Wind-Stereo Radio Time Delay Source Location Project

Notes

Sam Badman - 07/27/2017

1 Source Location from 2 Baselines

A single baseline time delay measurement specifies the source location (assuming isotropic emission and the same optical path length for rays reaching both spacecraft) to lie on one branch of a 2 sheet hyperboloid with it's two foci given by the location of the two spacecraft. This is because the source location is the locus of all points which have the same path length difference to the two spacecraft - this is precisely the geometrical definition of one sheet of a 2 sheeted hyperboloid.

A second independent baseline gives another hyperboloid and the intersection of these two surfaces is a line in 3D space which gives the locus of possible points where the source could be located. This line is a conic section with it's foci lying in the plane defined by the two baselines, and oriented perpendicular to this plane.

For sources located much further away than the baseline distance, the conic section is a hyperbola regardless of baseline geometry. It tends to a straight line at large source distances and so the source location can be reduced to a unit vector direction with the loss of distance information, to within reflection in the xy plane. For closer sources, the unit vector direction is dependent on the distance.

For closer sources, the baseline geometry can result in the solution locus being an ellipse or a hyperbola. For ellipses, there are no far field solutions to the time delay. In any case except when the source is directly aligned with one of the baseline, the possible locations are only constrained to a 1D curve in space.

If we know nothing apriori about the source, this is the best we can do with 2 baselines. A third independent baseline would reduce the 1D locus of positions to a single point. However, if we know information about the source, for example we know that Jupiter lies approximately in the ecliptic plane, we can use this information to intersect with the 1D locus and find the location to at least 2 single points.

With measurements from Wind, Stereo A and Stereo B, we have exactly this geometry and results. We know the absolute positions of these 3 objects so we know their direction relative to each other and their separation (baseline) distances.

For the purposes of parameterising the hyperboloids:

1. We take the location of Wind to be our origin as it is a common point in both baseline vectors.
2. We define our coordinate system reference axis to be the Wind-Stereo A separation vector
3. We define the coordinate system reference plane to be the plane containing the reference vector, and the Wind-Stereo B vector.
4. We make the choice that the z axis points in the direction defined by the cross product of the two baseline vectors. For the usual Wind-Stereo geometry which looks like a delta shape pointing away from the Sun, because the reference plane is approximately the ecliptic this choice means our z axis is approximately coaligned with the HEE z-axis, the z-axis defined by prograde rotation in the ecliptic plane.

2 Analytic Solution

2.1 Geometry

With the coordinate system defined above, with respect to WIND, (W) located at the origin, Stereo A, (A), Stereo B, (B) and the source point (S) are located at positions:

$$\vec{r}_{WA} = \begin{pmatrix} r_{WA} \\ 0 \\ 0 \end{pmatrix} \qquad \vec{r}_{WA} = \begin{pmatrix} r_{WB}\cos\theta_{AB} \\ r_{WB}\sin\theta_{AB} \\ 0 \end{pmatrix} \vec{r}_{WS} = \begin{pmatrix} r_x \\ r_y \\ r_z \end{pmatrix} \tag{1}$$

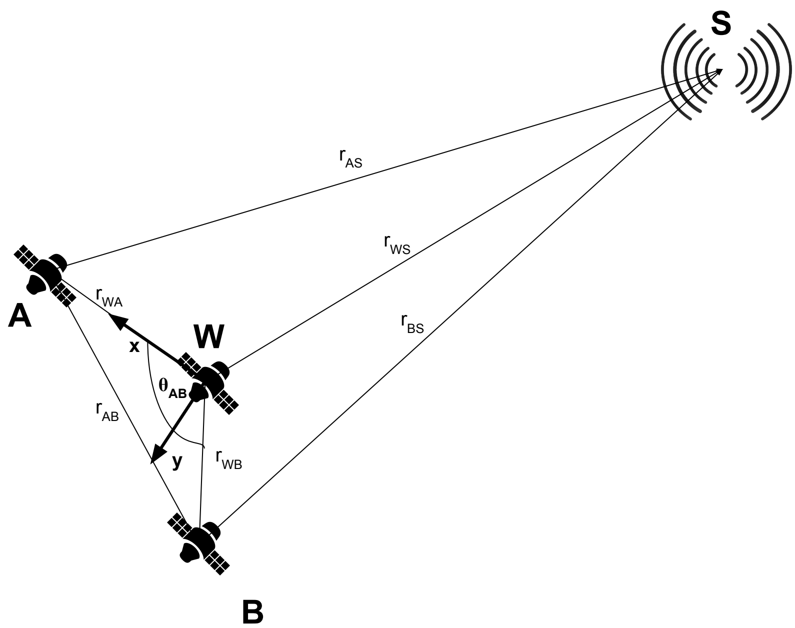
where for a vector $\vec{r}$, r is the magnitude, and where $\theta_{AB} \in \{0, \pi\}$ is the angle subtended by the WIND-StA and WIND-StB vectors. Note the baseline vectors both lie in the chosen x-y plane.

The source location vector has magnitude r_{WS}. This can be located anywhere in 3D space. As mentioned above, we will not in general be able to determined this as a 3D point, but can at least constrain it to a 1D curve in 3D space.

Note the following vector relations, which will be key in the subsequent analysis.

$$\vec{r}_{WS} = \vec{r}_{WA} + \vec{r}_{AS} \tag{2}$$
$$\vec{r}_{WS} = \vec{r}_{WB} + \vec{r}_{BS} \tag{3}$$

The geometry as viewed from along the z-axis is summarised in the figure below:



2.2 Time Delay

This whole project revolves around the fact that the radio wave propagation time from a given point source to each spacecraft is slighty different due to the spacecraft's different positions in space.

Let's say that at time $t = 0$ a monotone, isotropic radio pulse is produced at the source. It expands from the source as a spherical wave with expansion speed equal to the speed of light c.

Stereo A receives the pulse at time $t_A = r_{AS}/c$, Stereo B receives the pulse at time $t_B = r_{BS}/c$, and WIND receives the pulse at time $t_W = r_{WS}/c$. We see the arrival times are directly proportional to the distance from the respective spacecraft to the source, and so the spacecraft with a closest line of sight distance to the source receives the signal earliest and so on. However, *we do not know the absolute time of emission of the signal a priori.* We can only measure the difference in time between the signal arriving at two spacecraft, that is $t_A - t_W$ and $t_B - t_W$. With this noted, we can simplify by changing the above definitions of t_A, t_B, t_W, setting t_W to 0, and so letting t_A denote the time (positive or negative) that the signal arrives at A relative to W, and likewise for t_B. Then, we have $r_{AS} - r_{WS} = ct_A$ and $r_{BS} - r_{WS} = ct_B$

2.3 Solving for the source position

The driving goal here is to express the components of $\vec{r}_{WS}$ in terms of one parameter. Here, I choose this parameter to be $r_{WS} \in \{0, \infty\}$, essentially the absolute travel time from the source to WIND.

