transport is derived from the produced output. From an analytic viewpoint, the demand functions for transportation used by Oum and by Friedlander and Spady are obtained in a context of perfect competition using the lemma of Sheppard [12]. The use of cost functions, and not of production functions, can be explained by the dual relationship linking these two expressions

This approach is thus based on a single function, of the translogarithmic type, well adapted to aggregated data. It seems evident that it cannot be used directly on a network, composed of a large number of links and nodes on which a cost (or a cost estimation) has to be assigned. Nevertheless, the idea that the shipper tries to minimize his costs cannot be rejected.

Probabilistic models

This approach, after based on logit models, has been used by several authors.

The (multinomial) logit model takes the following form :

$$P_n(i) = \frac{e^{V_{in}}}{\sum\limits_{j \in C_n} e^{V_{jn}}}$$

[9] Sloss J., 1971, "The Demand for Intercity Motor Freight Transport: a Macroeconomic Analysis", The Journal of Business, January.

[10] Oum T. H., 1977, "Derived Demand for Freight Transportation and Inter-Modal Substitutibilities in Canada", Transportation Research Forum Conference Proceedings XVIII(1), 56-57.

[11] Friedlander A. F. and Spady R. H., 1981, "Freight Transport Regulation: Equity, Efficiency and Competition in the Rail and Trucking Industries", MIT Press, Cambridge, MA.

[12] Sheppard E. et Curry L., 1982, "Spatial Price Equilibria", Geographical Analysis 14, 279-304.

with :

$$0 \leq P_n(i) \leq 1$$

$$\sum_{i \in < c_n} P_n(i) = 1$$

where :

- $P_n(i)$: probability that the alternative i is chosen between n possibilities
- V_{in} and V_{jn} : systematic (representative) components of the utility function of i and j
- C_n: possible choices

The multinomial logit model is characterized by the fact that the deciding person has a choice between more than two alternatives among a number of C_n alternatives:

$$U_{in} = V_{in} + \epsilon_{in}$$

where ϵ_{in} represents a random noise.

Since the probabilistic models are also discussed in the section "disaggregated models", we will not pursue this subject in greater depth here. Moreover, the aggregated approach is not very suitable for studies based on transportation networks requiring an high level of detail. In addition, the network topology also plays an important role in the modal choice and this aspect is often, if not always, ignored at a high level of aggregation.

2.3.2 Disaggregate models

Probabilistic models

The probabilistic approach has been used on a disaggregated level as well. By introducing an uncertain component on the profit level, Daughety and Inaba [13] have formulated the logit models that have often been used [14]. This type

[13] Daughety A. F. and Inaba F.S., 1978, "Estimation of Service-Differentiated Transport Demand Functions", Transportation Research Record, 668, 23-30.

[14] For instance :

- Daughety A. F., 1979, "Freight Demand Transport Revisited: A Microeconomic View of Multi-Modal Multi-Characteristic Service Uncertainty and Demand for Freight Transport", Transportation Research, 13, 281-288.

of models presupposes that the shipper can be represented by a certain vector of characteristics S, that indirectly reflects his preferences. That shipper has the choice between several modes of transportation, that can be represented by vectors X of attributes. The utility function linked to a mode can be represented in the following way :

$$\cup = V(S, X) + \epsilon(S, X)$$

where V is a non stochastic function reflecting the representative preferences of the population while the particular character of a certain shipper is indicated by the random function, (.

If only two modes are used, if X^1 and X^2 are the respective characteristics of the modes 1 and 2, mode 1 will be chosen if :

$$V(X^1, S) + \epsilon(X^1, S) > V(X^2, S) + \epsilon(X^2, S)$$

Behaviour characterized by aversion to risks has been the object of an in-depth analysis by Wilson [15]. According to this author the aversion is not the same for all the modes of transportation. The hypothesis of independence of the stochastic error, necessary to develop a logit model, may be unrealistic. A more probable hypothesis of distribution of errors would necessitate the errors to be independently and normal distributed. The result of such a distribution leads to the definition of the probit model.

Models derived from inventory theory

The inventory approach aims at optimizing the behaviour of the shipper aspiring to make more profit. Baumol and Vinod [16] were forerunners in this field. Das [17], Constable and Whybark [18] followed in their footsteps. Although this approach is relatively old, it was not used in empirical models until about fifteen years ago, among others by Allen [19], Chiang and Roberts [20] McFadden and Winston

- Daughety A. F. and Inaba F.S., 1981, "An Analysis of Regulatory Change in the Transportation Industry", Rev. of Econ. and Stat., 53, 246-255.
- Levin R. C., 1981, "Railroad Rates, Profitability and Welfare under Deregulation", Bell J. of Econ., 12, 1-26.

[15] Wilson A. G., 1981, "A Disaggregate Model of the Demand for Intercity Freight Transportation", Econometrica, 49, 981-1006.

[16] Baumol W.J. and Vinod H.D., 1970, "An Inventory Theoretic Model of Freight Transportation Choice", Management Science, 16, 413-21

[17] Das C., 1972, "Choice of Transport Service: An Inventory Theoretic Approach", The Logistics and Transportation Review, 10, 181-87

[18] Constable G.K. and Whybark D.C.,1978, "The Interaction of Transportation and Inventory Decisions", Decision Science, 3, 688-99

[19] Allen W.B., 1977, "The Demand for Freight Transportation: A Micro Approach", Transportation Research 11, 9-14.

[20] Chiang Y.S. and Roberts P.O., 1980, "A Note on Transit Time and Reliability for Regular-Route Trucking", Transportation Research, 14B, 59-65.

[21]..

The model is based on a profit function that takes into account the inventory value of the goods at the beginning and at the end of the shipment. The aim is to determine the optimum quantity and frequency of the shipments, so as to maximize the profit of the firms. The basic expression of this approach is :

$$CT = C_s + C_t + C_i$$

where :

- C_s : direct annual transportation costs;
- C_t : annual financial costs due to the storage of the goods during the transportation;
- C_i : annual administrative costs + costs linked to the stockpile.

The drawing up of such a model requires of course a large amount of data, since the transportation costs depend on the size of the shipment, on the storage costs of the goods, on the demand for those particular goods, etc.

The approach is nevertheless extremely interesting since it explicitely takes the goods into account in the decision-making process. Furthermore, the disaggregate nature of this technique is well adapted to the network methods, because the different costs elements that constitute the model can be assigned to the links or the nodes of a network. The choice of a route, corresponding eventually to the transportation decision, can then be made by applying a shortest path algorithm.

Although, originally, this model was not developed to be used in a network, this technique is one of the easiest to adapt. The modal choice implemented in the virtual network will therefore be based on this theory.

2.3.3 Concluding comments on the modal choice techniques

Whether aggregated or not, the modal choice techniques are not entirely satisfactory as to the dynamics of the networks.

The aggregated techniques are not really suitable for networks. The same goes for the disaggregated probabilistic approach. In these two cases it is difficult to take the details of networks into account.

The approach derived from the inventory theory will serve as a source of inspiration during the development of the specific cost functions that will be used

[21] McFadden D. and Winston C., 1981, "Joint Estimation of Discrete and Continuous Choices in Freight Transportation", Meeting of the Econometric Society.

in the applications of the virtual network. Moreover, the models handled by NODUS will take the costs borne by the carriers into account by looking for the cheapest routes, whereas the original approach merely aims at maximizing the profit of the shippers.

It remains clear, however, that the criteria leading to the choice of a mode of transport cannot only be expressed in monetary terms. Very diverse elements such as the duration of the trip, the way in which the goods are loaded or the category of the goods to be transported. All these criteria affect the level of quality (in a broad sense) of a transportation mode.

2.4 Assignment techniques

The assignment methods, that aim at spreading the O-D matrixes over the network while minimizing the (generalized) costs, can be classified in four large categories, according to whether or not they take into account the capacity constraints linked to the network, and according to whether or not the objective function is linear or not (see table 2.7). This objective function can actually take different forms depending on the point of view that is adopted in a model. When the aim is to optimize an entire transportation system, the total costs have to be minimized. On the other hand, when the aim is to obtain a user-optimum model, the average costs have to be minimized.

		Non stochastic behavior	Stochastic behavior
Capacity constraints	No	All-Or-Nothing	Multi-flow
	Yes	Equilibrium	Mutli-flow equilibrium

Table 2.7: Assignment methods

In the equilibrium and multi-flow models, several routes are used between the same origins and destinations, whereas the all-or-nothing models find and use a single route between each O-D pair. These three assignment techniques, along with several alternative implementations, are available in NODUS. The multi-flow equilibrium is not implemented.

In comparison with the all-or-nothing model, the classical equilibrium model takes the existing traffic into consideration when trying to determine a route in the network, between cost functions of the cost-flow type. This boils down to implicitly taking the capacity into account.

The assignment techniques may or may not take the stochastic aspect of the behaviour of the users into account, independently of what has been indicated in the preceding schedule.

The stochastic methods have chiefly been developed for the transport of passengers [22]. Different travellers have indeed a different perception of their environment. A driver, for instance, may prefer a motorway to drive from one

[22] The most known models are those of :

point to another, whereas his colleague may prefer a national trunk road. This preference leads to the notion of utility. For the application of such models, the users are supposed to have an idea of the utility of each possible route. Often it is assumed that they tend to choose the route that maximizes their utility. The representation of such a behaviour makes use of more or less complex probabilistic models according to whether the user has a choice between two (logit, probit, ...) or more (multinomial logit, Dial, ...) routes.

Of course, the transportation of goods is also influenced by this kind of considerations, mainly when it concerns transportation in an urban environment, during which congestions can arise. Here, attention will rather be focussed on non stochastic methods since the multi-modal transportation problems are only conceivable on relatively long distances and on national or international networks.

2.4.1 The all-or-nothing method

The models applying an all-or-nothing assignment method presuppose that the network is not limited by capacity constraints. All the quantities to be transported included in the O-D matrix are assigned without any constraint. In spite of this limitation, the all-or-nothing methods are still amply used in certain stages of studies. They also suit very well to large networks that are not too congested or for visualising flows in non binding situations.

For macroscopic applications, it is not very important to know whether there is congestion at certain crossroads. What is important though, is to know that in order to traverse a given city a certain amount of time is needed. This kind of consideration actually becomes clear through the topology of the used network : the details of urban regions (streets, crossroads, ...) are not digitized when large networks, such as the European network, are being used. Congestion can then be accounted for by a supplementary cost (congestion, passage at some borders, ...) that is added to the nodes of a network, so that an all-or-nothing method may become acceptable. NODUS obviously offers the possibility of realizing this type of assignments.

2.4.2 Equilibrium methods

It seems evident that at a certain moment congestions appear. During the assignment phase it is therefore important to introduce a method able to solve the

- Dial R.B., 1969, "A Probabilistic Multipath Traffic Assignment Model wich Obviates Path Enumeration", Transportation Research, 5, 83-112.
- Gunarsson S.O., 1972, "An Algorithm for Multipath Traffic Assignment", PTRC Seminar Proceedings, Urban Traffic Model Research, 8-12 May.
- Randle J., 1979, "A Convergent Probabilistic Road Assignment Model", Trafic Eng. and Control, 20, 519-521. This model is better known as G.M.T.U. (Greater Manchester Transportation Unit).

constraints linked to the capacity of the network. One of the most used methods to solve congestion problems is the application of a Frank-Wolfe algorithm[23], that offers the possibility of re-assigning part of the flows to alternative routes as an application of the idea that the transportation costs on an link increases with the flow.

This algorithm, or variants of it, have been used many assignment models. Therefore, two of those variants are presented here.

Application of the Frank-Wolfe algorithm:

- n: iteration number
- C_a: cost on link a
- F_a: flows on link a
- V_a: flows on link a, weighted by parameter λ
- λ: balancing parameter

- Phase 1 : Initialization
  - Counter n = 0
  - Assign a cost $C_a^{(n)}$, to each link, equivalent to the cost when there is no congestion
  - Assign the traffic (all-or-nothing method). The flow on link a is $F_a^{(n)}$.
  - $V_a^{(n)} = F_a^{(n)}$
- Phase 2 : n = n + 1
- Phase 3 : $C_a^{(n)} = C_a(Va^{(n-1)})$
- Phase 4 : All-or-nothing assignment on the basis of those new costs and calculation of $F_a^{(n)}$.
- Phase 5 : $V_a^{(n)} = V_a^{(n-1)} + \lambda^{(n)}(F_a^{(n)} - V_a^{(n-1)})$

$\lambda^{(n)}$ can then be chosen in order to minimize the following objective function [24].

$$Z^{(n)} = Z(\lambda^{(n)}) = \sum_a \int_0^{V_a^{(n)}} C_a(v)dv$$

[23]Frank M. and Wolfe P., 1956, “An Algorithm for Quadratic Programming“, Naval Research Logistic Quarterly, 3, 95-110.

[24]There are several methods to make the calculations. In the example used here, an integral is applied in the calculation, which is implemented in a ”user equilibrium” approach through the average costs. This type of equilibrium is a rather ”egoistic” approach of the equilibrium ...

$$\text{for } 0 \leq \lambda^{(n)} \leq 1$$

- Phase 6 : If the end condition is not yet satisfied, go to phase 2.

Thomas [25] presents a complete algorithm and a numerical method for determining $\lambda^{(n)}$.

These methods and the modifications that have subsequently been made to them, lead to fairly good results, but they require much more computation time due to their iterative nature. Other much faster methods have been developed

One of these is the "incrementals" method, also known as the "Chicago"-method [26]. It consists in assigning only part of the O-D matrix before modifying the costs of the links. In that way, the assignment proceeds bit by bit, until the entire matrix is assigned (sequential loading). With this method the individual flows can be incorrect. The first elements of the O-D matrix will be systematically assigned first, and will not suffer from congestion. The last routes to be calculated, however, will always suffer the most harm, as they are assigned on a congested network. The obtained result is as a consequence often wrong. However, and although the techniques of partial assignment are no more often used [27], the global solution, namely the total flows on each separate link, can be a good estimation of the result based on the algorithms presented above.

In the Frank-Wolfe method, the time needed to compute $\lambda^{(n)}$ is sometimes nearly as long as the time needed to compute the shortes paths. We also know that the incremental method doesn't always provide correct solutions. An elegant solution is provided by the Method of Successive Averages [28] (MSA), which suggests to use $\lambda^{(n)} = 1/(1+n)$. With such a value, the algorithm often needs more iterations than the Frank-Wolfe approach, but the total computing time is nearby always faster, as the time needed to compute $\lambda^{(n)}$ is immediate.

One thing should be noted, though. When trying to link whatever assignment techniques on a network to the economic theory, it should be kept in mind that the assignment is always based on the search for a path [29] made of a succession of links. Each of these links has a certain cost, and the total cost of a route is simply the sum of the costs (linear or not) of the differents links used. The process is additive by nature, which may conflict with the principles of the economics. This way of computing the total cost on a travel route actually ignores possible economies of density.

[25] Thomas Roy, 1991, "Traffic Assignment Techniques", Avebury Technical

[26] Caroll J. D., 1959, "A Method of Traffic Assignment to an Urban Network", Highway Res. Bd. Bull. 224, Trip Characteristics and Traffic Assignment, 64-71.

[27] Modern computers can cope with equilibrium models on large network.

[28] Powell W.B, Sheffi Y., 1982, "The Convergence of Equilibrium Algorithms with Predetermined Step Sizes", Transportation Science 16,1 , 45-55

[29] Or, more precisely, a set of pathes.

2.4.3 Concluding comments on the assignment techniques

At this stage it is impossible to make a choice between the two described assignment techniques. It all depends, actually, on the problem that is to be solved. A flow analysis on a regional network may require an equilibrium approach, whereas problems linked to national or international networks may be solved with an all-or-nothing algorithm, and in which congestion-related cost can be introduced as fixed costs on certain nodes or links.

Chapter 3

The virtual network

Before analysing the concept of the virtual network, a few basic concepts of the graph theory have to be discussed. The following paragraphs are certainly not intended as a new contribution to that theory; they merely allow the reader who is not familiar with this subject to understand the basic concepts on which the virtual network is founded. These few pages are based on the excellent book by Minoux and Bartnik [1].

3.1 Concepts

3.1.1 Graph

A graph G = [X, U] is determined by :

- A set X of which the elements x ∈ X are called vertices or nodes. N = #X represents the number of nodes. The graph is said to be of the Nth order. The nodes are numbered i= 1..N.
- a set U of which the elements u ∈ U are ordered couples of nodes, called arrows. If u = (x_1, x_2) is an arrow of G, x_1 is the extremity at the beginning of and x_2 the extremity at the end of u. The number of arrows is indicated by M= #U.

[1] Minoux M. and Bartnik G., 1986, "Graphes, Algorithmes, Logiciels", Dunod informatique. Interested readers will certainly like the book by Gondrand M. , Minoux M., 1979, "Graphes et Algorithmes", Eyrolles. These two books have been chosen because they deal with and clearly explain most of the classical algorithms of the graph theory, both from a methodological viewpoint and from the viewpoint of computer science. The complexity of the algorithms presented is systematically calculated, allowing for a motivated choice between the different algorithms, according to the particular conditions of the model to be developed.

3.1.2 Successors and predecessors

x_2 is said to be a successor of x_1 if there is an arrow starting at x_1 and ending at x_2. The set of successors of a node x ∈ X is indicated by Γ_x.

x_2 is said to be a predecessor of x_1 if there is an arrow (x_2, x_1). The set of predecessors of x ∈ X can then be indicated by Γ_x^{-1}.