First, we use vector relations (2)-(3) :

$$
\begin{aligned}
\vec{r}_{WS} = \vec{r}_{WA} + \vec{r}_{AS} \implies r_{AS} &= |\vec{r}_{WS} - \vec{r}_{WA}| \\
&= \left| \begin{pmatrix} r_x \\ r_y \\ r_z \end{pmatrix} - \begin{pmatrix} r_{WA} \\ 0 \\ 0 \end{pmatrix} \right| \\
&= \sqrt{r_{WS}^2 - 2r_x r_{WA} + r_{WA}^2} \\
\vec{r}_{WS} = \vec{r}_{WB} + \vec{r}_{BS} \implies r_{BS} &= |\vec{r}_{WS} - \vec{r}_{WB}| \\
&= \left| \begin{pmatrix} r_x \\ r_y \\ r_z \end{pmatrix} - \begin{pmatrix} r_{WB}\cos\theta_{AB} \\ r_{WB}\sin\theta_{AB} \\ 0 \end{pmatrix} \right| \\
&= \sqrt{r_{WS}^2 - 2(r_x\cos\theta_{AB} + r_y\sin\theta_{AB})r_{WA} + r_{WA}^2}
\end{aligned}
$$

and subtract the distance from the source to WIND from both equations to get everything in terms of time differences which are known. We can then solve for r_x and r_y in terms of the parameter r_{WS} and other known quantities:

$$
\begin{aligned}
r_{AS} - r_{WS} = &\sqrt{r_{WS}^2 - 2r_x r_{WA} + r_{WA}^2} - r_{WS} \\
&\implies (ct_A + r_{WS})^2 = r_{WS}^2 - 2r_x r_{WA} + r_{WA}^2 \\
&\implies r_x = \frac{1}{2r_{WA}}(r_{WS}^2 + r_{WA}^2 - (ct_A + r_{WS})^2) \\
&\implies r_x = \frac{r_{WA}^2 - (ct_A)^2}{2r_{WA}} - \frac{ct_A}{r_{WA}} r_{WS} \\
&\implies \boxed{r_x(r_{WS}) = a_x + b_x r_{WS}}
\end{aligned}
$$

$$
\begin{aligned}
r_{BS} - r_{WS} = &\sqrt{r_{WS}^2 - 2(r_x\cos\theta_{AB} + r_y\sin\theta_{AB})r_{WB} + r_{WB}^2} - r_{WS} \\
&\implies (ct_B + r_{WS})^2 = r_{WS}^2 - 2(r_x\cos\theta_{AB} + r_y\sin\theta_{AB})r_{WB} + r_{WB}^2 \\
&\implies r_x\cos\theta_{AB} + r_y\sin\theta_{AB} = \frac{1}{2r_{WB}}(r_{WS}^2 + r_{WB}^2 - (ct_B + r_{WS})^2) \\
&\implies r_x\cos\theta_{AB} + r_y\sin\theta_{AB} = \frac{r_{WB}^2 - (ct_B)^2}{2r_{WB}} - \frac{ct_B}{r_{WB}} r_{WS} \\
&\implies r_y = \frac{1}{\sin\theta_{AB}}\left(\frac{r_{WB}^2 - (ct_B)^2}{2r_{WB}} - \frac{ct_B}{r_{WB}} r_{WS} - (a_x + b_x r_{WS})\cos\theta_{AB}\right) \\
\implies r_y = \frac{1}{\sin\theta_{AB}}&\left(\left(\frac{r_{WB}^2 - (ct_B)^2}{2r_{WB}} - \frac{r_{WA}^2 - (ct_A)^2}{2r_{WA}}\cos\theta_{AB}\right) - \left(\frac{ct_B}{r_{WB}} - \frac{ct_A}{r_{WA}}\cos\theta_{AB}\right) r_{WS}\right) \\
&\implies \boxed{r_y = a_y + b_y r_{WS}}
\end{aligned}
$$

and lastly, we obtain r_z:

$$\begin{aligned} r_{WS}^2 &= r_x^2 + r_y^2 + r_z^2 \\ \implies r_z &= \pm\sqrt{r_{WS}^2 - r_x^2 - r_y^2} \\ \implies & \boxed{r_z = \pm\sqrt{(1 - b_x^2 + b_y^2) r_{WS}^2 - 2(a_x b_x + a_y b_y) r_{WS} - (a_x^2 + a_y^2)}} \end{aligned}$$

Thus, in summary, we have a locus of possible source positions from a given measurement of t_A and t_B:

$$\boxed{\vec{r}_{WS}(r_{WS}) = \begin{pmatrix} r_x(r_{WS}) \\ r_y(r_{WS}) \\ r_z(r_{WS}) \end{pmatrix} = \begin{pmatrix} a_x + b_x r_{WS} \\ a_y + b_y r_{WS} \\ \pm\sqrt{(1 - b_x^2 + b_y^2) r_{WS}^2 - 2(a_x b_x + a_y b_y) r_{WS} - (a_x^2 + a_y^2)} \end{pmatrix}} \tag{4}$$

with

$$\begin{aligned} a_x &= \frac{r_{WA}^2 - (ct_A)^2}{2r_{WA}} \\ b_x &= -\frac{ct_A}{r_{WA}} \\ a_y &= \frac{1}{\sin\theta_{AB}} \frac{r_{WB}^2 - (ct_B)^2}{2r_{WB}} - \frac{r_{WA}^2 - (ct_A)^2}{2r_{WA}} \cos\theta_{AB} \\ b_y &= -\frac{1}{\sin\theta_{AB}} \left(\frac{ct_B}{r_{WB}} - \frac{ct_A}{r_{WA}} \cos\theta_{AB} \right) \end{aligned}$$

2.4 Transformation to heliospheric coordinates

The final task is to rotate this locus into a more useful/general coordinate system. The positions of STEREO A and B are found via SPICE kernels, which can be used to compute vectors in a range of coordinate systems. One particular choice which makes sense for solar system objects is Heliocentric Earth Ecliptic (HEE), where the reference plane is the ecliptic and the reference vector is the Sun-Earth line.

To do the mathematics above, we aligned the Wind-Stereo A vector with the x-axis, and asserted the z-axis to be generated by the cross product of the Wind-Stereo A and Wind - Stereo B vectors. Given the vectors in HEE coordinates : $\vec{r}_{WA}$ and $\vec{r}_{WB}$, the equations for the unit vectors in HEE are:

$$\hat{x} = \vec{r}_{WA}/r_{WA} \tag{5}$$

$$\hat{z} = \frac{\vec{r}_{WA} \times \vec{r}_{WB}}{|\vec{r}_{WA} \times \vec{r}_{WB}|} \tag{6}$$