3.1.3 Complete graph

A graph G = [X,U] is said to be complete if for each pair of nodes (x_1, x_2) there is at least one arrow (x_1, x_2) or (x_2, x_1).

A transportation network is, therefore, seldom, if ever, a complete graph.

3.1.4 Orientation of a graph

A non ordered pair (x_1, x_2), called the link (x_1, x_2), is connected to each arrow (x_1, x_2), to the ordered pair (x_1, x_2). A link is thus a non oriented arrow. A graph G = [X,U] is a non oriented graph if the set U is a set of links.

A transportation network can be oriented or not. In an urban network, for instance, it is important to take one-way traffic into account. This type of network is thus oriented. For large networks for national transportation, on the other hand, a non oriented graph can be used since each arrow can be traversed in two directions at the same cost.

However, although the virtual network is to be used on networks of the second type, it has to be oriented, for reasons that will be explained later.

3.1.5 Density of a graph

The density of a graph is the relationship between the number of arrows (links) of a graph and the number of arrows (links) of which a complete graph having the same number of nodes consists. In the case of an oriented graph, the density is :

$$\frac{M}{N(N-1)}$$

In the case of a non oriented graph, the density is expressed by :

$$\frac{2M}{N(N-1)}$$

A transportation network (as well as the virtual network that can be generated from) is therefore a graph with a low density. A node, indeed, is never directly linked to all the other nodes. As a general rule, a node is attached to 3 or 4 other nodes at the most. This characteristic can influence the choice of the algorithm that will be used to calculate the pathes.

These few basic concepts being clarified, the notion of multi-modal network remains to be defined. The adjective multi-modal covers two separate problems :

- The choice of the proper transportation mode (road, railway, inland waterways)
- The choice of the means of transportation (Is it an electric or a diesel-powered train? Is it a 300 or a 1000 ton ship?)

If the only transportation modes considered are the terrestrial ones (trucks, trains and ships), the multi-modal network is composed of three distinct mono-modal networks : the road network, the railway network and the network of inland waterways. Each network is characterized by one single transportation mode, but different means of transportation are possible. The different networks are interconnected by common terminals and/or ports.

The multi-modal network consists of a set of nodes and a set of links. Each link represents a trunk that can be used by only one transportation mode, but by different transportation means.

In the multi-modal network as it has been defined, it is possible to change of transportation modes and means, and to use several times the same mode or means of transportation to reach a destination. Imagine loading the goods into a lorry, subsequently using waterways, and again transferring the goods into a lorry to reach the final destination. The travel route consists thus of a set of trunks each one used by one transportation mean, and of several transshipments. For each of those routes it is important to try to minimize the total cost.

3.2 Intuitive approach

Let us say that R, T and E represent respectively road, railways and waterways.

Firstly, the costs linked to the expenses for loading and transshipping can be considered as weights to be connected to the nodes of a network whereas the costs linked to the shipping itself are connected to the links of the network.

It is, however, necessary to be able to connect a weight to a node in a conditional way. The cost of loading or transshipping is only borne if the goods are indeed loaded or transshipped. In other words, the transition from one link to another often takes place without having to change the mode or means of transportation.

Moreover, the weight of the nodes is almost never the same. To convince yourself you need only to observe any loading port along the Canal Albert/Albertkanaal (Belgium). The canal allows ships of up to 9000 tons but many of the ships on the canal are more modest ones. It goes without saying that, at the same port, the cost of loading a small ship differs from the cost of loading a large one.

The basic idea of the method and of the algorithm that will be proposed here, is to create a virtual network, starting from a real network. In this virtual network a link is created for all the elements to which a weight will be connected.

The solution can first be presented in an intuitive way, by using the example of a simple waterways network, as shown in figure 3.1.

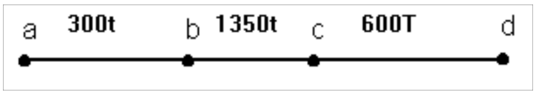

Figure 3.1: Simple network of waterways

This network consists of 4 nodes and 3 links. The links represent waterways that can support ships of 300, 1350 and 600 tons respectively.

To get from node a to node d, the least expensive route may imply a transshippment of the goods at node b to a ship of 1000 or 1350 tons, and transferring the goods to a ship of 600 tons at node c.

Another possibility is to start using ships of 600 tons at node b and keep using these to get to d.

Finally, an entire trip with ships of 300 tons would also be possible.

Although this network is a very simple one, it clearly reflects the problem of the modal choice.

The basic idea of the proposed method is to create a virtual network, starting from a real network. In the virtual network all the weights, whether linked to links or to nodes, will be assigned to links. The notation used for the identification codes of the nodes in this virtual network, makes it possible to immediately know the type of cost that has to be connected to each link. The virtual network derived from the real network presented above is represented in figure 3.2.

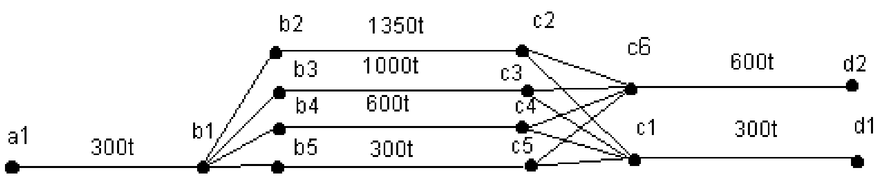

Figure 3.2: Corresponding virtual network

The solution implies the creation of a new set of nodes b1-b5, c1-c5, d1-d2 and a new set of links linking these new nodes.

In a first stage all the existing real links will be split into virtual links according to the possible means of transportation that can be used :

- The link (a,b) has not been split because no other ships can pass through a canal that can only support ships of 300 tons.
- The link (b,c) has been split into four parts. A canal of 1350 tons can indeed accept ships of 300, 600, 1000 and 1350 tons pass through
- The link (c,d) has been split into two parts, since it is possible to have ships of 300 and 600 tons pass through the canal.

The number of real links has now more than doubled, but they still need to be connected by means of other virtual links. These correspond to the transshippment operations.

The real node b, for instance, is represented in the virtual network by four virtual nodes and by four virtual links. On these links the costs of transshippment from a ship of 300 tons to ships of 600, 1000 and 1350 tons can be assigned. There is also a virtual link representing the passage from a segment of 300 tons to another segment of 300 tons. The weight of that link is zero; it corresponds to the simple transit of the ship through the (real) node b, without transshippment operations.

The same mechanism is used for the real node c. All modal combinations are also included in the form of virtual links and nodes. Two links without weight have been created for ships of 600 and 300 tons 'simple transit'.

In this way, the multi-modal network is represented by a mono-modal network on which each link has a unique weight representing either the cost of transportation over a certain distance, either the cost of a possible transshipment. When a cost is assigned to each link, the cheapest path can be computed by means of an algorithm such as the algorithm of Johnson. The resulting solution is an exact solution, taking all the possible choices into account and not an heuristic leading to an approximate solution.

3.3 Systematic development

The concept of the virtual network now remains to be developed and the algorithm for generating a virtual network starting from a real network, must be written. In order to make the following pages easier to read, the different stages of the development of the algorithm will be illustrated by a simple example of a network.

3.3.1 General method

Given the real network G = [X,U] of figure 3.3. This network is composed of 4 nodes (#X = N = 4) a, b, c and d and of 5 links (#U = M = 5) numbered from 1 to 5.

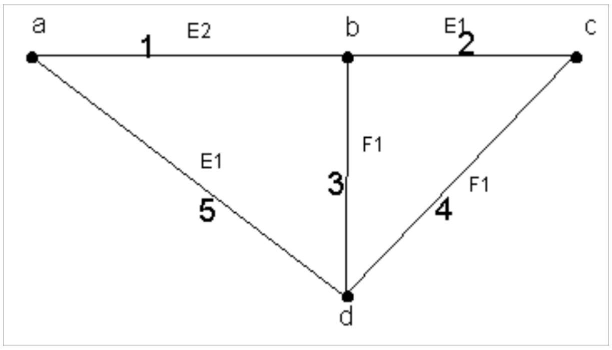

Figure 3.3: Multi-modal network

Two modes of transportation are present on the network : the waterways W and the railways T. For the waterways, there are canals (E1 and E2) supporting different sizes of ships, meaning canals of 300 and of 600 tons. On the links of type E2 it is possible to have ships of the types E1 and E2. There is only one type of train, F1 (diesel-powered). It is important to use a coherent notation for the means of transportation. For instance, if a link supports convoys of type E4, it has to be able to have convoys of the types E1, E2 and E3. For this reason the diesel-powered train is indicated by F1 and the electric train by F2, because diesel-powered trains can also make use of electrified lines, whereas the opposite is impossible.