$$\hat{y} = \hat{z} \times \hat{x} \tag{7}$$

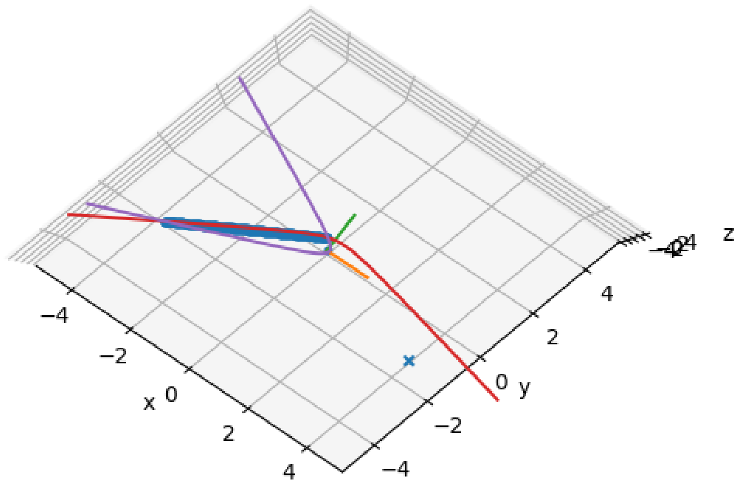
$$\tag{8}$$

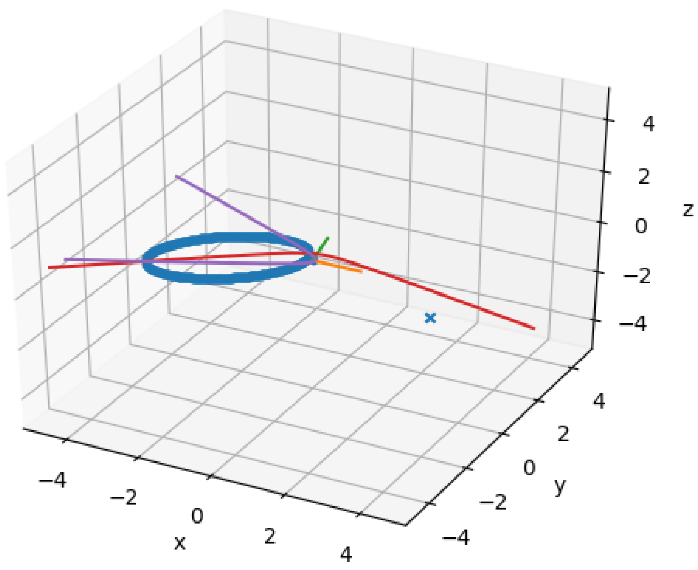
where θ_{AB} is found via the dot product of the baseline vectors.

Now we can produce a rotation matrix with these unit vectors and transform the hyperbola from the baseline frame (bf) into our chosen coordinate system (hee):

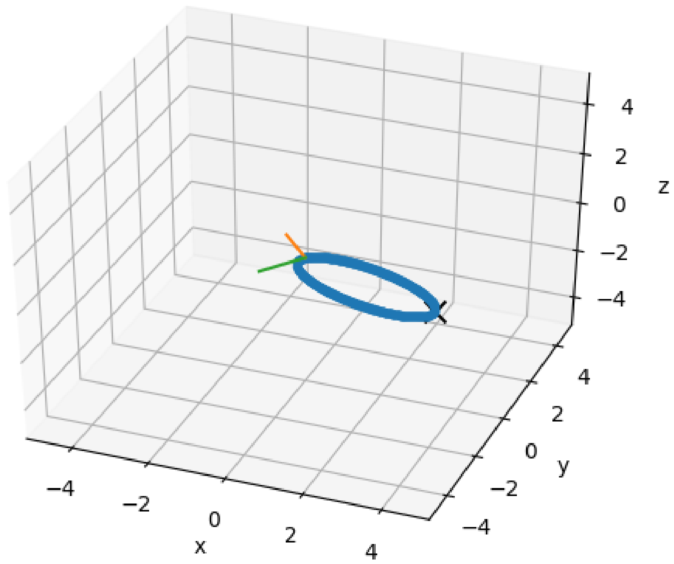
$$\vec{r}_{source,HEE} = (\hat{x}|\hat{y}|\hat{z}) \cdot \vec{r}_{source,bf} \tag{9}$$

The resulting fitted intersection is plotted below. The hyperbola are plotted using the geometry discussed in the next section. They show the possible loci in the baseline plane, and their intersections match up with the intersection of the 3d locus with the baseline plane. This is a good sanity check of the analytical result above.





With a fake observation of Jupiter on 2010/10/08, where the inputted time delays are calculated specifically from the known distances from Jupiter to the respective satellites, the following curve is found to coincide with the position of Jupiter, which is marked with an x on the plot.



These fake time delays are calculated as follows:

$$\begin{aligned} \text{Distance A} \rightarrow \text{J} &: 5.190AU \\ \text{Distance W} \rightarrow \text{J} &: 4.011AU \\ \text{Distance B} \rightarrow \text{J} &: 4.560AU \\ \text{TOF J} \rightarrow \text{A} &: 2590s \\ \text{TOF J} \rightarrow \text{W} &: 2001s \\ \text{TOF J} \rightarrow \text{B} &: 2276s \end{aligned}$$

Giving ideal time delay measurements $t_A - t_W = 589s = 9.82min$, and $t_B - t_W = 275s = 4.58min$

In the context of searching events for emission from Jupiter on this date, with this baseline geometry, the intersection of this locus with Jupiter's known orbital plane will give two solutions, one very close to the baselines, and the other further away which is the true solution. Other dates, when Jupiter and the spacecraft are in a different geometric configuration, may yield a locus which is a hyperbola. In this case, there will be a unique solution intersecting Jupiter's ecliptic plane.

A similar intersection will be able to be performed for events coming from the Sun, and for extrasolar sources, the far field limit of the hyperbolae will be the solution (will yield 2 points, each reflected in the basline plane.)

2.5 2D Analog

If we are just going to assume the source lies in the ecliptic plane (which is true of many sources in the heliosphere), then we can restrict our analysis to 2D vectors and simplify things considerably.

The analysis proceeds exactly as before, except at the last step where before we calculated r_z, we now require $r_z = 0$ and hence we find r_{WS} is constrained by the condition:

$$r_{WS}^2 = r_x^2 + r_y^2 = a_x^2 + a_y^2 + 2(a_x b_x + a_y b_y) r_{WS} + (b_y^2 + b_y^2) r_{WS}^2$$

which is just a quadratic equation for r_{WS} with the solutions:

$$r_{WS} = \frac{1}{2(b_x^2 + b_y^2 - 1)} \left[-2(a_x b_x + a_y b_y) \pm \sqrt{4(a_x b_x + a_y b_y)^2 - 4(b_x^2 + b_y^2 - 1)^2 (a_x^2 + a_y^2)} \right]$$