The table 3.1 represents the network :

Link number	Node 1	Node 2	Type of route
1	a	b	E2
2	b	c	E1
3	b	d	F1
4	d	c	F1
5	a	d	F1

Table 3.1: Notation of the real network

The first stage of the method for creating a virtual network is to examine the set of links U and to create (→) for each real link j ∈ U as many virtual links $\bar{u}_j^{tm}$ as there are possible means of transportation connected to the transportation mode t on that link :

$$\forall_{j \in U} \forall_m u_j \rightarrow \bar{u}_j^{tm}$$

Each virtual link $\bar{u}_j^{tm}$ has two virtual end-nodes $\bar{x}_o^{jtm}$ and $\bar{x}_d^{jtm}$ to which an identifier has been assigned, coded in four parts as follows :

- The identification code of the real node i from which the virtual node is generated,
- The identification code of the real link j that has been multiplied (when several means of transportation are possible on that link),
- The identification code of the mode of transportation t on the real link j,
- The identification code of the means of transportation m that is possible on the new virtual link that is generated from the real link j.

A virtual node therefore can be identified by $\bar{x}_i^{jtm}$.

Since each real link has an origin o and a destination d, a real link will generate the virtual links $\bar{u}_j^{tm} = (\bar{x}_o^{jtm}, \bar{x}_o^{jtm})$ (see table 3.2).

Real links	Origin of virtual links	Destination of virtual links
1	a1E2	b1E2
	a1E1	b1E1
2	b2E1	c2E1
3	b3R1	d3R1
4	d4R1	c4R1
5	a5E1	d5E1

Table 3.2: Moving links

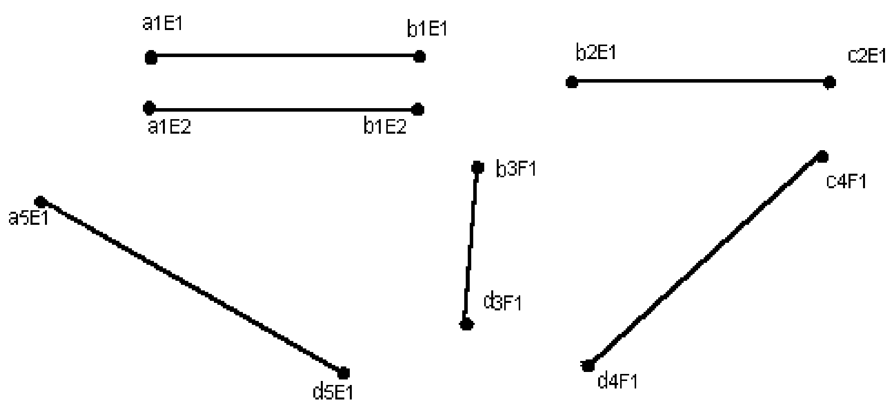

Figure 3.4: Separation of the transport means

Starting from this first result (figure 3.4), all the links representing possible transshipments have to be generated. This operation applies to all real nodes. In the first phase, all real nodes have been replaced by a set of virtual nodes.

$$\forall_{i \in X}, X_j \rightarrow \bigcup \bar{x}_i^{jtm}$$

All virtual nodes referring to the same real node have to be interlinked so as to represent the set of possible transshipments [2].

$$\forall_k \forall_{k,k \neq l} \rightarrow (\bar{x}_i^{ktm}, \bar{x}_i^{lt'm'})$$

For the real node b (figure 3.5) this gives the figure 3.3 :

Real node	Origin of virtual links	Destination of virtual links
b	b1E1	b1E2
	b1E1	b2E1
	b1E2	b2E1
	b1E1	b3R1
	b1E2	b3R1
	b2E1	b3R1

Table 3.3: Transshipment virtual links

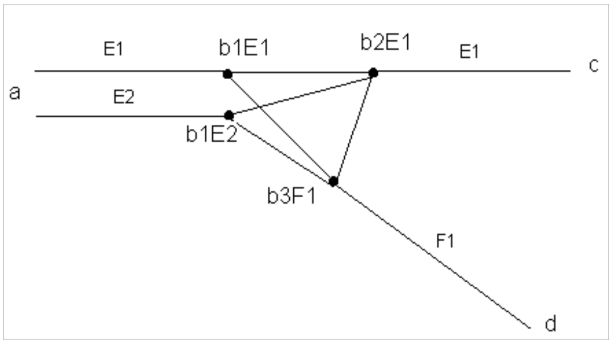

Figure 3.5: Transshipments

At this stage there are two types of virtual links :

- The virtual links representing a distance that has to be covered. These links represent the links of the real network, possibly multiplied if there are several possible means of transportation. Such a link has two virtual end-nodes originating from two different real nodes.
- The other virtual links represent all the possible transshipments. Each virtual node is interlinked with all the other virtual nodes of which the identification code refers to the same real node, excluding, however, the nodes generated by the same real link.

[2]Obviously, it is not because two links are connected that a transshipment is possible at the connexion. This case will be discussed later.

The last two columns of table 3.4 show the costs to be computed on each virtual link. In the case of a link that is generated directly from the real network, it concerns a transfer between two nodes and the weight is a function of the covered distance. 'Simple transit' costs are represented by a zero in the 'transshipment' column [3]. The costs of the type "E2→E1" represent the costs linked to a transshipment, in this case the transfer from ships of 600 tons to ships of 300 tons. There cannot be a cost in the columns "transshipment" and "shipping" at the same time.

Origin	Destination	Transshipment cost	Shipping cost
a1E1	b1E1	-	Cost = $f(distance)$
a1E2	b1E2	-	Cost = $f(distance)$
b1E1	b2E1	0	-
b1E2	b1E1	E2→ E1	-
b1E2	b2E1	E2→ E1	-
b1E1	b3R1	E1→ R1	-
b1E2	b3R1	E2→ R1	-
b2E1	b3R1	E1→ R1	-
b2E1	c2E1	-	Cost = $f(distance)$
b3R1	d3R1	-	Cost = $f(distance)$

Table 3.4: Possible operations around the real node b

If the same operation is repeated for all real nodes, the virtual network looks as the network represented by figure 3.6.

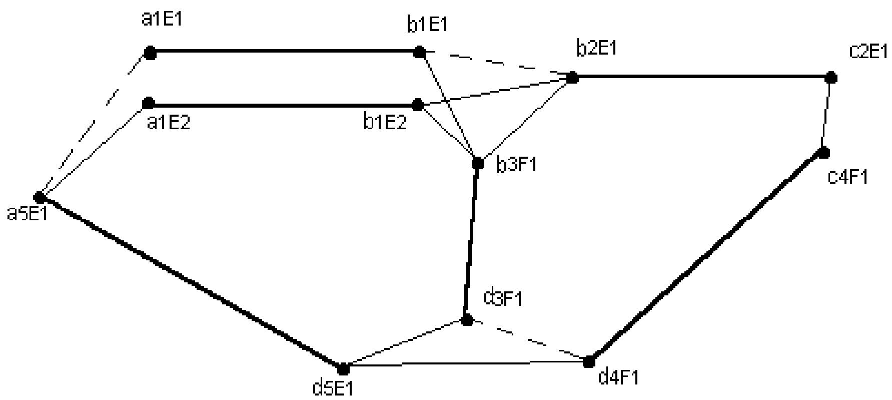

Figure 3.6: Partial virtual network

In this network, the boldfaced links represent the links of the real network, possibly split up. The links indicated by a dotted line represent the (simple transit(virtual links. The transshipments, finally, are indicated by a thin continuous line.

[3]In certain cases the cost is not really zero since this type of links can also serve to represent a cost linked, for instance, to the passage of a border or to the toll on a motorway.

This method can be transcripted into an algorithm (figure 3.7), in which the symbol # is used for the concatenation of the identification code.

```
DEFINE
tab1, tab2: arrays of virtual nodes
t1, t2 : indexes in these arrays

BEGIN
t1 ← 1
FOR j = 1 → mode ← transport mode on link j
    k ← number of transport means on link j
    FOR l = 1 → k
        n1 ←start node of link j
        n2 ←end node of link j
        node1 ← n1#j#mode#k
        node2 ← n2#j#mode#k
         Save link(node1, node2)
        tab[t1] ← node1
        tab[t+1] ← node2
        t1 ← t1 + 2
    END FOR i
END FOR j

FOR k = 1 → N
    tab2[ ] ← part of tab1[ ] generated from node k
    t2 ← size of tab2[ ]
    FOR i = 1 → t2
        FOR j = i+1 → t2
            Save link (tab2[i], tab2[j])
        END FOR j
    END FOR i
END FOR k
END
```

Figure 3.7: Basic algorithm

3.3.2 The entry-nodes

One problem remains unsolved : although it is possible to travel within the virtual network, it is not possible to enter it or to leave it ! The final user wants to find a travel route between the real nodes a and b and not between the virtual nodes axxx and bxxx. Moreover, there is a cost linked to entering and leaving the network since the goods have to be loaded and unloaded.

The virtual links and nodes again offer a solution. In the example of node b, for instance, it is sufficient to create a new virtual node b000 and to link it to all other virtual nodes generated for that real node. All the new links represent the costs of loading and unloading the goods.

$$x_i \rightarrow \bar{x}_i^{000}$$

$$\forall_i \forall_{ktm} \rightarrow (\bar{x}_i^{ktm}, \bar{x}_i^{000})$$

This lead, for the node b, to table 3.5 and figure 3.8:

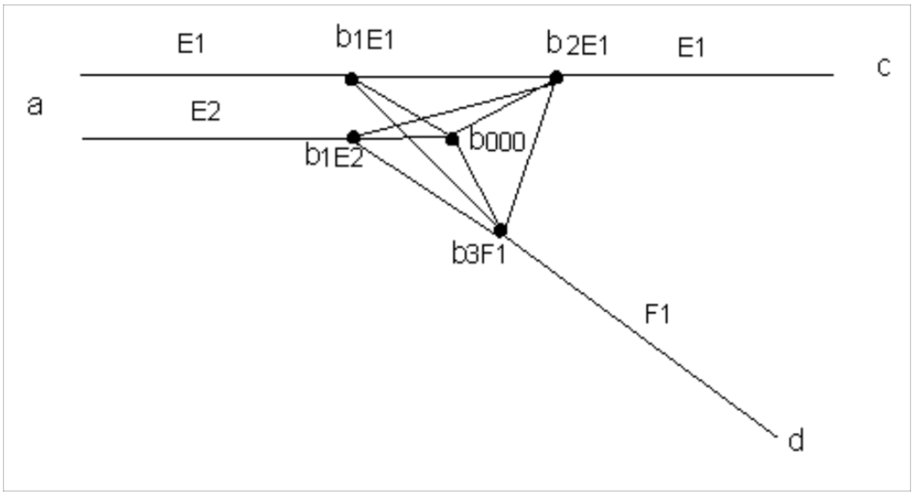

Figure 3.8: Loadings/Unloadings in a virtual network

Origin	Destination	Cost
b000	b1E1	0 → E1
b000	b1E2	0 → E2
b000	b2E1	0 → E1
b000	b3R1	0 → R1

Table 3.5: Costs on (un)loading virtual links

In table 3.5, the cost represents the initial loading of the goods or their final unloading.

The algorithm shown above has to be modified to take these new virtual nodes and links into account (see figure 3.9).

3.3.3 The simple transit nodes

At this stage, it is difficult to apply this method to a real network, as it often contains a serie of nodes that do not represent points where goods are being

```
DEFINE
tab1, tab2: arrays of virtual nodes
t1, t2: indexes in these arrays

BEGIN
t1 ← 1
FOR j = 1 → mode ← transport mode on link j
    k ← number of transport means on link j
    FOR l = 1 → k
        n1 ←start node of link j
        n2 ←end node of link j
        node1 ← n1#j#mode#k
        node2 ← n2#j#mode#k
         Save link(node1, node2)
        tab[t1] ← node1
        tab[t+1] ← node2
        t1 ← t1 + 2
    END FOR i
END FOR j

FOR k = 1 → N
    tab2[ ] ← part of tab1[ ] generated from node k
    t2 ← size of tab2[ ]
    FOR i = 1 → t2
        FOR j = i+1 → t2
            Save link(tab2[i], tab2[j])
        END FOR j
    END FOR i

    FOR i = 1 → t2
        node ← "node" part of tab2[i]#000
        Save link(node, tab2[i])
    END FOR i
END FOR k
END
```

Figure 3.9: Introduction of the (un)loading, virtual links

Real node	Real link	Mode	Means	Virtual node
b	1	W	1	b1E1
b	1	W	2	b1E2
b	2	W	1	b2E1
b	2	W	2	b2E2
b	3	W	1	b3E1

Table 3.6: Simple transit nodes

loaded/unloaded. The road network is characterized by a multitude of cross-roads that are also nodes, but where no goods are being loaded. In the same way the railway network contains some stations that are exclusively reserved for the passengers and where transshipments of goods are impossible.

For these nodes, no transshipment virtual links have to be generated.

In the example of figure 3.10, node b represents an intersection of a waterway of 600 tons (E2) and another of 300 tons (E1).

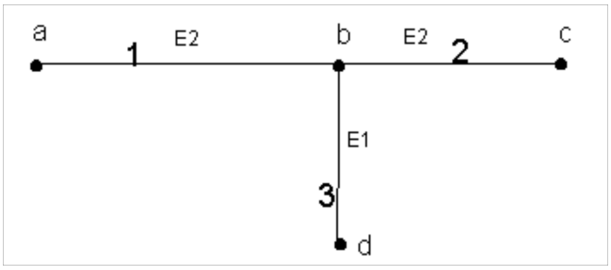

Figure 3.10: Intersection of two waterways

The trunks of type E2 also support convoys of type E1. A ship of 600 tons coming from segment 1 can therefore continue to segment 2, but not to segment 3.

The previously explained method leads thus to the creation of the virtual nodes listed in table 3.6) :

The virtual links now need to be generated. Again, shipping and transshipment virtual links must be generated. This time, however, only the nodes referring to the same combination mode-means of transportation (t = t' and m = m') may be interlinked. For this reason b1E2 and B2E2 will be interlinked, but not b1E2 and b3E1 (this last combination would suggest a transshipment from a ship of 600 tons to a ship of 300 tons and vice versa). In other words, only the "simple transit"virtual links are created. Moreover, since these "transit" nodes are not points where the network can be entered or left, the b000 node does not need to be created, nor do the virtual links that are generated from it.

$$\text{If (i = transshipment) or (t = t' and m = m')} \rightarrow \forall_k \forall_l \rightarrow (\bar{x}_i^{ktm}, \bar{x}_i^{lt'm'})$$
$$\text{If i is a transshipment node} \rightarrow \forall_i \forall_{ktm} \rightarrow (\bar{x}_i^{ktm}, \bar{x}_i^{000})$$

That leads to the creation of the set of links reported in table 3.7 (see also figure 3.11).

Origin	Destination	Transshipment cost	Shipping cost
a1E1	b1E1	-	$f(distance)$
a1E2	b1E2	-	$f(distance)$
b2E1	c2E1	-	$f(distance)$
b2E2	c2E2	-	$f(distance)$
b3E1	d3E1	-	$f(distance)$
b1E1	b2E1	0	-
b1E1	b3E1	0	-
b2E1	b3E1	0	-
b1E2	b3E2	0	-

Table 3.7: Virtual network at the intersection

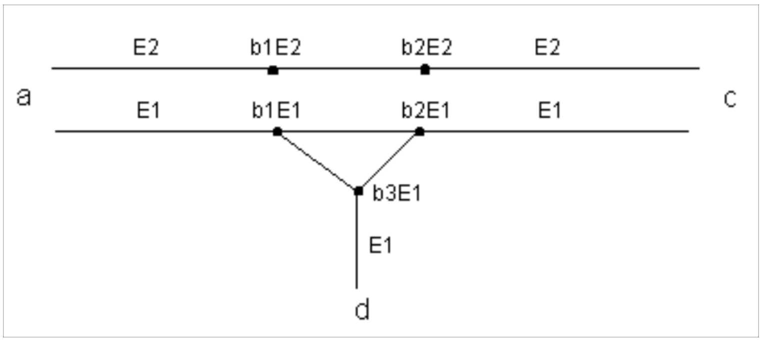

Figure 3.11: Virtual network at the intersection

The method differs according to whether the node is a place where goods are loaded/unloaded/transshipped or simply a place of transit. The nodes of the graph therefore have to be marked as belonging to one of those two categories.

This differentiated treatment (see figure 3.12) has a very interesting effect. Indeed, since the "transit" nodes generate less virtual links, the virtual network itself will consist of clearly less links than the network generated by the basic algorithm, an effect which will accelerate the processing time.

3.3.4 Orientation of the virtual network

The method hereby proposed leads to the generation of a non oriented virtual network. Since each link has to be balanced by a unique weight (a cost), the use of a non oriented graph creates the problem of the equivalence of weights when there are transfers from origin to destination or from destination to origin.