Thus the (x,y) coordinates given r_{WA}, t_A, r_{WB}, t_B and θ_{AB} are just:

$$\boxed{\vec{r}_{WS}(r_{WS}) = \begin{pmatrix} r_x \\ r_y \end{pmatrix} = \begin{pmatrix} a_x + b_x r_{WS} \\ a_y + b_y r_{WS} \end{pmatrix}} \tag{10}$$

with

$$\begin{aligned}
a_x &= \frac{r_{WA}^2 - (ct_A)^2}{2r_{WA}} \\
b_x &= -\frac{ct_A}{r_{WA}} \\
a_y &= \frac{1}{\sin\theta_{AB}} \frac{r_{WB}^2 - (ct_B)^2}{2r_{WB}} - \frac{r_{WA}^2 - (ct_A)^2}{2r_{WA}} \cos\theta_{AB} \\
b_y &= -\frac{1}{\sin\theta_{AB}} \left(\frac{ct_B}{r_{WB}} - \frac{ct_A}{r_{WA}} \cos\theta_{AB} \right) \\
r_{WS} &= \frac{1}{(b_x^2 + b_y^2 - 1)} \left[-(a_x b_x + a_y b_y) \pm \sqrt{(a_x b_x + a_y b_y)^2 - (b_x^2 + b_y^2 - 1)(a_x^2 + a_y^2)} \right]
\end{aligned}$$

If it turns out the source has a z component (i.e. lies out of the ecliptic), this manifests itself as an error in our calcualtion in r_{WS}, where the true value is:

$$r_{WS} = \frac{1}{2(b_x^2 + b_y^2 - 1)} \left[-2(a_x b_x + a_y b_y) \pm \sqrt{4(a_x b_x + a_y b_y)^2 - 4(b_x^2 + b_y^2 - 1)(a_x^2 + a_y^2 - r_z^2)} \right]$$

3 Numerical Cross Check

To check the above analytical results, we can follow a different route to obtaining the locus. First, we consider the 2d surface of points on which the source could lie due to each baseline, then we numerically intersect these surfaces and find a vector array describing this intersection. If the analytic curve above fits this array well, it is a good confirmation that we are doing the right thing.

3.1 Hyperboloid Geometry and Parameterisation

3.1.1 2D

In 2D, a hyperbola is constructed from all the points in the x-y plane which have the same difference in distance to the two foci, F_1,F_2.

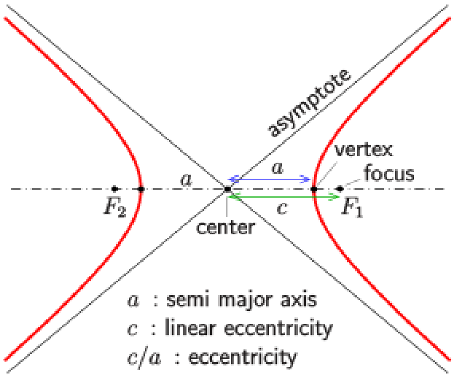


Figure 1: By Ag2gaeh - Own work, CC BY-SA 4.0, https://commons.wikimedia.org/w/index.php?curid=57270355

The equation describing this curve as sketched above is:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \tag{11}$$

where the origin is located at the center, the x axis points from the center to F_1 and the z axis points out of the page.

This can be parameterised:

$$x = \pm a \cosh u \tag{12}$$
$$y = b \sinh u \tag{13}$$
$$u \in \{-\infty, +\infty\} \tag{14}$$

3.1.2 3D

The 2D hyperbola considered above, may be used to construct a hyperboloid of rotation. There are two types : 1 sheeted, and 2 sheeted.

The 1 sheeted hyperboloid of rotation can be found by rotating the 2D hyperboloid about the y-axis, i.e. the direction perpendicular to the line joining the foci.

This shape is no longer the locus of points with equal path difference to the foci. This can be seen by realising that the one sheeted hyperboloid intersects the plane defined by the foci separation vector, and intesecting the origin. This plane is the locus of points equidistant from the foci and so these points have a different path difference to other points on the hyperboloid.

Instead, a 2 sheeted hyperboloid is obtained by rotating the 2D hyperbola about the foci axis. This, by contrast, *is* the locus of all points in 3D space which have the same path difference to the two foci.

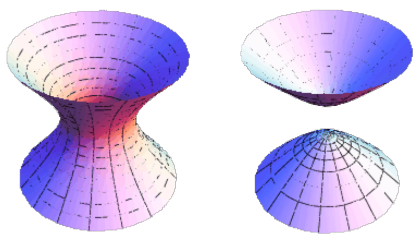


Figure 2: http://mathworld.wolfram.com/Hyperboloid.html , Left: Single Sheet, Right: Double Sheet

Assuming the center is at the origin and the primary focus still defines the x-axis, the 2 sheeted hyperboloid is described by the equation:

$$\frac{x^2}{a^2} - \frac{y^2+z^2}{b^2} = 1 \tag{15}$$

or parametrically:

$$x = \pm a \cosh u \tag{16}$$
$$y = b \sinh u \cos v \tag{17}$$
$$z = b \sinh u \sin v \tag{18}$$
$$u \in \{-\infty, +\infty\}, v \in \{0, \pi\} \tag{19}$$

The sheet with it's vertex at the primary focus is specified by $x = a \cosh u$, while the other sheet is specified by $x = -a \cosh u$

Finally, for the purposes of the geometric set up of baselines in the Wind-Stereo configuration, we find the parametric equation describing a hyperboloid where the secondary focus is located at the origin, and the primary is located at $\vec{r}_{base} = r_{base}(\cos\theta, \sin\theta)$.

Starting from the alignment used above (center at origin, x axis from origin to primary focus), we first translate the center so that the secondary focus is located at the origin, and then we rotate the whole locus about the z axis by an angle θ :

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \boldsymbol{R}(\hat{z}, \theta) \cdot \begin{pmatrix} \pm a \cosh u + r_{base}/2 \\ b \sinh u \cos v \\ b \sinh u \sin v \end{pmatrix} \tag{20}$$

$$= \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \pm a \cosh u + r_{base}/2 \\ b \sinh u \cos v \\ b \sinh u \sin v \end{pmatrix} \tag{21}$$

$$= \begin{pmatrix} (\pm a \cosh u + r_{base}/2) \cos\theta - (b \sinh u \cos v) \sin\theta \\ (b \sinh u \cos v) \cos\theta \pm (a \cosh u + r_{base}/2) \sin\theta \\ b \sinh u \sin v \end{pmatrix} \tag{22}$$

$$\tag{23}$$

Add figures showing hyperboloids for x-aligned and rotated baselines.

3.2 Applying the geometry to the problem

So now let's apply this mathematics to the Wind - Stereo time delay. Taking the position of Wind (W) as the origin and the Wind-Stereo A vector as the x-axis, then let the baseline distance be R_{WA}. Now, if a signal arrives at Wind at time t_W and at Stereo-A at time t_A, the path length difference is $\Delta r_{WA} = c(t_A - t_W)$. Note that Δr can be positive (if the signal arrives at stereo A (A) *later* than at Wind), and negative if the opposite is true. Δr_{WA} is the distance from A to the source minus the distance from W to the source. This quantity defines the vertex of the hyperboloid: a. If $a = 0$, the vertex is at the center and $\Delta r = 0$. If a is positive, i.e. the vertex is towards Stereo A, then the source is a distance a closer to A and a further from W. Therefore the path difference is $2a$, so $\pm a = \Delta r_{WA}/2$, where the sign is actually given by the value of Δr_{WA}, i.e. for a given time delay measurement, we only choose one sheet of the hyperboloid.