Such an equivalence is to strong hypothesis to be applied to a freight transportation network. Certain costs are a function of the direction on the link.

```
DEFINE
tab1, tab2: arrays of virtual nodes
t1, t2: indexes in these arrays

BEGIN
t1 ← 1
FOR j = 1 → mode ← transport mode on link j
    k ← number of transport means on link j
    FOR l = 1 → k
        n1 ←start node of link j
        n2 ←end node of link j
        node1 ← n1#j#mode#k
        node2 ← n2#j#mode#k
        Save link(node1, node2)
        tab[t1] ← node1
        tab[t+1] ← node2
        t1 ← t1 + 2
    END FOR i
END FOR j

FOR k = 1 → N
    tab2[ ] ← part of tab1[ ] generated from node k
    t2 ← size of tab2[ ]
    FOR i = 1 → t2
        FOR j = i+1 → t2
            IF not a transshipment node
                AND "mode" part of tab2[i] = "mode" part of tab2[j]
                AND "means" part of tab2[i] = "means" part of tab2[j])
            OR transshipment node
                    THEN Save link(tab2[i], tab2[j])
            END IF
        END FOR j
    END FOR i

    FOR i = 1 → t2
        IF transshipment node THEN
            node ← "node" part of tab2[i]#000
            Save link(node, tab2[i])
        END IF
    END FOR i
END FOR k
END
```

Figure 3.12: Introduction of the "simples transit virtual links"

This is, for instance, the case for loadings and unloadings : experience shows that it often takes more time to unload than to load a vehicle.

In practice, the algorithm of the virtual network will generate oriented links, which makes it possible to assign different costs according to the direction of the virtual link. In order to generate only one (oriented) arrow between two virtual nodes (to avoid unwanted turns during the cheapest path calculation), all the nodes are “doubled” by giving them a positive and a negative sign.

Using the notation used in section 3.3.1, the virtual network generated around node b and illustrated by figure 3.8 can be represented as in figure 3.13.

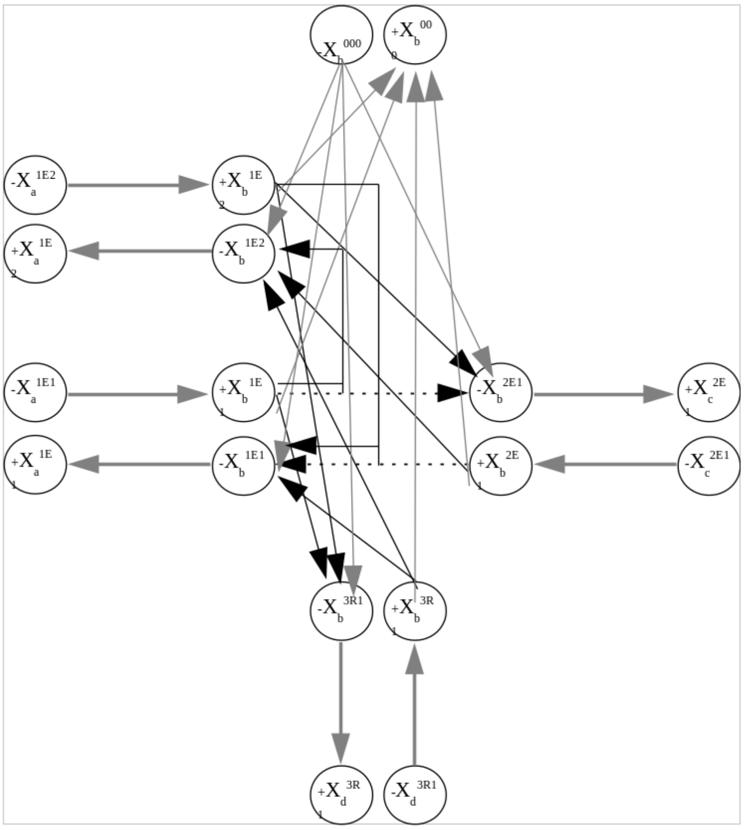

Figure 3.13: Oriented virtual network

3.3.5 Additional control on the generation of the virtual network

The virtual network, as it has been defined for the moment, is the result of an automatic procedure. This way of proceeding, however, implies certain restrictions, because some transshipment possibilities are generated automatically whereas these movements are not possible on the real network. This is, for instance, the case with private ports on inland waterways. A particular company may be authorised to load and unload goods there, but this port does not serve as a public place of transshipment. Therefore it is important to be able to control the generation of the virtual network.

This check becomes possible if exclusions lists are kept for the different nodes of the real network. In practice it must be possible to define, for each node of the network, the list of handling operations which would be generated automatically but which are not possible in reality because of the physical characteristics of the network at that location. The generation procedure of the virtual network is consequently adjusted, since it consults these lists of exclusions to know whether or not a virtual link can be created.

The final algorithm is is presented in figure 3.14.

3.4 Assignment of the costs on the virtual links

The virtual network makes use of four types of distinct costs : loading/unloading, transshipping, moving and simple transit. The type of cost to be assigned to each virtual link can be automatically deducted from the notation used for the two virtual end-nodes of the link.

For readability reasons, the notation used up to here has been a notation of the type "b1E1". It is clear that this type of notation cannot be used in practice. NODUS will therefore use a 10 digits integer to represent the number of the real node, another 10 digits integer for the number of the real link, two digits to represent the transportation mode and two digits to refer to the means of transportation.

As mentioned above, the virtual node is indicated in the following way (see also table 3.8) :

1. Moving (**"mv"**): the numbers of the real nodes are different in the two virtual nodes codes (case 1). Always from a - to a + sign.
2. Simple transit (**"tr"**): the number of the real links vary whereas the mode and the means of transportation have remained the same (case 2). Always from a + to a - sign.
3. Transshipment (**"tp"**): the mode and/or means of transportation vary (case 3). Always from a + to a - sign.
4. Loading (**"ld"**)/unloading (**"ul"**): one of the two real links is "0" (cases 4a and 4b). Always from a - to a - sign for loading and from a + to a + sign for unloading.

Different costs correspond to these different cases. In the following chapter, a general methodological framework will be provided for the development of specific cost functions for each application.

```
DEFINE
tab1, tab2: arrays of virtual nodes
t1, t2: indexes in these arrays

BEGIN
t1 ← 1
FOR j = 1 → mode ← transport mode on link j
        k ← number of transport means on link j
        FOR l = 1 → k
                n1 ←start of link j
                node1 ← +#n1#j#mode#k and node2 ← -#n2#j#mode#k
                Save link(node1, node2)
                tab[t1] ← node1; tab[t+1] ← node2
                t1 ← t1 + 2
                node1 ← -#n1#j#mode#k and node2 ← +#n2#j#mode#k
                Save link(node2, node1)
                tab[t1] ← node1; tab[t+1] ← node2
                t1 ← t1 + 2
        END FOR i
END FOR j

FOR k = 1 → N
        tab2[ ] ← part of tab1[ ] generated from node k
        t2 ← size of tab2[ ]
        FOR i = 1 → t2
                FOR j = i+1 → t2
                        IF not a transshipment node
                                AND "mode" part of tab2[i] = "mode" part of tab2[j]
                                AND "means" part of tab2[i] = "means" part of tab2[j]
                        OR transshipment node
                                        THEN IF non excluded transfer
                                                IF "node" part of tab2[i] ¿ 0
                                                        THEN Save link(tab2[i], tab2[j])
                                                        ELSE Save link(tab2[j], tab2[i])
                                                END IF
                        END IF
                END FOR j
        END FOR i

        FOR i = 1 → t2
                IF transshipment node AND non excluded transfer THEN
                        node ← -#"node" part of tab2[i]#000; Save link(node, tab2[i])
                        node ← +#"node" part of tab2[i]#000; Save link(tab2[i], node)
                END IF
        END FOR i
END FOR k
END
```

Figure 3.14: Algorithm of generation of a virtual network

Case	Node1	Link1	Mode1	Means1	Node2	Link2	Mode2	Means2
1	-1000	1000	1	1	+1001	1000	1	1
2	+1000	1000	1	1	-1000	1001	1	1
3	+1000	1000	1	1	-1000	1001	1	2
4_a	-1000	0	0	0	-1000	1001	1	1
4_b	+1000	1001	1	1	+1000	0	0	0

Table 3.8: Codification of the virtual links in NODUS

3.5 Concluding comments

The virtual network (and NODUS, the software that implements it can be presented as :

- An exhaustive representation of all possible movements and operations on a multi-modal transportation network.
- A systematic way to generate of the elements composing the network through an automatic procedure.
- A general representation of a network, adapted to the realisation of a wide range of different applications.
- A systematic and codified notation of the elements of the network, allowing to know the nature of the costs to be assigned to the different links. This same notation, based on numbers of virtual nodes coded in four parts, contains all the necessary information on the modes and means of transportation used. This information can be used, after an assignment on the network, to retrace the modes and means of transportation that have effectively been used on the different links composing the route. This is an important characteristic and a valuable aspect of the concept of the virtual network.

Chapter 4

General considerations on costs

Having briefly presented the theoretical concepts on which NODUS is based, the different costs that can be assigned to the links of the virtual network remain to be analysed. Unfortunately, and in spite of the existence of several network models, the literature presents very few specific cost functions. On the following pages, cost elements already published will be discussed and commented , in order to provide a general and concrete methodological framework, applicable to the virtual network.

Economic analyses based on a network model are only significant if the weights used on the different links of the network are credible. These weights can take different forms. They can concern prices, costs, time limits,... In order to be able to express these different forms of weights as monetary values, the concept of "generalised cost" is often used, a basic formulation of which was presented by Kresge and Roberts [1] :

$$C_{ij} = f_{ij} + b_1 s_{ij} + b_2 \sigma s_{ij} + b_3 w_{ij} + b_4 p_{ij}$$

where:

- f_{ij}: direct costs supported by the operator between the nodes i and j
- s_{ij}: travelling time between i and j
- σs_{ij}: variability of the travelling time
- w_{ij}: waiting time before the actual transportation

[1] Kresge, D.T. and Roberts, P.O.,1971, "Techniques of Transportation Planning: Systems Analysis and Simulation Models", Brooking Institution, Washington DC. A more depth discussion about generalised cost functions can be founded in Wilson A.G. and Bennet R.J., 1985, "Mathematical Methods in Human Geography and Planning", John Wiley & Sons, N-Y.

- p_{ij}: probability of having the shipment damaged or lost

In this formulation, the coefficients b_n that balance the different components of the function are generally proportional to the value of the transported goods. After the example of the approach of Baumol and Vinod, presented in an earlier chapter, the generalized cost offers the possibility to assign a cost to all the variables influencing the traffic on a network. This way, a shipment expressed in kilometres or a waiting time expressed in hours can be summed because expressed as monetary values. The uncertain terms in those formulations can, however, not be directly assigned on the network, except when they are incorporated as average values.

The virtual network requires the development of four types of cost functions. As mentioned above, the type of function is known through the notation used for the virtual nodes. A concrete illustration is presented in table 4.1.

- Moving (**"mv"**): The identification numbers of the real nodes vary.
- Simple transit (**"tr"**): The identification numbers of the real links vary whereas the mode and the means of transportation have remained the same.
- Transshipment (**"tp"**): The identification numbers of the real links vary, as well as the mode and/or means of transportation.
- Loading (**"ld"**) / unloading (**"ul"**): One of the two identification numbers of the real links is "0".

Case	Node1	Link1	Mode1	Means1	Node2	Link2	Mode2	Means2
1	-1000	1000	1	01	+1001	1000	1	1
2	+1000	1000	1	01	-1000	1001	1	1
3	+1000	1000	1	01	-1000	1001	1	2
4a	-1000	0	0	0	-1000	1001	1	1
4b	+1000	1001	1	1	+1000	0	0	0

Table 4.1: Examples of notation for virtual links

Now, the cost elements that can be used with the different cases remain to be determined, knowing that initially and intuitively, the total cost of transportation can be resolved in the " accountancy " way presented in figure 4.1:

Important remark: It is important to keep in mind that the functional form proposed at the end of the chapter should not be regarded as a unique and unalterable formulation, but rather as a solution well adapted to NODUS. The only methodological restriction imposed by the concept of the virtual network is the development of specific functions for the four cases mentioned above. The functional form of these functions can be freely chosen. Having specified this, it may be interesting to indicate a few lines of thought, a kind of "code of good behaviour" that has to be observed during the development of cost functions in a multi-modal environment:

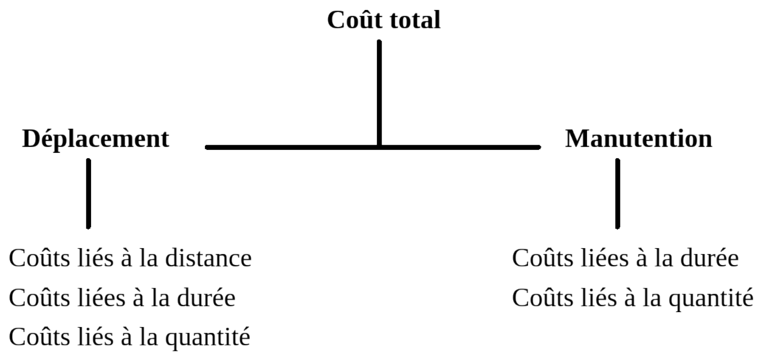

Figure 4.1: Transport costs

- Coherence of the viewpoints: a transportation can take place and be effected by the company itself or by an independent carrier. In the first case, all the costs borne by the company during the transportation process have to be analysed, whereas in the second case it is of great importance to know the charged tariffs. It is thus essential to adapt the same viewpoint for the different modes of transportation taken into consideration on the network.
- Coherence of the units: when a unit of measure is chosen (monetary units per ton, per kilometre, per ton/kilometre,...) for a mode of transportation on a type of virtual link, it obviously is important to use that same unit of measure for the other modes of transportation.
- Coherence of the variables: in the definition of the generalised costs, the same cost factors have to be used for the different modes of transportation. If, for instance, the duration of the trip or the financial costs are taken into account for one mode of transportation, these same costs also have to be introduced in the functions developed for the other modes.
- Coherence in time: the data used in the calculations have to be time-compatible (same year of reference) for the different modes of transportation.

Having made these observations, it is possible to define a general approach of the cost elements applicable on a virtual network.

1. Gathering of data while observing the principle of time-coherence
2. Development of cost functions (while observing the three other principles of coherence)
   - For all modes and means of transportation.
   - For the four types of virtual links.
3. Calculation of the costs on each virtual link.

In the literature, two large categories of cost functions are proposed. The first one is aimed at analysing the set of costs linked to the transportation activities. This is an aggregated approach which, by definition, does not apply to a network. Unlike the first categorie, the second type of cost functions analyses a movement from a given origin to a given destination. Unfortunately, this type of approach is rather less spread, but there are certainly sufficient references to be able to have a correct idea on the useful characteristics of transport-related costs.

The total cost of transportation consists of different parts that can be classified in the following categories:

- The actual transportation: all the costs connected to the moving of a vehicle between the points of origin and destination of the trip.
- Inventory value: costs entailed by the storage of the goods during a certain time span. This part comprises real expenses such as insurances, interest and opportunity cost (the transported goods represent a certain sum of money, which is tied up and could have been used in another way).
- Handling, storage: expenses linked to the manipulations of the goods beside the actual trip. It concerns packaging, stocking, loading and unloading.
- Indirect costs: costs entailed by activities subsidiary to transport (administrative services, ...). It is difficult to identify these costs for a particular trip.

Besides these costs, the costs linked to congestion and the impact of the quality of transport also need to be taken into consideration.

Congestion is indeed one of the factors influencing the cost of transportation. It seems evident that at some moment the users of a network will encounter congestion problems, and this has to be taken into account in the assignments. The aim is to introduce a method able to solve the constraints linked to the capacity of the network.

There are two fundamentally different ways to proceed:

- At a microscopic level, congestion leads to a slowing down of the flow and to a choice of alternative routes to avoid traffic jams. A solution for this problem can be found in the network equilibrium methods that recompute the cost functions while the flow increases. These dynamic processes explicitly take constraints such as the capacity of the roads or the congestion at terminals into account.
- At a macroscopic level, it is not always important to know whether certain crossroads are saturated at 8 o'clock in the morning, since those crossroads are not even introduced in the network. However, general considerations such as the passage through big cities or the waiting time at certain borders need to be taken into account. This microscopic viewpoint influences the

cost functions, but not necessarily the routes that are used. Actually, when a large network such as the trans-European one is digitized, the details of what is going on in a city are but rarely dealt with. In such a case, a digitized link can sometimes represent several parallel routes over which the traffic will be spread over. The cost related to congestion can then simply be assigned to the " simple transit " virtual link in the form of a fixed cost per unit of time lost.

The quality of the transportation should be looked upon in a very broad sense and it essentially represents the multi-products dimension of the transportation process. Actually, the distinction between the different products has not yet been made, except for their inventory value. This value, however, is not representative since different means of transportation can be used to transport commodities of different categories. Imagine two types of goods of which the weight expressed in tons is more or less the same, and which could be transported by the same mode of transportation, but which, in practice, would be transported by different modes of transportation and at different costs. The choice of the mode of transportation is thus also influenced by a certain quality inherent to that mode and difficult to express in monetary terms.

Let us take the example of a Belgian producer exporting clothes to France. He chooses road transport, although it would be cheaper by train. A truck, however, gives him two advantages:

- Door to door transport;
- Clothes are transported on hangers in the truck. In this way, the pieces of clothing do not wrinkle.

What value should be attributed to the fact that a piece of clothing does not wrinkle ?

Whereas factors such as time, frequence or operational safety are often found, the concept of quality cannot that easily be introduced in a cost function. Moreover, it is not easy to determine at what moment quality will play a role. Is this the case while loading or while shipping ? While very interesting, this question goes far beyond the scope of this methodological note.

Starting from the definition of the virtual network, that requires four types of cost functions, the table 4.2 reflects the types of costs which should be taken into consideration.