Once a is determined, b is given by $b = +\frac{1}{2}\sqrt{R_{WA}^2 - \Delta r_{WA}^2}$.

Putting this together, in terms of R_{WA}, t_W and t_A, the hyperboloid of possible source vectors defined by the measurement of Wind and Stereo A, as expressed in the given coordinate system is:

$$\begin{pmatrix} x_{WA} \\ y_{WA} \\ z_{WA} \end{pmatrix} = \frac{1}{2} \begin{pmatrix} (c(t_A - t_W) \cosh u + R_{WA}) \\ \sqrt{R_{WA}^2 - c^2(t_A - t_W))^2} \sinh u \cos v \\ \sqrt{R_{WA}^2 - c^2(t_A - t_W)^2} \sinh u \sin v \end{pmatrix} \tag{24}$$

$$u \in \{-\infty, +\infty\}, v \in \{0, \pi\} \tag{25}$$

Now adding the baseline of Wind (W) and Stereo B (B), with separation R_{WB} and with Stereo B located at $\vec{R}_{WB} = R_{WB}(\cos\theta_{AB}, \sin\theta_{AB}, 0)$, and let the signal arrive at B at time t_B, the corresponding hyperboloid is:

$$\begin{pmatrix} x_{WB} \\ y_{WB} \\ z_{WB} \end{pmatrix} = \frac{1}{2}\begin{pmatrix} (c(t_B - t_W)\cosh u + R_{WB})\cos\theta - (\sqrt{R_{WB}^2 - c^2(t_A - t_W)^2}\sinh u \cos v)\sin\theta \\ (\sqrt{R_{WB}^2 - c^2(t_A - t_W)^2}\sinh u \cos v)\cos\theta + (c(t_A - t_W)\cosh u + r_{base}/2)\sin\theta \\ \sqrt{R_{WB}^2 - c^2(t_A - t_W)^2}\sinh u \sin v \end{pmatrix} \tag{26}$$

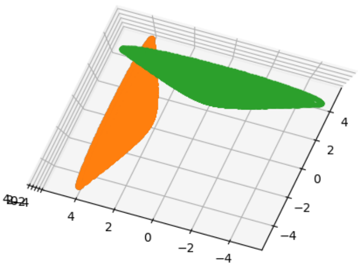
$$u \in \{-\infty, +\infty\}, v \in \{0, \pi\} \tag{27}$$

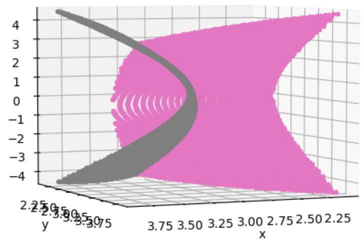
The location of the source is thus constrained to lie on the intersection of the above vectors, i.e. points satisfying

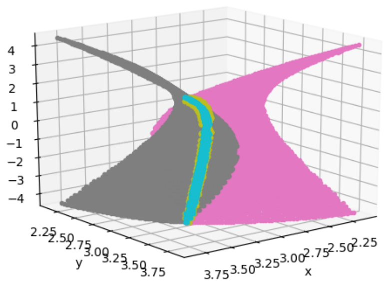
$$\begin{pmatrix} x_{WB} \\ y_{WB} \\ z_{WB} \end{pmatrix} = \begin{pmatrix} x_{WA} \\ y_{WA} \\ z_{WA} \end{pmatrix} \tag{28}$$

With 3 unknowns but only 2 free parameters, this problem is underconstrained and therefore the solutions are a subset of points, and not an individual one.

This problem is most tractable numerically. First, let's sketch the geometry, and solve the retrieval from a given source location. Choose the baselines to be 3 lightseconds long, their relative angle to be 90 deg and the source location to be at (3lightsecs,3lightsecs,0), i.e. so the 3 spacecraft and the source form a square in the xy plane. This results in the following hyperboloids and intersection locus:







This intersection we obtain is an ellipse or a hyperbola, which is symmetric in the baseline plane. For the case of a hyperbola, it asymptotes to the parallel rays approximated direction, and at closer distances tells us the distance to the source as well as the direction. This intersection can be overlayed on the above analytic solution to cross-check it's validity.

4 A real event: Jovian Decametric Radio Emission 2010/10/18

4.1 Motivation for suspecting Jupiter

Combining data from STEREO/WAVES and WIND/WAVES for an event observed on 2010/10/18 19:20:00 - 20:50:00, we get the following plots:

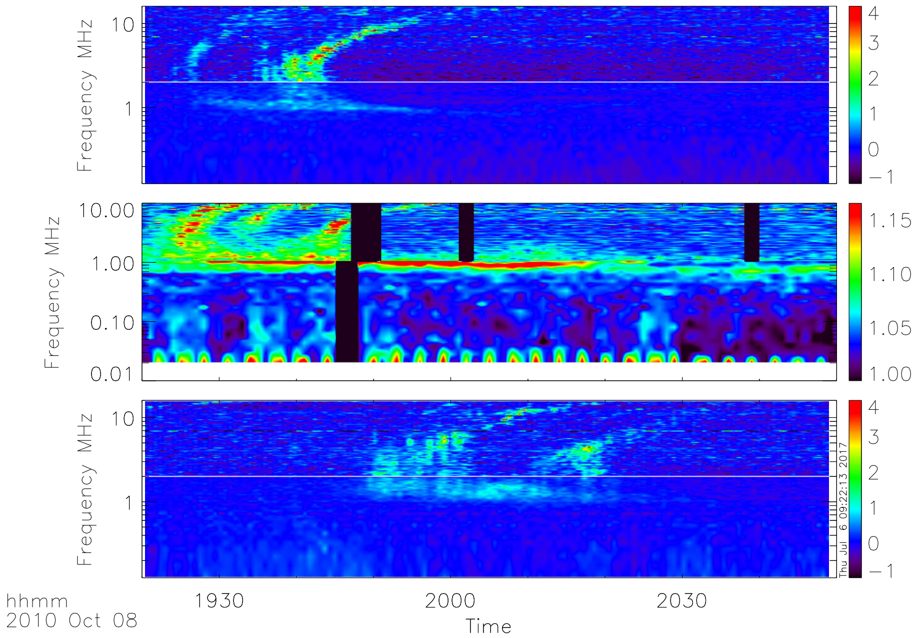


Figure 3: **Top:** Stereo A, **Middle:** WIND, **Bottom:** Stereo B

As can be seen, at all 3 instruments we see two 'fingers' of emission which arrive later with increasing frequency. A rough by eye estimation of this data implies that the burst arrives at WIND first, then Stereo A second a very short time later (1-3 min), and then finally it reaches Stereo B 30 minutes later. Given the baseline geometry at this time :