Type of virtual link	Nature of the costs
Shipment	Transportation
	Inventory value
	Indirects costs
(Un)Loading	Handling
	Inventory value
	Indirect costs
Transshipment	Handling
	Storage
	Inventory value
	Indirect costs
Simple transit	" Macroscopic " congestion

Table 4.2: Types of costs on the virtual links

Chapter 5

Some examples of specific cost functions

The following pages will present a complete set of cost functions for the different "classical" terrestrial modes of transportation, which are the roads, the railways and the waterways. These functions are *one* possible approach among many others. They have, however, the advantage of having shown a certain stability in several published practical applications.

5.1 The shipping costs

The transit costs are expressed in monetary units per weight-unit and per distance-unit covered.

5.1.1 The transportation

The transportation cost consists of different parameters of a more technical nature. This cost differs according to the modes of transportation.

Waterways

The ships Given:

- F : the fixed annual costs (constant capital annuity, insurances, maintenance and wages of the crew),
- u : number of working hours per year,

- T : load of the ship in weight-units,
- b : fuel consumption in monetary units per time-unit (hour),
- ϕ : average speed.

The shipping cost per weight-unit and per distance-unit is expressed by :

$$B = \frac{F + b.u}{u.\phi.T}$$

The barges Given:

- F_p: the costs linked to the towboat include a capital annuity, insurances, maintenance and the wages of the crew,
- F_b : the fixed costs linked to the barges only include a constant capital annuity, insurances and maintenance,
- u : the number of working hours per year,
- T : the load of the barge in weight-units
- b : the fuel consumption in monetary units per time-unit (hour),
- ϕ : the average speed.

$$B = \frac{F_p + F_b + b.u}{u.T.\phi}$$

Railways

As for the barges, the fixed costs consist of F_m and F_w, representing the fixed costs linked to the engine (constant capital annuity, insurances and wages of the crew) and those linked to the wagons (constant capital annuity, insurances and maintenance) respectively. Because the maintenance is a fixed cost for the wagons, it is incorporated in F_w. The maintenance costs of the engine, however, vary and are formulated distinctly.

Given:

- F_m : fixed cost linked to the engine,
- F_w : fixed cost linked to the wagons,
- u : the number of working hours per year,
- T : the load of the train in weight unit,

- b : the fuel consumption in monetary units per time-unit (hour),
- ϕ : the average speed,
- e : the maintenance cost of the engine monetary units per distance unit.

$$B = \frac{F_m + F_w}{T.u.\phi} + \frac{b+e}{T}$$

This comes down to:

$$B = \frac{F_m + F_w + (b+e).u.\phi}{T.u.\phi}$$

As the specific cost functions for ships and barges, these forms are very "technical" functional forms trying to take the most relevant parameters linked to the costs of a train into consideration.

This type of approach sometimes underestimates of the real costs, since the costs linked to a certain level of efficiency of the transportation systems, such as the labour costs of possibly redundant personnel, are not taken into account.

Roads

Contrary to the boats, the fixed costs do not include maintenance costs. The maintenance of a truck takes place after a certain number of distance-units (distance-units).

Given:

- u : the number of working hours per year,
- T : the load of the truck in weight units,
- c : the fuel consumption in monetary units per distance-unit,
- ϕ : the average speed,
- e : the maintenance cost in monetary units per distance-unit.

Therefore

$$B = \frac{F}{T.u.\phi} + \frac{c+e}{T}$$

This comes down to:

$$B = \frac{F + (c+e).U.\phi}{T.u.\phi}$$

5.1.2 Inventory value

The inventory value represents the opportunity cost entailed by the immobilization of the transported commodity during the trip. This value is expressed by the interest charged on the value of the load for a period corresponding to the total duration of the trip.

Given:

- V : value of the commodity (monetary units per distance-unit),
- R_i : interest rate that will be applied to the inventory value,
- D : duration of the trip

The expression $V.R_ii.D$ makes it possible to know the cost linked to the inventory value during the trip, in the form of an opportunity cost. Since the average speed of the convoy is known, it is possible to calculate the duration of the shipping on a " moving " virtual link.

5.1.3 Indirect costs

Because of their structure, the railways bear very high administrative and other diverse costs, which have to be included in the transportation costs. "Administrative and other costs" are not directly entailed by the activity of transportation of the company. It goes over:

- General services,
- Exploitation of the infrastructure,
- Marketing and sales departments,
- Other charges,
- Contributions for risks and accidents (the railway companies insure themselves).

These costs can, for instance, be reduced to a certain sum per weight unit per distance-unit.

5.2 The transshipment costs

The second type of links generated in a virtual network are the " transshipment link ". The cost that has to be assigned on these links also consists of several aspects.

5.2.1 Handling

Beside the costs linked to the investment and to the use of the infrastructures, the handling costs depend on the duration of this operation. A transshipment can be split up into the actions of unloading and loading, the durations of which can be estimated with the formula of Deming (see above). When a means of transportation is made up of several loading units (wagons or barges), the time of loading or unloading is calculated per loading unit. The total time of loading is obtained by multiplying the time per unit by the number of loading units. This way of proceeding is explained by the non linearity of the function of Deming. It actually takes more time to load ten wagons of 30 weight units than to load 300 weight units at once.

5.2.2 Inventory value

The duration of the handling, obtained thanks to the formula of Deming, can be expressed as a fraction of one year. That expression is $D1$. If V is the value of the commodity and R_i the interest rate to be applied, $V.D1.Ri$ expresses the interest borne during the loading and/or the unloading of one weight unit of a commodity value V.

5.2.3 Indirect costs

In this section the costs linked to the immobilization of the vehicles during the handling operations will be dealt with.

Waterways

The ships Given :

- F: the fixed annual costs (constant capital annuity, insurances, maintenance and wages),
- L : the time needed for loading and unloading,
- u : the number of working hours per year,
- T : the load of the ship expressed in weight units,
- n : the number of persons needed for loading and unloading.

The fixed cost per weight unit can then be expressed by:

$$A = \frac{F.\frac{L}{N}}{u.T}$$

The barges Given:

- Fp : the fixed costs linked to the towboat,
- F_b : the fixed costs linked to the barges,
- t : the time needed to form the convoy,
- L : the time of loading and unloading,
- u : the number of working hours per year,
- T : the load of the ship in weight units,
- n : the number of persons needed for loading and unloading.

The fixed costs linked to the towboat include a constant capital annuity, insurances, maintenance and wages. The fixed costs linked to the barges only include a constant capital annuity, insurances and maintenance.

$$A = \frac{F_p.t + \frac{F_b.L}{n}}{u.T}$$

Railroads

As for the barges the fixed costs can be split up into F_m and F_w.

- L : the time needed for loading and unloading,
- u : the number of working hours per year,
- T : the load of the train in weight units,
- n : the number of persons nedeed for loading and unloading,
- t : the time needed to form the convoy in the shunting-yard.

Therefore:

$$A = \frac{F_m.t + \frac{F_w.L}{n}}{u.T}$$

Roads

Given:

- F: fixed costs,

- L : the time nedeed for loading and unloading,
- u : the number of working hours per year,
- T : the load of the truck in weight units,
- n : the number of persons nedeed for loading and unloading.

$$A = \frac{F.\frac{L}{n}}{u.T}$$

5.3 The (un)loading costs

A cost structure closely linked what has been defined for the transshipments is implemented for this third type of virtual links.

In this case, a transit through a warehouse sometimes has to be taken into account.

5.3.1 Handling

Please refer to the functions presented in the section "transshipment costs" section.

5.3.2 Storage

It is very difficult to develop a general cost function for the storage. The storage actually varies according to several parameters: each warehouse is different and functions in a different way. In order to simplify the problem a fixed amount per weight unit can be introduced, although this way of proceeding can be criticized.

5.3.3 Inventory value

See "Transshipment costs".

5.3.4 Indirect costs

See "Transshipment costs".

5.4 Simple transit costs

There is a fourth and last type of virtual links left, the one representing the "simple transit", on which a cost linked to congestion can be assigned, if no equilibrium model is applied.

Other costs can also be assigned to the "simple transit" virtual links : the costs entailed by the passage of borders or those entailed by technical constraints (different track-gauge for instance) can be taken into consideration. This type of costs is introduced in the functions by means of a fixed cost, connected to certain nodes of the network.

These (real) nodes actually generate (virtual) links on which it is possible to assign a cost function. Part of these virtual links consist of "simple transit" virtual links, i.e. The case of the passage of a border or of the tollage on a motorway clearly illustrate the use that can be made of simple transit links. Actually, the passage of a border does not imply a change of mode/means of transportation. There often are, however, administrative formalities that can take a certain time and therefore cost money. This type of cost can very well be assigned to a "simple transit" virtual link. This way of reasoning can also be applied in order to take the costs linked to the tollage on motorways into consideration.