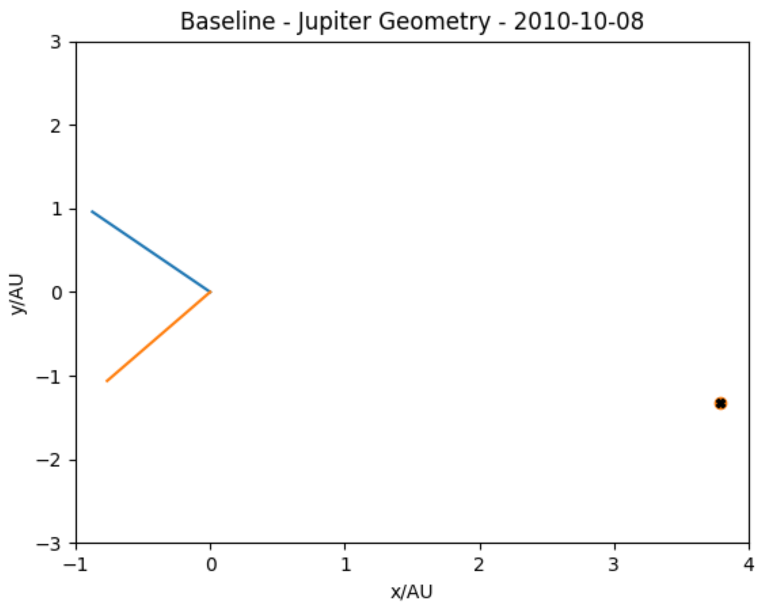


Figure 4: Geometry of the spacecraft (Wind-STEREO A in Blue, Wind-Stereo B in orange), and the relative position of Jupiter (Red circle)

The spacecraft are separated by of order 1AU which is approximately 8.3 light minutes. Therefore the fact that the signal reaches STEREO B 30 minutes later makes no sense if the emission is isotropic. In this time, light would travel 3.5 times further than the separation between WIND and STEREO A. Therefore the emission must be more complex.

4.2 Jovian Decametric Emission (DAM)

The Jovian decametric emission is a complicated interaction between volcanic dust in the orbital path of Jupiter's closest moon Io, and Jupiter's own magnetic field.

One important feature of the emission is that it is narrowly beamed into space in a hollow cone, and that it rotates over time. Many references [¿¿ref,ref,ref¡¡] refer to an Io-component and non-Io-component in the emission. The former refers to emission corotating with Io - with a period of 1.76 days. The latter refers to emission which instead corotates with the magnetic field of Jupiter, which has a period of just 9.9 hours. Another component worth noting is the beat period of Io and Jupiter's rotation, which is approx 12.95hr [Carr et al.,1983, Kaiser,1993;Queinnec and Zarka, 1998, Panchenko 2010]

4.3 Application to Observation

In the baseline geometry diagram above, we can visualise a cone of radiation being swept anticlockwise by Jupiter's rotation. As the beam sweeps down past the spacecraft, it first hits Stereo A, then WIND, then lastly Stereo B. We can work out the angle subtended by each pair of spacecraft at Jupiter and hence the time it takes the beam to sweep from one spacecraft to the other. On 2010/10/18 19:20:00 we obtain: $A\hat{J}W = 6.859^o$ and $W\hat{J}B = 15.977^o$. These translate to a sweep time from A to W of 679.0s or 11.3min and from W to B of 1581.7s or 26min.

If this is the source of our observation, then there are two competing effects causing the time difference (ref http://onlinelibrary.wiley.com/doi/10.1029/2010GL042488/full). The observed time delay can be written as:

$$t_{obs} = t_{ltt} + t_{sweep}$$

where ltt stands for light travel time. We can imagine that if the spacecraft were close enough, or light fast enough that the light travel time from the source to the spacecraft was instantaneous, then the spectral signatures would be exactly separated by the sweeping time. A visual comparison again shows that the sweeping time alone cannot explain the observations as the A and W observations are much closer in time than 11 minutes.

To visualise both effects working together, let's step through a single event at one frequency as recorded by two of the spacecraft. At time $t = 0$, let's say the cone exactly intersects the position of Stereo A. At this time, a photon emitted at Jupiter along the cone will travel along a straight line, and will reach Stereo A at exactly the light travel time from the source to Stereo A, $t_A = 0 + t_{ltt,A}$.

Now, as time advances from $t = 0$, the beam sweeps around, emitting photons in straight lines which all eventually pass through empty space in between Wind and Stereo A. Finally, at time $t = t_{sweep,A\rightarrow W}$, it intersects the position of Wind and at this time, a photon is emitted along a straight line which passes through Wind. The photon then reaches Wind at time $t_W = t_{ltt,W} + t_{sweep,A\rightarrow W}$

For our event, we know that $t_{ltt,W}$ is around 10 minutes longer than $t_{ltt,A}$, and that the sweep time (assuming the beam corotates with Jupiter's magnetosphere) is around 11 minutes. This means that the observed time delay between the sources is:

$$\begin{aligned} t_A - t_W &= t_{ltt,A} - t_{ltt,W} - t_{sweep,A\rightarrow W} \\ &\approx -1\text{min} \end{aligned}$$

i.e. the signal actually reaches Wind 1 minute after it reaches Stereo A, even though geometrically it is 10 light minutes closer to Jupiter.

With this in mind, if we suspect an observation is coming from Jupiter as with the above data, then a coarse procedure we can follow is to correct the plots with respect to the sweeping time, then estimate the time delay from the corrected plots, then feed it into the analytical solution and see if the locus passes through the location of Jupiter. Given there are other rotational periods associated with DAM emission, we can also try correcting with respect to these, and see if they lead to a better match with Jupiter.

Correcting our data with respect to Jupiter's magnetosphere rotation rate means we leave Stereo A unchanged, we move Wind back 11.3 minutes, and move Stereo B back 26+11.3 minutes. This produces the following corrected dynamic spectra:

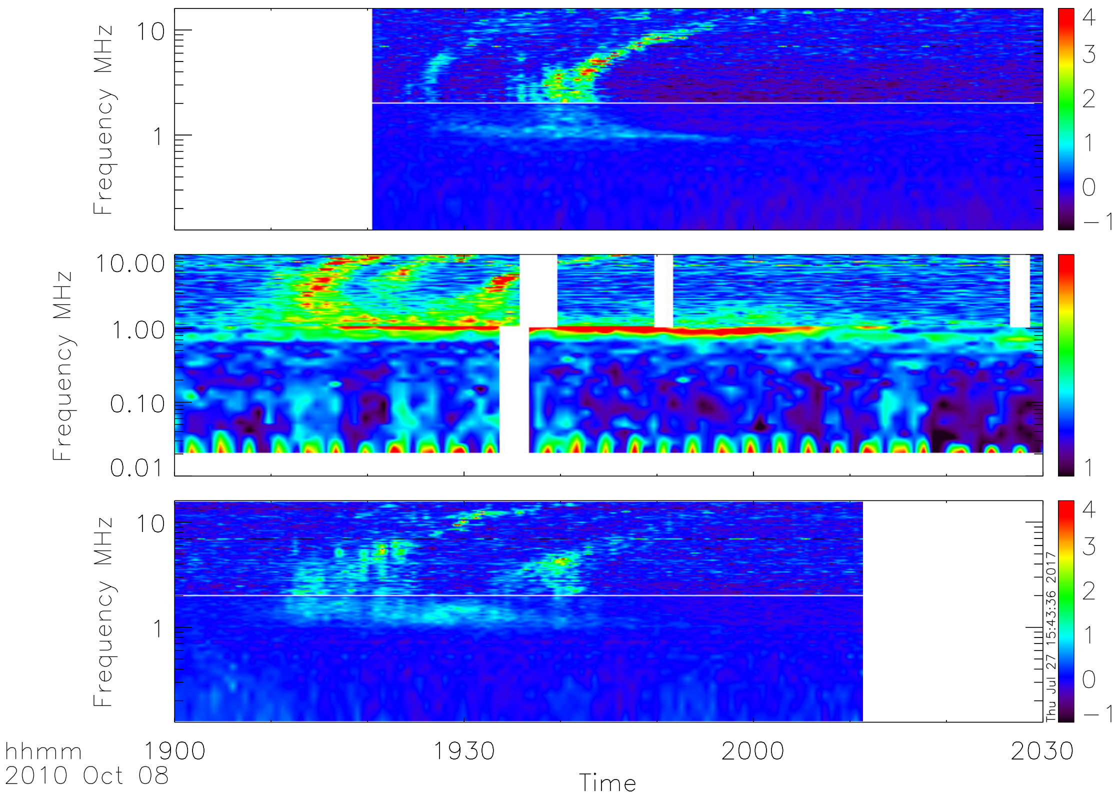
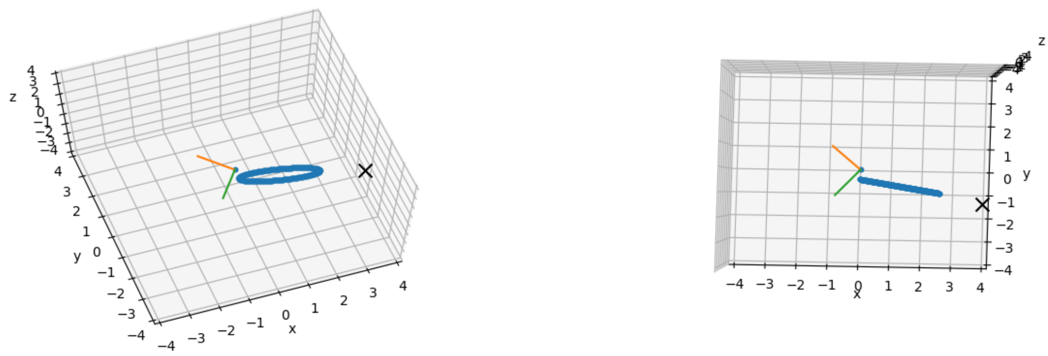
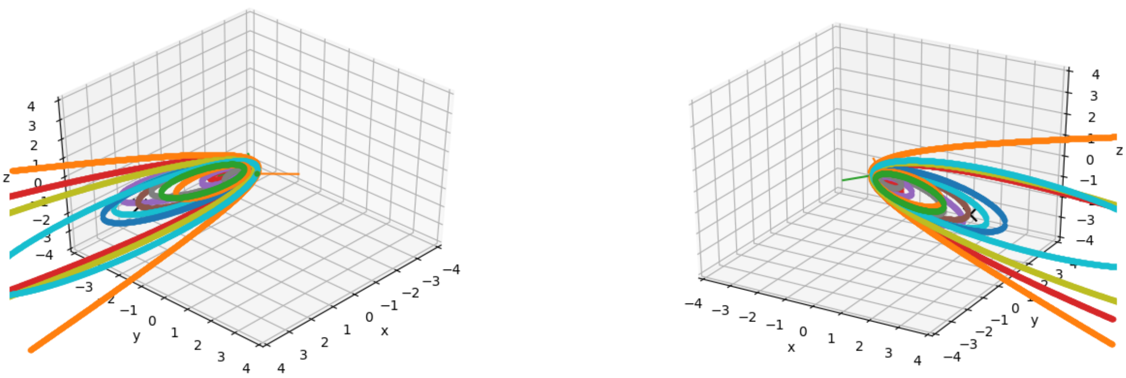


Figure 5: Dynamic Spectra of Decametric Emission recorded at Stereo A (Top), Wind (Middle), Stereo B (Bottom) on Oct-08 2010 starting at 1900 UT, shifted in time by the beam sweep time for the Jovian magnetic field rotation period.

Now a by eye estimate of the time delays looks more reasonable, the burst first registers at Wind centered at approximately 19:15, Stereo A around 19:25 and Stereo B around 19:20. So $t_A \approx 10\text{min}, t_B \approx 5\text{min}$. Plugging these numbers into the analytical solution, we obtain the following result:



Clearly, this solution is far from accurate, but is still in the correct direction - it seems we are on the right track. Also, the by eye estimate is very rough. Let's estimate a narrow uncertainty of time delay around 1min. This gives $t_A = 10 \pm 1\text{min}, t_B = 5 \pm 1\text{min}$. This gives the following range of loci:



The direction of the loci are always pretty accurate and the solution of the position in the jupiter orbital plane moves a lot. The locus is clearly quite unstable to errors.

Next, let's get more accurate estimates of the measured time delay. To do this, we will fit a gaussian peak to the data in each frequency bin. The peak to peak difference between spacecraft will be the best estimate of the time difference, and an uncertainty range can be calculated from the FWHM of these fits.

5 Location of Jupiter assuming ecliptic location

In this section, we start from the same event as above, but assume all baseline and source vectors are 2D - constrained to the ecliptic plane.

First, since the observation of the decametric emission was taken at a known time, and the jovian and spacecraft orbital elements are all known, we can calculate the true time delay that we would expect to measure between free streaming emission arriving at each of the three spacecraft, assuming the radiation has exactly the same time profile at each s/c, the beaming of the emission source is correctly accounted for, and that we can measure it's onset exactly. This was calculated above and we found that the signal would arrive at Wind first at $t = 0s$, followed by Stereo B at $t = 275s = 4.58min$ and lastly Stereo A at $t = 589s = 9.82min$.

Here we retrieve the position of Jupiter using these idealized measurements:

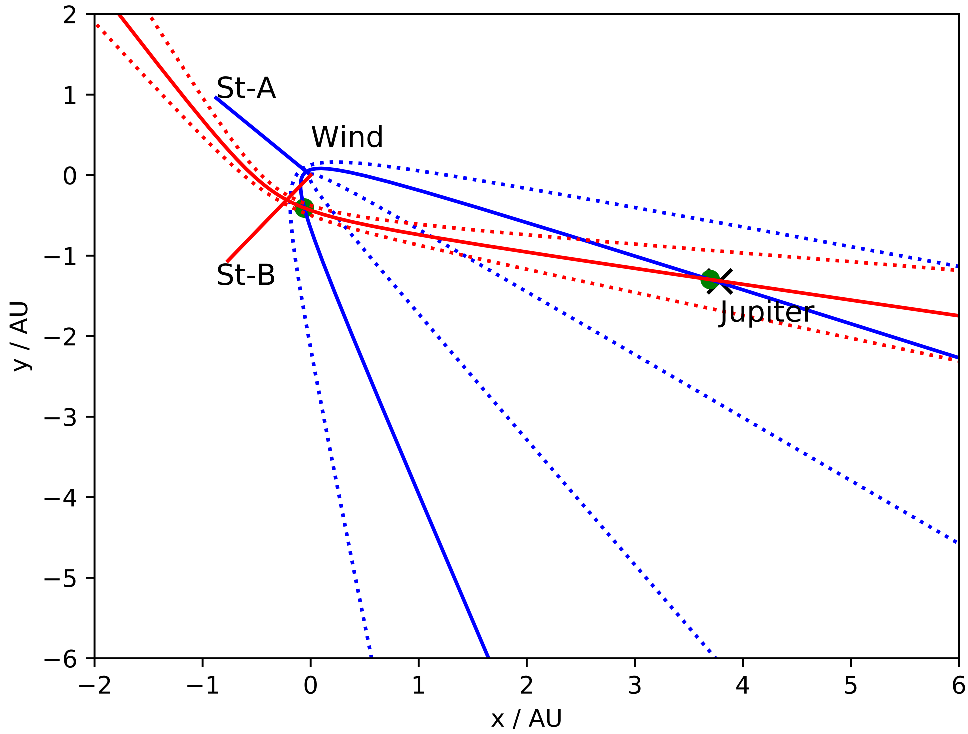


Figure 6: Jupiter's position in the ecliptic plane is retrieved via intersecting two hyperbola generated from idealized time delay measurements of receipt of a signal at the Stereo A, Wind, Stereo B spacecraft on 2010/08/10. The straight blue line is the Stereo A-Wind baseline, the solid blue parabola is the resulting hyperbola from the time delay measurements between these two spacecraft, and the dotted blue lines is the envelope of how the hyperbola could vary within the error caused by the time cadence of the the Wind (60s) and Stereo (40s) measurements. The red lines and curves are the corresponding results for the Wind-Stereo B baseline and time delay measurements. The large black cross represents the true position of Jupiter on the day in question. The small green dots are the analytic intersection of the two hyperbolae. As can be seen, one is almost exactly at the location of Jupiter, the other is an alias result. The small offset in the first intersections position is due to the assumption that Jupiter lies in the ecliptic plane. In reality, at the time of measurement it had an out of ecliptic component of $0.1AU$. The region of uncertainty is the dotted boundary surrounding the true location of Jupiter. We see that for the geometry in question, this uncertainty region is narrow but very elongated (the true distance of Jupiter is poorly constrained), but the direction of the location of Jupiter is well constrained.

Next, we repeat this retrieval but this time we give a first attempt at measuring a time delay directly from the data. Referring to figure 5, the burst as it appears in each instrument appears to be separated into 3 fingers. The first two are quite messy and merge/disappear into each other. The third (i.e. latest) is clearer and so we use this to obtain a time measurement. To do this, on each panel, I overplotted a horizontal line at 5MHz and 10MHz. I then used the cursor to find the inner edge of this finger as it intersected these horizontal lines and read out the time value for each spacecraft. Then subtracting them to give a time difference and finally averaging between 5MHz and 10MHz to get:

$$t_A - t_W = 9.1 min t_B - t_W = 5.25 min$$

While these are not the known true values of the time delay, the sum of the errors of Stereo and Wind is 50s which is enough to make these measurements consistent with the true values.

Inserting these time delay values into the jupiter position retrieval code, we obtain:

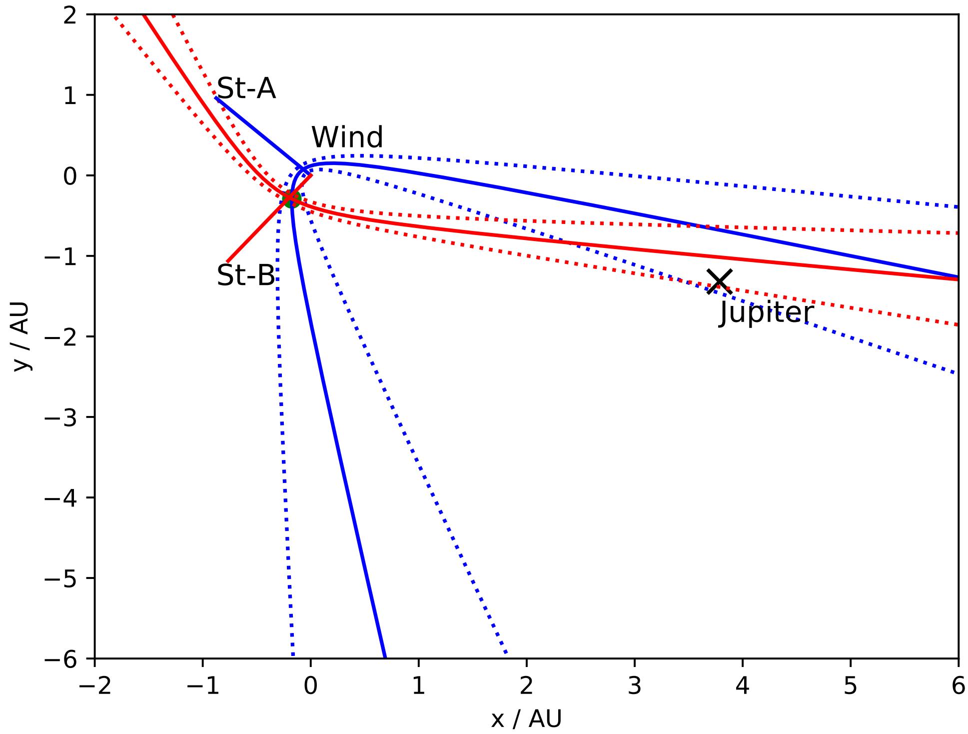


Figure 7: Jupiter's position in the ecliptic plane is retrieved via intersecting two hyperbola generated from real time delay measurements of receipt of a signal at the Stereo A, Wind, Stereo B spacecraft on 2010/08/10. The lines are the same as describe in Figure 6. above. We see the true location of Jupiter is now far from the retrieved solution, but this difference is mainly in distance from the origin, the angular position of Jupiter on the sky is well described despite the coarse method of obtaining the time delay and the complicated nature of the Jupiter source both in the actual data (3 messy fingers) and external factors such as the beaming of the source. Importantly, the true location of Jupiter lies within the boundaries of uncertainty of the hyperbolae.

6 To Do

1. Get a first estimate of position of Jupiter from analytic solution and by eye estimate of time delay with sweeping correction. Assign an error to the estimate and get bounds on the locus. **Done:** Needed an artificially small error for range of solutions to look reasonable.

2. Do this again more rigorously by fitting gaussians to the data in each frequency band. See if there is any systematic information about the time delay between channels.

3. Find a strong solar radio burst. Repeat the above work: Show we find the Sun in a coarse grain. See if fitting can locate sources to off limb, and see if they follow the parker spiral with decreasing frequency.

4. Look for other sources in the data / consult catalog of GRBs and see if any can be located in data.